Isaac Newton Institute for Mathematical Sciences

Annual Report
1996 – 97

UNIVERSITY OF CAMBRIDGE

Contents

Director's Preface

It was with a great sense of honour, and not a little trepidation, that I inherited the mantle of Director of the Isaac Newton Institute for Mathematical Sciences from Sir Michael Atiyah, who retired on 30 September 1996. Sir Michael had guided the Institute from its early planning in 1990, and together with Peter Goddard as Deputy Director, the advice of a gifted Scientific Steering Committee and the commitment of a supportive Management Committee, had successfully established the Institute as the outstanding centre for fundamental research in the mathematical sciences that it has become today. The Newton Institute, and the standards that it represents, owes a great debt of gratitude to their devoted service.

I wish also to pay tribute to Sir Peter Swinnerton-Dyer who acted as Executive Director from June 1995, and who also retired on 30 September 1996, having helped steer the Institute through a difficult transitional period.

The simultaneous retirement of two such powerful figures in mathematics presents, it need hardly be said, a particularly difficult situation, and range of responsibilities, for the incoming Director. Fortunately I am aided by Noah Linden, who has long experience as Assistant Director and now assumes the burden of Deputy Director; and by Colin Sparrow, who held the position of Hewlett-Packard Senior Research Fellow at the Institute till 30 June 1996, and who now fills the new role of Liaison Officer, the liaison in question being particularly that with the UK mathematical community. I am moreover aided and supported by an exceptionally devoted staff, whose unstinting efforts make the Institute an ideal environment for research interaction and creativity. The many tributes of those who have worked here testify to the truth of this statement!

There can be no doubt that we face difficult and challenging times ahead. Much of the initial funding of the Institute was for an initial five-year period. This 'honeymoon' comes to an end with this report at 30 June 1997, at which stage the Institute has completed a remarkable, indeed wonderful, range of 21 programmes spanning a vast range of topics in Mathematics and its applications in Physics, Astrophysics, Geophysics, Engineering and Environmental Science, Computer Science, Biology and Finance. Our future programmes are already chosen until the year 2000 and our funding is now reasonably secure into the new millennium; the longer-term financial security of the Institute, however, requires the gradual establishment of an endowment fund. The Isaac Newton Trust has recognised this need, and has provided a most welcome loan on terms which yield valuable leverage in our inevitable and pressing ongoing fund-raising efforts.

It will be evident that this year's Annual Report is more concise than those of previous years; this is deliberate, in the hope that it will also be more readable. Statistical information, and detailed lists of participants in the various programmes, are relegated to separate Appendices which are available on request from the Institute.

Keith Moffatt
30 June 1997

Professor Keith Moffatt, FRS (Findlay Kember)

Highlights

Programmes

During the year July 1996/June 1997, a total of 1061 visitors took part in the Institute's programmes and workshops. There were 22 workshops, over 650 seminars were presented in the Institute, and 220 papers have been produced or are in preparation by participants. The programmes that took place were:

Mathematics of Atmosphere and Ocean Dynamics
July to December 1996
Organisers: JCR Hunt (UK Met Office), ME McIntyre (Cambridge), J Norbury (Oxford), I Roulstone (UK Met Office)

Mathematical Modelling of Plankton Population Dynamics
29 July to 6 September 1996
Organisers: J Brindley (Leeds), M Fasham (Southampton), J McGlade (Warwick)

Four-Dimensional Geometry and Quantum Field Theory
4 November to 13 December 1996
Organisers: Sir Michael Atiyah (Cambridge), IM Singer (MIT)

Non-Perturbative Aspects of Quantum Field Theory
January to June 1997
Organisers: DI Olive (Swansea), P van Baal (Leiden), P West (King's College, London)

Representation Theory of Algebraic Groups and Related Finite Groups
January to June 1997
Organisers: M Broué (Paris), RW Carter (Warwick), J Saxl (Cambridge)

The short (six-week) programmes on *Mathematical Modelling of Plankton Population Dynamics* and on *Four-Dimensional Geometry and Quantum Field Theory* were experimental in nature, previous programmes having been invariably of six months' duration. The *Plankton* programme interacted well with the larger parallel programme on *Mathematics of Atmosphere and Ocean Dynamics,* while the *Four-Dimensional Geometry* programme acted as a precursor for the six-month programme on *Non-Perturbative Aspects of Quantum Field Theory* which followed. In terms of enthusiastic participation, both short programmes were an unqualified success. The programme on *Mathematics of Atmosphere and Ocean Dynamics* had increased participation during the period between the short programmes, and arranged a total of no less than ten workshops and conferences over the total six-month period.

Direction

Sir Michael Atiyah retired as Director of the Institute on 30 September 1996, having served in this capacity since the inauguration of the Institute in 1992. Sir Peter Swinnerton-Dyer simultaneously retired as Executive Director, having served during the transitional period since June 1995. Professor Keith Moffatt was appointed Director, with Dr Noah Linden as Acting Deputy Director, and Dr Colin Sparrow as Liaison Officer, from 1 October 1996.

Dr Noah Linden
(Findlay Kember)

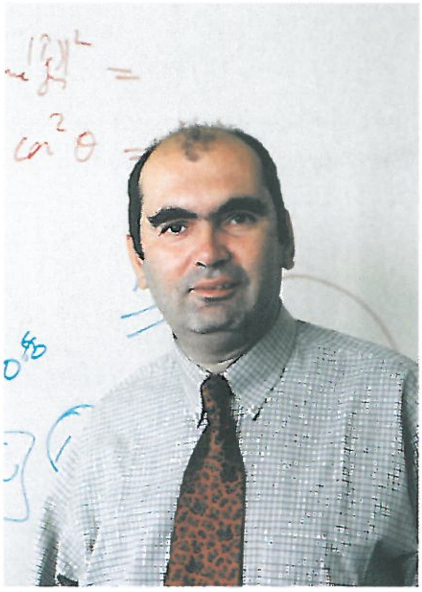
A retirement event was held in honour of Sir Michael and Sir Peter on 4 October 1996. This was attended by a large number of distinguished scientists who have connections with the Institute, as well as representatives of its funding bodies. Two lectures were given at the Institute, by Professor Friedrich Hirzebruch from the Max-Planck Institute Für Mathematik in Bonn on *Working with Michael* and by Professor John Tate from the University of Texas at Austin on *The Arithmetic of Plane Cubics.* These were followed by the unveiling of John Robinson's sculptures, *Creation* and

Sir Michael Atiyah OM, FRS and Professor Sir Peter Swinnerton-Dyer at the retirement celebrations in October 1996
(Neville Taylor)

Genesis, which take their place, together with *Intuition,* in the forecourt of the Institute; and then by a reception at the Institute, and dinner at Jesus College.

On the recommendation of the Management Committee, Sir Michael has been elected a Senior Fellow of the Institute for three years from 1 October 1996; and Sir Peter's tenure as a Senior Fellow has been continued for a further three-year period from 1 July 1997.

Dr Sandu Popescu



Hewlett-Packard Fellow

Dr Sandu Popescu took up his appointment as Hewlett-Packard Reader in Quantum Mechanics on 1 October 1996. Sandu has worked on the foundations of quantum mechanics, and more particularly on measurement theory, geometrical phases, topological effects and Bell inequalities; he has now turned his attention to quantum information, quantum computation and quantum cryptography, areas of great potential interest to Hewlett-Packard as well as to the Newton Institute.

Special Appointments

The following 'visiting' appointments were made during the year:

- Rothschild Visiting Professors:
  Professor Brian Hoskins (University of Reading)
  Mathematics of Atmosphere and Ocean Dynamics

  Professor Ashoke Sen (Mehta Institute, Allahabad)
  Non-Perturbative Aspects of Quantum Field Theory

- Institute of Physics Fellows:
  Dr Andrei Marshakov (PN Lebedev Physics Institute, Moscow)
  Professor Ludwig Faddeev (Steklov Institute, St Petersburg)
  Non-Perturbative Aspects of Quantum Field Theory

- Gabriella and Paul Rosenbaum Fellows:
  Dr Alex Mahalov (Arizona State University)
  Dr Vadim Bondarevsky (University of Minnesota)
  Mathematics of Atmosphere and Ocean Dynamics

  Dr Pei-Ming Ho (Lawrence Berkeley Laboratory)
  Four-Dimensional Geometry and Quantum Field Theory

  Dr Kay Magaard (Wayne State University)
  Representation Theory of Algebraic Groups and Related Finite Groups

  Dr David Pierce (University of North Carolina)
  Non-Perturbative Aspects of Quantum Field Theory

Young Scientists

The Newton Institute is keen to encourage the participation of young researchers and postdocs. A Junior Membership scheme (see below) was introduced during 1996/97, and seven instructional conferences (listed on page 5), aimed specifically at young people, were held at the Institute. Although exact figures for the ages of workshop attendees are not available, a survey at a typical workshop shows an average age of 33 years.

Young scientists visiting the Institute for an instructional conference (Findlay Kember)

Junior Membership

The Institute has set up a Junior Membership scheme to encourage the participation of young UK scientists (graduate students or those within five years of obtaining a PhD) in all its scientific activities. This initiative both recognises the important contribution that Junior Members have to make to Institute research programmes, and seeks to maximise the contact between members of the UK community and the large number of senior overseas visitors to the Institute.

Junior Members receive regular information about Institute activities, and can apply for special grants to enable them to visit Institute programmes for up to two weeks to work and study with longer-term participants. Grants are also available to support participation in Institute workshops or conferences.

Each Institute programme is allocated £5,000 from Institute funds to fund junior member participation. Since the scheme was launched in early 1997, nearly 150 young scientists (from over 40 UK institutions) have joined and numbers continue to grow at a rapid rate.

Home institutions of the Junior Members include: Aberystwyth; Aston; Bath; Birkbeck College, London; Birmingham; Bolton Institute; Bradford; Bristol; Brunel; Cambridge; Chester; Cranfield; Edinburgh; Goldsmiths' College, London; Imperial College, London; Keele; King's College, London; Leicester; Liverpool; Loughborough; Manchester; Napier; Newcastle; Oxford; Paisley; Queen Mary and Westfield College, London; Reading; Robert Gordon; Royal Holloway and Bedford New College, London; Sheffield; Southampton; Stirling; Strathclyde; Sussex; Swansea; Teesside; University College, London.

Further information about the Junior Membership scheme is available via WWW at

http://www.newton.cam.ac.uk/junior.html

or email

jmember@newton.cam.ac.uk

with enquiries and applications.

Instructional Conferences for Young Scientists

Within the five programmes, a number of conferences were held which were instructional in character, and aimed particularly at graduate students and young postdocs. These were:

NATO Advanced Study Institute/Euroconference:

- Modular Representation Theory and Subgroup Structure of Algebraic Groups and Related Finite Groups

Poster session at a NATO Advanced Study Institute (Findlay Kember)

NATO Advanced Study Institute:

- Confinement, Duality and Non-Perturbative Aspects of QCD

Euroconferences (EC Human Capital and Mobility scheme):

- Modelling the Role of Zooplankton in the Marine Food Chain
- Dynamical Systems Approach to Atmosphere/Ocean Sciences
- Mathematical Problems in Atmosphere and Oceanic Data Assimilation

Summer Schools (EC Training and Mobility of Researchers scheme):

- Instructional Conference in Representation Theory of Algebraic Groups and Related Finite Groups
- Duality and Supersymmetric Theories

Cambridge Philosophical Society Bursaries

These bursaries for young people were awarded to:

- Ms AK Griffith (University of Reading)
  Mathematics of Atmosphere and Ocean Dynamics
- Mr AM Edwards (University of Leeds)
  Mathematical Modelling of Plankton Population Dynamics
- Dr K Bremke (University of Bielefeld)
  Representation Theory of Algebraic Groups and Related Finite Groups
- Dr P Sutcliffe (University of Kent)
  Non-Perturbative Aspects of Quantum Field Theory

UK Participation

Every effort is made to ensure that the UK scientific community receives maximum benefit from the Institute's activities. Information about workshops and seminars is publicised widely via posters, email and World Wide Web, and outreach visits (described below) are made by participants. In addition, the Directorate (Keith Moffatt, Noah Linden and Colin Sparrow) have made a number of visits to UK institutions to inform and receive feedback about the Institute's activities. Other means of disseminating information about the Institute's scientific work include making available videos of seminars given at the Institute, and the publication of books arising from Institute programmes (a complete list to date is given on page 10). Following constructive discussions with LMS, and in order to increase the involvement of the UK mathematical sciences community, the Management Committee has agreed to aim at a minimum target of 20% UK participation (not counting Cambridge participants) in future programmes of the Institute.

Outreach

Institute participants gave 162 seminars in departments outside Cambridge. UK universities and other institutions at which participants have talked during 1996/97 include: Aberystwyth; Bidston Observatory; Birmingham; Bristol; British Meteorological Office; Durham; East Anglia; European Centre for Medium Range Weather Forecasting; Exeter; Hewlett-Packard Basic Research Institute in Mathematical Sciences, Bristol; Imperial College, London; Kings College, London; Leeds; Leicester; Liverpool; Manchester; Open; Oxford; Queen Mary Hospital; Queen Mary and Westfield, London; Royal Holloway and Bedford New College, London; Sheffield; Southampton; Surrey; Sussex; UMIST; University College, London; Warwick; York.

Electronic Information

In February 1997 the Institute's World Wide Web site was substantially redesigned and restructured to improve the accessibility of programme information, as well as providing a clearer and more attractive interface. The Website is one of the most effective means of publicising the Institute's activities, and includes

- complete weekly seminar lists for all programmes
- a 'News' page highlighting new and updated information
- workshop announcements and schedules
- information on the Junior Membership scheme
- details of the Institute's electronic mailing lists
- general information and contact details for past, current and future programmes
- list of invited participants for each programme (updated daily)
- contact details for staff and current participants
- the latest 'Call for Proposals' for future programmes
- scientific policy statement

The Institute's Website can be found at

http://www.newton.cam.ac.uk

Programme and seminar information is also widely distributed via electronic mailing lists, with subscribers all over the world. The total number of subscribers continues to increase and has now reached over 1000. The mailing lists are open to all, and information on how to subscribe can be found on the Website above. Alternatively, send an email to

majordomo@newton.cam.ac.uk

containing the message 'lists' (leaving the subject field blank) for more details.

Institute Seminars

During term-time the Institute holds a number of 'Institute Seminars' on Mondays at 5pm. These are talks by distinguished mathematicians and mathematical scientists, aimed at a general mathematical audience. During 1996/97, the following seminars were presented:

28 Oct T Davies (UKMO)
Numerical Weather Prediction and Climate Modelling

11 Nov E Witten (IAS Princeton)
Duality and Three Manifolds

25 Nov S Popescu (Newton Institute)
What is Quantum Computation?

17 Feb J Ball (Oxford)
The Calculus of Variations After 300 Years

24 Feb D Ruelle (IHES)
New Theoretical Ideas in Nonequilibrium Statistical Mechanics

3 Mar A Sen (Mehta Research Institute)
Introduction to String Dualities

10 Mar M Broué (Paris 7)
Complex Reflection Groups

28 Apr J Mirrlees (Trinity College)
The Mathematical Analysis of Incentive Problems

19 May Sir Christopher Zeeman (Oxford)
Geometric Unfolding of a Difference Equation

26 May R Carter (Warwick)
Representations of Simple Lie Algebras - Modern Variations on a Classical Theme

2 Jun AS Fokas (Imperial)
Integrability - from D'Alembert to Lax

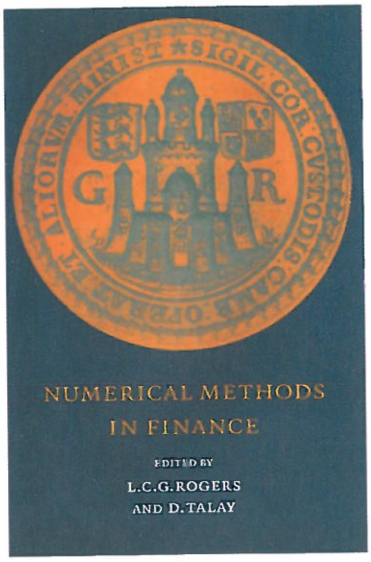
9 Jun L Faddeev (Steklov Mathematical Institute)
Algebraic Aspects of Quantum Integrable Models on Discrete Space-Time

Professor Ed Witten, who gave one of this year's most popular Institute Seminars (Robert P Matthews)

Newton Institute Publications

Three further volumes have appeared in the Cambridge University Press series, *Publications of the Newton Institute*. These are:

- *Contact and Symplectic Geometry*
  CB Thomas (Ed)
  CUP 1996, 310 + viii pp
- *Semantics and Logics of Computation*
  AM Pitts and P Dybjer (Eds)
  CUP 1997, 361 + xii pp
- *Numerical Methods in Finance*
  LCG Rogers and D Talay (Eds)
  CUP 1997, 326 + x pp



'Numerical Methods in Finance', latest in the series 'Publications of the Newton Institute'. Jacket: The Last Seal of the Mint Corporation, engraved by John Croker, c.1709

A complete list of Newton Institute Publications, and of other books emerging from Newton Institute Programmes, is given on page 10.

Video-Recordings of Lectures

The following videos of selected Lectures delivered at the Institute are available for hire or for purchase from the Institute:

F Hirzebruch (October 1996)
Working with Michael

J Tate (October 1996)
The Arithmetic of Plane Cubics

E Witten (November 1996)
Duality and Three Manifolds
Supersymmetric Gauge Theories in 3-dimensions
Relation to String Theory

J Ball (March 1997)
The Calculus of Variations after 300 Years

A Sen (March 1997)
String Dualities

JA Mirrlees (April 1997)
The Mathematical Analysis of Incentive Problems

Royal Society/Japan Society for the Promotion of Science

The RS/JSPS scheme entitled *Infinite Analysis and Geometry,* and administered by the Newton Institute and by RIMS, Kyoto, has been extended to March 1998. The scheme has funded numerous visits of Japanese scientists to UK universities, and reciprocal visits by UK scientists to Japanese universities; the UK universities involved in 1996/7 were Bath, Cambridge, Imperial and King's Colleges London, Nottingham, Oxford, Swansea and Warwick.

European Post-Doctoral Institute

The European Post-Doctoral Institute for the Mathematical Sciences (EPDI), a joint initiative of the Institut des Hautes Etudes Scientifiques (Bures-sur-Yvette, France), and the Max-Planck Institut für Mathematik (Bonn, Germany), together with the Newton Institute, was inaugurated in October 1995 in order to create a European mathematical consortium of excellence for young post-doctoral Fellows. Five Fellows were appointed during the first year of the initiative, each with two years' tenure. A further five have been selected to take up appointment during the academic year 1997/8. (One of these, Dr J Marklof, will be a participant in the Programme *Disordered Systems and Quantum Chaos.*) Continuation of the initiative will be conditional on attracting funding from European Community, or other, sources.

Funding News

The University of Cambridge has made a non-recurrent grant of £40K *per annum* for three years to the Director, to finance partially the appointments of Deputy Director and Liaison Officer and to cover costs associated with fund-raising.

In January 1997 the Isaac Newton Trust made a loan of £1M to the Institute, the interest on this loan to provide infrastructure support, and the loan to be convertible to endowment if, within four years, a matching contribution to endowment can be raised from other sources.

The Leverhulme Trust has announced a grant of £210K (£70K *pa* for three years from 1 July 1997) towards the funding of participants from Eastern Europe, the former Soviet Union, Mexico, South America, China and Israel.

A new contract has been agreed with CNRS (Centre National de la Recherche Scientifique) which will provide continuation of the funding of French participants at Newton Institute Programmes for a further five-year period. The sum allocated for 1997/98 is 400,000 FF.

The London Mathematical Society (LMS) agreed to roll on its support of the Institute at a level of £15K *pa* for two years from 1 July 1997.

Individual programmes attracted additional support from various sources: from NERC and GLOBEC for *Mathematical Modelling of Plankton Population Dynamics;* from the Met Office for *Mathematics of Atmosphere and Ocean Dynamics;* and from LMS for *Four-dimensional Geometry and Quantum Field Theory* and for *Representation Theory of Algebraic Groups and Related Finite Groups* (Spitalfields Days were also supported by LMS for these two programmes).

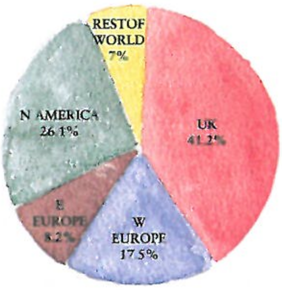
Professor Keith Moffatt, University Vice Chancellor Professor Alec Broers and Dr A Déroulède (CNRS) at the signing of the new CNRS funding agreement (Fletcher Morgan)

Programme Participation

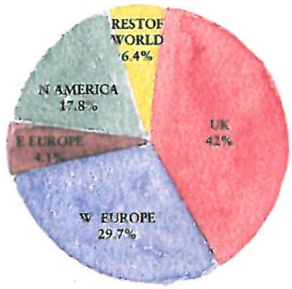
A total of 1061 visitors was recorded for 1996/97. This included 258 long-stay participants, each staying between two weeks and six months (eight weeks on average) and 341 short-stay participants who stayed for two weeks or less. Of these, 39 were affiliated participants, graduate students who accompanied participants and stayed for periods ranging from several days to the full six months. Within the four programmes there were 22 workshops in total, which were periods of more intense activity on specialised topics, and these attracted an additional 462 visitors to the Institute. Undoubtedly there were many others who attended occasionally for lectures, workshops or Institute seminars.

The first programme variation from the Institute's original pattern of four six-month programmes took place this year, producing a different pattern of participation. The two short programmes naturally had fewer long-stay participants than an average six-month programme. The *Mathematics of Atmosphere and Ocean Dynamics* programme had the highest number of long-stay visitors and the highest average length of stay. The total number of long-stay participants in each programme was between 27 (short programme) and 73. The statistics for long-stay participants in the 1996/97 programmes are given in the following table:

The pie-charts below show the percentages of long-stay and short-stay participants broken down by country of residence. There has been a noticeable increase in UK long-term participation (28% in 1995/96, 41% in 1996/97)



Countries of residence of long-stay participants



Countries of residence of short-stay participants

All of this is against the background of regular series of seminars in each programme. During the year 1996/97 there were over 650 lectures and seminars given in the Institute.

Participants notify the Institute of publications either submitted or in preparation. The number of publications notified to the Institute in 1996/97 was 220.

Programme	*Long-Stay Participants*	*Average long stay (days)*
Mathematics of Atmosphere & Ocean Dynamics	73	90
Mathematical Modelling of Plankton Population Dynamics	27	29
Four-Dimensional Geometry & Quantum Field Theory	34	34
Representation Theory of Algebraic Groups & Related Finite Groups	61	85
Non-Perturbative Aspects of Quantum Field Theory	63	55

In addition to the workshops which serve to widen UK participation in the programmes, the programme organisers are encouraged to organise less formal special days, short meetings or intensive lecture series which can attract daily or short-term visitors, so further opening the activities of the Institute to the UK mathematical community.

Institute Publications

An important part of the scientific work of the Institute is the publication of proceedings, survey texts etc. Below is a list of books arising from Institute programmes to date; further titles are due to be published in the near future.

***Publications of the Newton Institute* series, Cambridge University Press:**

CM Brown and D Terzopoulos (Eds)
Real-Time Computer Vision, 1994
223 + xvi pp

JM Charap (Ed)
Geometry of Constrained Dynamical Systems, 1995
331 +xiv pp

BT Grenfell and AP Dobson (Eds)
Ecology of Infectious Diseases in Natural Populations, 1995
521 + xii pp

V Isham and G Medley (Eds)
Models for Infectious Human Diseases, 1996
490 + xxiii pp

D Mollison (Ed)
Epidemic Models, their structure and relation to data, 1995
424 + xvii pp

AM Pitts and P Dybjer (Eds)
Semantics and Logics of Computation, 1997
361 + xii pp

MRE Proctor and AD Gilbert (Eds)
Lectures on Solar and Planetary Dynamos, 1994
366 + xii pp

MRE Proctor, PC Matthews and AM Rucklidge (Eds)
Solar and Planetary Dynamos, 1994
375 + xiii pp

LCG Rogers and D Talay (Eds)
Numerical Methods in Finance, 1997
326 + x pp

CB Thomas (Ed)
Contact and Symplectic Geometry, 1996
310 + xviii pp

NATO ASI series:

H Osborn (Ed)
Low-dimensional topology and quantum field theory
Plenum, 1993
NATO ASI Series B, Physics; v. 315
324 + viii pp

G Grimmett (Ed)
Probability and Phase Transition
Kluwer, 1994
NATO ASI Series C, Mathematical and physical sciences; v. 420
322 + vxi pp

A McKane, M Droz, J Vannimenus *et al* (Eds)
Scale Invariance, Interfaces, and Non-equilibrium Dynamics
Plenum, 1995
NATO ASI Series B, Physics; v. 344
344 + viii pp

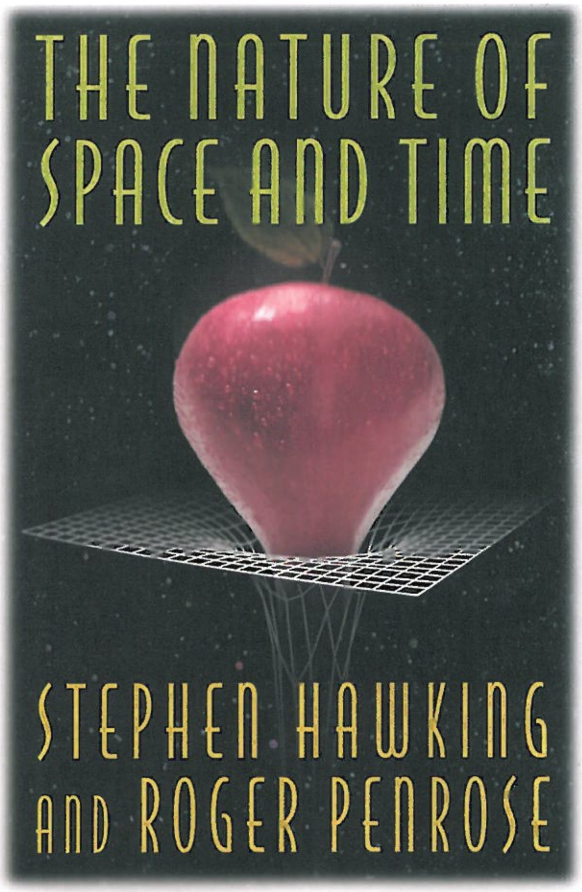
A-C Davis and R Brandenberger (Eds)
Formation and Interactions of Topological Defects
Plenum, 1995
NATO ASI Series B, Physics; v. 349
397 + viii pp

Other titles:

R Anderson (Ed)
Information Hiding
Springer-Verlag, 1996
Lecture notes in Computer Science; v. 1174
277 pp

R Anderson (Ed)
Personal Medical Information Security, Engineering, and Ethics
Springer-Verlag, 1997
250 + vi pp

D Gollmann (Ed)
Fast Software Encryption
Springer-Verlag, 1996
Lecture Notes in Computer Science; v. 1039
218 + x pp



'The Nature of Space and Time', jacket illustration created by Marek Antoniak, based on an original design by Simon Gill and Stephen Hawking

S Hawking and R Penrose
The Nature of Space and Time
Princeton University Press, 1995
The Isaac Newton Institute Series of Lectures
137 pp

M Lomas (Ed)
Security Protocols
Springer-Verlag, 1997
Lecture Notes in Computer Science v. 1189
202 + xiii pp

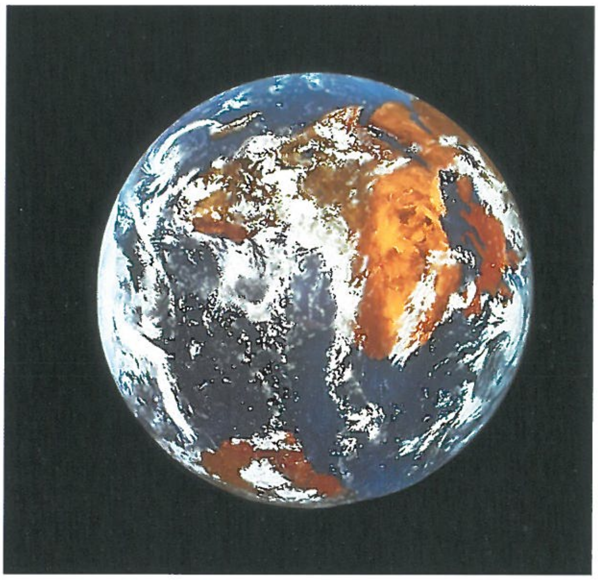
D Pitt, DE Rydehead and P Johnstone (Eds)
Category Theory and Computer Science
Springer-Verlag, 1995
Lecture Notes in Computer Science v. 953
252 pp

Mathematics of Atmosphere and Ocean Dynamics (July to December 1996)

Report from the Organisers:
JCR Hunt (UKMO); ME McIntyre (Cambridge);
J Norbury (Oxford); I Roulstone (UKMO)

Scientific Background

The past few years have witnessed a number of exciting parallel developments in both the mathematical aspects and the phenomenology of stratified, rotating fluid dynamics, with the promise of practically important spin-offs including the analysis and prediction of weather systems. Recent mathematical advances have brought a new geometric viewpoint to these problems, in particular a new appreciation of the central role of potential vorticity and its connection with the symplectic geometric structure of the underlying equations of motion regarded as a hamiltonian dynamical system. The profound consequences that this may have, for such practical matters as weather forecasting and climate modelling, is a prime example of the interplay of geometry and physics.



A Meteosat false colour image of the Earth (Copyright European Space Agency)

The aim of this programme was to explore the deeper problems of atmospheric dynamics, numerical weather prediction (NWP) and oceanography within the framework of analysis and differential geometry. The very complex flow of the atmosphere and ocean (as illustrated above) is believed to be described accurately by the classical equations of fluid motion. In the asymptotic regimes most relevant for weather and climate forecasting, it can be shown that the solutions of the fluid equations stay close to the solutions of much simpler dynamical systems. These approximate models seek to describe flows in which there is a dominant balance between the Coriolis, buoyancy and pressure-gradient forces. Such approximations to Newton's second law are commonly referred to as *balanced models*. Some of these models have hamiltonian structure, and can be shown to have well-behaved solutions, including the coherent structures important for weather forecasts. They can also be derived by applying constraints to the complete fluid equations. It is then possible to derive properties of the weather, viewed as a dynamical system, from the simpler equations.

Idealised canonical vortex structures have been identified with solutions of the Monge-Ampère partial differential equation. Their interaction and evolution, which play a major role in weather developments and in the behaviour of ocean eddies, have been much studied using members of the hierarchy of balanced models, for instance the semi-geostrophic equations. The semi-geostrophic equations are among the balanced models that have been shown to possess hamiltonian structure.

The canonical vortex structures and their interaction and evolution may be described, in the semi-geostrophic model, by solutions to a non-standard optimisation problem. A solution may be singular, with a typical singular surface being identified with a weather front. It has been shown, for example, that the singularities arise from the convexifications of multivalued Legendre dual functions, such as the swallowtail. Balanced evolution, which in this model entails the complete absence of fast inertia-gravity waves, is generated by a hamiltonian such that the solution is a sequence of minimum energy states, in a certain sense. The model evolution preserves symplectic structure and corresponds, within the model approximations, to the fundamental meteorological-cum-oceanographic fluid-dynamical principle of conservation of potential vorticity.

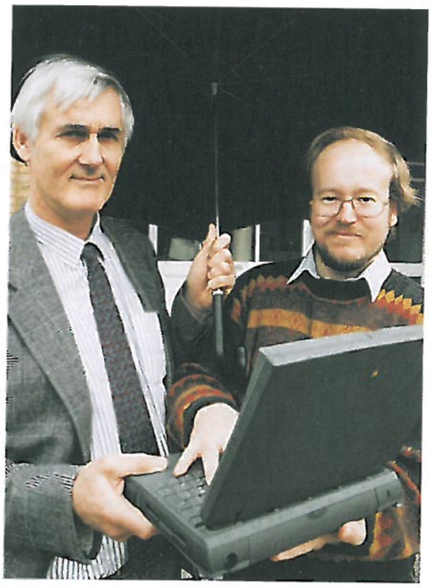
Weather forecasts are routinely computed for up to ten days ahead, based on large quantities of wind, temperature and humidity data that are collected continuously and used to modify the computations. The data are of course insufficient to determine the exact state of the atmosphere. Since they are very expensive to obtain there is a premium on their optimal exploitation. Therefore it is of the highest importance for numerical weather prediction to identify the dominant processes and flow features that determine how the large scale weather patterns develop. By then ensuring that the continuous assimilation of data is consistent with these features, the accuracy of the forecasts is greatly increased. Ocean modelling is beginning to develop similar data assimilation techniques.

A major challenge for the programme was to bring ideas from geometry, analysis and the theory of dynamical systems to bear on the practical and urgent problems of weather forecasting, ocean and climate modelling.

Organisation

The overall planning was carried out by Ian Roulstone and John Norbury in consultation with Julian Hunt and Michael McIntyre. The day-to-day administration of the programme was carried out by Ian Roulstone.

Professor Julian Hunt and Dr Ian Roulstone; the programme attracted media attention (photo by kind permission of Cambridge Evening News)



Many of the participants played a crucial role in the organisation of workshops during the programme: George Craig *Non-conservative processes;* Shouhong Wang and Edriss Titi *Dynamical systems approach to ocean/atmosphere sciences;* Rupert Ford *Ocean dynamics;* Andrew Lorenc *Variational data assimilation systems;* Olivier Talagrand *Mathematical problems in atmospheric and oceanic data assimilation;* Mike Baines and Mike Cullen *The numerical mathematics of weather dynamics;* Onno Bokhove *Low-order models and balanced dynamics;* Marie Farge and Nicholas Kevlahan *Statistics and dynamics of vortices in geophysical flows;* Peter Read and Lenny Smith *Predictability;* Sylvie Gravel and Peter Haynes *Transport and mixing.*

The basic strategy was to organise workshops and seminars with the aim of bringing the mathematicians and meteorologists together in an environment conducive to the cross-fertilisation of ideas. To that end, the first three weeks were devoted to an open forum of introductory lectures, review lectures and research seminars. The aim of the review lectures was to provide something of interest to all participants. The timetable allowed for informal interaction and impromptu contributions. By the end of this opening session the Institute was hosting what must have been one of the most vigorous and stimulating workshops in theoretical fluid dynamics ever staged. Both long-term and short-term goals had been identified and areas ripe for development had been uncovered.

On 2nd/3rd October a presentation of the aims and achievements of the programme was given to a number of senior staff from various European meteorological services. Julian Hunt gave an overview of the programme and Mike Cullen reviewed the work in progress with a lecture on *How mathematicians can help numerical weather prediction.* A general discussion followed with critical reviews of research and development plans for the future.

A one-day meeting of the Dynamical Problems Specialist Group of the Royal Meteorological Society was held at the Institute on 30th October. This out-of-town meeting was attended by about 80 visitors and participants.

The final session of talks from 9th-18th December on *Changing approaches to modelling the atmosphere and oceans* was well attended, with a number of participants who had been present earlier in the programme making a return visit. The talks summarised the plans for continuing collaborative work that had been initiated or consolidated by the programme.

Work is in progress preparing a collection of review articles by various participants, covering a whole range of pure and applied mathematical themes of the programme. Cambridge University Press have indicated that they would be interested in publishing such a volume.

Finally, the support given by the Institute staff was outstanding. Nothing was too much trouble. Everyone felt very much 'at home' and this was, of course, absolutely crucial to the success of the scientific programme.

Participation

Over 250 participated in the programme as a whole. There was a core group of about 35 long-term visitors staying for at least two months, which ensured the programme had a strong backbone. It was evident that, through the longer-term participants, issues raised at various times during the programme, for example during the first three weeks, were raised later in the meeting at other conferences and workshops. The longer-term visitors were, broadly speaking, divided into two groups: Those who interacted as much as possible and those who used the time to actively pursue research interests. This mix worked well - at the end of the programme we had established many new collaborations and a number of papers had been drafted or even submitted.

The personal reports appeared very positive. Several senior members of the programme commented that this had been the most stimulating period of their scientific career.

Meetings

Non-conservative processes

From 19 - 25 August, a workshop was held on the subject of non-conservative processes. The purpose was to consider various processes in the atmosphere and ocean, such as dissipation and convection, that are frequently ignored in theoretical fluid dynamics, and to consider how they may be included in the new mathematical and numerical treatments being developed during the AOD programme. The workshop was well attended by both long-term participants and visitors. There was considerable discussion during the workshop, as the approaches to including non-conservative processes were as varied as the physical phenomena they described.

Dynamical systems approach to ocean/atmosphere sciences

The main topics covered in this workshop were dynamical systems aspects of the atmosphere/ocean, including the construction of models, mathematical analysis and numerical simulations of their solutions, the existence and approximations of global attractors, slow manifolds and inertial manifolds, Hamiltonian balanced models, etc.

Predictability

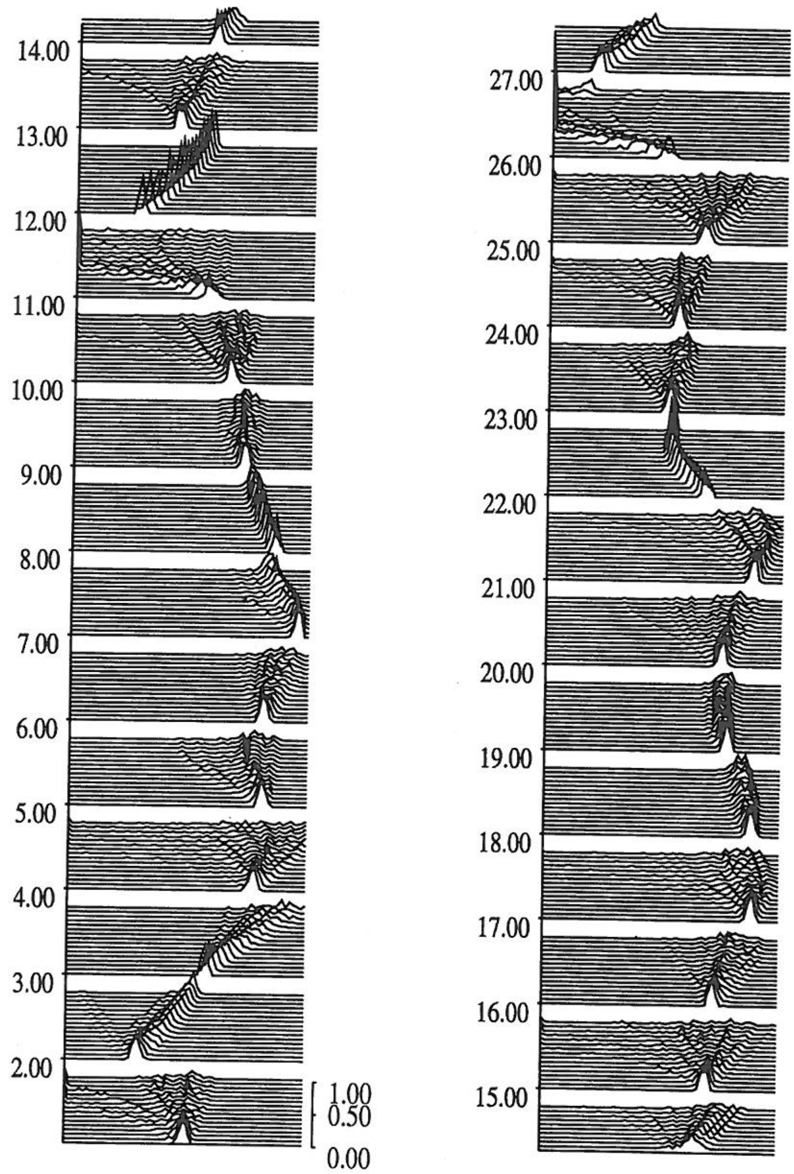
The aim was to conduct a small meeting focused upon formulating the open problems in predictability and determining how best to marshal the resources and insights now available from nonlinear dynamical systems theory, statistics, physical experiments and operational numerical weather forecasting toward solving these problems.

Inasmuch as the formulation of specific questions was the desired output of the meeting, twelve addressable questions were identified. These were based on the following selection of broad questions of interest to the organisers: Given an arbitrarily large observational data set, can we formulate an optimal method for deciding between various ensemble formulation schemes? and if so, to what extent can we expect results to generalise from laboratory systems to the atmosphere? or even between different models of the same system? Can we compute uncertainty due to model error in addition to uncertainty due to imperfect initial conditions? What insights, if any, from low-dimensional dynamical systems can be usefully applied in the context of the dynamics of operational models?

Statistics and dynamics of vortices in geophysical flows

The workshop focused on the challenge of developing new statistical theories of turbulent and other geophysical flows incorporating recent discoveries of the role played by coherent vortices in these flows. The goal of the conference was to try to understand how to combine the statistical and coherent structure approaches in order to create new statistical theories that take advantage of the detailed knowledge we now have of the structure and dynamics of vortices. These hybrid theories should overcome some of the limitations of traditional statistical theories (eg the Kolomogorov theory) and find applications in a wide variety of geophysical flows.

The workshop was divided into two weeks. The theme of the first week was the statistical theories of two-dimensional turbulence: point-vortex versus vortex-patch models. In the second week the talks and informal discussions dealt with the coherent structures encountered in geophysical flows: stability, dynamics and how to describe them statistically. The workshop was very flexible and informal. There was only one speaker each morning who gave a talk on one of the themes of the workshop. The speakers had as much time as they wished (usually 2-3 hours) and lively discussions were common. The afternoons were devoted to shorter one-hour talks, either prepared in advance or stimulated by other presentations. The participants represented many different specialities and approaches, from applied mathematicians to physicists to oceanographers, and this mix promoted many fruitful cross-disciplinary discussions.



The evolution of the probability density functions reflecting ensemble forecasts of temperature in a rotating annulus experiment. Time increases upwards and the gaps are inserted to mark the time of each new observation. With each new observation, the uncertainty collapses back to a Gaussian distribution. (Courtesy of LA Smith, 'The Maintenance of Uncertainty', International School of Physics 'Enrico Fermi' Course CXXXIII)

Low-order models and balanced dynamics

This workshop focused on three aspects: low-order, ie finite-dimensional, dynamical models of atmospheric and oceanographic phenomena; hamiltonian balanced models; and vortex-gravity wave interactions. The low-order model session offered new qualitative insights into the initialisation, assimilation and predictability problem in weather forecasting, including the slow-manifold issue. The balanced models session offered new results on spherical extensions, and (non-) linear stability theorems of semi-geostrophic dynamics and L_1-dynamics. The vortex-gravity wave session offered new results on the generation of gravity waves, including asymptotic stability criteria, by geophysical flows and rigorous mathematical theories in small Froude- and anisotropic Rossby-number limits.

Starting with an overview lecture on balance, the workshop was structured logically: insights from low-order modelling, ie from KAM-tori, assessments of balanced models and definitions of slowest invariant manifolds, found echoes in the subsequent sessions: in small-divisor problems, (in)stabilities in hamiltonian balanced models, in wave action and phase, and in weak-wave interactions. The informal proceedings with 12 extended abstracts were much appreciated and provide a lasting record of the workshop. Of the 20 speakers, five came only for the workshop, while the majority were long-time participants. The workshop thus served as a soundboard for ideas developed during the Newton programme.

Variational data assimilation systems

The theoretical promise of variational data assimilation, for providing an optimal fit to observations and constraining model equations, has been known for many years. However, it is only in recent years that the technology has been available to make the approach practicable for operational numerical weather prediction. Several centres have implemented, or are developing, operational variational systems using observations at a synoptic time (3DVAR). For these, the key design problem is the representation of the prior constraints on balance and smoothness, for numerical models with very many degrees of freedom. The extension to observations for a time period (4DVAR) necessary for many of the theoretical advantages, poses another major practical problem, namely the cost of many forecast and adjoint mode integrations over the time period.

This workshop, part of the European collaboration in short-range numerical weather prediction, was designed to exchange experience and foster collaboration in developing the techniques whereby the theoretical methods of variational assimilation could be applied in operational forecast systems. There were presentations about many of the systems being developed in operational NWP centres (ECMWF, NCEP, UK Met Office, Meteo France, HIRLAM), plus discussion sessions about techniques and problems from 3DVAR and 4DVAR. A principal goal of the discussion groups was to initiate collaboration. Topics included:

- Requirements for, and use of, software packages such as large-scale minimisation and Lanczos algorithms
- The exchange of results for standard test cases, such as FASTEX
- Exploitation of o-b and o-a statistics for tuning and validation of assimilation algorithms
- Software standards, and methods for exchanging and sharing code
- Observation preprocessing
- Balance conditions

In the majority of these areas, new collaboration was initiated.

Ocean dynamics

Presentations and discussions in this Euroconference focused on: large-scale circulation models using the thermocline equations; topographic effects, ranging from topographic control of ocean gyres to dynamical processes on the coastal shelf; parametrisation of eddies in the ocean and the internal variability of ocean circulations.

A particular feature of the discussion on large-scale circulation models was the recent application of symmetry methods to the solution of the thermocline equations, and their use in the interpretation of numerical simulations aimed at understanding the structure of the oceanic thermocline. On the topic of eddy parametrisation, several speakers showed that substantial improvements could be made to ocean circulation models by including an effective eddy transport process, in addition to an eddy diffusion process, in large-scale models. The small scale of oceanic eddies compared to the large-scale circulation makes this imperative (since computational resources required to resolve individual eddies in a global model are prohibitive) but also possible (in that it provides, at least to some extent, the necessary scale separation). The scale separation is perhaps not as well defined as one might like, but the conference has further stimulated the inclusion of eddy parametrisations into ocean models which explicitly exclude eddies, as the sternest test of the ideas.

It was proposed that ocean circulation models should be viewed as dynamical systems, whose dependence on certain parameters should be viewed in terms of bifurcation theory. It was suggested that variability of thousands of years could be expected, even in the absence of convective overturning.

Finally, various numerical innovations were presented, primarily concerned with the accurate representation of topography in numerical models of the ocean, which remains one the most challenging technical problems in the field.

Mathematical problems in atmospheric and oceanic data assimilation

Balance conditions are an important constraint for the new generation of data assimilation schemes using variational principles. These seek to minimise cost functions based on the fit of observations to four-dimensional integral curves of the equations of motion. This is usually viewed as an inverse problem, inverting the prediction of observed values from calculated model values. For practical models, there is insufficient data to determine all the modes (ie types of motion) that the model can represent. This can be resolved by a number of strategies, including the use of balance constraints which characterise features required for the purposes of forecast users. This Euroconference involved scientists from both universities and meteorological services. There were five invited lectures and 18 shorter talks. A number of graduate students presented their work, and others attended on an educational basis. The week concluded with a lecture and discussion on the possibilities for future collaboration between mathematicians, physicists and weather forecasters.

The numerical mathematics of weather dynamics

Topics included numerical methods with invariance properties, constrained hamiltonian systems, the approximation of inertial manifolds, symplectic integration, adaptive grids and front tracking, non-smooth optimisation, data assimilation and singular vector analysis. Numerical sessions associated with the Euroconferences of the previous two weeks in *Ocean Dynamics* and *Atmosphere and Ocean Dynamic Assimilation* formed part of this workshop.

Brickworks at dusk (Woburn, Bedfordshire). By day, smoke is carried freely up and down by the motion of the air when there is any thermal convection from the ground. At sunset, when the cooling of the ground has stabilised the lower layers of air, they are much less turbulent. The heat of the gases carries the plumes of smoke upwards in the neutrally stratified air at higher levels and no odour reaches the ground.
(Photograph copyright Diane Wylley)

Transport and mixing

The need for an accurate description and better understanding of transport and mixing in the atmosphere and oceans has been underlined by the growing interest in the role of chemistry in climate, along with the feasibility of fine scale atmospheric and oceanic numerical simulations. A characterisation of transport and mixing in the geophysical context is made difficult because of its highly inhomogeneous nature, as is exemplified by the existence of barriers (or quasi-barriers). This subject is one of the most interdisciplinary of all the subjects in atmospheric and oceanic dynamics. The workshop combined the presentation of observational and simulated results, along with a review of classical and more recent theoretical approaches to the study of transport and mixing (eg contour advection and chaotic mixing in periodic and aperiodic flows). Substantial discussion sessions punctuated the meeting, and helped to clarify certain key concepts.

Achievements

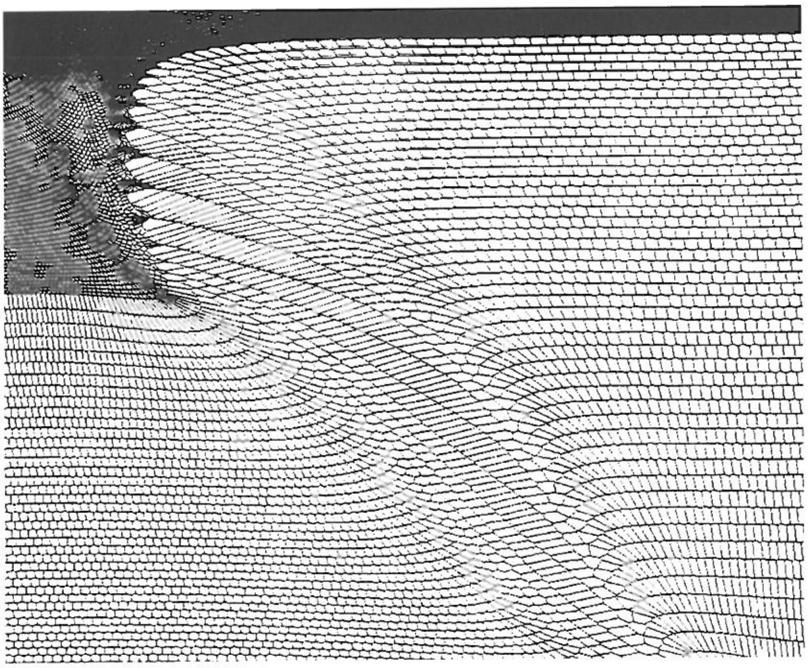
3D Euler, global regularity and integrability

During the Cambridge programme the work was focused on mathematically rigorous asymptotic analysis of geophysical dynamics described in the context of Boussinesq equations for different Burger number regimes. This was a continuation of the work on the small anisotropic Rossby number limit of 3D rotating Euler equations presented during the Newton Institute Programme *From Finite to Infinite Dimensional Dynamical Systems* in July - December 1995. On the physical side, regimes of geophysical dynamics presenting the global picture for small Froude or Rossby numbers were described summarising the physical implications of our mathematical analysis. As the Burger number increases from zero to infinity, it was demonstrated that a gradual unfreezing of energy cascades for ageostrophic dynamics occurs. On the mathematical side, the challenge was to prove that dynamics of limit equations uniformly closely approximate geophysical turbulence. Such theorems were proven with much less regularity required for the initial data, with the Sobolev space H_7 being the worst and H_4 being the best case for 3D Euler-Boussinesq. Existence of attractors for the "primitive" equations of geophysics was proven.

In addition to the programme seminars, many participants organised small working groups which were conducive to cross-fertilisation

Hamiltonian balanced models

Progress was made in a number of key areas: (a) a classification of which features of hamiltonian balanced models, like semi-geostrophic theory, are special and which are general, and how such models can be generalised to arbitrary accuracy without losing hamiltonian structure, (b) how such generalisations can be constructed, at least formally, by inserting into the hamiltonian framework any of the highly accurate balance conditions used in recent studies of accurate potential vorticity inversion, and (c) how, in a certain class of models, canonical coordinates, analogous to geostrophic-momentum coordinates, can be found. Quaternionic and Kähler structure has been identified within the class of such models.



An example of a geometrical configuration comprising roughly 10,000 finite elements of a semigeostrophic model of two-dimensional frontogenesis. The picture shows an upper "stratospheric" layer of elements distributed with a relatively low potential density above a "troposphere" of greater potential density (low potential vorticity). Fast geometrical algorithms currently under development will enable three-dimensional dynamically evolving finite element solutions to be constructed by analogous methods and used to elucidate various qualitative and quantitative aspects of extratropical meteorological flows where the spontaneous formation of discontinuities obstructs investigation by the more conventional numerical approaches.
(Courtesy of Dr J Purser)

Studies of semi-geostrophic flow focused on particular finite dimensional solutions. The purpose was to understand the Lie group significance of semi-geostrophic models. These finite dimensional solutions are reminiscent of heavy top solutions. With semi-geostrophic theory at one end of complexity and a heavy top at the other, one can trap many other types of finite dimensional Lie group dynamics in between. For 2 x 2 real matrices a 3D Nambu-type system was found. The polar decomposition of matrices $M = PK$ (here P is symmetric positive definite and K is an $O(2)$ matrix) allows this system to decouple into two systems. One system is for P and is expressible as $\dot{P}^2 = [J, P^2 - P]$, where J is the symplectic 2 x 2 matrix and [,] is the matrix commutator. The other system is for K, driven by P; so it is just a quadrature for K. The case of 2 x 2 Hermitean matrices gave the same interesting form which for Hermitean matrices is also of Nambu type and just misses being Poisson-Lie by a conformal rescaling of time. These systems need to be further investigated from this point of view and their Lie group theoretic significance understood.

A spin-off from the above was a study of another class of low-dimensional particular solutions of semi-geostrophic theory. This is the case of two finite elements interacting in a rectangular region undergoing semi-geostrophic flow. This problem has an interesting discrete symmetry (Z_2 x Z_2 acting on R^5) which has a profound effect on its heteroclinic orbits and on its bifurcations (the aspect ratio of the rectangle is the bifurcation parameter). Numerical code has been written for calculating the trajectories of the motion to check the results for the jumps the trajectories take when they cross the diagonals of the rectangle which defines the fluid region.

Finally, a study of the finite dimensional version of semi-geostrophic theory, useful in numerical simulations, was undertaken. This problem is intimately related to the convexity properties of the momentum map and - after a re-formulation - can be re-expressed as a minimisation problem relative to the Kostant polytope.

Geophysical fluid dynamics

Around the main theme of the programme on variational methods, and analysis and derivation of balanced models for the ocean and atmosphere, a tremendous number of fascinating issues arose for those interested in smaller (meso-) scale motions. The various one- and two-week symposia were perfect for bringing in researchers who could attend for only a short time. The Ocean Modelling week, for example, brought in most UK dynamical oceanographers as well as US visitors. Presentations of numerical integrations for coastal flows, in particular, stimulated much work on applying the general theme of variational methods to these smaller scale flows. Similarly, presentations on bottom-trapped currents stimulated work on non-separable instability problems. Many of the participants whose primary interest was large-scale flow brought with them interesting questions about smaller-scale flow (eg in shelf-wave/Rossby-wave interaction) and throughout the programme there were always stimulating and productive discussions of meso-scale physics. In each of the examples given here, the flows are now far better understood than they would have been had the programme not taken place. The contribution to GFD was positive and has led to new research and collaborations that endure now the programme is over.

Rearrangement theory

Polar factorisation of vector-valued functions

Subject to certain technical assumptions on f, an integrable function $f : X \to \mathbf{R}^n$ defined on a measure interval X was shown to have a polar factorisation $f = f^{\#} \circ \sigma$ through Y for any Y a measurable set in $\mathbf{R}^n$, where $f^{\#}$ (the *monotone rearrangement* of f) is equal a.e. in Y to the gradient of a $\overline{\mathbf{R}}$-valued convex function defined on $\mathbf{R}^n$, and $\sigma : X \to Y$ is a measure-preserving map. One sufficient hypothesis is that, after the removal from X of all level sets of f having positive measure, the inverse image under f of any set of zero measure again has zero measure. This extends and simplifies the pioneering work of Brenier on polar factorisation.

Rearrangements and quasi-geostrophic vortices

An existence theorem was proved for a steady vortex of bounded extent in an ambient three-dimensional quasi-geostrophic shear flow, having the property that in each horizontal plane, the potential vorticity anomaly is a rearrangement of a prescribed function. The shear of the ambient flow was assumed to be a linear function of height, the PV anomaly was assumed to have the same sign as the ambient shear, and the depth of the fluid was assumed infinite. The proof of the existence theorem was achieved by maximising an energy functional over an appropriate class of functions, called *stratified rearrangements.*

Rearrangements and stability

Work continued on the stability of steady solutions of the vorticity equation of hydrodynamics, in a planar simply-connected region. By a solution (steady or unsteady) is meant a weak *solution* having vorticity in L^∞, as studied in Judovič's existence and uniqueness theorem for the initial-value problem. By *stability* is meant nonlinear stability in the L^2-norm on the space of vorticities, where the steady state and the admissible perturbed states are solutions as defined above. It was shown that strong maximisers or minimisers of kinetic energy (or, for a circular region, angular momentum), are stable. For a circular region, the result establishes stability of all solutions with circular symmetry for which the vorticity is a bounded monotone function of the radius (extending results of Dritschel). For a more general region, the unique energy minimiser on the class of rearrangements of an arbitrary non-negative L^∞-function (shown to exist by Burton and McLeod) was shown to be stable.

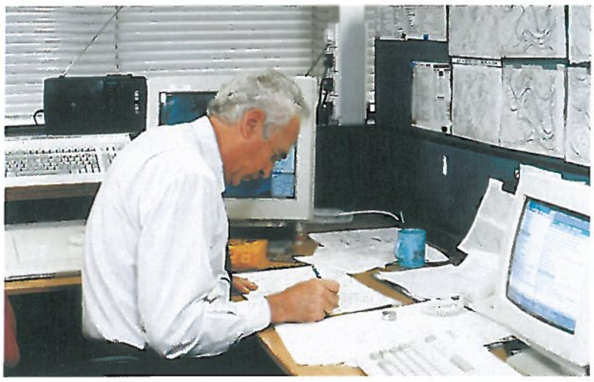
Energy minimising solutions of the semi-geostrophic equations

For a solution to the semi-geostrophic equations on an f-plane, the *Cullen-Norbury-Purser* principle states that at each fixed time, the fluid particles arrange themselves to minimise geostrophic energy. This yields a variational problem: minimise energy over the set of rearrangements of a given vector-valued function X_0, where the minimiser (if it exists and is unique) gives the actual state of the fluid. It was proved that for any square integrable X_0, the unique minimiser is the *monotone rearrangement* of X_0, equal a.e. to the gradient of a convex function. In this way, solutions can be viewed as a sequence of minimum energy states. This result shows that the corresponding Monge mass transport problem has a unique solution even when the measure on the co-domain fails to be absolutely continuous with respect to Lebesgue measure. Work to extend these results to spherical geometry, where f varies, was started.

Potential benefits to forecasting

Numerical weather prediction and ocean modelling is a problem of mathematical physics. The aim is to improve the present methods of atmosphere and ocean forecasting which utilise the largest computers in the world and 100 million observations per day costing, according to the World Meteorological Organisation, more than $1 billion per year. The result is that increasingly reliable weather forecasts are regularly issued out to five days, approximate climate change predictions are provided to governments for periods of up to 100 years, and seasonal to yearly variations of some features of the main ocean currents are predicted.

However, even with the largest computers envisaged, it is impossible to solve these equations accurately. At the Institute, new results have been found which show to what level of accuracy the complete equations can be



Senior forecaster based at the National Meteorological Centre, Bracknell (Crown Copyright, reproduced by permission of the Controller of HMSO)

represented by simpler systems which can be solved exactly and understood more completely.

The fact that the atmosphere evolves close to a dynamical system with high predictability both explains the current success of operational predictions, and suggests that useful further progress can be made by exploiting this closeness more fully in the design of numerical prediction systems. Results presented at the meeting from both atmosphere and ocean models suggested that it was well worth making efforts to reduce the generation of other types of solution by computational errors or model design which could not support the simpler solutions effectively.

Atmosphere and ocean models include very sophisticated models of sub grid scale processes and physical forcing. Considerable progress was made in showing how some sub grid scale processes can be represented as a transport, and how this transport can play a key role in maintaining the large scale atmosphere/ocean circulations. This has substantial implications for model design, since these transports have to be properly integrated with the rest of the dynamics.

Both atmosphere and ocean forecasts can only be made with an analysis of the limited observations available. A promising new technique requires fitting the model to data over a time period. At the meeting, ideas from control theory were used to suggest better ways of doing this. Knowledge that the solution is close to a simple state can be exploited in using the data. Similarly, the knowledge that typical model errors can largely be explained by Lagrangian displacements can be exploited in designing analysis methods that use the observations to correct this type of error. Some areas of the atmosphere are much more sensitive than others, and properties of the local stability of the simple underlying dynamical system can be used to optimise the analysis in these critical regions. Perturbations to those areas are also the most effective way of generating ensembles for use in making probability forecasts.

A recurring controversy during the programme was the issue of whether future progress would be best achieved through faster supercomputers but without any major scientific advances, or whether the new theoretical concepts discussed at the Institute will have to be used to achieve the biggest practical impact. This was aired at a special meeting involving scientists on the programme and national meteorological services held in October. For the atmosphere there is a strong case for exploiting the new methods, but the timescale on which they can be applied is not yet clear. For ocean modelling there is wider agreement that the new methods discussed at the Institute will be necessary, especially in order to exploit the very limited amount of observations available for the deep ocean.

Several computational results were presented during the programme which showed that methods based on fundamental new results in geometry and dynamics can indeed give results almost impossible to obtain by traditional methods. We can hope for substantial advances in our ability to predict weather and climate over the next few years.

Edinburgh at sunset, showing cirrus and cirrus spissata cloud formations (Photograph copyright SD Burt)

Follow-up meeting

A follow-up meeting will be held at the Newton Institute from 1-5 December 1997. It is anticipated that about 30-40 scientists will attend, many of whom participated in the 1996 programme. The meeting will provide a forum for reviewing recent progress in the areas listed above under 'Achievements'.

Mathematical Modelling of Plankton Population Dynamics (29 July to 6 September 1996)

Report from the Organisers:
J Brindley (Leeds), M Fasham (Southampton), J McGlade (Warwick)

Organisation and Character

The main aim of the programme was to bring mathematicians into close contact with leading representatives of various scientific disciplines involved in plankton dynamics, including biologists, oceanographers, ecologists and geochemists. The structure of the programme was planned to allow them to talk about their own interests and contributions to the field, and to encourage them to establish new collaborations of the breadth and depth required for scientific progress in this highly interdisciplinary area.

The goal of developing new collaborations is best attained by allowing adequate time for interactions between two or more individuals and for informal discussion. This dictated the general pattern of activity within the programme, which was of course the first to depart from the "standard" six-month format; thus there was one, occasionally two general seminars each day with a more intensive day once each week devoted to a specific topic. Discussion time was generous and well filled; differences in culture between the participants ensured lively and highly instructive exchanges.

One week was occupied by a conference on *Modelling the Role of Zooplankton in the Marine Food Chain,* sponsored by GLOBEC, and constituting a Euroconference of the Newton Institute. This was intensive and contained about 40 talks. Many of the 50 or so attendees at the conference spent a few extra days on the programme, often giving more leisurely expositions of their work. Their attendance was aided by generous financial support from GLOBEC and NERC. This conference was arranged in response to the advice of the Scientific Committee of the Newton Institute to give consideration to the relationship to the fishing industry of plankton population dynamics.

Participants

Recent rapid developments in the subject to a certain extent dictated the desirable spread of participants, and the organisers brought together a very interdisciplinary group. Outside the conference week, the number of full participants in the visitor programme to the Institute varied from 15 to 24. Numbers in residence were, however, significantly higher than this, with a regular attendance of 8-10 research students, resulting in a ratio of senior to junior participants somewhat different from that in most other programmes. This different balance was a deliberate objective of the organisers; in the event, not only did the students gain much from their attendance, but to an unexpected degree the programme gained much from their own contribution to seminars and discussion. All of them have now embarked on new collaborations, and in one or two cases papers are in a fairly advanced state of preparation. Their presence was widely welcomed and appreciated and, given appropriate funding, a big "if", they will form the core of a vigorous next generation of UK researchers in this field.

Other participants had stays varying from a few days to six weeks. Most of the key individuals identified by the organisers were able to participate for some of the time; the only frustration was that unavoidably off-set timings of visits sometimes meant that desirable interaction was lost.

The geographical distribution of visitors is set out below. It inevitably reflects the global nature and importance of the problem of plankton dynamics, and its breadth ensures the widest opportunity for future UK participation and contribution. Participation from the former Soviet Union and Eastern Europe, India, China and Japan was particularly welcome, as was that from the southern hemisphere. The shortness of the programme and the time of year mitigated against "outreach" visits by overseas participants, but one or two of them did visit other UK establishments.

Participants came from: USA 8, Denmark 1, Germany 2, India 1, Netherlands 1, Russia 3, Japan 2, Turkey 1, Ukraine 1, Canada 2, Poland 2, New Zealand 1, Spain 1, Norway 2, Italy 1, China 1, South Africa 1, France 1, together with 37 from the UK (including students).

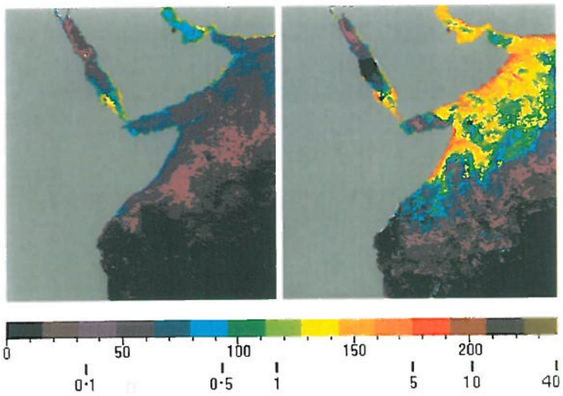
Scientific Programme

The programme aimed to explore and define the many-faceted nature of the theme, and at the same time allow participants the maximum time for active research. The daily seminars quickly exposed not only wide gulfs in specific knowledge and expertise between various participants, but also tension (exploited creatively) between the generalising desires of mathematicians and the biologists' perception of the importance of detail. Over the six-week period a number of features of commonly agreed importance and concern were recognised; a few of them were intensively discussed. They and others will form the basis for future collaboration. Some of the more important are:

- **sparsity** of data and expense of obtaining it - hence need for improved methods of collection (eg satellite), interpretation, assimilation, parameter estimation

- wide **variance** of behaviour revealed in real data - need to develop methods for handling/predicting
- this **importance of structure** (age, size, fitness etc) in population dynamics driving a need for models and mathematics capable of handling this
- **interaction** of biological models with fluid flow on all scales, from picoplankton(1μm) to mesocale oceanic (1000 km), including its influence on population distribution/patchiness,
- **evolutionary strategies,** eg exploiting niches in turbulence spectrum,
- **model** construction, calibration and testing,
- need for wider development of **stochastic** modelling.

All these areas pose exciting mathematical opportunities demanding imaginative modelling and the development of new mathematics in an atmosphere of real interdisciplinary challenge. The presence of participants on the concurrent *Mathematics of Atmosphere and Ocean Dynamics* programme, and their interactions with PPD participants, served to emphasise the extent to which methodologies developed there should be valuable in PPD in the future.



Winter (left) and Summer chlorophyll levels in the Arabian Sea and Persian Gulf. Chlorophyll level is a measure of phytoplankton density, and these satellite images indicate very clearly the large seasonal variations and spatial structure, partly but not wholly explicable in terms of ocean currents and turbulence, or other physical forcing. Mathematical modelling seeks to develop understanding of both the forcing effects, and also the underlying "free" and very rich biological/physical dynamics.

Outcomes and Achievements

The principal outcomes of a programme as short as six weeks are inevitably matters of future promise rather than of completed achievement. Nevertheless a large number of participants reported that they had embarked on, progressed with, or completed the writing and submission for publication of research papers during their stay, and it is fair to assume that all this output will gain from the interactions of the authors at the Newton Institute.

Much more important though is the foundation for future work laid during the programme. The focusing of attention on several areas of mutually agreed urgency, as indicated above, will ensure appropriate concentration of effort and collaboration, and plans established at the Newton Institute should greatly enhance the effectiveness and minimise unnecessary duplication of that research effort. A number of future informal group meetings have been planned, and, slightly more formally, a mini-symposium on the interaction of plankton population dynamics with fluid dynamics is proposed for the SIAM meeting on Applied Dynamical Systems in Utah in May 1997. A good deal of the momentum of the Newton Institute meeting will also be maintained by the JGOFS Modelling Symposium (Oban, May 1997) and future GLOBEC meetings.

The organisers decided against publication of conference proceedings; they felt that the interests of the subject would be met much better by a focused book written by a small number of people, rather than by a collection of perhaps disparate papers. Such a book is now planned and discussions are in hand with Cambridge University Press for publication at the end of 1997. Another outcome which will, we trust, yield tangible results, will be the impetus which the existence of the programme will give to applicants for the various national and international funding bodies for future support for the field, and indeed for the creation of whole new funding programmes (eg PRIME 2, UK GLOBEC). An early instance of this is a bid to the EU MAST 3 Programme by Professor McGlade and several colleagues on "Physical/Biochemical Models of the NW Approaches to European Shelf", which has already received substantial support from NOAA.

Four-Dimensional Geometry and Quantum Field Theory (4 November to 13 December 1996)

Report from the Organisers:
MF Atiyah (Cambridge), I Singer (MIT)

Organisation

This was an experimental six-week programme and many lessons were learnt about how to run such a programme. In the first place, since it took place during the normal academic working period it was not realistic to expect many long-term senior visitors. One can take a sabbatical term, but hardly a sabbatical six weeks.

This meant that the programme had to be structured around a number of key speakers, each of whom came for a week and gave a set of three lectures. The programme then built around these by providing introductory or supplementary material.

To make the best use of this format it was decided to encourage as many younger visitors as we could manage, both PhD students and young postdoctorals. We advertised widely and offered financial support where possible, and we were grateful to the London Mathematical Society for making us an additional grant for this purpose.

We were very successful in achieving this objective and one of the main benefits of the programme will have been the stimulating effect on a large number of younger mathematicians and physicists (mainly from the UK).

Participation

There were 34 invited long-stay participants who visited the Institute for 15 days or more, not necessarily concurrently, and 52 invited short-stay participants who visited for an average of six days each. In addition, many others attended specific talks or series of lectures. There were a large number of visitors during the time of the Witten talks. Naturally this period had the highest concentration of visitors of the programme.

Local Involvement

The subject of this programme was one in which Cambridge has very great strength. Many senior members of both departments were actively involved and helped out by giving lectures. Since the audience contained many younger UK Mathematicians from elsewhere this was a case where Cambridge contributed (at no extra cost) to the whole programme.

Longer Term Visitors

Exceptions to the statement that long-term senior visitors from abroad were not available was provided by two Japanese mathematicians (Furuta and Nakajima) who came for the whole time and Nakajima in particular was a very useful contributor, giving a number of significant lectures. Tyurin from Moscow also came for an extended period and was a valuable member.

Scientific Programme

Each of the first four weeks had a key lecturer focusing on one theme. These were as follows:

Week 1
David Morrison (Duke University and the Institute for Advanced Study, Princeton)
Mirror Manifolds

Week 2
Edward Witten (Institute for Advanced Study, Princeton)
Quantum Field Theory and Invariants of 3-Manifolds

Week 3
Clifford Taubes (Harvard)
The Seiberg-Witten equations and Symplectic Manifolds

Week 4
Peter Kronheimer (Harvard)
The Seiberg-Witten equations applied to 3-Manifolds

The last two weeks were more varied but included key lectures by a number of physicists on *String Theory* (G Moore, R Dijkgraaf, M Duff, M Douglas). Several lectures were also given on *Monopoles.* Segal reviewed his work with Selby on the topology of monopole moduli spaces, explaining how this provides evidence for duality in supersymmetric Yang-Mills theory. Jarvis presented a new 1-1 correspondence between monopoles and rational functions defined on the 2-sphere of half-lines emanating from a chosen point. This correspondence is consistent with the known topology of monopole moduli spaces, and gives new insight into the possible discrete rotational symmetries of monopoles. Subsequent to the programme, Jarvis's ideas have stimulated further work on symmetric monopoles and on monopole scattering processes, and they have been applied to Skyrmions, a type of three-dimensional soliton which provides a model of atomic nuclei. In the final week there was a one-day Spitalfields Meeting.

The topicality of the whole meeting was fully justified by the continuing new results being made and presented. For example Morrison's lectures dealt with very recent new ideas on mirror manifolds, developed by Yau and others, while Witten's lectures presented material that had not yet appeared anywhere.

Witten's lectures were undoubtedly the high point of the programme. Up to 200 people attended his lectures (overflow rooms with CCTV were arranged). Videos of his lectures were also made and there has been much demand for these (both on loan and on sale). In addition to his lectures at the Newton Institute Witten gave two lectures at DAMTP and one to the annual meeting of the LMS in London.

The main topic of the programme was 'duality' as it has emerged in quantum field theory, and its applications to geometry. In fact duality seems now to have revolutionised string theory and the main thrust of the meeting was to expose mathematicians to these exciting developments in physics.

When preparing the programme the main mathematical breakthrough that was to be our focus was the work of Seiberg and Witten, centred round their now famous differential equation. The duality ideas that emerged from the physics predicted that the Seiberg-Witten theory should be essentially equivalent to the Donaldson theory of 4-manifolds. Subsequent work has fully justified these expectations.

When the programme actually took place things had moved on and Witten's lectures dealt with the application of similar ideas of duality to 3-manifolds. Witten showed that the invariants of 3-manifolds that had been obtained from the perturbation expansion of Chern-Simons quantum field theory had counterparts arising from other theories. In these the role of Lie groups was replaced by hyperkähler manifolds. This was totally novel and unexpected and will clearly become the focus of mathematical attention. This term a seminar is being run on the subject in Cambridge (using the video of Witten's first lecture as an introduction). There is also parallel activity at the IHES in France.

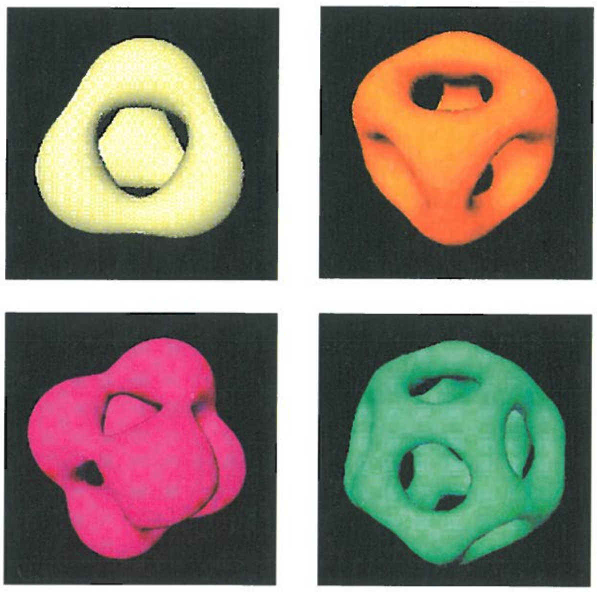
A motivating special case of Witten's new theory identifies the Casson invariant of a 3-manifold with the invariant arising from the 4-dimensional hyperkähler manifold that parametrises the SU(2)-monopoles of charge 2(modulo translation). This identification relies on the deep duality ideas of Seiberg and Witten and provides a fascinating connection between two apparently disparate topics, both of which have been extensively studied by mathematicians.

The higher charge monopole moduli spaces also play an important role as examples of hyperkähler manifolds in Witten's theory. This has given new impetus to attempt to better understand the geometry of these manifolds. Some properties of the hyperkähler metrics have been obtained using quantum field theoretic methods, by a collaboration of programme participants, (Dorey, Khoze *et al*). Hyperkähler manifolds arise in the context of super-symmetric quantum field theories and Witten once again urged mathematicians to get to grips with the technology of supersymmetry.

Witten's third lecture explained the latest ideas of duality in the context of string theory. Although the mathematics of this is poorly understood it was quite clear that most aspects of duality had their most natural interpretation in terms of string theory. Understanding what this all means in mathematical terms will be one of the great challenges of the future.

Logistics

A short programme with a large turnover of short-term visitors puts a considerable strain on the Institute staff. As a scientific organiser I was protected from their problems and, as far as I could judge, everything ran very smoothly. However there are no doubt lessons that can be learnt on the administrative side.



Surfaces of constant energy density for magnetic monopoles with platonic symmetry
(PM Sutcliffe, University of Kent)

Representation Theory of Algebraic Groups and Related Finite Groups (January to June 1997)

Report from the Organisers:
M Broué (Paris), RW Carter (Warwick), J Saxl (Cambridge)

Introduction and Objectives

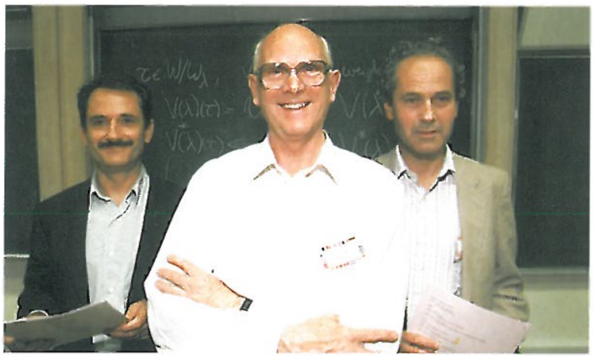
The leading pioneer in the development of the theory of algebraic groups was C Chevalley. Chevalley's principal reason for interest in algebraic groups was that they establish a synthesis between the two main parts of group theory - the theory of Lie groups and the theory of finite groups. Chevalley classified the simple algebraic groups over an algebraically closed field and proved the existence of analogous groups over any field, in particular the finite groups of Lie type.

Since Chevalley's pioneering work in the 1950s the representation theory both of the algebraic groups and of the related finite groups has developed rapidly, and remains at the centre of interest today. The aim of the 6-month meeting was to advance this representation theory in a number of directions.

The first such direction can be described as modular representation theory in defining characteristic. Here we take a simple algebraic group over an algebraically closed field of prime characteristic p, and consider representations of the group over this same field. There is no known character formula for the irreducible modules, but there is a conjectured character formula due to G Lusztig when p is not too small. Attempts to prove Lusztig's character formula have led to spectacular progress over recent years.

The second direction is modular representation theory in cross characteristic. Here one takes the finite groups of Lie type over a field of characteristic p and considers representations over an algebraically closed field of characteristic l different from p. Let B be a block of the group G with defect group D and B_D the corresponding block of the normaliser $N_G(D)$. Broué has conjectured that, if D is abelian, the derived categories of the finitely generated B-modules and B_D - modules are equivalent. Attempts to prove Broué's conjecture have led to major advances in the cross characteristic theory.

The third direction is the relationship between the modular representation theory and the subgroup structure of algebraic groups. For simple algebraic groups over the complex field definitive results on subgroup structure were proved by Dynkin in the 1950s, but the corresponding results for groups in prime characteristic are much more difficult, and intimately related to the modular representation theory.



Programme organisers M Broué, R Carter and J Saxl (Findlay Kember)

Organisation

Most of the world leaders in algebraic groups attended the programme for some period during the six months. The organisers aimed to encourage a stimulating but informal research atmosphere. This was done by bringing participants together in the Newton Institute at certain regular times. The open structure of the Institute greatly assisted this process. Conversations between participants regularly took place over morning coffee and afternoon tea, and at occasional wine receptions.

A regular series of seminars was organised. These seminars usually took place twice a week, on Tuesdays and Thursdays, and provided an opportunity for participants to speak in turn about their work. Care was taken not to organise too many such seminars, so as to leave participants with plenty of time for their research work. The names of speakers in the RAG seminars and the titles of their talks are given in the Appendices, available separately from the Institute.

In addition there was a series of six seminars given by RW Carter on the recently published paper of Lusztig, *Unipotent representations of simple p-adic groups* which attracted a regular attendance of some 20 participants.

The organisation of the meeting was greatly assisted by the dedicated work of the staff of the Newton Institute.

Meetings

Instructional Conference (6 - 11 January)
The six-month programme began with an instructional conference, at which the lecturers were M Broué, RW Carter, S Donkin, MJ Geck, JC Jantzen, B Keller, MW Liebeck, JC Rickard and R Rouquier. Each lecturer gave a series of three one-hour lectures. J Michel substituted for M Broué on one occasion.

This conference was supported by grants from the European Union and the London Mathematical Society. There were 93 participants, from the following countries: UK 34, Germany 16, France 12, USA 8, Greece 4, Italy 3, Portugal 3, Australia 2, Russia 2, Austria 1, Canada 1, Denmark 1, Holland 1, Ireland 1, Israel 1, Kazakhstan 1, Romania 1 and Sweden 1.

The aim of the conference was to introduce the ideas and techniques used in the representation theory and subgroup structure of algebraic groups and finite groups of Lie type.

The response to this instructional conference was enthusiastic, and many favourable comments from the participants were received.

It has been decided to produce a monograph based on the lectures given at the conference, entitled *Representations of Reductive Groups* and published by Cambridge University Press.

Spitalfields Day (4 April)
This meeting was supported by the London Mathematical Society and was attended by about 50 participants. Lectures were given by GD James, JC Jantzen, M Kashiwara and G Malle. Unfortunately R Guralnick had to withdraw as a speaker and M Kashiwara kindly agreed to lecture at short notice. The lectures proved both instructive and entertaining.

Modular Representation Theory in Non-defining Characteristic (7-12 April)
This meeting was organised by G Robinson and supported financially by the London Mathematical Society and the Isaac Newton Institute. It was attended by 53 participants from the following countries: UK 23, France 8, USA 8, Germany 6, Japan 3, Denmark 2, Australia 1, China 1 and Ireland 1. The lecturers were M Broué, M Cabanes, JF Carlson, R Dipper, S Donkin, K Erdmann, M Geck, G Hiss, M Kashiwara, S König, G Lehrer, O Mathieu, G Malle, J Michel, JC Rickard, R Rouquier, LL Scott, T Shoji, B Srinivasan, JG Thompson and A Zalesskii.

Modular Representations and Subgroup Structure of Algebraic Groups and Related Finite Groups (24 June - 4 July)
The director of this Advanced Study Institute was HH Andersen and the organisers were M Broué, RW Carter and J Saxl. The meeting was supported by NATO, the European Union, the Leverhulme Foundation, the London Mathematical Society and the Isaac Newton Institute. There were about 100 participants, of whom 15 were ASI lecturers. The participants came from the following countries: UK, USA, France, Germany, Denmark, Russia, Turkey, Italy, Ukraine, Canada, China, Japan, Portugal, Romania, Vietnam, Ireland, Kazakhstan, Malaysia and Norway.

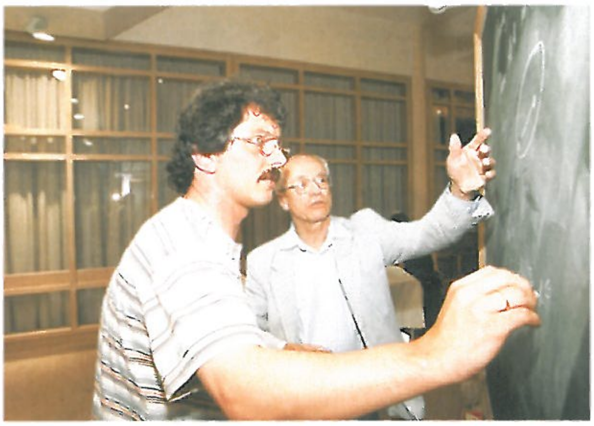
The ASI lecturers were HH Andersen, M Aschbacher, M Broué, RW Carter, R Guralnick, GD James, A Kleshchev, MW Liebeck, P Littelmann, G Lusztig, R Lyons, G Robinson, J Saxl, GM Seitz and J-P Serre.

Talks were also given by A Borovik, J Brundan, F Digne, S Donkin, P Fong, R Gow, JE Humphreys, K Magaard, G McNinch, A Premet, G Röhrle, LL Scott, R Steinberg, I Suprunenko, T Tanisaki and PH Tiep.

The contributions of G Lusztig, J-P Serre and R Steinberg were particularly appreciated. A list of seminars given at the ASI can be found in the Appendices, available separately from the Newton Institute.

Two poster sessions were held as part of the meeting, in which some 20 posters were displayed and discussed.

A volume is to be produced based on this ASI meeting entitled *Algebraic Groups and their Representations* and published by Kluwer Academic Publishers.



J-P Serre and colleague making use of one of the Institute's many chalkboards at the NATO ASI reception
(Findlay Kember)

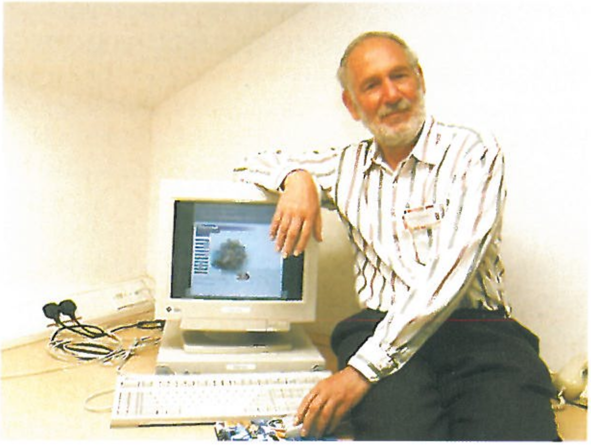
Participants

There were 61 long-stay participants in the RAG programme with an average stay of 85 days. There were also a number of short-stay participants coming on average for about a week. One of the highlights of the programme was the one-month visit of George

Lusztig, who has made so many fundamental advances in the theory of algebraic groups. At the ASI meeting Lusztig gave three lectures, each on a very different aspect of representation theory.

The visit of Jean-Pierre Serre to the ASI meeting was much appreciated. Serre gave a series of three lectures on completely reducible homomorphisms into reductive groups. He also gave a series of two lectures on finite subgroups of Lie groups.

Another notable visitor to the RAG programme was Masaki Kashiwara, who visited the Newton Institute for two weeks in April. Kashiwara gave an expository lecture on crystal bases of quantum groups, and also a talk on his recent ideas for obtaining crystal bases in affine quantum groups.



Robert Steinberg
(Findlay Kember)

It was a particular pleasure to have Robert Steinberg as a visitor for a one-month period. Steinberg, one of the pioneers in the theory of algebraic groups, gave a talk outlining a new and much shorter proof of the isomorphism theorem for reductive groups.
It was also a great pleasure to have Walter Feit as a visitor for one month. Well-known as one of the great figures in the representation theory of finite groups, Feit lectured on his recent work on integral representations of Weyl groups.

The Rosenbaum Fellow for the RAG meeting was Kay Magaard of Wayne State University, USA. Magaard stayed for the full six-month period and was involved in three collaborative research efforts, each of which is now nearly complete. He also started three further collaborative projects with other visitors to the programme.

Achievements

It is not possible in limited space to give an exhaustive account of what has been achieved, but we concentrate on describing achievements in particular areas of interest.

Modular representations in defining characteristic
One of the main aims in the modular representation theory of algebraic groups has been to prove Lusztig's conjectured character formula. The proof falls into four main parts, of which the fourth is as yet incomplete. The first part was achieved by Kazhdan and Lusztig in 1980, the second by Kashiwara and Tanisaki in 1994, the third by Kazhdan and Lusztig in 1994, and the fourth by Andersen, Jantzen and Soergel in 1994. Andersen, Jantzen and Soergel were able to show that if p is sufficiently large the characters of the irreducible modules for the algebraic group are the same as for the quantum group at a p^{th} root of unity, but were not able to establish how large p needs to be for this equality of characters to hold. This point must be settled to complete the proof of Lusztig's conjecture.

Andersen has been working to try to settle this point during his visit to the Newton Institute. This has not yet been completed. Andersen reports that the key to the comparison between algebraic groups and quantum groups seems to involve the study of Ext groups which appear in his recent proof of a sum formula for tilting modules. Further study of these Ext groups is continuing in the hope that this will settle the Lusztig conjecture for algebraic groups. The study of tilting modules is currently of great interest, and JE Humphreys, O Mathieu and T Tanisaki are among those involved in this study.

Lusztig himself has recently been working in a different direction. He has been considering irreducible modules for the Lie algebra of a simple algebraic group over an algebraically closed field of characteristic p. The characters of these irreducible modules are unknown, indeed the problem of determining the characters of the irreducible modules for the group is a special case of that for the Lie algebra. Irreducible modules for the group correspond to irreducible modules for the Lie algebra with p-character zero.

In his final lecture at the Newton Institute Lusztig proposed a conjecture for the irreducible characters of the Lie algebra for modules with arbitrary p-character. This is related to the alcove geometry of the affine Weyl group and contains his conjecture for the irreducible characters of algebraic groups in characteristic p as a special case.

G Lusztig in informal discussion with participants after his final Institute lecture (Findlay Kember)

Lusztig's conjectured character formula for simple Lie algebras in characteristic p will no doubt be the focus of attention for some time to come.

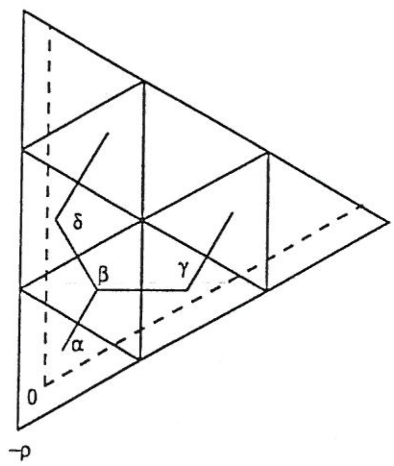
JC Jantzen, one of the leading workers in this area, visited the Newton Institute for three months. During this visit he continued his work on non-restricted representations of certain simple Lie algebras of classical type.

The classification of simple Lie algebras over algebraically closed fields of small characteristic is still incomplete. Difficulties remain in the cases p=2,3,5,7. A Dzhumadil'daev, during his visit, found a new series of simple Lie algebras in characteristic 5 given by an exceptional deformation of the Hamiltonian algebra H_1, and a new series in characteristic 3 given by an exceptional deformation of the special Lie algebra S_3.

Modular representations in cross characteristic
Work towards a proof of Broué's conjectures for blocks of finite reductive groups has led to two kinds of problems, which in turn have become goals in themselves.

The first problem consists in determining the l-adic cohomology complexes of Deligne-Lusztig varieties attached to some 'nice' regular elements of the Weyl group, in order to check that these complexes do define equivalences of derived categories as predicted by Broué's conjectures. A first approximation to this problem consists in computing the l-adic cohomology groups of the Deligne-Lusztig varieties and checking that they are disjoint modules for the finite reductive group G^F. Following earlier work of Lusztig, this question was studied by Digne, Michel and Rouquier, who continued their fruitful collaboration during the RAG programme. Another aspect of this problem consists in studying the commuting algebra of G^F acting on the graded module given by the direct sum of the l-adic cohomology modules in different degrees. This commuting algebra is conjectured to be a cyclotomic Hecke algebra as introduced by Broué and Malle. At the Newton Institute Broué, Digne, Malle, Michel and Rouquier continued their collaboration on these questions.

The second main problem consists of the quest for algebraic group-like objects, called 'strange creatures' by Lusztig and 'spetsoids' by Broué, Malle and Michel, whose Weyl groups should be complex reflection groups. Lusztig had noticed quite some time ago that one could define 'generic degrees of unipotent characters' attached to non-crystallographic Coxeter groups, which behave as if they were character degrees of a finite reductive group. These observations were extended to non-real finite complex reflection groups by Broué, Malle and Michel, who continued their work in this direction at the Newton Institute. During one of his talks at the ASI workshop, Lusztig explained how to determine the sizes of the 'special pieces' associated to some non-crystallographic Coxeter groups. In the case of Weyl groups these special pieces are certain unions of unipotent classes of the corresponding finite reductive group. Geck and Malle were able to apply and extend Lusztig's ideas to groups of type H_3 and then to find an algorithm which should provide a polynomial giving the cardinality of a special piece, both for Weyl groups and for some complex reflection groups.



Lusztig's conjectured character formula is expressed in terms of alcove geometry. The weight lattice is divided into bounded regions called alcoves and there are reflection operators mapping weights into neighbouring alcoves. The figure shows the alcove geometry for the group SL_3

In order to carry out this work it is necessary to extend to finite complex reflection groups many properties of Coxeter groups. For example Broué, Malle and Rouquier determined the structure of the fundamental group of the hyperplane complement of the irreducible complex reflection groups (up to finitely many open cases) and studied the associated generalised Hecke algebras. There was further work in this direction by Bessis, Bonnafé and Lehrer.

Geck and Hiss investigated the decomposition matrices which appear in the cross-characteristic representation theory of finite reductive groups, and continued to make progress in the direction of trying to show that these matrices always have unitriangular form. The decomposition matrices appear to be closely related to decomposition matrices of q-Schur algebras and their generalisations. Such algebras were studied by Dipper, James and Mathas as well as by Du and Scott. The theory of cellular algebras developed by Graham and Lehrer is used to investigate such generalized q-Schur algebras. All of these workers engaged in fruitful conversations on q-Schur algebras and their generalisations during their visits to the programme. Rickard studied the Alvis-Curtis duality map on generalised characters of finite reductive groups in cross characteristic. He showed that the fact that Alvis-Curtis duality commutes with Harish-Chandra induction and restriction is true at the level of functors between derived categories as well as at the character-theoretic level. Such commutation is always valid in the cross characteristic situation.

Other aspects of representation theory

The problem of finding natural bases for modules over algebraic groups and quantum groups is currently of much interest. P Littelmann has found such monomial bases in recent years using his path model for representations. During his stay at the Newton Institute he made a further substantial advance. He constructed bases of dual Weyl modules, indexed by paths with a given end point, which are compatible with restriction to Demazure submodules and also satisfy the other conditions required of bases in standard monomial theory. Littelmann's method uses the Frobenius map for quantum groups at a root of unity, and avoids the case-by-case arguments previously used in standard monomial theory.

The canonical bases for quantum groups introduced by Lusztig and Kashiwara remain the subject of great interest. Carter and Marsh showed that for each reduced word for the longest element of the Weyl group of type A_4, there is a family of monomials in the canonical basis for the quantum group of type A_4 and that these families of monomials fill out exactly those regions of linearity for Lusztig's piecewise-linear function which are defined by the minimum number of inequalities. This gives many, but not all, of the canonical basis elements in type A_4. An example of NH Xi shows that the same ideas cannot be used without modification in type A_5.

CW Curtis and K Shinoda enjoyed a felicitous collaboration since, on arrival at the Newton Institute, they realised that they had both formulated the same conjectures on Gauss Sums and Kloosterman Sums arising in the representation theory of finite reductive groups. They had reached these conjectures from different viewpoints but were able to make considerable joint progress and will write joint articles on *Unitary Kloosterman sums and Gelfand-Graev representations of SL_2* and *Gauss sums attached to finite reductive groups.*

One of the main aims in the modular representation theory of abstract finite groups is to establish Alperin's weight conjecture - that the irreducible modules should be parametrised by objects called weights. At the beginning of February 1997 H Pahlings produced what appeared to be a counter-example to one of the variants of Alperin's conjecture, and this led EC Dade to conclude that the automorphism group McL2 of MacLaughlin's sporadic simple group was a counter-example to Alperin's conjecture itself. This seemed highly unlikely to G Robinson, who was working at the Newton Institute. Robinson asked Rouquier to test whether Alperin's conjecture was valid in McL2. Making intensive use of the system GAP, Rouquier and his collaborators were able to show that Alperin's conjecture did indeed hold in this group. Thus the presence at the Newton Institute of experts with the necessary mathematical and computational skills led to the rapid and decisive rebuttal of the false claim that McL2 was a counter example to Alperin's conjecture.

Robinson gave lectures on Alperin's conjecture at the ASI workshop, when he described various surprising arithmetical conditions on character degrees which would follow from Alperin's and related conjectures. On hearing these lectures Geck was able to prove some of these arithmetical conditions directly for finite groups of Lie type, thus giving another example of the many unpredictable but fruitful interactions between the participants.

There were also useful discussions involving Robinson, Michel, Carter and Thompson on the character theory of the group of unitriangular matrices over a finite field.

The modular representation theory of the symmetric groups continues to generate great interest as the characters of the irreducible modules remain unknown.

This subject is closely related to the representation theory of general linear groups, and to the theory of q-Schur algebras. A Kleschchev, JW Brundan, GD James and A Mathas held intensive discussions in this field and several forthcoming articles are planned. Kleshchev gave a series of lectures on this subject at the ASI workshop, including an outline of his proof of Mullineux's conjecture on tensoring with the sign character.

Subgroup structure of simple algebraic groups
The subgroup structure of exceptional simple algebraic groups over the complex field is well understood by classical results of Dynkin. The situation over algebraically closed fields of positive characteristic is much more complicated. However, at least if the characteristic is large enough (with explicit small bounds), much has been done recently, especially by Liebeck and Seitz. It is desirable to remove the restrictions on characteristic - this is one of the projects Liebeck and Seitz were engaged in during their visit to the Newton Institute. Such results have consequences concerning maximal subgroups of finite simple groups of Lie type.

In classical groups (both finite and algebraic over an algebraically closed field), when investigating subgroup structure one is led to investigate those subgroups which are almost simple modulo the scalars. This leads to questions about irreducible linear representations of almost simple groups. It is then important to know which representations give rise to maximal subgroups of the corresponding classical group. This has been an active area of research. For example Magaard and Malle, and Guralnick and Seitz have considered special cases while at the Institute, and Kleshchev reported on a special case during his ASI lectures on symmetric groups.

The problem of classifying subgroups in classical groups becomes much easier if we insist on the presence of certain elements in the subgroup. For example Guralnick and Saxl obtained a classification of groups generated by bireflections. This has an application to the Brauer k(GV) problem, and a possible further application in invariant theory was mentioned by Malle in his Spitalfields Day lecture.

There are good lower bounds on degrees of faithful irreducible representations of groups of Lie type in cross characteristic. In several problems investigated during the meeting it became necessary to understand explicitly the representations of degree less than or equal to that of the alternating or symmetric square of a representation of minimal degree. This has now been achieved for most of the finite families of finite classical groups by Guralnick and Tiep, Hiss and Malle, and Guralnick, Magaard and Saxl.

One of the basic results concerning simple algebraic groups is the fact that there are only finitely many double cosets B\G/B. Liebeck, Guralnick, Seitz and Brundan, are investigating other cases of closed subgroups H,K with finitely many double cosets H\G/K. Seitz discussed this during his lectures at the ASI workshop. A related question has been investigated by Röhrle.

A related area is that of determining multiplicity-free representations and distance-transitive graphs. Cohen, Guralnick, Ivanov, Liebeck, Magaard and Saxl worked on various aspects of this problem during the meeting.

The classification of finite simple groups is of fundamental importance in group theory. Work on the revision project, started by Gorenstein and others, will continue for many years. This work is strongly influenced by the theory of simple groups of Lie type, since these provide most of the examples. Lyons discussed the revision project in his ASI lectures. There is one area in particular which has attracted much attention recently - this is the case of quasithin groups of even characteristic. Aschbacher is working on this area with Smith and Meierfrankenfeld. He gave three lectures at the ASI discussing this project, which is likely to continue for another year or so.

Non-Perturbative Aspects of Quantum Field Theory (January to June 1997)

Report from the Organisers:
DI Olive (Swansea), P van Baal (Leiden), P West (Kings College London)

Introduction

It has been recognised for more than sixty years that quantum field theory is the theoretical framework that welds together two of the greatest and best established physical ideas of the twentieth century; relativity and quantum mechanics. As such it underlies all attempts to explain the observed patterns and behaviours of elementary particles as revealed by large particle accelerators. Nevertheless it has persisted in suffering a number of shortcomings. Here is a partial list:

(1) Physical predictions are often based on "perturbative" approximations in which a certain parameter is taken as small, sometimes with little justification

(2) Quantum effects often produce divergent expressions whose interpretion is debatable

(3) Geometrical principles imply non-linearities in the equations of motion that can be responsible for solutions of a solitonic nature whose physical role must be evaluated

(4) The incorporation of Einstein's theory of gravity is necessary but leads to more extreme difficulties of types (2) and (3)

(5) It is difficult to explain why the constituents of nuclear particles, known as quarks, should be "confined" ie never produced individually in particle collisions

Three years ago Sen, Seiberg and Witten produced strong evidence of a mathematical nature for the validity of an idea proposed in 1977 but originally unverifiable. This was the conjecture of Montonen and Olive that a form of electromagnetic duality holds in a rather specific field theory, namely a spontaneously broken gauge theory with supersymmetric features.

Since this idea involved interchange of electrically charged particles with magnetic monopole solitons and hence small values of the coupling parameter with large values, it addressed two of the above difficulties while even newer developments address the remaining ones, with supersymmetry always retaining a crucial role. Hence the above list now appears remediable in a unified way using duality.

One of the first fruits was a much enhanced understanding of the invariants of possible four-manifolds furnishing the background space-time for the quantum field theories with electromagnetically dual properties. This development has already been highlighted in previous programmes run at the Newton Institute, *Symplectic Geometry* in 1994 and *Four-Dimensional Geometry and Quantum Field Theory* in 1996, and has bolstered confidence in the validity of the general idea of electromagnetic duality.

Strategy

The organisers therefore decided to concentrate more on fostering and developing the new light shed on the questions listed above. Thus they planned a wide-ranging programme, encompassing a gamut of topics running from the mathematical analysis of multimonopole solutions to advanced computer analyses of quark confinement in the context of QCD (quantum chromodynamics).

Programme organisers P West, P van Baal and DI Olive (Findlay Kember)

This spectrum of topics running from the more pure mathematical to the more down-to-earth physical was also mirrored in the chronology of the six-month programme. This enabled the mathematical part to link up with the preceding programme on *Four-Dimensional Geometry*, while the middle three-month period covered duality, supersymmetry and generalisations to superstring theory, thereby incorporating gravity. The essential role of higher dimensional objects such as membranes, 5-branes and D-branes was also included.

Participants were invited from a wide range of disparate background interests within the subject in order to encourage a stimulating mix of expertise at any given time. For this reason overlaps between the parts of the general timetable were encouraged, and proved fruitful.

In order to leave time for discussions and work it was decided to aim at two seminars a week, timed not to clash with other theoretical physics seminars in the university.

Because the subject is so new and yet has aroused such world-wide interest it has very quickly grown so large that it is difficult for researchers to keep abreast of all the developments. It is also true that progress has been so rapid that the subject has lacked consolidation: it is often very difficult to find satisfactory proofs of various statements in the literature, however persuasive they may be.

To remedy this situation and to aid entrants to the field, funds were sought to mount conferences of a pedagogical and expository nature as detailed below.

In addition two series of lectures were organised on different aspects of string duality. These were timed to run over a single week period in order to facilitate attendance by outsiders. A Sen (Mehta) spoke on string duality and J Schwarz (Cal Tech) discussed aspects of M-theory. These speakers are renowned both for their key contributions to the subject and their expository talents.

The interested community is particularly attuned to accessing various electronic bulletin boards for notices of forthcoming meetings. Accordingly attempts were made to use these to advertise all these activities very widely. Special funds were made available by the Newton Institute to facilitate participation by young UK researchers and these helped in attracting large audiences. Unfortunately funds were not available for video records.

Conferences

David Olive and Peter West organised a two-week EC Summer School entitled *Duality and Supersymmetric Theories* between 7-18th April. The meeting comprised an advanced school that presented an account of electromagnetic duality and its generalisations. Included were introductory lectures on supersymmetric gauge theories, supergravity theories, solitons and string duality as well as a more advanced course on monopole moduli spaces, Seiberg-Witten theory, D-branes, and M-theory.

The lecturers were C Bachas (Ecole Polytechnique), T Eguchi (Tokyo), J Gauntlett (QMW, London), G Gibbons (Cambridge), J Harvey (Chicago), C Hull (QMW, London), N Manton (Cambridge), DI Olive (University of Wales Swansea), B Zumino (Berkeley) and P West (King's, London). They gave courses varying between two and five lectures.

Pierre van Baal, assisted by a committee consisting of I Drummond (Cambridge), M Shifman (Minnesota) and P West (King's, London), organised the NATO Advanced Study Institute, likewise of two weeks' duration, entitled *Confinement, Duality and Non-perturbative Aspects of QCD*. This formed the culmination of the programme running from 23 June to 4 July. Here the lectures compared the two lines of approach to quark confinement with duality and condensation of monopoles as common themes, but with one emphasing supersymmetry and the other involving 't Hooft's concept of abelian projections and related numerical studies.

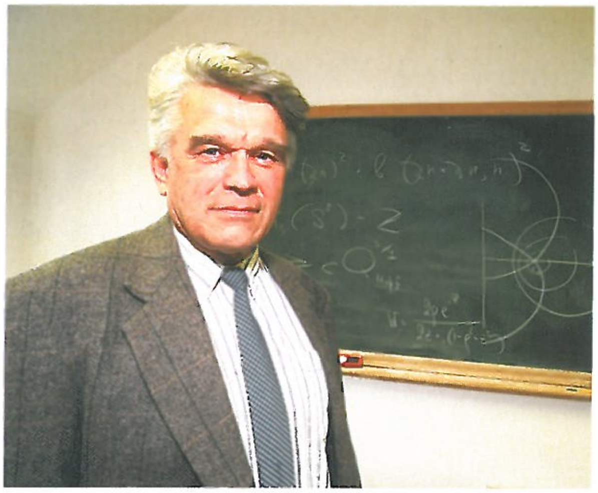
G 't Hooft at the NATO ASI
(Findlay Kember)

The lecturers were G 't Hooft (Utrecht), C Michael (Liverpool), P West (King's London), P Weisz (MPI Munich), P Hasenfratz (Bern), M Teper (Oxford), C Wetterich (Heidelberg), M Shifman (Minnesota), L Faddeev (St Petersburg), P Lepage (Cornell), P Landshoff (Cambridge), D Zwanziger (New York), E Shuryak (Stonybrook), M Polikarpov (ITEP, Moscow), R Kenway (Edinburgh), P van Baal (Leiden), T Suzuki (Kanazawa), R Perry (Ohio State), A Schwimmer (Weizmann), J Verbaarschot (Stonybrook) and A Di Giacomo (Pisa). This time the number of lectures delivered varied between one and three. The attendance for each meeting was about one hundred.

Participation

There were 63 long-stay participants who stayed for an average of 55 days, and 56 short-stay participants staying for an average of 11 days. In addition, there were at least 150 visitors who took part in workshops for up to two weeks.

Sen was appointed as Rothschild Distinguished Professor. Visiting professorships were also awarded to Academician L Faddeev, A Schwimmer and A Marshakov.



Academician L Faddeev
(Findlay Kember)

David Pierce was Rosenbaum Fellow. Attendance fluctuated during the programme, peaking during the university vacation periods.

Achievements

Less than a month since the end of the programme is too early for a full assessment of its achievements. Besides the expository role already mentioned, one of the goals has been to stimulate new research directions and new collaborations. Judging by the enthusiastic reports of the participants, this has succeeded, with a large number of papers in progress and in draft.

However it is possible to give a flavour of the different parts of the programme by highlighting some of the real research achievements already documented.

Certain first-order differential equations, known as Bogomolny equations, play an important role in the supersymmetric gauge theories displaying electromagnetic duality properties. They resemble the self-dual or instanton equations but have a different physical interpretation, describing a number of similar static magnetic monopole solitons. The manifold of solutions has a number of geometric properties whose detailed understanding is crucial to duality. For example, physics implies it has a Riemannian metric which is in fact hyperkähler. For magnetic charge two, it was determined explicitly by Atiyah and Hitchin in terms of elliptic functions. Sen's vital contribution was to realise that the validity of duality hinged on the cohomology of this manifold and to use the metric to determine it. The result was quite surprising but fully supported the Montonen-Olive conjecture and so sparked the work of Seiberg and Witten as well as the avalanche that followed.

A different set of nonlinear equations, the Skyrme equations, thought to be applicable in nuclear physics, displays unexpectedly similar sets of solutions. Two of the participants (Manton and Sutcliffe) together with Houghton, a student of Manton, were able to establish a remarkable explanation. This exploited a connection of both equations to rational maps between a pair of Riemann spheres. This refined and developed an observation originally due to Donaldson.

As a result, much more accurate approximation to various Skyrme configurations were found. New insights into the crucial cohomological properties of the Bogomolny solution manifolds are indicated.

Discussions with E Weinberg had stimulated this work, while Weinberg himself was able to further his investigations of the Bogomolny solutions spaces for gauge groups larger than SU (2).

Sen was able to apply Seiberg-Witten techniques involving elliptic fibrations to establish dual equivalences between an F-theory compactified on a Calabi-Yau (n+1)-fold to an orientifold of type *IIB* theory compactified on an auxiliary complex n-fold, thereby extending and unifying previous results.

The key insight of Seiberg and Witten into the duality of gauge theories with $N = 2$ supersymmetry was the identification of the role of Riemann surfaces. Families of surfaces of genus one are parametrised by a complex quantity identified with a parameter in the gauge theory. The natural transformation of this modular parameter with respect to the group of modular transformations is reflected in the electromagnetic duality property of the gauge theory. Despite the appeal of this, it is desirable to clarify the logical structure of these connections. Werner Nahm was able to make significant progress in this and, in particular extend his previous work exploiting the theory of modular functions, by adding masses to the quarks.

One of the triumphs of the Seiberg-Witten theory from a physical point of view is that for a certain value of its moduli parameter the Riemann surface degenerates, resulting in a monopole losing its mass. By a natural slight modification of the theory, reducing the supersymmetry from $N = 2$ to $N = 1$, these massless monopoles then condense into the vacuum, and so confine electric charge by the dual Meissner effect. It has long been felt that such a mechanism could occur in QCD without supersymmetry and explain quark confinement. Indeed enormous computer power has been expended to check this and some members of the associated community attended the latter part of the programme as well as the NATO ASI.

Mikhail Shifman stayed for two periods, thereby interacting with two communities of participant, those studying duality in supersymmetric gauge theories and those working on quark confinement in pure QCD without supersymmetry. As well as completing his authoritative review on non-perturbative dynamics in supersymmetric gauge theories, he worked on exact results for domain walls in $N = 1$ supersymmetric gauge theories, finding results unexpectedly analogous to those known for $N = 2$ and thereby finding a bridge. These results were subsequently confirmed by Witten from a different point of view using D-branes.

Edward Shuryak and Ariel Zhitnitsky collaborated on estimating the gluon and charm content in the eta-prime meson using instanton gas methods. They found a relatively large contribution, nicely explaining the large branching ratio for B-meson decay into eta plus anything found experimentally.

Finally we should like to thank the staff of the Newton Institute for their unstinting work, unfailing helpfulness and devotion to the well-being of the programme.

Scientific Planning and Future Programmes

Scientific Policy

From its inception, it has been intended that the Newton Institute should be devoted to the Mathematical Sciences in the broad sense. In this respect the Institute differs significantly from similar institutes in other countries. The range of sciences in which mathematics plays a significant role is enormous, too large for an Institute of modest size to cover adequately at any one time. In making the necessary choices, important principles are that no topic is excluded *a priori* and that scientific merit is to be the deciding factor.

One of the main purposes of the Newton Institute is to overcome the normal barriers presented by departmental structures in Universities. In consequence, an important, though not exclusive, criterion in judging the 'scientific merit' of a proposed research programme for the Institute is the extent to which it is 'interdisciplinary'. Often this will involve bringing together research workers with very different backgrounds and expertise; sometimes a single mathematical topic may attract a wide entourage from other fields. The Scientific Steering Committee therefore works within the following guidelines:

(a) the mixing together of scientists with different backgrounds does not *per se* produce a successful meeting: there has to be clear common ground on which to focus;

(b) each programme should have a substantial and significant mathematical content;

(c) each programme should have a broad base in the mathematical sciences.

Research in mathematics, as in many other sciences, tends to consist of major breakthroughs, with rapid exploitation of new ideas, followed by long periods of consolidation. For the Newton Institute to be an exciting and important world centre, it has to be involved with the breakthroughs rather than the consolidation. This means that, in selecting programmes, a main criterion should be that the relevant area is in the forefront of current development. Since the Institute's programmes are chosen two to three years in advance, it is not easy to predict where the frontline will be at that time. The best one can do is to choose fields whose importance and diversity are likely to persist and to choose world leaders in research who are likely to be able to respond quickly as ideas change.

Although the novelty and the interdisciplinary nature of a proposed programme provide important criteria for selection, these must be subject to the overriding criterion of quality. With such a wide range of possibilities to choose from, the aim must be to select programmes which represent serious and important mathematical science and which will attract the very best mathematicians and scientists from all over the world. However, the Institute is receptive also to proposals of an unorthodox nature if a strong scientific case is made.

Although the Institute operates on a world-wide basis and contributes thereby to the general advancement of mathematical science, it must also be considered in the context of UK mathematics. A natural expectation of all those concerned is that each programme will be of benefit to the UK mathematical community in a variety of ways. If the UK is strong in the field, UK scientists will play a major part in the programme; if the UK is comparatively weak in the field, the programme should help to raise UK standards, and instructional courses, aimed primarily at younger researchers and research students, will play a vital role here.

Because of the wide base of support for the Newton Institute in the EPSRC and elsewhere, the Institute's programmes shall as far as possible represent an appropriate balance between the various mathematical fields. In order to retain the backing of the mathematical and scientific community, the Institute will run programmes over a wide range of fields and, over the years, achieve this balance. Such considerations, however, are secondary to the prime objective of having high quality programmes.

Professor Sir Christopher Zeeman, FRS, Chairman of the Scientific Steering Committee

The membership of the Scientific Steering Committee on 30 June 1997 was:

Professor Sir Christopher Zeeman, FRS	Oxford	GB, Chairman
Professor HK Moffatt, FRS	Newton Institute	Director
Sir Michael Atiyah, FRS	Cambridge	Co-opted
Professor Sir Michael Berry, FRS	Bristol	GB
Professor J-M Bismut	Orsay	GB
Professor M Cates	Edinburgh	GB
Professor S Donaldson, FRS	Oxford	EPSRC
Professor TWB Kibble, FRS	Imperial	PPARC
Professor J Moser	ETH Zürich	GB
Professor A Newell	Warwick	GB
Professor BD Ripley	Oxford	EPSRC
Professor AFM Smith	Imperial College	EPSRC
Professor JF Toland	Bath	LMS
Professor CTC Wall, FRS	Liverpool	LMS

The Committee is required to meet once per year but in practice meets twice per year, usually in April and October. The Scientific Steering Committee perceives its role as involving both the consideration of proposals received and the stimulation of proposals in the areas of mathematical sciences which it considers to be potentially particularly suitable for the Institute. The Institute advertises its willingness to receive proposals in a variety of ways which have included the annual distribution of a poster containing a "Call for Proposals" to over 500 departments and institutions concerned with mathematical sciences in the UK and abroad, and publicity by email and World Wide Web. At meetings the Committee regularly considers in which areas it should stimulate proposals and the Director, Deputy Director or individual Committee members then assume responsibility for taking action in particular areas.

Future programmes

Disordered Systems and Quantum Chaos

July to December 1997

Organisers: JP Keating (Bristol), DE Khmelnitskii (Cambridge), IV Lerner (Birmingham), P Sarnak (Princeton)

The quantum properties of disordered systems have been the focus of considerable attention in many branches of physics, principally nuclear physics and condensed matter physics. Recently it has been recognised that many of the same phenomena also occur in deterministic systems which possess only a few degrees of freedom, but which are chaotic in the classical limit. Even more surprisingly, the theories developed in these areas also have natural counterparts in a number of topics in mathematics; for example, in the study of spectral properties of random operators and random matrices, in the theory of Fourier integral operators, in harmonic analysis (specifically in the theory of the Riemann zeta-function and related L-functions). In the past few years an extremely stimulating and productive cross-fertilisation between the above fields has slowly been developing. The aim of the programme is to accelerate the already significant rate of progress on some of the important common problems which occur, in different guises, in each area. The main topics upon which the programme will focus are localisation, fluctuation statistics, and trace formulae; with a particular emphasis on their role in the theory of mesoscopic systems.

Neural Networks and Machine Learning

July to December 1997

Organisers: CM Bishop (Aston), D Haussler (UCSC), GE Hinton (Toronto), M Niranjan (Cambridge), LG Valiant (Harvard)

Research in machine learning has advanced significantly in recent years, stimulated in part by the emergence of a range of successful, large-scale applications. Examples include optical character recognition, classification of sleep stages from EEG signals, cervical smear screening, and real-time tokamak plasma control. At the same time there have been many impressive developments in the theoretical foundations of this field, arising from several complementary approaches. Concepts from statistical pattern recognition have been used to formulate a general framework for machine learning based on statistical inference. Parallel developments in computational learning theory have led to a characterisation of computational and sample-size requirements for learning problems, while also resulting in powerful new algorithms. In addition, concepts from information theory, differential geometry and statistical mechanics have been exploited to give alternative insights into neural networks. The principal aims of this programme are to promote greater inter-disciplinary collaboration between researchers with different theoretical perspectives, to strive for a more unified mathematical framework for neural networks and machine learning, and to stimulate the development of new algorithms for practical applications.

Dynamics of Astrophysical Discs
January to June 1998
Organisers: J Goodman, JCB Papaloizou (QMW); JE Pringle (Cambridge); JA Sellwood (Rutgers)

Many astrophysical systems, over a vast range of length scales, consist of matter organised in differentially rotating, centrifugally supported discs. Such systems include planetary rings, protostellar discs (which provide the environment from which planets may form), close binary star systems, and normal and active galaxies. Understanding the structure and evolution of astrophysical discs is therefore of central importance in astronomy. Discs may sometimes be modelled as collections of discrete particles, and this leads to the study of collisionless many body problems. Other studies treat the disc as a differentially rotating turbulent fluid, possibly containing a magnetic field. The effects of self-gravitation may also need to be taken into account. The programme will bring together experts from relevant areas in astrophysics, and mathematicians and scientists familiar with appropriate analytic methods and numerical simulation techniques, including the solution of differential or integro-differential equations. The aim is to make progress with the different approaches, and, in particular, to advance our understanding of the connections between them.

Arithmetic Geometry
January to June 1998
Organisers: J-L Colliot-Thélène (Orsay), J Nekovář (Cambridge), C Soulé (IHES)

The origin of this subject was the study of solutions of Diophantine Equations - that is the the search for integer or rational solutions of systems of polynomial equations - using geometric methods. Today, Arithmetic Geometry has expanded to cover a wide range of topics central to Number Theory, Algebraic Geometry and other branches of pure mathematics. The programme will highlight several areas of this vast subject, including: arithmetic of algebraic cycles, motivic cohomology, rational points on algebraic varieties, Arakelov theory, and regulators and special values of *L*-functions.

Biomolecular Function and Evolution in the Context of the Genome Project
July to December 1998
Organisers: P Donnelly (Oxford), W Fitch (Irvine), N Goldman (Cambridge)

There is a long and productive history of interplay between genetics on the one hand and mathematics and statistics on the other. The "molecular revolution" over the last 15 years, and in particular the impetus of genome projects, has transformed the field to one with an abundance of data and a paucity of relevant mathematical models and techniques. By 1998, the maturation of genome projects will make data on DNA, proteins, gene duplications and gene arrangements on the chromosomes widely available. As a consequence of recent advances in computational statistics, vast improvements in the quality of statistical analyses of these data are possible. They will have a profound impact on the practice of biological research, and, ultimately, medical diagnostics and preventive medicine. The driving force of the present programme is the opportunity offered by genome sequence research to understand biomolecular function and evolution at a much more complete level than hitherto possible and to sustain recent progress in a number of relevant mathematical areas. Problems in analysing the flood of molecular genetic sequences and structures raise a range of challenging biomathematical research topics. This inter-disciplinary programme will bring together mathematicians and computer scientists working on subjects such as probabilistic modelling, stochastic processes, geometry, statistical data analysis, computational complexity, neural networks, genetic algorithms and expert systems; and molecular biologists working in medical and biological fields.

Nonlinear and Nonstationary Signal Processing
July to December 1998
Organisers: WJ Fitzgerald (Cambridge), RL Smith (North Carolina), A Walden (Imperial), PC Young (Lancaster)

The classical theory of signal processing is based on models which are stationary, linear and in many cases also assume that signals have Gaussian amplitude distributions. In recent years there has been a rapid growth in the applications of signal processing in many modern areas of engineering, communications and computing, as well as in financial time series, macro-economics, the environmental and biological sciences, physiology, etc; parallel advances in the theory have introduced many new models and methods. Among these are nonlinear autoregressive and state-space models; models with time-varying or state-dependent coefficients as representations of nonstationary and nonlinear series; adaptive methods of forecasting, interpolation and smoothing; linear non-Gaussian methods, and methods derived from the theory of dynamical systems. The purpose of this programme is to bring together statisticians, engineers and other researchers who use signal processing methodology to develop a general framework to unify existing methods, and to identify areas which may benefit from the application of methods developed for other purposes or where new methodology is required.

Turbulence
January to July 1999
Organisers: GF Hewitt (Imperial College), PA Monkewitz (Lausanne), N Sandham (QMW), JC Vassilicos (Cambridge)

Prediction and control of turbulent fluid flow is a scientific problem with immense practical importance. Turbulence is characterised by a large number of weakly correlated motions interacting on a wide range of length- and time-scales which appears disordered and chaotic. However, it also contains organised flow features such as shear layers and long-lived vortex tubes and vortices. Fundamentally different approaches ensue from stressing either the space-filling disorder or the localised order in the flow. A major limitation on the accuracy of prediction of fluid flow in industrial applications is the relative inaccuracy of the turbulence models used in the calculations. This programme will bring together turbulence experts from different communities (mathematical sciences, fundamental fluid mechanics and turbulence modelling) to explain their different approaches and to develop new understanding of both practical and fundamental problems. The objective of the programme will be to determine the extent to which different properties of turbulence may be considered universal. Topics that will help elucidate this question and will be given special attention are transition and control, coherent structures, the relations between geometry, structures, dynamics and statistics, intermittency in turbulence and other dynamical systems, and the mathematical and numerical properties of closure models.

Mathematics and Applications of Fractals
January to April 1999
Organisers: RC Ball (Cambridge), KJ Falconer (St Andrews)

This programme is concerned with topical aspects of the mathematics and applications of fractals. Our major aims are to encourage interaction between mathematicians and scientists with different approaches and viewpoints, to increase the awareness of scientists of the mathematics that is already available, and to focus the attention of mathematicians on areas where further theoretical development is needed. In particular it is hoped to emphasise the following areas: problems of a 'dynamic' nature relating to physical problems, such as diffusions and PDEs on fractals or domains with fractal boundary; multifractal theory and its applications; geometric measure and integration theory, especially recent techniques such as tangent measures; characterisation and measurement of fractals, including alternatives to dimension (eg lacunarity); the identification of mathematically based protocols to identify that a structure is fractal, along with how the mathematical theory 'in the limit' relates to finite scales; and fractal aspects of the distribution of galaxies.

Complexity, Computation and the Physics of Information
May to August 1999
Organisers: A Albrecht (Imperial), RM Solovay (Berkeley), W Zurek (LANL)
The study of information is linked to a wide range of interdisciplinary research which is defining new frontiers in physics and mathematics. Mathematical notions such as "algorithmic entropy" (or "algorithmic complexity") are central to the fundamental problem of constructing rigorous definitions of entropy and information, while the importance of these concepts to key physical processes (including quantum cosmology, the arrow of time, exotic quantum systems and signal transmission) attracts the attention of theoretical and experimental physicists. An exciting application is that of "quantum computation", which defines challenging problems in mathematics, materials physics, and

computer science. The programme will bring together members of all the relevant scientific communities to focus on the physics, mathematics and applications of information and entropy.

Structure Formation in the Universe
July to December 1999
Organisers: VA Rubakov (Institute for Nuclear Research, Moscow), PJ Steinhardt (Pennsylvania), NG Turok (Cambridge)
Understanding how structure emerged in the universe provides one of today's great scientific challenges. Huge quantities of new astronomical data, including maps of the cosmic microwave sky fluctuations and of the distribution of galaxies, are providing stringent constraints on possible theories. At the same time, the results of new particle physics experiments are beginning to imply very strong constraints on the possible nature of the dark matter. The two structure formation theories investigated in most detail so far involve quantum fluctuations generated during inflation, and cosmic defects produced at symmetry breaking phase transitions. Both theories involve physics beyond the standard model, and if either is proven correct, there will be important implications for high energy theory.

The programme will begin with discussions of the latest observational data, including the statistical techniques needed to analyse the new data sets, with the aim of fitting the observations together in a coherent framework. Extensions and variants of current theories, as well as entirely novel approaches will then be considered. During the programme, fundamental questions regarding the big bang and inflationary theory will be addressed, as well as connections to string theory and quantum gravity.

Mathematical Developments in Solid Mechanics and Materials Science
September to December 1999
Organisers: K Bhattacharya (Pasadena), PM Suquet (Marseille), JR Willis (Cambridge)
There is great current interest in how the microscopic structure of a solid material influences its macroscopic response to stress. Conversely, the application of stress can influence microstructure. Microscopic damage may occur, leading ultimately to the formation of large cracks and structural failure. Phase transformations occur in some materials, creating structures at various length scales which evolve with stress.

The challenge, both for mathematics and physical modelling, is to comprehend relationships between models at different length scales. This has led already to a well-developed theory of "homogenization" when the scales are widely separated, and has both exploited and stimulated advances in the calculus of variations. When the scales are separated but still comparable, there is a need for a micromechanical rationale for including scale effects in macroscopic models. The phenomena may be unstable, at least at the microscopic level, and, even if stable, may admit multiple equilibria. Study of the kinetics of the processes is a key requirement, making demands both for modelling and for the analysis of partial differential equations. In particular, the (possibly hierarchical) development of large-scale patterns is an open problem.

The main focus of the programme will be on microstructure formation and evolution, as related to phase transformations, damage development and fracture. Each subject has its own group of specialists (in various mixes, mathematicians, physicists, engineers, materials scientists). There are already overlaps, both between subjects and disciplines, and it is intended that the programme will exploit and extend these, to common advantage.

The following further programmes have been agreed (more details will be available in the near future):

Ergodic Theory, Geometric Rigidity and Number Theory
January to June 2000
Organisers: A Katok (Penn State), G Margulis (Yale), M Pollicott (Manchester)

Singularity Theory
July to December 2000
Organisers: VI Arnold (Moscow and Paris IX), JW Bruce (Liverpool), D Siersma (Utrecht)

Management

The management of the Institute is the responsibility of the Management Committee. This consists of the Director, the Deputy Director, the Heads of the Department of Applied Mathematics and Theoretical Physics (DAMTP) and the Department of Pure Mathematics and Mathematical Statistics (DPMMS), five persons appointed by the General Board of the Faculties (of whom one is nominated by the Council of the School of Physical Sciences (CSPS), one by the Faculty Board of Mathematics and one by the School of Technology), a Chairman appointed by the General Board, one person appointed by each of St John's College, Trinity College and the EPSRC and one additional person co-opted at the discretion of the committee.

Professor Sir Martin Rees, FRS, Chairman of the Management Committee

The membership of the Management Committee at 30 June 1997 was:

Professor Sir Martin Rees, FRS	CSPS & General Board, Chairman
Professor HK Moffatt, FRS	Director
Professor A Baker, FRS	Trinity College
Professor D Brannan	General Board
Professor JH Coates, FRS	Head DPMMS
Professor DG Crighton, FRS	Head DAMTP
Professor AP Dowling	School of Technology
Dr N Linden	Acting Deputy Director
Professor FP Kelly, FRS	General Board
Dr GLI Richards	EPSRC
Professor G Segal, FRS	St John's College
Dr C Sparrow	Faculty of Mathematics
Professor Sir Christopher Zeeman, FRS	General Board

Sir Michael Atiyah was also a member of the Management Committee until his retirement at the end of September 1996.

During the year 1996/97 the Management Committee met once during each University term. At these meetings it received reports on the Institute's finances, administration, publicity, housing arrangements and fund-raising efforts. It approved the scientific programmes which the Director proposed to it on the advice of the Scientific Steering Committee and it received the minutes of that Committee. It was responsible for recommending elections to Rothschild Visiting Professorships to the General Board on the advice of the Director. It received regular detailed reports on the progress of planning for those programmes which had been approved and final reports on those programmes which had been completed.

Building

Lease

In October 1996, the University acquired a lease of the Isaac Newton Institute for a term of 175 years, this lease being renewable for a further term of 175 years.

Centre for Mathematical Sciences

Plans have been announced [Cambridge University Reporter, 14 May 1997,703 - 705] to move the Department of Applied Mathematics and Theoretical Physics and the Department of Pure Mathematics and Mathematical Statistics to a new Centre for Mathematical Sciences on the Clarkson Road site, adjacent to the Newton Institute. A major fund-raising campaign is in progress, and it is hoped that building work will begin in April 1998.

Professor Peter Goddard, Chairman of the Fundraising Committee for the Centre for Mathematical Sciences and former Deputy Director of the Newton Institute, with the architect's model of the new site. (Findlay Kember)

Library Gallery

A new gallery, generously financed by a grant from the Thriplow Trust, has been built in the Library, and provides vital space for the ever-growing collection of books and journals. The gallery was opened at a Reception held on 17 February following John Ball's Institute Seminar *The Calculus of Variations after 300 years.*

Librarian and Information Officer Andrea Le Core, in front of the new library gallery (Findlay Kember)

Financial Report

Major Donations in Cash

SERC/EPSRC	£4,199k over 8 years
Isaac Newton Trust*	£1,400k over 10 years
St John's College	£750k over 5 years
Cambridge University	£508k over 9 years
Hewlett-Packard	£490k over 5 years
NATO	£484k over 6 years
Centre National de la Recherche Scientifique	£420k over 10 years
Leverhulme Trust	£375k over 6 years
NM Rothschild & Sons	£333k over 5 years
Rosenbaum Foundation	£330k over 7 years
European Union	£300k over 7 years
London Mathematical Society	£108k over 7 years
Gonville and Caius College	£100k
Prudential Corporation plc	£100k over 4 years
Institute of Physics	£68k over 7 years
Nuffield Foundation	£57k
British Met Office	£37k
AFCU (Hamish Maxwell) $50,000	£32k
AFCU (Anonymous Donation) $50,000	£32k
Emmanual College	£30k
Jesus College	£30k over 6 years
Daiwa Anglo-Japanese Foundation	£26k over 2 years
Corporate Members (FIN)	£22k
Cambridge Philosophical Society	£15k over 10 years
Bank of England	£12k
Applied Probability Trust	£10k over 3 years
Trinity College	£10k
National Environment Research Council	£10k
Unilever	£10k
Schlumberger UK	£9k
Wellcome Trust	£8k
DSM (Netherlands)	£6k
GLOBEC	£6k
Christ's College	£5k
Harlequin Software	£5k
British Society of Rheology	£3k
Agricultural and Food Research Council	£2k

*The Isaac Newton Trust donated £1,050,000 to the Institute over the first five years of its operation and has given a loan of £1,000,000 for the second five years. This loan will be converted to endowment if a matching sum can be raised.

Donations in Kind

Computer equipment has been donated by Hewlett-Packard, Sun Microsystems (who have also sold equipment to the Institute at a very substantial discount) and Apple UK (total value of computer hardware donations in excess of £1,000,000); software has been donated by NAG, Claris and Wolfram Research.

A large number of books and journals have been donated by publishers and individual members of the mathematical community.

Summary Accounts

Income	95/96	96/97
Grant income - Revenue	1,248,070	1,487,560
Grant income - Workshop	270,754	228,774
Donations - Revenue	4,237	669
Donations - Capital	0	0
General income	39,973	48,975
Housing	7,714	8,604
Total	**1,570,748**	**1,774,583**
Expenditure		
Scientific Salaries	247,750	331,267
Scientific Travel and Subsistence	305,039	423,832
Scientific Workshop	221,485	179,616
Other scientific costs	5,982	9,166
Staff costs	291,437	284,576
Computing costs	37,326	41,474
Library costs	15,754	16,208
Audio-Visual	6,862	6,647
Building - Capital	1,708	0
Building - Rent	184,000	184,000
Building - Repair and maintenance	12,308	14,360
University Overheads	34,335	35,158
Consumables	51,931	49,389
Equipment - Capital	4,838	23,653
Equipment - Repair and Maintenance	3,737	10,009
Publicity	6,667	8,581
Recruitment costs	10,388	9,153
Miscellaneous	42,815	0
Reprovision	90,000	145,000
Total	**1,574,362**	**1,772,089**

Notes

Capital Expenditure
A new library gallery was built in December 1996 with a generous grant of £20,000 from the Thriplow Trust.

Scientific Salaries
Expenditure on scientific salaries increased in 1996/97 because fewer participants were sponsored by their own institutions to come on Institute Programmes.

In 1995/96, a large number of participants on the *Computer Security, Cryptology and Coding Theory* programme were sponsored by their companies.

Scientific Travel and Subsistence
Additional income, in particular the grant from the Met Office, meant that additional funding was available to spend on scientific travel and subsistence.

Equipment - Capital
This included the purchase of new photocopiers for use by staff and participants.

Grant Income - Revenue

This breaks down as follows:

	Year 4	Year 5
EPSRC salaries	259,604	379,391
EPSRC travel and subsistence	225,988	249,566
Isaac Newton Trust	210,000	212,807
St John's College	150,000	150,000
Hewlett-Packard	98,000	98,000
NM Rothschild & Sons	55,383	15,788
CNRS	50,655	44,434
Rosenbaum Foundation	45,155	45,095
Prudential	25,000	0
Leverhulme Trust	43,093	66,907
Leibniz	28,573	22,698
Info Systems Committee	16,119	16,119
Royal Society	12,299	9,752
London Mathematical Society	10,000	17,500
Institute of Physics	10,000	10,000
Jesus College	5,000	5,000
Thriplow Trust	2,200	0
Cambridge Philosophical Society	1,000	1,000
Met Office	0	37,359
Director's Fund	0	40,000
University of Cambridge	0	66,144
Total	**1,248,069**	**1,487,560**

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Front cover photograph.
The Newton Institute with
John Robinson's sculpture
'Genesis' in the foreground
(Neville Taylor)