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APPENDICES

Please note that the following statistical information may be obtained, on request, from the Institute or from **http://www.newton.cam.ac.uk/reports/9900/appendices.html**

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Director's Report

This year has seen exceptionally vigorous activity at the Newton Institute over a very wide range of fields, not only in pure mathematics, but also in computer science, astrophysics, materials science and condensed matter physics. Five programmes have been completed, and short reports on these are given below; the full reports can be found on pp 18 to 45.

The Scientific Steering Committee met in October 1999, and selected two new programmes: a three-week programme on *Surface Water Waves* (13-31 August 2001) and a four-month programme on *New Contexts for Stable Homotopy Theory* (September to December 2002). The complete pattern of programmes accepted up to December 2002 is shown on page 14.

During the course of this year I was very glad to welcome Professor Kostya Khanin as our new Hewlett-Packard Senior Research Fellow (succeeding Professor Sandu Popescu); and Dr Robert Hunt as the new Deputy Director of the Newton Institute (succeeding Dr Noah Linden). Both Sandu and Noah moved to Bristol University in September 1999, with the very good wishes of the Institute.

With great sadness, I have to record the death on 12 April 2000 of Professor David Crighton, who, through his membership of the Management Committee, had given unwavering support for the activities of the Institute since its foundation eight years ago. David's ever-cheerful and positive presence at meetings of the Management Committee and at many activities of the Institute will be very sorely missed.

The first 'Satellite Workshop' of the Newton Institute was held at the University of Warwick in April 2000 (see p 33). We were also involved in the organisation of a major international meeting on *Mathematics and Theoretical Physics: Challenges for the Twenty-First Century*, held in Singapore 13-17 March 2000.

The Faulkes Gatehouse is now under construction on the site adjacent to the Newton Institute, and will be completed and ready for occupation by June 2001; this will provide a marvellous additional resource for the Institute. The new Betty and Gordon Moore Library for the Physical Sciences, Technology and Mathematics will also be available to Institute members from October 2001.

I have been very glad to visit the Universities of Edinburgh, Glasgow, Leeds, Birmingham and Swansea during the course of the year, both to give seminars, and to make presentations and receive feedback concerning the work of the Newton Institute. One important innovation this year is that we are now placing selected keynote Institute lectures on the Web (in the form of a soundtrack coupled with scans of transparencies). We expect to make an increasing proportion of Institute lectures available in this way in the future.

On the funding front, both EPSRC and PPARC have now confirmed the level of their support for the Institute up to 2005 and 2004 respectively, and we can therefore confidently plan programmes for the next quinquennium. Continuing support for the Institute has also been announced by the Leverhulme Trust, the London Mathematical Society and the Isaac Newton Trust; to all of these bodies I wish to express profound gratitude.

Our contribution to World Mathematical Year 2000 through the 'Posters in the Underground' project is proceeding according to schedule; six posters have now appeared (see pp 46-47); six more are at various stages of preparation. I am greatly indebted to Dr Andy Burbanks, whose creative skills have been vital to the accomplishment of this project. There has been a huge demand for copies of the posters from schools, and the World Scientific Publishing Company has generously agreed to print 500 additional copies of each poster, free of charge, in order to meet this demand. It is intended also to produce sets of postcards which should be available early in 2001.

Keith Moffatt

30 June 2000

Brief Scientific Report on Programmes

Programme 30: Complexity, Computation and the Physics of Information (May to August 1999)

This programme was concerned with the role of quantum coherence in superpositions, and especially entangled states, when embedded in a decoherent environment. The programme brought together experts in quantum coherence, quantum information theory, decoherence and complexity theory. It was the first major programme in this rapidly developing area. Major new insights were obtained. In particular:

- The need for entanglement in the speed up of quantum computing, and the way that NMR quantum computing works.
- The role that quantum teleportation plays in secure quantum communication.
- The construction of bounds on the amount of entanglement within a particular mixed state.
- The use of a new mathematical technique of majorisation in quantum information theory.
- The emergence of classicality though the intervention of decoherence, and the role of 'erasure'.

Programme 31: Structure Formation in the Universe (July to December 1999)

This programme was extremely timely. A number of observational results had just come out, or came out during the programme, including supernovae redshift surveys, cosmic microwave sky maps and dark matter surveys. On the theoretical side there were also a number of new developments and challenges, particularly in the ideas of 'brane worlds'. There was also a need for intense discussion and debate of longstanding theoretical puzzles, such as the quantum mechanics of inflation, for which it was very useful to have the main proponents present for an extended period. The comments of participants were uniformly positive and all reported valuable discussions and new collaborations.

The programme was an international event of high significance for the field. UK scientists benefited strongly through establishing new contacts and collaborations. There was continuous strong pedagogical content, and many students attended from across the UK.

Significant work was carried out during the programme on the following topics:

- New models of dark matter stimulated by the latest discoveries that cold dark matter fails to reproduce observed structure of galaxies.
- Brane worlds and gravity in such scenarios.
- Particle physics implications of brane worlds.
- Quantum cosmology and inflation.
- Gravitational clustering and statistical descriptions.
- Cosmic microwave anisotropies and statistical descriptions.
- Models of quintessence and the cosmological constant.

This last work was reported in an article in the *New York Times* in February 2000.

Programme 32: Developments in Solid Mechanics and Materials Sciences (September to December 1999)

This four-month programme focused on microstructure, its formation and evolution, and the influence of microstructure on macroscopic properties in the context of phase transformations, damage development and fracture. It brought together specialists in these subjects from diverse disciplines including mathematics, materials science, engineering and physics. It thus provided a forum for the exchange of ideas (both between subjects and disciplines), and facilitated the identification of common issues and exploitation to mutual advantage of the advances in different areas.

The immediate outcome of the programme is that participants have made new contacts across disciplinary divides that previously had not been bridged so explicitly and deliberately: interactions which are likely to last into the future have been established between mathematicians with a rigorous approach to the homogenization of partial differential equations with rapidly varying coefficients, engineers concerned with modelling the development of damage during service of materials, and physicists who approach the modelling of materials from an atomistic or quantum theoretic standpoint. Seeds have been sown and the full extent of the benefit will be apparent in a year or two, when contacts established during the programme have had the opportunity to mature into productive collaborations.

Programme 33: Ergodic Theory, Geometric Rigidity and Number Theory (January to July 2000)

Ergodic theory is an area of mathematics with all of its roots and development contained within the 20th century. Strands of the modern theory can be traced back to the work of Poincaré, but the subject began to take a more recognizable form through the seminal work of von Neumann, Birkhoff and Kolmogorov. The impetus to these developments was the important concept of 'ergodicity' in dynamical systems - by which the temporal evolution of the system, though averaging over typical orbits (almost every orbit in the measure theoretic sense), corresponds to spatial averages over the system. An important concept in physical systems, it also set the foundation for applications to other branches of mathematics, most notably geometry and number theory.

This programme brought together both established experts in the field, and younger researchers, from home and abroad, in an effort to promote scientific research and training of the highest quality. To this end it was remarkably successful, with progress being made on a large number of problems, in a diverse number of different directions. Particularly good progress was made in the area of intersection between Lie groups and Ergodic theory. There was also good progress in applications to number theory over different fields, and on geometric problems using ergodic theoretic approaches. There was progress also in understanding which groups act on a circle, and on the difficult problem of counting asymptotically the number of periodic trajectories of a billiard on a polygonal table.

In summary, this programme made significant contributions to the study of Ergodic Theory and its applications to a range of important areas.

Programme 34: Strongly Correlated Electron Systems (January to June 2000)

The general framework for understanding the behaviour of electrons in metals is that of Fermi-liquid theory (which has been rigorously established within a convergent perturbation theory) where the electrons are described in terms of renormalized quasi-particles. However, in recent years an increasing number of metallic systems have been discovered where the quasi-particle picture appears to have broken down.

The aim of the programme has been to bring into dialogue the wide range of expertise in quantum many-body and condensed matter physics on these problems. New results were achieved in the following areas: the theory of quantum Hall ferromagnets; topological effects in short antiferromagnetic Heisenberg spin chains; the effects of interactions and disorder on mesoscopic conductance fluctuations in the two-dimensional chiral metal; the thermoelectric properties of quantum dots in the Coloumb blockade regime; creep and pinning in disordered media; tunnelling between Luttinger liquids. This programme also brought together a number of experts in integrable models with different backgrounds.

Programme Participation

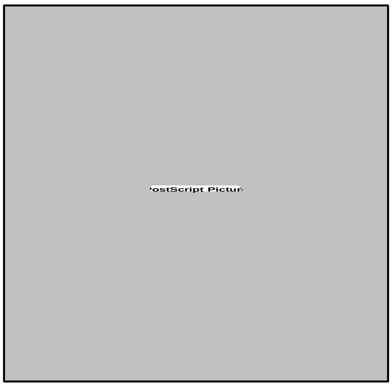
A total of 1592 visitors was recorded for 1999/2000. This includes 282 long-stay participants, each staying between two weeks and six months (7 weeks on average) and 358 short-stay participants who stayed for two weeks or less.

Within the five completed programmes there was a total of 22 workshops (periods of intense activity on specialised topics) which attracted an additional 794 visitors to the Institute. In addition, 158 visitors were registered as having taken part in the special events held outside the Institute programmes, and there were many others who attended occasionally for lectures, workshops or Institute Seminars. Within all the programmes, workshops and special events, around 1200 seminars were given in total at the Institute during the year.

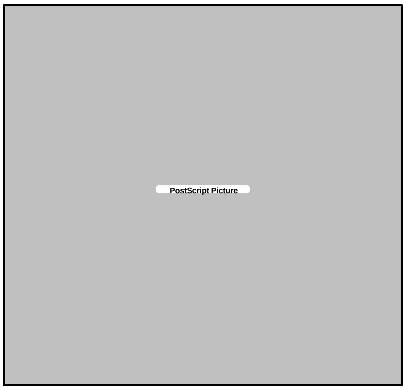
In addition to the workshops, which serve to widen UK participation in programmes, the programme organisers are encouraged to organise more informal special days, short meetings or intensive lecture series which can attract daily or short-term visitors, so further opening the activities of the Institute to the UK mathematical community. Furthermore, over 130 seminars were given at other institutions by Institute participants.

Programme	Long-stay participants	Mean stay (days)	Short-stay participants	Mean stay (days)
Structure Formation in the Universe	59	61	73	9
Developments in Solid Mechanics and Materials Science	48	45	49	10
Ergodic Theory, Geometric Rigidity and Number Theory	60	48	75	9
Strongly Correlated Electron Systems	71	44	109	7
Complexity, Computation and the Physics of Information	44	47	52	14

The pie charts below show the percentages of long-stay and short-stay participants broken down by country of residence:

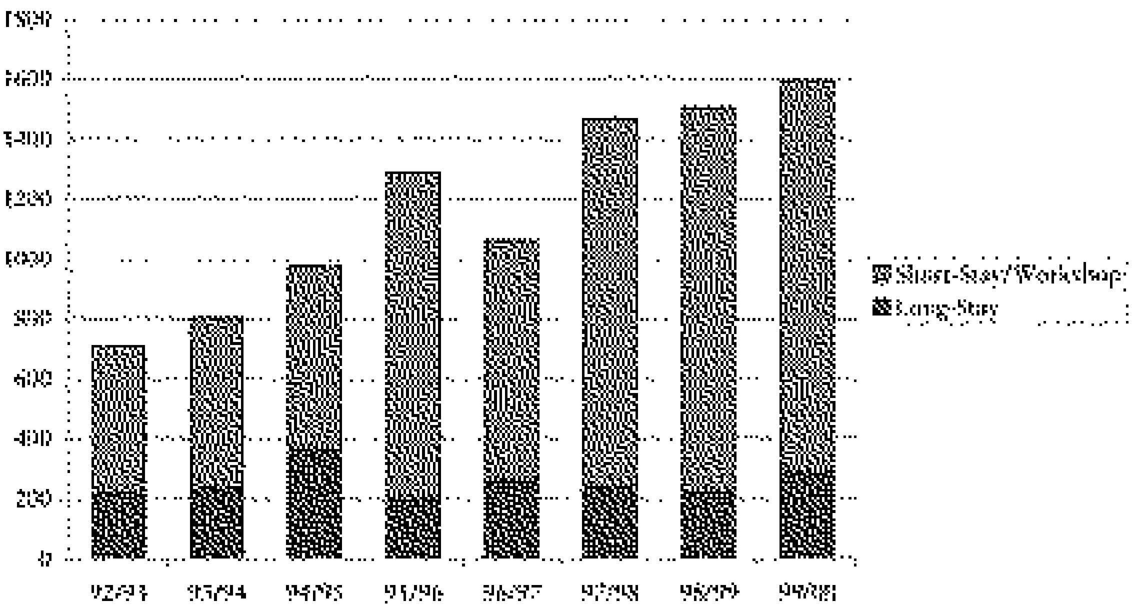


Long-stay participants



Short-stay, workshop and special event participants

The following chart summarises the total figures for long and short-stay participation since the Institute began its programmes:



The median age for long and short-stay participants combined is 40 years, with an interquartile range 33-42 years.

More detailed statistics, including visit dates and home institutions of participants, and a complete list of seminars and papers, are given in the Appendices, available separately from the Institute or at

http://www.newton.cam.ac.uk/reports/9900/appendices.html

Other Institute News

Academic Appointments

Dr Robert Hunt was appointed as Deputy Director from 1 January 2000.

The appointment of Dr Colin Sparrow as the Institute Liaison Officer was renewed for a further three years from October 1999.

Dr Konstantin Khanin took up his appointment as Hewlett-Packard Senior Research Fellow on 1 October 1999.

The Management Committee of the Institute renewed the Senior Fellowships of the Institute awarded to Sir Michael Atiyah OM FRS, Professor Sir Peter Swinnerton-Dyer FRS and Professor Peter Goddard FRS.

Rothschild Visiting Professors

The following Rothschild Visiting Professors were appointed during the year within the programmes indicated. Rothschild Visiting Professors normally give several 'Rothschild Visiting Lectures' at other UK institutions during their visits. These are as indicated.

Professor A Guth (MIT), *Structure Formation in the Universe:*

> Lectures at Edinburgh, Oxford and Portsmouth Universities and at Imperial College

Professor RD James (University of Minnesota), *Developments in Solid Mechanics and Materials Sciences:*

> Lectures at Oxford, Nottingham and Bath Universities

Professor TV Ramakrishnan FRS (Indian Institute of Science, Bangalore), *Strongly Correlated Electron Systems:*

> Lectures at Manchester, Oxford and Birmingham Universities and at Imperial College

Professor H Furstenberg (Hebrew University of Jerusalem), *Ergodic Theory, Geometric Rigidity and Number Theory*

Gabriella and Paul Rosenbaum Fellows

The Gabriella and Paul Rosenbaum Foundation has generously supported the work of the Institute through funding one fellow from the USA for each programme over the last seven years. This grant came to an end in December 1999. We wish to record our grateful thanks to the Trustees of the Foundation for this most welcome support during the formative years of the Institute.

The Fellows appointed during 1999 were:

Dr B Schumacher (Kenyon College), *Complexity, Computation and the Physics of Information*

Dr R Caldwell (Princeton), *Structure Formation in the Universe*

Dr VB Shenoy (Brown), *Developments in Solid Mechanics and Materials Sciences*

EPDI Fellows

The Newton Institute is one of the participating Research Institutes in the European Post-Doctoral Institute (EPDI). The other participating Institutes at present are:

- Institut des Hautes Etudes (Bures-sur-Yvette)
- Max-Planck-Institute für Mathematik (Bonn)
- Max-Planck-Institute für Mathematik in den Natürwissenschaften (Leipzig)
- Institut Mittag-Leffler, Swedish Academy of

Sciences (Djursholm)
- Stefan Banach International Mathematical Center (Warsaw)
- Erwin Schrödinger International Institute for Mathematical Physics (Vienna)

EPDI offers a number of 2-year postdoctoral Fellowships each year to enable the holders to spend some time at the participating Institutes, and at other approved research centres - for details please see the link to EPDI on our website.

Two EPDI fellows participated this year in the programme *Ergodic Theory, Geometric Rigidity and Number Theory.* They were:

- Dr Y Shalom (Yale)
- Dr B Klingler (IHES)

Awards

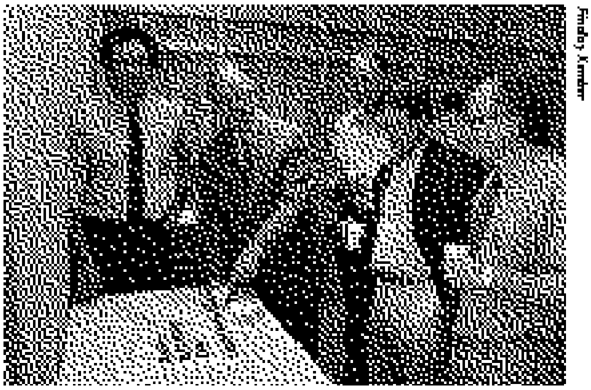
Prof David Crighton FRS was awarded Honorary Doctorates from Loughborough University, UMIST and the University of Crete.

The following were elected Fellows of the Royal Society:

- Prof G Gibbons (Organiser of the *Geometry and Gravity* programme)
- Prof WT Gowers (Scientific Steering Committee)
- Prof R Mackay (ex-Management Committee)
- Prof MS Raghunathan (participant in *Ergodic Theory, Geometric Rigidity and Number Theory*)
- Prof TV Ramakrishnan (participant in *Strongly Correlated Electron Systems*)

An encryption algorithm for the 21st century designed in an international collaborative effort, including Ross Anderson (Organiser of the *Computer Security, Cryptology and Coding Theory* programme, 1996), is one of 5 finalists in the US National Institute for Standards and Technology (winner to be announced in 2001).

Sir Martin Rees (Management Committee) was awarded the Bruno Rossi prize (shared with Prof P Mezaros, Penn State), by the High-Energy Astrophysics Division of the American Astronomical Society.



James Hartle's 60th Birthday Celebration. Left to right: N Linden, J Hartle, T Kibble, R Penrose, G Gibbons

Special Events

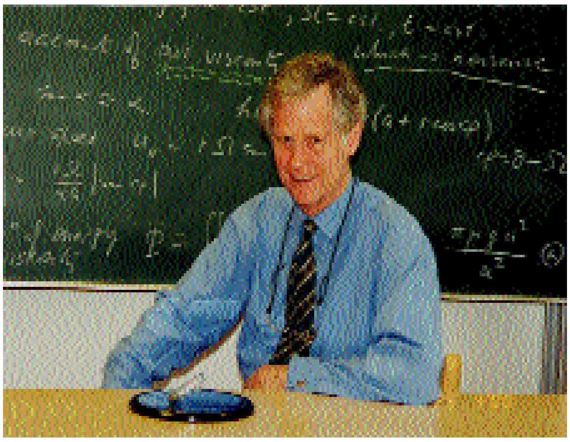
A special meeting on *Quantum Mechanics and Cosmology* was held on 2 September 1999 in honour of James Hartle's 60th Birthday. The speakers were C Isham (Imperial); Sir Roger Penrose (Oxford); GW Gibbons (Cambridge) and SW Hawking (Cambridge). Audio files and OHP transparencies for these lectures can be found on our website at
http://www.newton.cam.ac.uk/webseminars

The UK Cosmology Meeting was held at the Newton Institute as part of the programme on *Structure Formation in the Universe*. Speakers included M Parry (Imperial); F Viniegra (Oxford); A Green (QMW); C Charmousis (Durham); W Naylor (Newcastle); and T Garagounis (Sussex).

The Inaugural Mary Cartwright Lecture, organised by the London Mathematical Society, took place at the Institute on 4 March 2000. WK Hayman (Imperial) spoke on *Recollections of Miss Cartwright,* and Caroline Series (Warwick) gave the Inaugural Mary Cartwright Lecture on *Exploring the Space of Quasifuchsian Groups*.

DM Edwards (Imperial) spoke on *Electrons in Disguise* as part of SET 2000 on 25 March 2000.

The MASAMB - X 2000 meeting on *Mathematical and Statistical Aspects of Molecular Biology* took place at the Institute 4-5 April 2000. This was a sequel to the highly successful programme on *Biomolecular Function and Evolution in the Context of the Genome Project* which ran from July to December 1998. Speakers at the meeting included N Cristianini (Bristol); J Chalker, O Pybus, B Griffiths, K Strimmer (Oxford); J Vilo, A Robinson (European Bioinformatics Institute); N Burroughs (Warwick); S Zöllner (MPI, Leipzig); P Nicodeme (Paris V); J Hein (Aarhus); T Hwa (UCSD); P Higgs (Manchester); H Shimodaira (Stanford); S Whelan (Cambridge); G McGuire (Reading); and T Warnow (Texas).



Andrew Thompson

The Director published a brief communication in *Nature* (**404**, 833-834, 2000) on *Euler's Disk and its Finite-Time Singularity*. This attracted a great deal of attention for the Institute from the media. Professor Moffatt was interviewed by BBC Radio 4's *Today* programme and there was also an item on BBC News 24. Articles appeared in the *Daily Telegraph*, *New Scientist*, *New York Times*, *International Herald Tribune*, *Santa Barbara News-Press*, *Süddeutsche Zeitung*, *Gazeta Wyborcza*, the BBC web site, IOP's web site and a Portuguese web site, *Redacao*.

John Latham was appointed Artist-in-Residence at the Newton Institute from November 1999 to February 2000 with support from the Institute of Visual Arts. As well as displaying and producing various works on-site, he gave a talk entitled *The Universe as Event-Structured.*

Newton Institute Papers and Preprints

Over 100 papers were produced or in preparation at the Institute during 1999/2000 (a complete list is given in Appendix F). Many of these are included in the Newton Institute's preprint series, to which participants are encouraged to submit papers. A web page giving details of Newton Institute preprints is available at **http://www.newton.cam.ac.uk/preprints.html**

Books arising from Newton Institute Programmes

The following titles were published during 1999/2000:

• DB Keim and WT Ziemba (Eds.)
Security Market Imperfections in Worldwide Equity Markets
April 2000, 531pp, ISBN: 0 521 57138 3 (Hbk)

• PR Voke, ND Sandham and L Kleiser (Eds.)
Direct and Large-Eddy Simulation III
November 1999, 456pp, ISBN: 0 792 35990 9 (Hbk)

A complete list of books published as a result of Newton Institute Programmes is available at **http://www.newton.cam.ac.uk/inibooks.html**

UK and International Liaison

Membership of the National Advisory Board as at 30 June 2000:

Professor AFM Smith	Queen Mary and Westfield College (Chair)
Professor HK Moffatt, FRS	Director, Newton Institute
Dr RE Hunt	Deputy Director, Newton Institute
Professor Sir Michael Berry, FRS	University of Bristol
Professor J Brindley	University of Leeds
Professor KA Brown	University of Glasgow
Professor EB Davies, FRS	Kings College London
Professor PJ Diggle	University of Lancaster
Professor CM Elliott	University of Sussex
Professor NJ Hitchin, FRS	University of Oxford
Dr M Sheppard	Schlumberger Cambridge Research Ltd
Professor JR Whiteman	Brunel University

National Advisory Board and Consultative Group

Following discussions with EPSRC, a National Advisory Board (NAB) for the Institute was established during this year, and held its first meeting on 20 March 2000. The remit of the NAB is:

"To advise the Director in all matters relating to the role of the Newton Institute as a National Institute for the Mathematical Sciences."

The membership, as at 30 June 2000, is given in the table above.

Anyone with views about the national role of the Institute is invited to make these views known to any member of the NAB.

The Consultative Group for Symposia Activities in the Mathematical Sciences had its third annual meeting on 6 March 2000. The group agreed that although much of its function would be taken over by the NAB it should continue to exist and consult by email.

As agreed by the group, the Institute continues to maintain the list giving details of symposia activities in the Mathematical Sciences, from short workshops to long-term research programmes. This can be found at:
http://www.newton.cam.ac.uk/symposia.html

Satellite workshops

The Institute is now encouraging organisers of longer (4 or 6-month) programmes to cooperate with local organisers in holding workshops at UK universities and institutions outside Cambridge.

Workshops are on a theme that is scientifically strongly related to the Institute programme, and involve a significant number of longer-term overseas participants from the Institute programme. They also, crucially, draw in and involve UK mathematicians and scientists who might not otherwise have been able to participate substantially in the Institute programme.

Costs for satellite workshops are typically approximately £10,000 (excluding the overseas travel costs of Institute participants) and are shared approximately 50/50 between the Institute and the host institution. The EPSRC and LMS mathematics programme have both agreed that the host institution is eligible to apply for workshop or conference grants to cover its share of the costs (subject to the usual review procedures). We are grateful that the LMS in particular has expressed a willingness to half-fund up to two such workshops a year.

The first of these workshops, *Ergodic Theory of* $\mathbb{Z}^d$ *Actions,* was held at Warwick University, 3-7 April 2000, part-funded by EPSRC, in connection with the *Ergodic Theory, Geometric Rigidity and Number Theory* programme. Four more workshops are planned in connection with Institute programmes in the 2000/2001 academic year. Proposals will be developed for 2001 and beyond.

Institutions interested in holding such workshops should contact either the Institute's Liaison Officer, Dr Colin Sparrow (c.t.sparrow@newton.cam.ac.uk), or the organisers of the relevant programme.

Seminars

All participants in Newton Institute programmes are strongly encouraged to visit other UK institutions during their time at the Institute, and many did so during 1999/2000. A total of 135 seminars were given outside Cambridge.

Audio files of selected Institute seminars with accompanying transparencies have been published on the web for the first time during 1999/2000. These include lectures given at the James Hartle Birthday meeting (see p 7) and most of the Monday evening seminars. They can be found at: **http://www.newton.cam.ac.uk/webseminars/**

BAMC

The Deputy Director attended the British Applied Mathematics Colloquium (BAMC 2K), held in Manchester in late April, to make a presentation about the work of the Institute, to encourage participation in its programmes and activities, and to receive feedback. Conference participants were shown a wide range of information about the Institute, including the list of future programmes, details of how to submit proposals for programmes, the composition of the Scientific Steering Committee and the new National Advisory Board, and copies of those World Mathematics Year 2000 posters which had appeared to date. A robust but useful discussion followed in which Dr Alasdair Rose, Manager of the Mathematics Programme at EPSRC, also took part. Amongst the issues raised were the proportion of Institute participants from Cambridge itself (answered with the use of the diagrams which appear on p 4); the balance of subject specialisms among the members of the SSC, and the possibility of imbalance in Institute programmes; and the role of Cambridge University in making appointments to the SSC (some speakers thinking it should play a larger part, to correct any potential imbalance, others thinking it should play no part whatever). Dr Rose emphasised that the Institute exists to benefit the UK mathematical sciences community and relies on their input and participation.

The Deputy Director undertook to bring these matters to the attention of the National Advisory Board at its meeting in October, where the suggestions and comments would be considered in the wide context of UK mathematics.

Young Scientists

The Institute holds a number of events each year which are specifically targeted at young scientists. In 1999/2000 these events included:

- NATO Advanced Study Institute on *Structure Formation in the Universe* (2-6 August 1999)
- NATO Advanced Study Institute/EC Summer School on *New Theoretical Approaches to Strongly Correlated Electron Systems* (10-20 April 2000)
- EC Summer School on *Connecting Fundamental Physics and Cosmology* (16-20 August 1999)
- EC Summer School on *Mathematical Developments in Modelling Microstructure and Phase Transformation in Solids*
- EuroWorkshop on *Computational Quantum Many-Body Physics* (18 - 21 February 2000)
- EuroConference on *Strongly Correlated Electron Systems - Novel Physics and New Materials*
- Workshop on *Non-Fermi Liquid Effects in Metallic Systems with Strong Electronic Correlation*, sponsored by the European Science Foundation (ESF)
- EuroWorkshop on *Rigidity in Dynamics and Geometry* (27-31 March 2000)
- Euroconference on *Ergodic Theory, Riemannian Geometry and Number Theory* (3-7 July 2000), supported by ESF

The following young scientists were recipients of bursaries from the Cambridge Philosophical Society (given specifically for young people) in 1999/2000:

Complexity, Computation and the Physics of Information
- J Anglin (University of Innsbruck)
- M Nielsen (CALTECH)

Structure Formation in the Universe
- U Pen (CITA, Toronto)
- U Seljak (Princeton)

Developments in Solid Mechanics and Materials Sciences
- F Otto (UCA, Santa Barbara)
- J Chapman (Oxford)

Ergodic Theory, Geometric Rigidity and Number Theory
- T Ward (East Anglia)
- C Walkden (Manchester)

Strongly Correlated Electron Systems
- ACM Green (Imperial)
- C Pepin (Saclay)

The Institute recognises that junior researchers have much to contribute to and much to gain from Institute programmes and events. In order to maximise the information available to junior researchers, and to facilitate their involvement in Institute activities, we introduced in 1997 a category of Junior Membership of the Newton Institute. To be eligible for Junior Membership of the Institute you must be a Research Student or within 5 years of having received a PhD (with appropriate allowance for career breaks) and you must work or study in a UK University or a related research institution.

Junior members receive regular advance information about programmes, workshops, conferences and other Institute events via a Junior Members' Bulletin, detailed information about any workshops of an instructional or general nature likely to be of special interest to young researchers, and information about suitable sources of funding or support for visits to the Institute, when available.

The Institute makes available some of its general funds specifically to support junior researchers in Institute activities. The types of involvement supported include (but are not limited to) attendance at workshops, conferences etc, and visits of up to 2 weeks to work or study with longer-term participants in the Institute's programmes.

Those interested in becoming Junior Members should consult the Institute's web site at **http://www.newton.cam.ac.uk/junior.html**

BRIMS

Report from the Hewlett-Packard Senior Research Fellow, Kostya Khanin

Findlay Kember

My main research interests are in the theory of dynamical systems and its interactions with the theory of turbulence, probability theory, statistical mechanics and ergodic theory. A very extensive period of development of the modern theory of dynamical systems in the last 30 years has led to a much better understanding of nonlinear dynamics and chaos. During this period, the progress was mostly restricted to a finite-dimensional world of mappings and ordinary differential equations.

However, in the last few years there has been a significant increase in mathematical research aiming to apply the ideology and methods of the theory of dynamical systems to chaotic phenomena in partial differential equations and, in particular, to turbulence theory. In the past year I have been working together with Renato Iturriaga (CIMAT, Mexico and Isaac Newton Institute) on the problem of Burgers turbulence. This problem is closely related to a problem of random Lagrangian dynamics. In a very general situation we have been able to construct stationary distributions for solutions of Burgers equation driven by random force. In an ongoing research project, we hope to be able to study the leading singularities of typical solutions and their impact on physically important correlation functions.

In a quite different line of research I have been working on multi-dimensional generalisations of continued fractions. The classical algorithm of one-dimensional continued fractions is used for construction of rational approximations to irrational numbers. Continued fractions are remarkable objects from many points of view. In particular, they provide one of the most beautiful applications of ergodic theory to number theory. In joint work with David Hardcastle (Heriot-Watt University) we have discovered an n-dimensional Gauss automorphism which generalises the famous one-dimensional Gauss transformation and inherits many of its beautiful ergodic properties.

The last research topic on which I have been working is connected with rigidity theory in one-dimensional dynamics. The main aim of rigidity theory is to show that two mappings with the same topological behaviour and the same structure of singularities have the same asymptotic metrical properties, i.e. they are smoothly conjugate to each other. My work in this area is focused on a particular type of singular behaviour, so-called break singularity. We hope in this case to construct a full rigidity theory, and also to develop methods which can be used for other types of singularities.

Grants and Funding

In 1999/2000 the Institute was successful in attracting support from the following organisations:

EPSRC: The Institute's application for funding for 2002 to 2008 was assessed by an EPSRC Review Panel which met at the Institute on 15 December 1999 and recommended the award of the travel and subsistence grant in full, and the salaries grant with minor amendments.

EPSRC confirmed the level of funding it is to offer to the Institute for the period 2002 to 2008 as follows:

£1,842,750 for travel and subsistence support;

£1,140,771 for scientific and related salaries for the period March 2002 - February 2005. The salary grant will continue in principle to 2008 but the rates will be reviewed at the end of this initial period.

PPARC: PPARC confirmed that its 'rolling grant' to the Institute will continue until at least 2004.

Leverhulme Trust: The Leverhulme Trust granted funding for a further three years, from July 2000 to June 2003, at an increased rate of £80,000 pa. This funding supports participants from the former Soviet Union, Eastern Europe, South America, India, China, Mexico and some other countries.

Hewlett-Packard: Hewlett-Packard gave a further generous donation of computer equipment to the Institute.

NSF: Both the *Structure Formation in the Universe* and the *Ergodic Theory, Geometric Rigidity and Number Theory* programmes benefited from funding obtained from the National Science Foundation in the US.

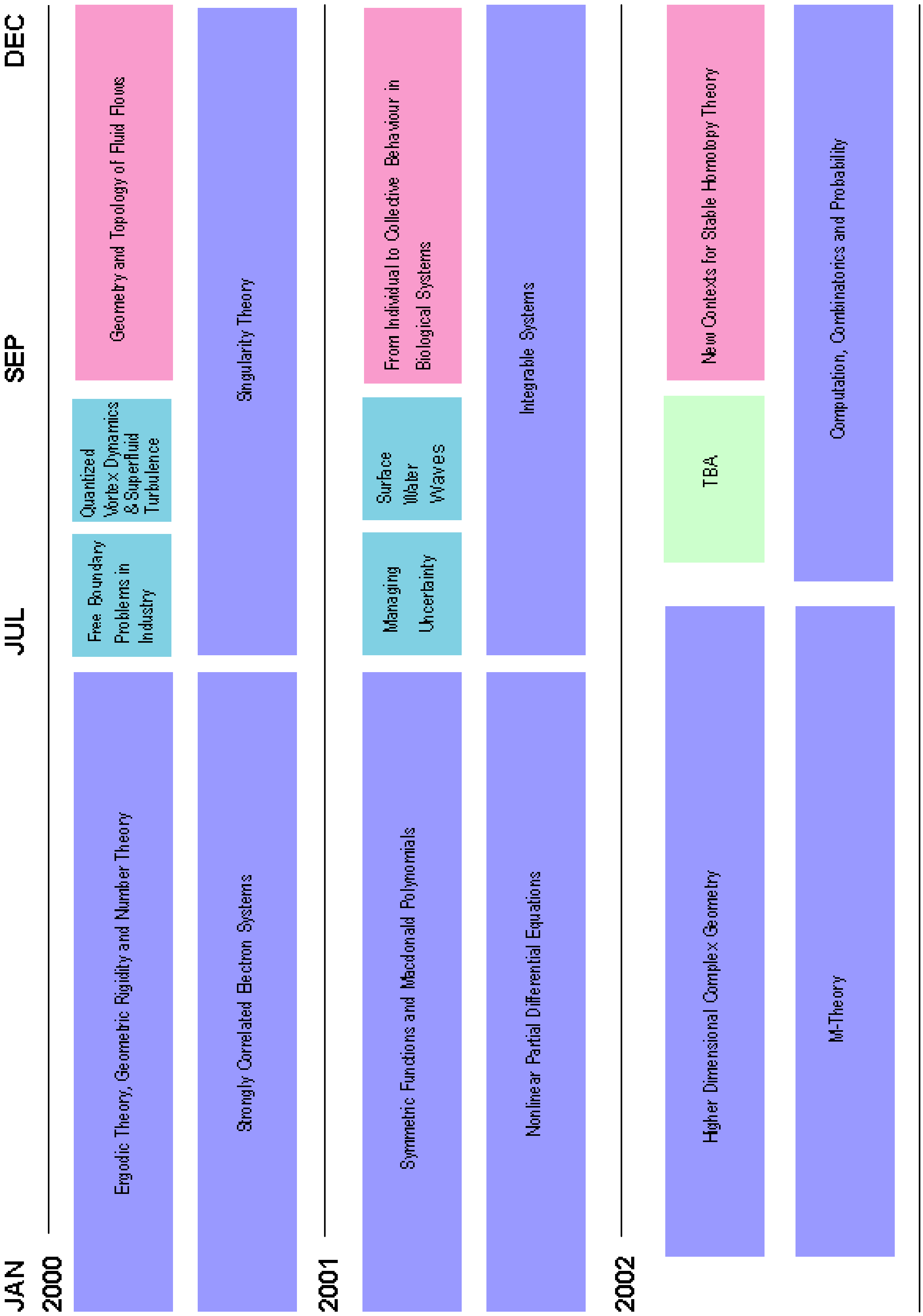
NATO: NATO funded an Advanced Study Institute on each of the *Structure Formation in the Universe* and *Strongly Correlated Electron Systems* programmes.

EC: Funding from the EC supported workshops on the *Structure Formation in the Universe*; *Developments in Solid Mechanics and Materials Sciences*; *Ergodic Theory, Geometric Rigidity and Number Theory*; and *Strongly Correlated Electron Systems* programmes.

Autonomy Systems Ltd: This company made a generous donation to the *Nonlinear and Nonstationary Signal Processing* programme, acknowledgement of which was unfortunately omitted from last year's report.

Future Programmes

The following schematic diagram shows the forthcoming programmes which have been selected by the Institute's Scientific Steering Committee. Full details are available at **www.newton.cam.ac.uk/programs**



Management Committee

Membership of the Management Committee at 30 June 2000 was as follows:

Professor HK Moffatt, FRS	Director, Newton Institute
Dr RE Hunt	Deputy Director, Newton Institute
Professor WBR Lickorish	Head, DPMMS, Cambridge
Professor TJ Pedley	Head, DAMTP, Cambridge
Professor FP Kelly, FRS (Chair)	General Board
Professor Sir Martin Rees, FRS	Council of the School of Physical Sciences
Professor N Fleck	Council of the School of Technology
Dr CT Sparrow	Faculty of Mathematics
Professor EB Davies, FRS	LMS
Professor NJ Hitchin, FRS	Chairman of SSC
Dr PT Johnstone	St John's College
Dr PMH Wilson	Trinity College
Dr AEA Rose	EPSRC
Professor J Brindley	Co-opted at the discretion of the Committee

The Management Committee is responsible for overall control of the budget of the Institute, and for its short-term and long-term financial planning and control. The Director is responsible to the Management Committee, which provides essential advice and support in relation to fund-raising activity, employment of staff at the Institute, appointment of Organisers of programmes, housing, library and computing facilities, publicity, and general oversight of all activities of the Institute. Its aim is to facilitate to the fullest possible extent the smooth and effective running of the visitor research programmes of the Institute and all related activities.

The Committee is especially concerned with the interactions between the Institute and its funding bodies, particularly the UK Research Councils, Cambridge University, the Cambridge Colleges, the London Mathematical Society, the Leverhulme Trust, and others.

It generally meets three times a year. Two of its members also serve on the National Advisory Board.

Professor FP Kelly, FRS,
Chair of the Management Committee

Scientific Policy Statement

From its inception, it has been intended that the Newton Institute should be devoted to the Mathematical Sciences in the broad sense. In this respect the Institute differs significantly from similar institutes in other countries. The range of sciences in which mathematics plays a significant role is enormous, too large for an Institute of modest size to cover adequately at any one time. In making the necessary choices, important principles are that no topic is excluded *a priori* and that scientific merit is to be the deciding factor.

One of the main purposes of the Newton Institute is to overcome the normal barriers presented by departmental structures in Universities. In consequence, an important, though not exclusive, criterion in judging the 'scientific merit' of a proposed research programme for the Institute is the extent to which it is 'interdisciplinary'. Often this will involve bringing together research workers with very different backgrounds and expertise; sometimes a single mathematical topic may attract a wide entourage from other fields. The Institute's Scientific Steering Committee therefore works within the following guidelines:

(a) the mixing together of scientists with different backgrounds does not *per se* produce a successful meeting: there has to be clear common ground on which to focus;

(b) each programme should have a substantial and significant mathematical content;

(c) each programme should have a broad base in the mathematical sciences.

Research in mathematics, as in many other sciences, tends to consist of major breakthroughs, with rapid exploitation of new ideas, followed by long periods of consolidation. For the Newton Institute to be an exciting and important world centre, it has to be involved with the breakthroughs rather than the consolidation. This means that, in selecting programmes, a main criterion should be that the relevant area is in the forefront of current development. Since the Institute's programmes are chosen two to three years in advance, it is not easy to predict where the front line will be at that time. The best one can do is to choose fields whose importance and diversity are likely to persist and to choose world leaders in research who are likely to be able to respond quickly as ideas change.

Although the novelty and the interdisciplinary nature of a proposed programme provide important criteria for selection, these must be subject to the overriding criterion of quality. With such a wide range of possibilities to choose from, the aim must be to select programmes which represent serious and important mathematical science and which will attract the very best mathematicians and scientists from all over the world. However, the Institute is receptive also to proposals of an unorthodox nature if a strong scientific case is made.

Although the Institute operates on a world-wide basis and contributes thereby to the general advancement of mathematical science, it must also be considered in the context of UK mathematics. A natural expectation of all those concerned is that each programme will be of benefit to the UK mathematical community in a variety of ways. If the UK is strong in the field, UK scientists will play a major part in the programme; if the UK is comparatively weak in the field, the programme should help to raise UK standards, and instructional courses, aimed primarily at younger researchers and research students, will play a vital role here.

Because of the wide base of support for the Newton Institute in the EPSRC and elsewhere, the Institute's programmes shall as far as possible represent an appropriate balance between the various mathematical fields. In order to retain the backing of the mathematical and scientific community, the Institute will run programmes over a wide range of fields and, over the years, achieve this balance. Such considerations, however, are secondary to the prime objective of having high quality programmes.

Scientific Steering Committee

The Institute invites proposals for research programmes in any branch of mathematics or the mathematical sciences. The SSC meets in April and October each year to consider proposals for programmes (of 3-week, 4-month or 6-month duration) to run two or three years later. Proposals to be considered at these meetings are submitted by 31 January or 31 July respectively. Successful proposals are usually developed in a process of discussion between the proposers and the SSC conducted through the Director, and may well be considered at more than one meeting of the SSC before selection is recommended. Proposers may submit a 'preliminary' proposal in the first instance with a view to obtaining feedback from the SSC prior to the submission of a full 'definitive' proposal.

The scientific planning and organisation of each programme are the responsibility of a team of three or four Organisers (aided in some cases by an Advisory Committee). The Organisers recommend participants in the programme, of whom up to twenty can be accommodated at any one time; they also plan short-duration workshops and conferences within the programme, to which many more participants may be invited. Each programme is allocated a budget for salary support, subsistence allowances and travel expenses.

Membership of the Scientific Steering Committee at 30 June 2000 was as follows:

Professor NJ Hitchin, FRS (Chairman)	University of Oxford
Professor HK Moffatt, FRS (Secretary)	Director, Newton Institute
Professor R Anderson, FRS	University of Oxford
Professor RH Dijkgraaf	University of Amsterdam
Professor CM Elliott	University of Sussex
Professor WT Gowers, FRS	University of Cambridge
Professor AJ Macintyre, FRS	University of Edinburgh
Professor MA Moore, FRS	University of Manchester
Professor A Newell	University of Warwick
Professor EG Rees	University of Edinburgh
Professor AFM Smith	Queen Mary and Westfield College
Professor J Stark	University College London
Professor S White, FRS	Max-Planck Institut für Astrophysik
Professor D Zagier	Max Planck Institut für Mathematik

The Scientific Steering Committee met on 18 October 1999. On this occasion Professor PJ Cameron (QMW) attended in place of Professor WT Gowers, and Professor D Rand (Warwick) attended in place of Professor A Newell.

Complexity, Computation and the Physics of Information

10 May to 20 August 1999

Report from the Organisers: PL Knight (Imperial College London), RM Solovay (UC Berkeley), W Zurek (LANL), A Albrecht (UC Davis)

Scientific Background

The remarkable increase in the power of computers over the last few decades creates the illusion that this process will continue indefinitely. However, it is clear that in the next 20 years a limit will be reached which is dictated by fundamental physical processes: within this time-scale the size of individual components etched onto micro-chips will approach atomic dimensions. In order to allow progress to continue a fundamentally new approach will be needed. Interestingly, at this fundamental component-size limit, the quantum mechanical nature of the interactions within the computer would become important. These considerations have led researchers to question whether a new type of computer could be constructed - a quantum computer - which would take advantage of quantum behaviour in order to perform computing tasks in different and vastly more efficient ways. At the heart of these ideas is the notion of entanglement in which the quantum states of different particles or fields become linked in a fundamental way leading to surprising non-local phenomena. Theoretical work has shown that a quantum processor may be able to perform certain important tasks much more efficiently than its classical counterpart because of the intrinsically parallel nature of quantum computations.

The emerging field of quantum information processing has grown at a staggering rate over the last few years. Up until now much of the activity has been theoretical but a few seminal experiments have been performed. These early experiments have highlighted just how difficult it will be to build a working quantum computer, by identifying decoherence in quantum systems as a key issue in practical implementations. The quantum superpositions used in this type of processor are very fragile and can be destroyed by a wide variety of sources of dissipation and noise that may prove to be very difficult to isolate and control. As a result, the study of decoherence has become a vital ingredient for current and future experimental programmes.

Thus far, only two technologies have been used to demonstrate simple quantum gates: single trapped atomic ions and nuclear magnetic resonance (NMR) in macroscopic samples. These are shown in Fig 1 (below), from the ion trap quantum computing group of Prof Blatt in Innsbruck, in which laser-cooled trapped ions are held in space by confining electrodes, and Fig 2 (opposite) from Dr Laflamme's discussion of his NMR work during the programme.

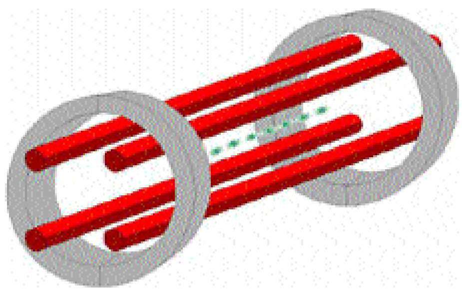


Figure 1: Trapped atomic ions

There is currently a debate under way as to whether the NMR experiments truly qualify for 'quantum processor' status since an ensemble of approximately 1023 systems is used. This methodology also suffers in that it is not scaleable to large numbers of quantum bits (qubits). By contrast, the trapped ion work is less developed but clearly generates 'true' quantum gates since it is intrinsically a single atom technique.

Both approaches have shown the deleterious effects of decoherence. Fortuitously, it has been shown that quantum information processing may be possible even in the presence of a certain amount of decoherence through quantum error correction procedures. Much work was devoted

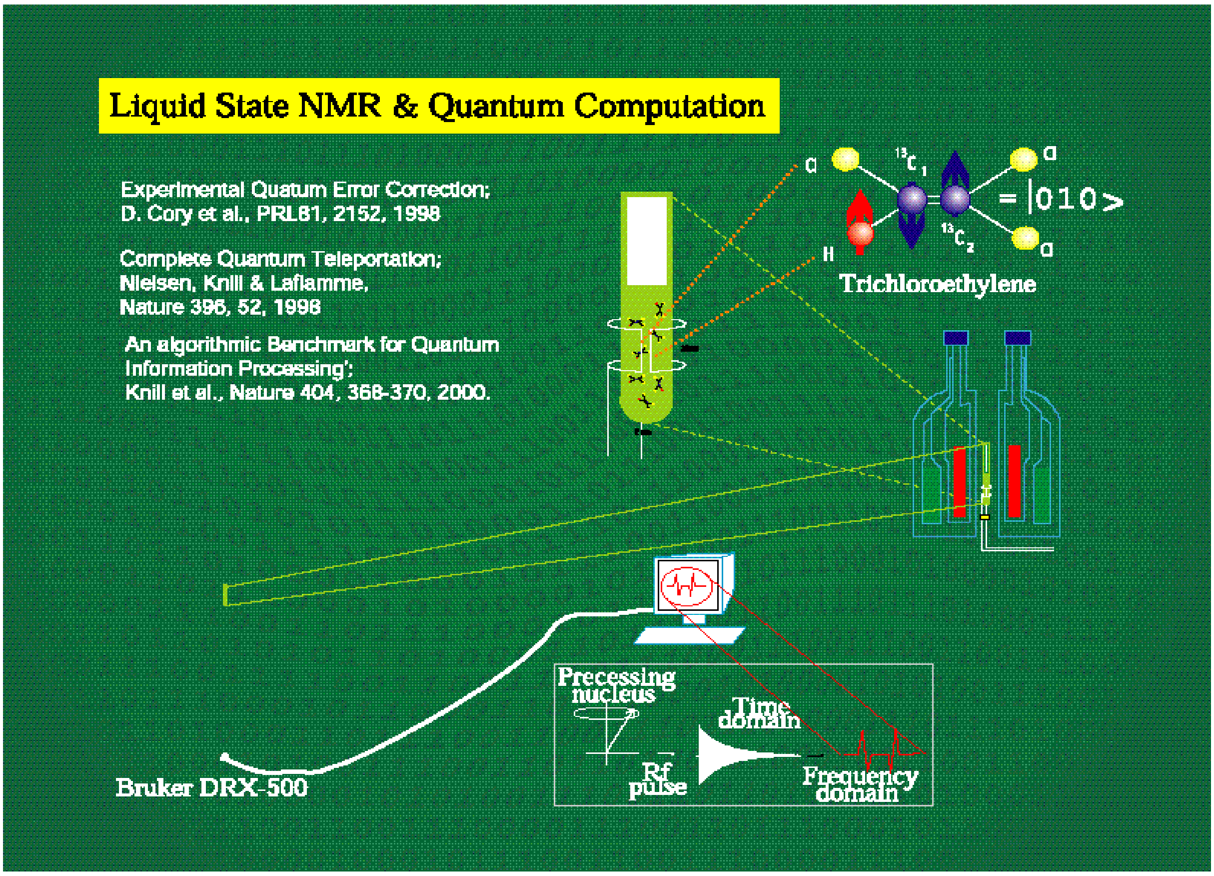


Figure 2: Liquid State NMR and Quantum Computation

to this topic at the programme, with some emphasis being given to a comparison of the merits of using quantum error correction or using the newly identified 'Decoherence-Free Subspaces'.

In Fig 3 (right), we show (from Kwiat et al) the angular distribution of correlated photons from an optical parametric amplifier: the rings correspond to different frequencies emitted. The sum of the upper and lower beam frequencies must add up to the pump laser frequency; this plus momentum conservation generates maximally entangled pairs of photons used in much of quantum information processing (including quantum teleportation, a central topic of discussion at the programme).

In Fig 4 (overleaf), we show the image of two laser cooled trapped ions: these can become entangled and used for 2-bit quantum gates at NIST in Boulder.

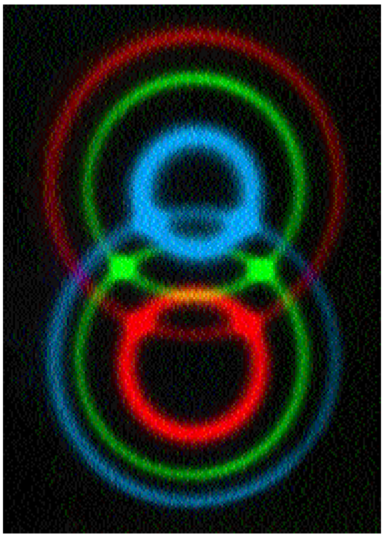


Figure 3: Correlated photons emitted from an optical parametric amplifier

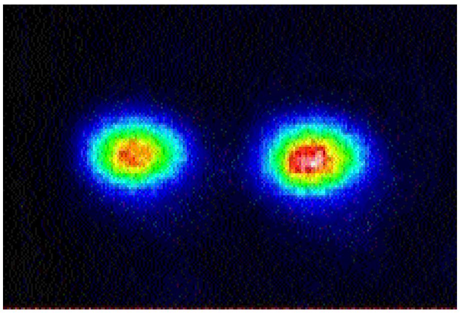


Figure 4: Two laser cooled trapped ions

Organisation

The core of the programme was concerned with the role of quantum coherence in superpositions, and especially entangled states, when embedded in a decoherent environment. The programme brought together experts in quantum coherence, quantum information theory, decoherence, complexity theory, and the like. It was the first major workshop in this rapidly developing area and was, in our view, highly successful. The Workshop organising team consisted of the Programme Organisers plus workshop organisers R Jozsa (Plymouth - now Bristol) and M B Plenio (Imperial College).

In addition, we were helped in many ways by the presence of two local experts who were then at Newton Institute, S Popescu and N Linden.

The initial proposer and organiser, Prof A Albrecht, moved from Imperial to UC Davis in 1998, and Prof P L Knight from Imperial then joined the organising team to maintain a UK resident organiser.

The long-term programme ran at the Newton Institute, but the workshops were transferred rather late in the day (due to building activities and associated disturbance) to New Hall, where some practical difficulties were experienced.

The finances of the workshops were somewhat precarious and only the co-location of the European Science Foundation Network initial meeting (coordinated by Dr Plenio) with the INI workshops made possible the high level activity we eventually enjoyed.

The facilities provided for us at the INI were outstanding. If we were to make minor criticisms, we should say that it is incredible that a modern building with so many windows does not have an effective air-conditioning system.

We were very grateful to the local staff of the INI for their professional assistance before and during the meeting. Their help was invaluable.

Meetings and Workshops

The week-by-week programmes of the workshops were outstanding. In particular, the second week of the programme consisted of a workshop on *Entanglement and Quantum Information Processing*, which was organised in conjunction with the ESF programme on *Quantum Information Theory and Quantum Computation* (launched at the beginning of 1999).

About 100 researchers from all over the world participated. Exchange of ideas was intense and stimulating: a number of publications posted to the Los Alamos preprint server acknowledge the workshop, and more work is in preparation. In the conference program, leading researchers as well as young researchers (PhD students and young postdocs) were given the opportunity to present their latest results. It was encouraging to see that a substantial fraction of the talks reported work that had not been published in any journal nor had appeared on the Los Alamos preprint server. This enhanced the active, workshop atmosphere of the meeting.

The scientific standard was high and a number of exciting new results, both experimental and theoretical, were reported. On the experimental side for example the work presented by Haroche (Paris), which appeared in *Nature* after the meeting, and the impressive progress on quantum cryptography by Gisin (Geneva) and Hughes (Los Alamos), generated much interest. The theoretical study of quantum entanglement of finite systems received a strong stimulus from work reported by Nielsen (Caltech) who introduced a new mathematical structure for this situation. Plenio (London) further demonstrated that entanglement can be used in a catalytic way without consuming it, thereby demonstrating an entirely new quality of entanglement. Both results will appear in PRL. The workshop had a strongly interdisciplinary nature, with participants from chemistry, experimental and theoretical physics (quantum optics, solid state physics and even cosmology), and mathematics to computer science. The resulting interdisciplinary activities are exemplified by the work on quantum computation in Nuclear Magnetic Resonance (Laflamme, Knill and Jones, as reported in PRL and *Nature*).

It was the general feeling that the meeting was very successful and marked the most important event in the field of quantum information theory in 1999. The scientific quality of the meeting demonstrated that the field is healthy and progressing well.

Achievements

Major new insights were obtained. In particular:

- The need for entanglement in the speed up of quantum computing, and the way that NMR quantum computing works.
- The role quantum teleportation plays in secure quantum communication.
- The construction of bounds on the amount of entanglement within a particular mixed state.
- The use of a new mathematical technique of 'majorisation' in quantum information theory.
- The emergence of classicality though the intervention of decoherence, and the role of 'erasure'.

Conclusions

The programme fulfilled the aims of the initial proposal. Scientists from different disciplines were able to interact and share insights. New results were obtained and a number of important papers written. The informal seminar series was valuable in making participants known to each other and in encouraging interaction. Many of the European participants were able to join forces in formulating collaborative applications for funding to the European Union for further support in this area. (We learnt that the EU has committed 16 million euros to this field and essentially every senior participant in our programme is now involved in this new collaboration for the future.)

Publications are being collected by the INI Information Officer. I have noted many papers appearing on the quant-ph Los Alamos server with INI acknowledgments.

Structure Formation in the Universe

19 July to 17 December 1999

Report from the Organisers: V Rubakov (Russian Academy of Sciences), PJ Steinhardt (Princeton), NG Turok (DAMTP, Cambridge)

This programme was extremely timely. A number of observational results had just come out, or came out during the programme, including supernovae redshift surveys, cosmic microwave sky maps and dark matter surveys. On the theoretical side there were also a number of new developments and challenges, particularly in the ideas of 'brane worlds'. There was also a need for intense discussion and debate of longstanding theoretical puzzles, such as the quantum mechanics of inflation, for which it was very useful to have the main proponents present for an extended period. The comments of participants were uniformly positive and all reported valuable discussions and new collaborations. The support of the staff at the INI was uniformly excellent and was absolutely essential to the smooth running of the programme.

Left to right R Caldwell, N Turok, R Crittenden, P Steinhardt, M Bucher, V Rubakov

The programme began with a very intense burst of schools and workshops, which had a major international impact.

The NATO school on *Structure Formation in the Universe* was a high quality school reviewing the subject and all the main areas of interest and importance.

The workshop on *The Statistical Analysis of Cosmological Data Sets* was a success, with some of the most exciting experimental results of the day (the BOOMERANG measurement of the cosmic microwave sky) being revealed for the first time.

The EC School on *Connecting Fundamental Physics and Cosmology* turned out to be the most exciting event of the summer internationally in the field of theoretical cosmology, as there is a lot of interest in the cosmology of M theory and in particular in large extra dimensions. We heard state-of-the-art talks from the leaders in this field and there were a large number of experts here to add comments and criticisms. A very large number of collaborative projects resulted from the workshop, involving UK participants with many of the overseas speakers. We were told by a number of distinguished physicists that this was one of the best meetings they have ever been to. Again the INI handled all the organisational details beautifully, showing considerable diplomatic skills even when faced with a set of physicists almost totally oblivious to normal rules of etiquette (skipping lunch ticket queues etc!).

Following the intensive summer, from October on, the main activity was seminars, an average of two a week. Two colloquia, by Rubakov on *Baryogenesis* and by Prof C Frenk on *Galaxy Formation*, were held which attracted reasonable audiences.

The Spitalfields Day involved 3 INI speakers (Guth, Crittenden, Turok) and one from Oxford (Lucas). There was an audience in excess of 80 and the meeting was a success.

Turok organised a small one-day meeting called *Futuristic Detectors for Cosmology* which brought experimentalists and theorists to the INI on 11 November 1999. The number of participants was small, around 40, but the discussion was very stimulating and useful in terms of establishing links between researchers across the UK working to detect cosmic backgrounds: photons, neutrinos, gravitons, and dark matter.

The final workshop of the programme, in the first week of December, turned out to be excellent, with review talks by a number of speakers (Moore, Bond, March-Russell, Turok) setting the scene for each day, followed by contributed talks. The audience was in excess of 80 for much of the workshop.

Participants discuss possible implications of having large extra dimensions

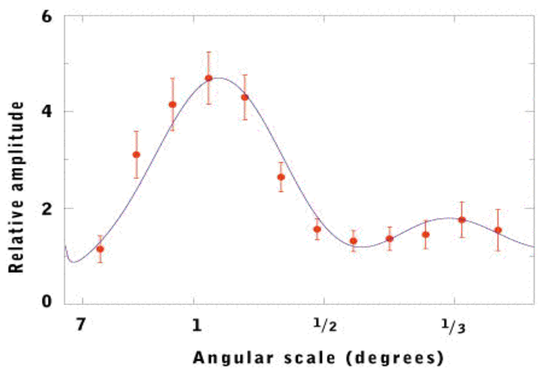
In general terms, the programme was an international event of high significance for the field. UK scientists benefited strongly through establishing new contacts and collaborations. There was continuous strong pedagogical content, and many students attended from across the UK.

Significant work was carried out during the programme on the following topics:

- New models of dark matter stimulated by the latest discoveries that cold dark matter fails to reproduce the observed structure of galaxies.
- Brane worlds and gravity in such scenarios.
- Particle physics implications of brane worlds.
- Quantum cosmology and inflation.
- Gravitational clustering and statistical descriptions.
- Cosmic microwave anisotropies and statistical descriptions.
- Models of quintessence and the cosmological constant.

This last work was reported in an article in the *New York Times* in February 2000.

Our only real problem with the programme was that it was surprisingly much easier to get overseas visitors to come for extended periods than it was for UK academic staff. Many of the US participants (eg Guth, Steinhardt) made several transatlantic trips to spend a week or two at the INI. It seems harder to persuade UK academics to make a similar effort, probably due to their above-average teaching and administrative load as compared to their overseas counterparts.



The power spectrum of the fluctuations implied by the BOOMERANG data. The CMB data provides the most convincing evidence that the geometry of the universe is flat (theoretical curve). This is an encouraging sign for inflationary models of the early universe, which were another central topic of the programme.

For the future I recommend the INI consider how to fund ‘lecturer fellowships’, or persuade funding councils to fund them, or other similar arrangements to free UK researchers from their University duties in order to attend INI programmes.

Developments in Solid Mechanics and Materials Sciences

6 September to 17 December 1999

Report from the Organisers: K Bhattacharya (Caltech), PMJH Suquet (LMA, Marseille), JR Willis (DAMTP, Cambridge)

Virtually every solid material contains features that are different at different length scales. For example, even the simplest piece of metal is made up of many crystallites (grains), which in turn are made up of many atoms. This complexity is compounded in sophisticated modern materials. The microscopic structure of a solid material influences its macroscopic response to applied stress, magnetic field and other macroscopic stimuli. Conversely, the macroscopic applied loads and fields affect the microscopic structure. Microscopic damage may occur, leading ultimately to the formation of large cracks and structural failure. Phase transformations occur in some materials, creating structures at various length scales which evolve with stress.

The challenge, both for mathematics and for physical modelling, is to comprehend relationships between models at different length scales. This has led already to a well-developed theory of 'homogenization' when the scales are widely separated, and has both exploited and stimulated advances in the calculus of variations. When the scales are separated but still comparable, there is a need for a micromechanical rationale for including scale effects in macroscopic models. The phenomena may be unstable, at least at the microscopic level, and, even if stable, may admit multiple equilibria. Study of the kinetics of the processes is a key requirement, making demands both for modelling and for the analysis of partial differential equations. In particular, the (possibly hierarchical) development of large-scale patterns is an open problem.

The four-month programme focused on microstructure, its formation and evolution, and the influence of microstructure on macroscopic properties in the context of phase transformations, damage development and fracture. It brought together specialists in these subjects from diverse disciplines including mathematics, materials science, engineering and physics. It thus provided a forum for the exchange of ideas (both between subjects and disciplines), and facilitated the identification of common issues and exploitation to mutual advantage of the advances in the different areas.

There were three periods of organised activities: an EC summer school and concentration, and two workshops. These consisted of detailed talks (of one hour or longer) with plenty of time for discussion. These periods attracted increased levels of visitors. Smaller groups were in residence during the rest of the programme engaged in detailed collaboration, seminars, tutorials and exchange of ideas.

Finella Kember

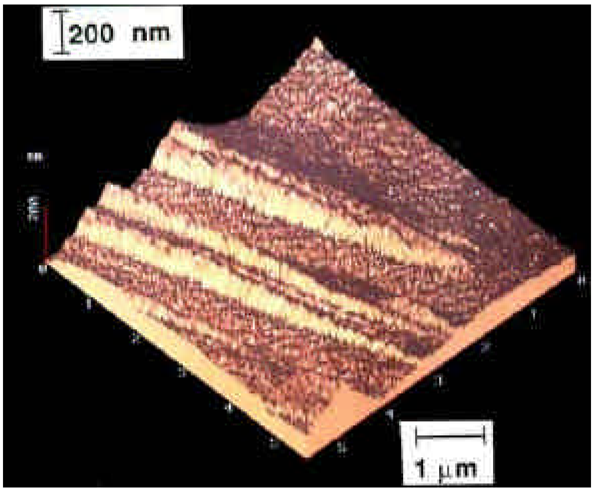
An informal seminar given by G Francfort

Workshops

EC Summer School and Concentration on *Mathematical Developments in Modelling Microstructure and Phase Transformations in Solids*

The programme started with a two-week EC summer school. Recently there have been some exciting advances in developing a mathematical framework for characterising microstructure, and for understanding the link between microstructure and macroscopic properties. These developments were introduced in four expository series of lectures during the first week. The four series dealt with

(a)



(b)

(a) A new steel was designed for use as rail track (b) through the study of the bainitic microstructure; Cambridge University Phase Transformations Group. Courtesy: Dr HKDH Bhadesia.

- Homogenization theory which describes the effective properties when there is a wide separation of scales.
- The hierarchy of atomistic models of solids starting from *ab initio* quantum mechanical methods based on density functional theory all the way to empirical potentials.
- The multitude of defects or failure mechanisms in solids including dislocations, damage and fracture and the array of models that describe them.
- The mathematical methods for describing and analysing microstructures and understanding how they arise in phase transforming solids.

The second week of the EC workshop was devoted to detailed lectures on current research activities. There is a mature, though incomplete, understanding of equilibrium microstructures in phase-transforming solids based on models with non-convex energy and results in the calculus of variations. Some talks reported interesting applications of this theoretical framework, while others discussed open questions surrounding the notion of 'quasiconvexity'. Much less is understood about the evolution of the microstructure, and this was addressed at some length in various lectures. A few talks reflected the recent interest in thin films: they dealt with the morphology and defect structure of epitaxial films of electronic materials, and the microstructure of films of active materials with an eye towards microactuators. Methods for understanding the effective behaviour of nonlinear heterogeneous solids, and for obtaining a combined atomic-continuum description were also discussed. Finally, two talks described how many of these ideas have led to the development of real materials.

The final two days of the EC workshop were held jointly with the annual meeting of the European (TMR) network on 'Phase transitions in crystalline solids'. The network annual meeting continued for an extra day and a half which was open to EC summer school participants, and during which over 25 scientists gave short talks describing their current research.

The EC summer school was followed by a concentration week during which many experts were in residence in the Newton Institute. There were a few lectures held in an informal style, but many more smaller discussions on emerging ideas. A topic which received much attention during this week was the extension of the methods of statistical mechanics to studying the highly correlated microstructure of phase transforming solids.

The introductory lectures at the EC summer school were given by: G Allaire, M Finnis, E Van der Giessen and S Müller. Other speakers were RD James, M Ortiz, D Kinderlehrer, R Abeyaratne, V Nesi, MB Luskin, L-Q Chen, TA Abinandanan, HKDH Bhadeshia, P Ponte Casteñeda, M Berveiller, PH Leo, B Spencer, K Bhattacharya, EKH Salje, G Friesecke, AP Sutton, M Pitteri, D Schryvers, JM Ball, A DeSimone and B Kirchheim.

Speakers during the concentration: LB Freund, M Rao, S Sengupta, OB Naimark, VP Smyshlyaev, GW Milton and AP Sutton.

Findlay Kember

RD James, Rothschild Professor, in discussion with J Willis.

Workshop on *Defect Mechanics and Non-Locality*

This workshop explored the world of defects at several different scales. Special attention has been paid to homogenization methods which not only allow for passing from one scale to another, but also could make allowance at any given scale for detail at an adjacent scale.

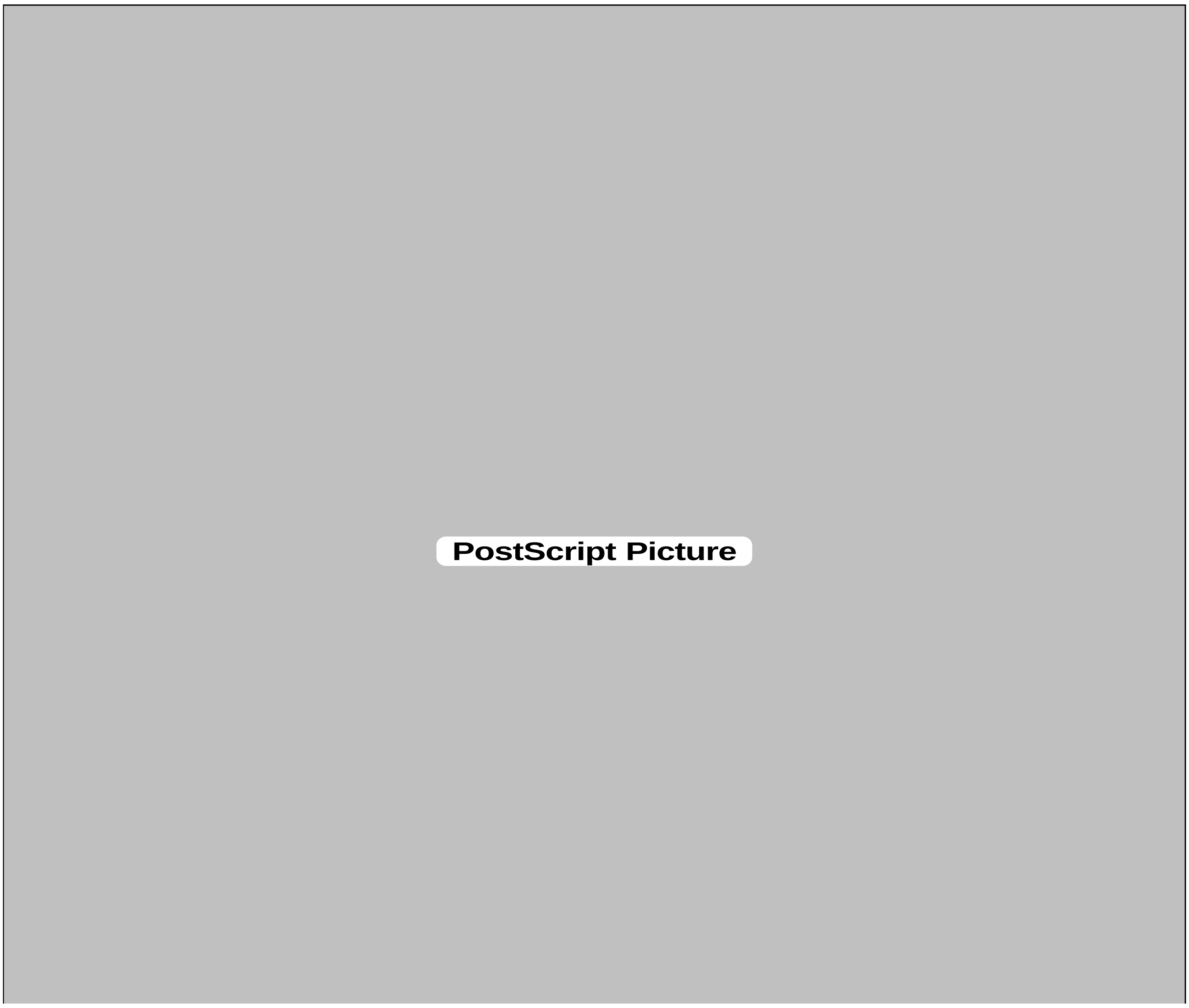
There are experimental observations which, to be understood, require such interactions between scales, often in the form of characteristic lengths. Formation of patterns of a given size in evolving microstructures, grain size effects in polycrystalline materials, influence of the underlying lattice to understand the Nabarro-Peierls stress, and size effects observed in the nonlinear range, provide a few examples of such situations. Another complementary motivation for coupling scales comes from the numerics. It is indeed a common observation that in the framework of classical models, strain or damage can localise in bands of arbitrarily vanishing width which makes the computations extremely sensitive to mesh size and orientation. All these observations have motivated the introduction of "non-local'' terms, primarily to explain patterns or to stabilise the numerics, though with an underlying assumption that such terms really do represent the physics. It is believed that these strain-gradients account for the fact that there is no strict separation of length scales between the micro- and macro-structure. However attempts to derive rational theories from microscopic considerations by expanding and truncating fields at the desired order almost inevitably give rise to ill-posed problems.

Experimental motivations, numerical approaches, and theoretical foundations of a rationale for these higher-order effects were presented during the workshop. Some of the results were obtained just before or even during the workshop, showing the timeliness of the topic and the extent of work to be done, for which the workshop has provided directions. One example of the many discussions which arose is the stabilising or destabilising effect of higher-order terms which is still a hotly debated question.

The speakers were: E Aifantis, Y Bréchet, JL Chaboche, SJ Chapman, N Fleck, GA Francfort, M Frémond, Y Huang, RD James, R. Luciano, RS MacKay, AB Movchan, O Naimark, A Needleman, GP Parry, P Ponte Castañeda, V Smyshlaev, C Stolz, N Triantafyllidis, JR Willis and V Zhikov.

Workshop on *Models of Fracture*

This workshop concentrated on problems of fracture which are at present being approached by specialists other than engineers, although the problems are of concern to the latter group as well. The community of physicists interested in a range of nonlinear phenomena (from experimental and theoretical viewpoints) was represented, and questions relating to the stability of propagating cracks were extensively discussed, mainly between this group and the engineers and applied mathematicians. Perhaps the most novel and



Prediction of a discrete dislocation analysis for the dislocation distribution and the opening stress in the immediate neighbourhood of a crack tip (from HHM Cleveringa, E Van der Giessen and A Needleman).

exciting development is the recognition of 'crack front waves' and the role that these play in phenomena such as the development of crack front disorder. Direct computational modelling is now possible, employing finite elements linked across each element boundary by cohesive forces. Such modelling yields very specific results of great complexity, and efforts are being made by certain pure mathematicians to model systems of this type employing the tools of homogenization. Allowance for the development of microcracks throughout the medium requires the use of functions of bounded variation and introduces a significant new component in the asymptotic analysis. Some promising results have been obtained for a one-dimensional idealisation, for which it has been shown that the "Cantor'' contribution to the variation is not required, thus restricting consideration to the space SBV. Current research is directed towards two and three dimensions. New ideas are needed, and access to guidance from the engineering community about what assumptions may be realistic is one of the by-products of the workshop.

The speakers were: KB Broberg, LB Freund, JR Rice, F Lund, Y Bréchet, M Adda-Bedia, H Nakanishi, J-B Leblond, G Dal Maso, G A Francfort, M Falk, J Sivaloganathan, A Braides, G Buttazzo, E Sharon, E Ching, V Kovtunenko, L Truskinovsky, K Ranjith, L I Slepyan, WJ Drugan, AB Movchan, R Craster, JR Willis and V Shenoy.

Outcome and Achievements

The immediate outcome of the programme is that participants have made new contacts, across disciplinary divides that previously had not been bridged so explicitly and deliberately: interactions which are likely to last into the future have been established between mathematicians with a rigorous approach to the homogenization of partial differential equations with rapidly varying coefficients, engineers concerned with modelling the development of damage during service of materials, and physicists who approach the modelling of materials from an atomistic or quantum theoretic standpoint. Seeds have been sown and the full extent of the benefit will be apparent in a year or two, when contacts established during the programme have had the opportunity to mature into productive collaborations. It is not possible to delineate the full range of possibilities here. However, certain activities were already in evidence during the programme; a selection is described below.

Phase transformations. One of the major problems concerning phase transformations is the development of a realistic model of the kinetics of transformation. Almost universally, across disciplines, descriptions based on first-order kinetics are adopted. Even within this framework, there is at present no credible model which would describe, for example, the development of the type of complex microstructure displayed by shape-memory alloys, and even the propagation of a single interface has only been modelled phenomenologically. RD James has proposed a more microscopic description in terms of a gradient flow in a 'wiggly energy' field. This has the potential, for example, to explain hysteresis. The analysis was performed in a one-dimensional realisation only. Now, Smyshlyaev and James together are trying to develop two- and three-dimensional theory, combining Smyshlyaev's expertise in high-frequency asymptotics acquired through the study of diffraction problems, with James' appreciation of the physics of the process, and the expertise of both in homogenization.

Another interaction in the area of phase transformations is between Bhattacharya and Castañeda. The former has already found estimates of the range of stress-free strain of polycrystalline shape-memory materials, while the latter has developed what is probably the most useful and accurate method for homogenization of a nonlinear composite material. The project is to develop and apply this methodology to obtain further insight into the stress-free strain régime.

Thin films. Thin films of ferroelectric material have potential use as actuators, and also as memory devices. One study, motivated by the latter application, is under way between Bhattacharya, Shenoy (INI participants) and Scott (Cambridge, Earth Sciences). The thin film is sandwiched between metal plates. If the film is too thin, boundary layers usually ignored in modelling the ferroelectric switching occupy a large fraction of the volume of the ferroelectric and compromise the performance. The project is to model the boundary layers so that the limiting thickness can be quantified. Another project, between Bhattacharya, Francfort and Fonseca, is concerned with modelling the decohesion of a thin layer deposited on a substrate, so that it is under stress either through pseudomorphic alignment or through thermal mismatch. If the layer is too thick, decohesion is favoured energetically. Variational methods are being developed, to provide a rigorous bound for critical thickness.

Non-locality. There is at present great interest in non-local models, which may describe the deformation of a material with microstructure in the case that the scale of the macroscopic variation is greater than that of the microstructure, but not so much greater that the homogenization limit applies. There are many phenomenological models of unproven validity, a few models based on analysis of micromechanisms, and virtually no rigorous theory. One promising interaction is between Frémond and Zhikov, which will address the modelling of media with extensive ties which link distant particles. Their approach will make use of

Zhikov's recent development of homogenization in terms of measures. Analysis at the less rigorous level associated with mechanics is being pursued by Drugan, Luciano and Willis in the context of randomly inhomogeneous media (there is more precise related work by Smyshlyaev and Cherednichenko in the simpler context of media with periodic microstructure). This research has led to explicit non-local constitutive relations which cannot be quantified exactly but whose operators can be bounded in Fourier transform space. Gradient approximations follow from small wavenumber expansions. There is at present a real difficulty in determining the range of validity of these gradient approximations. There is also a difficulty in determining in advance which gradient formulation (if any) will lead to a well-posed problem. Such questions, together with related concerns for identifying boundary conditions that are physically correct as opposed to mathematically convenient, have major implications for the modelling of the development of damage (growth of voids or microcracks during service). There is already quite a major industry of including non-locality into finite element codes, with parameters chosen so as to smooth out localization of deformation. The research in progress will assist in identifying when such smoothing is a physically realistic phenomenon and when it has simply the status of a mathematical device.

Fracture. The programme stimulated interactions between mathematicians, engineers and physicists interested in fracture problems. One such interaction involves Movchan, Rice, Sharon and Willis. Rice observed by computation a new type of wave, a "crack front wave", that propagates without attenuation or dispersion along the front of a propagating crack. Its existence has been confirmed analytically from a solution of Movchan and Willis, and experimentally by Sharon. Two aspects are under further development: Movchan and Willis are investigating the influence of viscoelastic dissipation (already seem in experiments of Sharon on PMMA), and Rice and Willis are investigating whether crack front waves can exist under conditions of shear loading, as occurs during an earthquake.

Ergodic Theory, Geometric Rigidity and Number Theory

5 January to 7 July 2000

Report from the Organisers: A Katok (Penn State), G Margulis (Yale), M Pollicott (Manchester)

Scientific background and objectives

The central scientific theme of this programme was the recent development of applications of ergodic theory to other areas of mathematics, in particular, the connections with geometry, group actions and rigidity, and number theory. The potential of ergodic theory as a tool in number theory was emphasized by Furstenberg's proof of Szemerdi's theorem on arithmetic progressions.

Ergodic theory is an area of mathematics with all of its roots and development contained within the 20th century. Strands of the modern theory can be traced back to the work of Poincaré, but the subject began to take a more recognizable form through the seminal work of von Neumann, Birkhoff and Kolmogorov. The impetus to these developments was the important concept of 'ergodicity' in dynamical systems - by which the temporal evolution of the system, though averaging over typical orbits (almost every orbit in the measure theoretic sense), corresponds to spatial averages over the system. An important concept in physical systems, it also set the foundation for applications to other branches of mathematics, most notably geometry and number theory.

Foremost amongst the recent contributions of ergodic theory to number theory is the solution of the Oppenheim Conjecture, a problem on quadratic forms which had been open since 1929. This conjecture was solved by Margulis, and a particular special case is the following:

The Oppenheim Conjecture. *Consider an indefinite ternary quadratic form, for example, the quadratic form*

$$Q(x,y,z) = ax^2 + by^2 - cz^2$$

in three variables, with $a,b,c > 0$ *for the purposes of illustration. The original conjecture of Oppenheim was that the values* $Q(l,m,n)$ *can be made arbitrarily close to* 0 *by taking choices of non-zero triples of integers* $(l,m,n) \in \mathbb{Z}^3 - (0,0,0)$*. This problem was extensively studied by Davenport, using purely number theoretic methods. The final solution of the conjecture was achieved by Margulis by using a reformulation of the problem into the ergodic theory of homogeneous flows on lattices.*

Of equal importance is the role of ergodic theory in geometry and the rigidity of actions. In recent years there have been diverse results, including rigidity results for higher rank abelian groups, and results on the classification of geodesic flows on manifolds of non-positive curvature. This is a quickly evolving area of research. In more recent years, the richness of the applications to geometry have become more apparent. This is illustrated by the famous Mostow rigidity theorem, by which the geometry of certain manifolds is completely determined by their topology (i.e. two manifolds with the same fundamental groups are isometric).

Rigidity of Anosov actions. *In the context of actions, there is a very well developed programme of Katok, Spatzier, and others, to show local* C^∞ *rigidity of algebraic Anosov actions of* $\mathbb{Z}^k$ *and* $\mathbb{R}^k$ *on compact manifolds as well as orbit foliations of such actions. More precisely, two actions of a group G agree up to an automorphism if the second action can be obtained from the first one by composition with an automorphism of the underlying group. Call a* C^∞*-action of a Lie group G locally* C^∞*-rigid if any perturbation of the action which is* C^1*-close on a compact generating set is* C^∞*-conjugate up to an automorphism. Katok and Spatzier proved* C^∞*-local rigidity of most known irreducible Anosov actions of* $\mathbb{Z}^k$ *and* $\mathbb{R}^k$ *(as well as the orbit foliations).*

A related theme is that of paucity of invariant measures.

The Furstenberg Conjecture. *Consider the two transformations on the unit circle* $S, T : K \to K$

defined by $Sz = z^2$ and $Tz = z^3$. The only ergodic invariant probability measures which are simultaneously preserved by both S and T are either Haar measure, or measures supported on finitely many points.

Whereas this famous conjecture remains open, it is known (by work of Rudolph) that the conclusion is true if we restrict to measures of positive entropy. This result was generalized to $\mathbb{Z}^n$-actions by Anosov toral automorphisms, and other more general settings, by Katok and Spatzier.

A well-known cross-discipline application lies in the connection with Quantum Chaos. In particular, a quantitative version of the Oppenheim Conjecture gives a proof of the Berry Conjecture on the eigenvalues of the Laplacian on flat tori.

The programme explored these and other emerging applications of ergodic theory. It brought together both national and international experts in ergodic theory and related disciplines, as well as other members from the wider UK and international mathematical communities. In particular, a major aim of the programme was to bring together people with different interests and backgrounds, and to promote the use of ergodic theory techniques.

Organisation

The overall planning for the programme was undertaken by all three of the organisers. The day-to-day organization of the programme was undertaken by M Pollicott from January to early April, and again from late May to the end. During this absence, his duties were undertaken by G Margulis.

There was also very able assistance from A Eskin and M Burger for specific workshops within the programme.

In addition to the workshops and meetings within the programme, there was a regular research seminar on Tuesday afternoon, and a more informal seminar on Thursday morning. There were also other seminars scheduled as required by the participants or the organisers.

In May there was a Spitalfields Day, with talks by A Eskin, A Katok and G Margulis.

During the month of June there was a one-day meeting in *Dynamical Systems* (sponsored by the LMS and organised by Sharp and Walkden, from Manchester), with talks by R Itturiaga (INI and Heriot-Watt), M Urbanski (North Texas) and T Ward (UEA).

In June the frequency of talks increased and the last week of the month was designated a Special Emphasis Week, prior to the final Euro-workshop in July. During this special week there were 3 talks each day by participants, focusing particularly on results obtained by longer term visitors.

A Katok and H Furstenberg also spoke in the Institute's Monday Seminar series, addressing a wider audience.

Findlay Kember

A Nevo (Princeton) speaking with Rothschild Professor H Furstenberg (Jerusalem).

Participation

The programme hosted in excess of 50 long term visitors, at various times, and 75 short term visitors. In addition, there was very strong participation in the workshops and other activities during the programme. The three organisers each spent a substantial period of the programme in residence. In addition, H Furstenberg was a Rothschild Professor and made an invaluable contribution to the programme during his month in residence.

As the participant list shows, the majority of experts in the field visited the Institute during this programme, and a large number of leading experts in related fields attended. There was a strong presence from Europe and North America, as well as a substantial presence from the former Soviet Union helped by the generous support from the Leverhulme Trust.

Many of the meetings and individual talks attracted mathematicians from both Cambridge and other British universities. The Junior Membership scheme, support from the NSF and support from the EU for two of the workshops was particularly useful in encouraging participation from PhD students and younger mathematicians. A number of participants went to give talks in other UK departments (e.g. Manchester, Surrey, QMW and Warwick).

Meetings and workshops

Lectures on Ergodic Theory, Geometry and Lie Groups (10-14 January 2000): A Katok (Penn State) and M Pollicott (Manchester)

The first meeting was designed to provide a firm foundation for the programme, and to help set the agenda for subsequent activities. It was also intended to provide an introduction to the subject for a broad audience, particularly younger mathematicians and non-experts from related areas.

The meeting consisted of five short lecture courses by acknowledged experts in the area. These were: M Burger (ETH Zurich), *Cohomological aspects of lattices and applications to products of trees*; R Feres (St. Louis), *Topological superrigidity and differential geometry*; H Furstenberg (Jerusalem), *Ergodic theory and the geometry of fractals*; A Katok (Penn State), *Dynamics and ergodic theory of smooth actions of higher tank abelian groups and lattices in semi-simple Lie groups*; and D Kleinbock (Rutgers), *Interactions between homogeneous dynamics and number theory*.

The meeting successfully achieved all of its aims. Participants were mathematicians, predominantly graduate students, postdoctoral fellows and younger researchers.

Euroworkshop: Rigidity in Dynamics and Geometry (27-31 March 2000): G Margulis (Yale), assisted by A Eskin (Chicago) and M Burger (ETH Zurich)

The second workshop was devoted to applications of ergodic theory to locally symmetric spaces, geometric rigidity, and number theory. This represented one of the most intense periods of activity during the programme.

The subjects covered in the meeting included such important recent developments as the classification of the actions of higher rank groups, unipotent flows on homogeneous spaces and the Oppenheim conjecture.

The meeting consisted of 33 lectures from an international audience. Those participating were a broad mix of senior mathematicians and younger researchers and students.The main speakers included: D Gaboriau (ENS, Lyon); B Kleiner (Michigan); F Labourie (Orsay); A Zorich (Rennes I); L Mosher (Rutgers), B Weiss (SUNY, StoneyBrook); H Pajot (Cergy-Pointoise); N Monod (ETH Zurich); A Iozzi (Maryland); B Leeb (Tubingen);A Karlsson (Yale); G Knieper (Bochum); A Furman (Illinois); B Goldman (Maryland); A Adams (Minnesota); H Abels (Bielefeld); M Skriganov (Steklov); A Stepin (Moscow State); M Dodson (York); V Kaimanovich (Rennes/Manchester); Y Guivarc'h (Rennes); G Tomanov (Claude Bernard, Lyon), Benoist (ENS, Paris), D Witte (Oklahoma State), A Parreau (Orsay); B Remy (Henri Poincaré, Nancy); F. Paulin (Orsay); Y Shalom (Yale); A Török (Houston); A Nevo (Technion); D Fisher (Yale); H Oh (Princeton); A Zuk (ENS, Lyon).

Ergodic Theory of $\mathbb{Z}^d$-actions (3-7 April 2000) : M Pollicott (Manchester), K Schmidt (Vienna) and P Walters (Warwick)

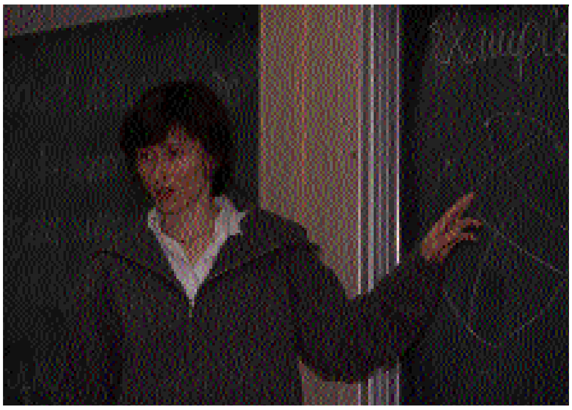
The third meeting in the programme was held in collaboration with the Mathematics Institute at the University of Warwick, and the venue was Warwick University. This meeting specialized more in the specific topic of $\mathbb{Z}^d$-actions, an area in which there was a very successful symposium at Warwick in 1993-94. This meeting focused on developments over the intervening six years, and showed that the area was still very active. There were 27 talks, all of 45 minutes. The total number of participants exceeded 75.

Speakers included: Aaronson (Tel Aviv), Auslander (Maryland), Bergeleson (Ohio), Bufetov (Moscow), Burton (Ohio), Dani (Tata), Einsiedler (UEA), Feres (Washington), Friedland (Chicago), Hurder (Illinois), Johnson (UNC), A Katok (Penn State), S Katok (Penn State), Kaminski (Krakow), Kitchens (IBM), Lind (Seattle), Margulis (Yale), Mozes (Jerusalem), Petersen (North Carolina), Putnam (Victoria), Schmidt (Vienna), Shah (Tata), Thouvenot (Paris), Tuncel (Seattle), Vershik (St. Petersburg), Ward (UEA).

Euro-conference on Ergodic Theory, Riemannian Geometry and Number Theory (3-7 July 2000) : A Katok (Penn State), G Margulis (Yale) and M Pollicott (Manchester)

This was the final meeting of the programme and served, in part, to review the achievements made during the previous six months. A number of speakers took the opportunity to present work which they had carried out while in residence at the Institute. The meeting encompassed much research activity, and marked the culmination of the programme.

There were a total of 29 talks, all of 45 minutes duration. The meeting attracted more than 100 participants. The speakers included: H Furstenberg (Jerusalem); G Margulis (Yale); H Masur (UIC), A Windsor (Penn State); A Gamburd (Dartmouth); D Burago (Penn State); E Ghys (ENS Lyon); B Klingler (INI); M Burger(ETH Zurich); C Drutu (Lille, MPI Bonn); G Soifer (Bar-Ilan); A Lubotzky (Jerusalem); L Clozel (Paris Sud); D Kleinbock (Rutgers); J Marklof (Bristol); M Kanai (Nagoya); U Hamenstadt (Bonn); M Babillot (Orleans); P Pansu (Paris Sud); B Kalinin (Penn State); C Walkden (Manchester); F Dal'bo (Rennes); Y Cheung (UIC CMI); J Schmeling (Berlin); C Connell (UIC); R Sharp (Manchester); S Katok (Penn State); U Bader (Technion); R Zimmer (Chicago).



U Hamenstadt (Bonn), presenting a seminar on the ergodic theory of geodesic flows during the final conference.

The organisers: G Margulis (Yale), M Pollicott (Manchester) and A Katok (Penn State)

Outcome and achievements

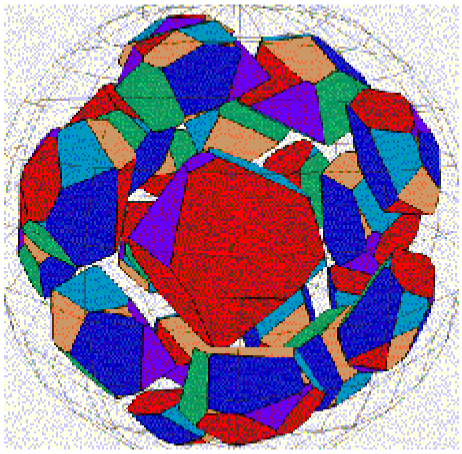
The main achievement of this programme was that it brought together both established experts in the field, as well as younger researchers, from home and abroad, in an effort to promote scientific research and training of the highest quality. To this end it was remarkably successful, with progress being made on a large number of problems, in a diverse number of different directions.

One of the topics where there was most progress was in the area of intersection between Lie groups and Ergodic theory. Abels, Margulis and Soifer worked intensively on the classical Auslander and Milnor conjectures and considerable progress was made on these problems. In addition, Abels also made progress, with Margulis, on another classical problem (due to Siegel) regarding metrics on reductive groups.

At a more algebraic level, Shalom made substantial progress in understanding to what extent lattices in higher rank Lie groups differ from lattices in the rank one Lie groups $Sp(n,1)$, in terms of their representation theory. On the face of it all of these lattices share Kazhdan's property (T), which dominates their behaviour. However, looking at uniformly bounded, rather than unitary, representations on Hilbert spaces reveals some fundamental differences.

Adams and Witte studied the classification of the homogeneous spaces of $SO(2,n)$ and $SO(1,n)$ that have Lorentz forms. This is an important ingredient in showing that $SL(2, \mathbb{R})$ is the only simple Lie group that can act non-tamely on a Lorentz manifold. Witte also collaborated with Lifschitz, one of the younger participants, on establishing rigidity of lattices in nilpotent algebraic actions over local fields of positive characteristic. He also completed work with Iozzi describing the Cartan-decomposition subgroups of $SU(2,n)$. They also completed a related project on tesselations of homogeneous spaces of $SU(2,n)$.

Another area which was emphasized during the



A Dirichlet fundamental domain of a lattice in three-dimensional hyperbolic space.

programme was the action by groups on manifolds. Dani worked on ergodic $\mathbb{Z}^d$-actions on Lie groups by their automorphisms. Skriganov worked with Margulis on proving ergodic theorems for submanifolds generated by nilpotent subgroups in $SL(n)$. These are results which arise naturally in applications of ergodic theory to lattice point problems.

The period spent by Witte and Zimmer at the INI allowed them to complete a long-term project describing actions of semi-simple Lie groups on compact principal bundles. Zimmer also took the opportunity to develop work with Nevo on properties of smooth projective factors for actions with stationary measure, with Fisher on actions on compact principal bundles, and with Labourie and Mozes on the study of locally homogeneous spaces with symmetry.

An important recurrent theme in the meeting was the application of ideas from the ergodic theory of homogeneous flows to number theory. Of particular timeliness was the refinement of the ergodic theoretic proof of the following famous conjecture of Baker and Sprindzhuk.

The Baker-Sprindzhuk Conjecture. *Given* $\mathbf{x} \in \mathbb{R}^n$, *let us write* $\Pi(\mathbf{x}) = \prod_{i=1}^{n} |x_i|$. *We say that a vector* $\mathbf{x} \in \mathbb{R}^n$ *is very well multiplicatively approximated if for some* $\varepsilon > 0$ *there are infinitely many* $q \in \mathbb{Z}$ *and* $\mathbf{p} \in \mathbb{Z}^n$ *such that* $\Pi(q\mathbf{x} + \mathbf{p})|q| \leq |q|^{-\varepsilon}$. *For the curve* $M_0 = \{(t, t^2, \ldots, t^n),\ t \in \mathbb{R}\} \subset \mathbb{R}^n$, *Baker* *conjectured that almost all points on M_0 are not very well approximated. For n = 2 this was proved by Schmidt in 1964, and for n = 3 by Beresnevich-Bernik in 1996. More generally, let $f_1,\dots,f_n$ be real-analytic functions on a domain $U \subset \mathbb{R}^d$ which, together with 1, are linearly independent over $\mathbb{R}$. A stronger conjecture (formulated by Sprindzhuk) states that almost all points of M_0 are not very well multiplicatively approximated. This was proved recently by Kleinbock-Margulis.*

The proof of the full conjecture by Kleinbock and Margulis uses the ergodic theory of homogeneous actions on the space $SL(n+1, \mathbb{R})/SL(n+1, \mathbb{Z})$. More precisely, given $\mathbf{y} \in \mathbb{R}^n$ we can define a lattice $\Lambda_{\mathbf{y}} = \begin{pmatrix} I_n & \mathbf{y} \\ 0 & 1 \end{pmatrix} \mathbb{Z}^{n+1}$. Then, for any vector $\mathbf{t}=(t_1, \dots, t_n)$, $t_i \geq 0$, we can denote $t = \sum_{i=1}^{n} t_i$ and $g_{\mathbf{t}} = \mathrm{diag}(e^{t}, e^{-t_1}, \dots, e^{-t_n}) \in SL(n+1, \mathbb{R})$ and can consider the family of lattices $g_{\mathbf{t}}\Lambda_{\mathbf{y}}$.

The dynamical characterization used by Kleinbock and Margulis for $\mathbf{y} \in \mathbb{R}^n$ to be very well multiplicatively approximated is that there exist $\gamma > 0$ and infinitely many $\mathbf{t} \in \mathbb{Z}^n_+$ such that $\delta(g_{\mathbf{t}}\Lambda_{\mathbf{y}}) \leq e^{-\gamma t}$.

This approach has been used to give ergodic reformulations (and potentially accessible versions) of many important conjectures in number theory (e.g. the Littlewood Conjecture in simultaneous Diophantine approximation). In particular, Kleinbock, Bernik and Margulis developed further these techniques and were able to establish Khintchine-type theorems on manifolds (more precisely the convergence cases for the standard and multiplicative versions). Using more classical techniques, Velani and Pollington obtained estimates on the size of the badly simultaneously approximable set.

During his extended stay at the Institute, Dani also worked on applications of flows on homogeneous spaces to Diophantine approximation, related to earlier work of Margulis on the Oppenheim conjecture. In particular, results were obtained concerning diophantine solutions of quadratic inequalities, again related to the Oppenheim Conjecture, in narrow strips in the associated quadratic space.

There was also good progress in applications to number theory over different fields. Kleinbock and Tomanov successfully worked on proving the natural *p*-adic and *S*-arithmetic versions of problems in Diophantine approximation.

A surprising application of these number theoretical results is to spectral theory on certain special manifolds. Eskin, Margulis and Mozes made substantial progress in studying quadratic forms of signature (2,2) and the difficult eigenvalue spacing on flat 2 tori. This is closely related to the well known Berry Conjecture in quantum chaos.

In another direction, there was also substantial progress on geometric problems using ergodic theoretic approaches. For example, Klingler worked on finding a criterion for arithmeticity of complex hyperbolic lattices, which is still in progress. He also collaborated with Maubon to work on the well-known Katok conjecture that the topological entropy of a compact manifold is always strictly larger than the Liouville entropy, unless the metric is locally symmetric.

During his visit, Furstenberg worked on the use of ergodic theory in the geometry of fractals and geometric Ramsey theory. This constitutes the most substantial progress on these problems since

F Labourie (Orsay) discussing with F Ledrappier (Ecole Polytechnique, Paris).

his own seminal paper in 1969.

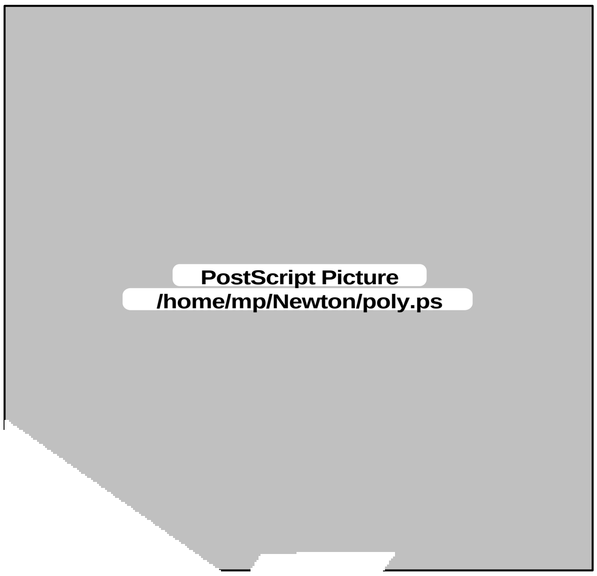
One of the geometers who participated in the programme, Burago, made progress with two problems: establishing 'kick-stability' (in the sense of Polterovich) of a parabolic subgroup of $SL(2,\mathbb{R})$, and examples of metrics on non-negatively curved manifolds with positive metric entropy. This involves products of hyperbolic matrices whose stable-unstable decompositions are not coherent. This is a delicate problem, the main difficulty being to destroy the degenerate situation by certain perturbations.

Sharp, one of the long-term British participants, worked on applications of ergodic theory to geometry. This included the study of minimizing measures for geodesic flows on negatively curved manifolds and new results on sector estimates for orbit counting for Kleinian groups. Three of the other younger participants, Feres, Fischer and Benveniste worked on stratified rigid structures and, in particular, on the construction of examples of rigid geometric structures in the sense of Gromov, with specified types of degeneracies.

There was also considerable progress in terms of understanding which groups act on a circle. R Feres and D Witte extended recent work of Ghys on groups actions on the circle, to actions by automorphisms of foliations of co-dimension one. Shalom and Witte initiated work on the problem of showing that Kazhdan groups cannot act smoothly on the circle. This work is a long term project, but which is already beginning to bear fruit.

Another topic which attracted considerable interest during this meeting was that of polygonal billiards. A difficult problem is that of counting asymptotically the number of periodic trajectories of a billiard on a polygonal rectangular table, where the polygon is rational (i.e. all angles are rational multiples of π). The trajectories correspond to the motion of a particle inside the polygon with elastic collision at the boundaries.

Eskin and Mazur obtained results using a geometric approach. Gluing together several



An orbit for a polygonal billiard.

copies one can associate a surface S with a flat structure, and counting families of periodic trajectories of the billiard is equivalent to counting cylinders of closed geodesics on the associated surface. For large T, the number of cylinders of closed geodesics of length at most T is shown to be asymptotic to $\pi C T^2$, for some $C > 0$. The value of C can be computed for some specific surfaces.

The programme also brought together a number of leading experts in abstract ergodic theory, and it was natural that progress was also made on important problems in this underlying subject. For example, Thouvenot worked on the structure of measure-preserving transformations in the positive entropy case, which is related to splitting measure-preserving transformation. This may even prove relevant to the other themes of the programme, since 'rigidity' in abstract ergodic theory can be thought of as having a 'trivial centralizer' (whereas an irrational rotation can induce a rigid transformation, this is impossible starting from a Bernoulli shift). He made progress in understanding the spectral theory of $\mathbb{R}$ actions of the rationals, in particular, the connection with the property of Lebesgue spectrum.

Two other long term visitors, Goldsheid and

Guivarc'h, collaborated to show estimates on the dimension of the Gaussian law with products of random matrices. This is closely linked to a classical problem studied by Furstenberg on such random products. In a somewhat different direction, Goldsheid also worked with Khoruzhenko on the distribution of eigenvalues in the Non-Hermitian Anderson model, an important classical model.

Another of the British long term visitors, Nair, showed that for various natural probability measures on the space of increasing sequences of integers almost all sequences are multiplicatively intersective. In connection with this problem, Nair and Weber (Strausbourg) are continuing to investigate the stability of multiple intersectivity under perturbation by integer-valued independent identically distributed random variables.

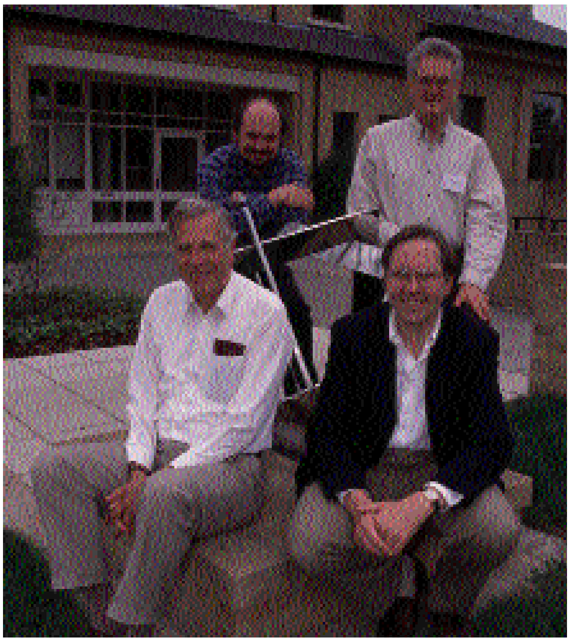
Finally, Kaimanovich and Schmidt took the opportunity to continue their work on the ergodicity of horocycle foliations of certain non-negatively curved manifolds, by extending the types of covers of compact manifolds for which they can show the horocycle action is ergodic. In particular, they have developed an approach which generalizes earlier results of Babillot-Ledrappier and Pollicott (all of whom participated in the programme). During the programme, Ledrappier and Pollicott extended some of these results to the case of stable manifold foliations of frame flows.

In summary, this programme made significant contributions to the study of Ergodic Theory and its applications to a range of important areas.

Strongly Correlated Electron Systems

5 January to 30 June 2000

Report from the Organisers: DM Edwards (Imperial), AC Hewson (Imperial), PB Littlewood (Cambridge), AM Tsvelik (Oxford)



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The organisers: David Edwards (l, seated), Alexei Tsvelik (l, standing), Alex Hewson (r, standing) and Peter Littlewood (r, seated).

Introduction

The aim of condensed matter theory is to understand the macroscopic behaviour of systems, in all their rich diversity, starting from a detailed description of the individual particles and the way they interact. This enterprise of relating the microscopic to the macroscopic has had some remarkable success stories, such as the understanding of phase transitions in the framework of the renormalization group, and the BCS theory of superconductivity. The general framework for understanding the behaviour of electrons in metals is that of Fermi-liquid theory (which has been rigorously established within a convergent perturbation theory) where the electrons are described in terms of renormalized quasi-particles, which are in one to one correspondence with the single particle excitations of the non-interacting systems, plus collective excitations which arise from the residual inter-particle interactions. However, in recent years an increasing number of metallic systems have been discovered where this framework does not provide a satisfactory description of their behaviour, and the quasi-particle picture appears to have broken down. This is the case in an important class of metallic systems, the high temperature (cuprate) semiconductors. Though they were discovered experimentally more than ten years ago, there is no generally accepted theory of their superconductivity. Such a theory is unlikely to emerge without a prior understanding of how and why Fermi-liquid theory breaks down in the normal state of these materials, and this has become an important theoretical question to solve.

Apart from the high temperature superconductors, Fermi-liquid theory is known to break down in other types of metallic systems. It breaks down in one-dimensional metals, and this is well understood. The Luttinger liquid framework replaces the Fermi-liquid one, and the low-lying excitations are purely collective ones, rather than quasi-electrons, which obey bose rather than fermi statistics. It has also been observed to break down in certain three-dimensional metallic compounds under applied pressure, and in some alloys at particular concentrations, at the point at which long range magnetic order can no longer be sustained. This is known as quantum critical behaviour and may be relevant for an understanding of the cuprate superconductors.

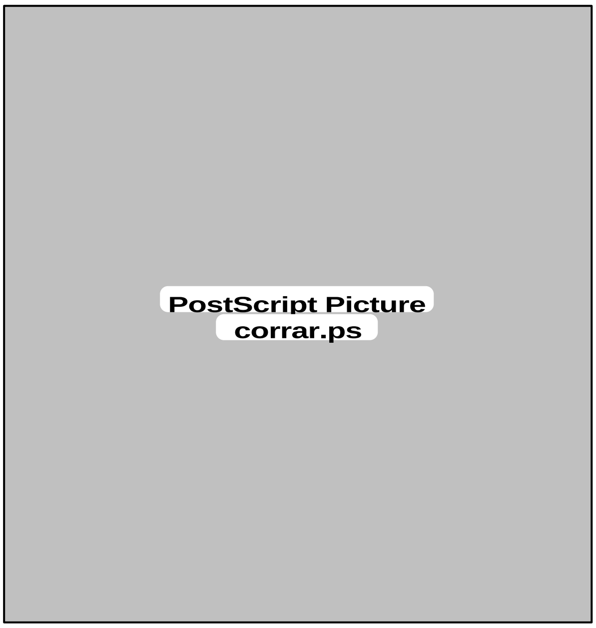
Unusual excitations have also been observed in quantum Hall systems, which are two-dimensional electron gases in which the motion of the electrons is further restrained by a strong magnetic field. They display anomalies in their conductivity, dependent on the number of electrons, which are known as the ‘integer’ and ‘fractional quantum Hall effect’. The common feature of these situations is the constrained motion of the electrons as a result of restricted dimensionality and/or the strong inter-electron interactions.

These constraints induce complicated correlations in the motion of the electrons, and hence the term which is used to describe them collectively: *Strongly Correlated Electron Systems*.

Explicit mathematical models have been put forward to explain these various manifestations of strongly correlated electron behaviour. The simplest models proposed for an understanding of the breakdown of Fermi-liquid theory in the cuprate superconductors are the Hubbard and t-J models, which describe the motion of the electrons in the CuO_2 planes of these materials and the strong inter-electron interactions they experience at the Cu sites. Standard many-body techniques for predicting the behaviour of these models, such as perturbation theory and mean field theory, are not applicable. Perturbation theory is inappropriate because the electron-electron interactions are very strong, and mean field theory takes no account of the subtle inter-electron correlations which are induced. Quantum Monte Carlo, and other numerical techniques, are restricted to small cluster sizes and of limited value as the results cannot be used for comparison with experiment.

A great variety of non-perturbative techniques have been developed for tackling quantum many-body problems. These range through quantum field theory approaches, Bethe ansatz, conformal field theory, renormalization group, variational, dynamic mean field theories, slave bosons, 1/*d* and 1/*N* expansions (*d*: dimensionality; *N*: number of degrees of freedom), plus a range of purely numerical techniques. All these techniques have their strengths and limitations, and considerable progress has been made on many fronts.

They have provided insights and quantitative descriptions of the behaviour of heavy fermions, metal-insulator transitions, transport in one-dimensional conductors, integer and fractional quantum Hall systems, and many aspects of the behaviour of high temperature superconductors. However, some of the most important questions, such as ‘why are the cuprates superconductors?’, remain to be answered. What is more, there is no consensus as to what are the physically relevant models to provide a basis to answer this question. Quite a number of plausible ‘scenarios’ have been proposed but a fully quantitative theory is lacking.



*The spin-spin correlation function S(**q**) and the density-density correlation function N(**q**) for the two-dimensional t-J model (J/t=0.4, doping=0.2) over the full square Brillouin zone, from the calculations of Bill Putikka (Ohio State), which were presented during his seminar.*

The aim of the workshop has been to bring into dialogue the wide range of expertise in quantum many-body and condensed matter physics to these problems. No single technique is likely to provide the answers to all the questions we have about these complex systems and their subtle and, at times, strange behaviour. However, we felt that by pooling our resources, we could clarify the issues, formulate the appropriate models, and develop the effective mathematical techniques for dealing with this class of many-body systems.

Experimental techniques for probing strongly correlated electron materials, such as photemission, are developing rapidly and are providing new and more accurate information on the behaviour of these systems. These results are putting theories to a more severe test, and revealing surprising new features, such as localization in the form of stripes in the cuprate superconductors. Many new materials are also being developed, in which the electrons are constrained in areas of mesoscopic dimensions, such as in quantum dots. These provide particularly good systems for testing theory against experiment as the parameters can be controlled and varied by applied voltages. Hence we felt it important to have some input from experimental work in the field at points within the programme.

Organisation

The programme was planned by the organisers, David Edwards, Alex Hewson, Peter Littlewood and Alexei Tsvelik, who then shared the responsibilities for different parts of the programme. Apart from the conferences and workshops, there was a regular programme of seminars, running at the rate of three per week, which was arranged so as to give each participant an opportunity to talk about their work early on during their stay. We also had a few open discussion periods which focused on specific controversial issues.

To convey the flavour of our programme to a wider mathematical audience AM Tsvelik and M Stone gave Institute Seminars on *Quantum Integrable Models* and *Path Integrals for Spin: From Molecular Cluster to Holomorphic Line Bundles*. To reach a much more general audience DM Edwards gave a well-attended lecture during National Science Week in which he explained why our work at the Newton Institute on strongly correlated electrons was interesting and important and how it fitted into the wider context of the history of the Universe.

Interaction was encouraged between participants and the members of research groups in Cambridge, both theoretical and experimental, involved in similar work. Many from these groups attended our seminars, and several of them gave talks within our seminar programme.

Participation

We had 71 long-stay participants, with an average stay of six weeks, and 109 short-stay participants, from a total of 23 countries during the course of the programme. We were very grateful to the administrative staff at the Institute for making this all possible. Arrangements were made not only for these visits, but also for all the extra conferences and workshops, with remarkable efficiency and good humour. It lightened our task as organisers, making it an enjoyable and rewarding experience.

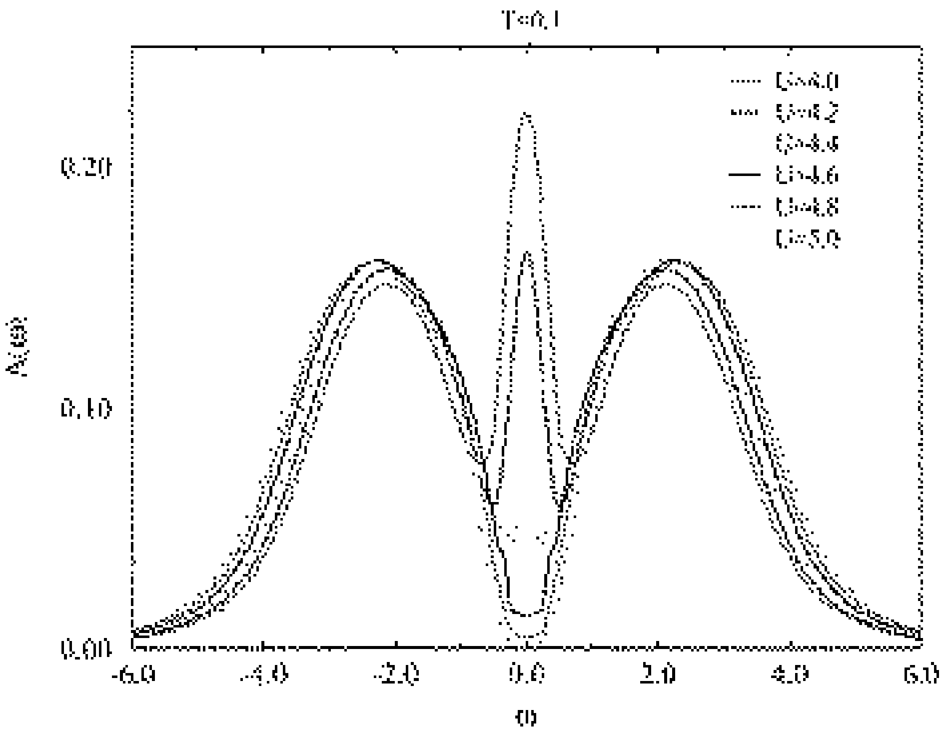
As well as a significant UK participation in the programme, 47 in total, there was an encouraging amount of interaction between visitors and groups within the UK, and 35 seminars were given by our participants at venues outside the Institute. Many collaborations were initiated or continued during the workshop, and there was much appreciation of the layout and organisation at the Institute in creating a stimulating atmosphere and promoting informal discussions.

Conferences and Workshops

Newton Institute-ESF Conference: *Non-Fermi Liquid Effects in Metallic Systems with Strong Electron Correlations*

Organisers: David Edwards, Peter Littlewood and Hilbert von Löhneysen

This conference, held during the period 5-8 January, was an excellent way to start the programme. It was organised in conjunction with the European Science Foundation Programme *Fermi-liquid instabilities in correlated metals* (FERLIN) which has a strong emphasis on experiment. The breakdown of Fermi liquid theory is perhaps the dominant central issue in strongly correlated electron systems and it was a good beginning to confront existing theory with new experimental data. The talks centred around four main topics: high temperature superconductors, heavy fermion materials near quantum critical points, fractional quantum Hall effect and low dimensional systems.



Finite temperature (T) results of the one electron spectral density for the infinite dimensional Hubbard model, taken from the numerical renormalization group calculations of Ralph Bulla (Augsburg) which he presented in his talk at the computational workshop. The development of a gap at the Fermi level $\omega = 0$ is seen clearly on increasing the interaction strength U. This results in metal to insulator transition at T = 0.

The speakers were: GG Lonzarich (Cambridge), F Steglich (Dresden), P Wölfle (Karlsruhe), O Stockert (Bristol), P Coleman (Rutgers), A Georges (ENS, Paris), M Girvin (Indiana), JH Smet (Stuttgart), N Read (Yale), M Rozenberg (Buenos Aires), AO Gogolin (IC, London), J Nicholls (Cambridge), PM Chaikin (Princeton), AJ Millis (Rutgers), A Freimuth (Köln), A Chubukov (Wisconsin), BD Rainford (Southampton), Q Si (Rice), M Long (Birmingham) and M Zhitomirsky (ETH, Zürich).

More than 80 people attended from 15 different countries and 18 of them contributed to a very successful poster session. This was preceded by an oral session in which each contributor had five minutes to advertise his poster.

Computational Many-body Physics

Organisers: Peter Littlewood and Philippe Monthoux

This Euro-workshop was held over the long weekend, 18-21 February. There were 21 invited speakers and the talks were spread out over four days. It was very much an international event with participants coming from Argentina, China, Japan and the United States, as well as many parts of Europe. Computational many-body methods constitute an important class of non-perturbational techniques for tackling problems involving strong correlation. The talks covered the main areas of activity in this field: renormalization group, quantum Monte Carlo, correlated wavefunctions, high temperature series, and Feynman diagram summation. The general consensus was that it was very beneficial to be able to get together to concentrate on specifically computational approaches, to discuss their

The current predicament as presented by Alexei Tsvelik in his introduction to the NATO ASI.

relative strengths and limitations, and to devise strategies for future work. Several of the participants were able to stay on for a longer period, so we were able to have more extended discussions, and additional seminars.

New Theoretical Approaches to Strongly Correlated Systems

Organiser: Alexei Tsvelik

This EC Summer School and NATO Advanced Study Institute was held during the period 10-20 April. Five broad areas of current activity in the theory of strongly correlated electron systems were covered in the ten days of the programme. These were: field theoretic methods, including conformal field theory, with particular application to one-dimensional systems; problems relating to disorder and localization; dynamic mean field theory; systems with quantum critical points; doped Mott insulators and, in particular, problems relating to the high temperature superconductors. Some of the highlights were the outstanding lectures by Affleck on quantum field theory approaches, Saleur on conformal field theory and integrable systems, Cardy on problems with disorder, Kotliar and Georges on dynamic mean field theory and its applications, Sachdev on quantum critical points, Giamarchi on low dimensional systems and disorder, and Kivelson on theories of the cuprate superconductors.

Though the emphasis was on the theoretical methods for tackling this class of difficult many-body problems, there were discussions on relating theory to experiment in many of the talks, particularly those by Essler, Giamarchi, Kotliar, Georges and Kivelson. The whole programme conveyed a good picture of the theoretical problems in this field and the diversity of the approaches which are being used to tackle them.

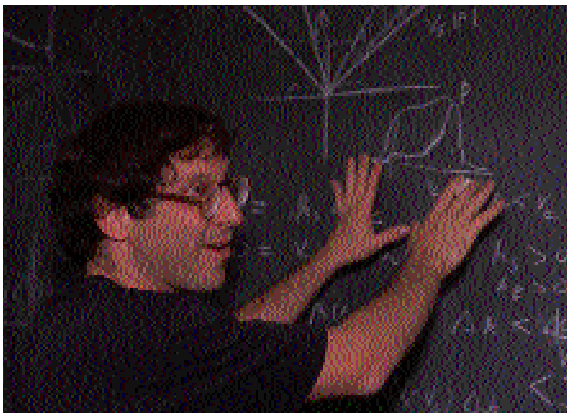
Strongly Correlated Electron Systems - Novel Physics and New Materials

Organiser: Peter Littlewood

This final conference of the programme, which received financial support from the EC, was held 26-30 June, with 100 participants. In recent years, more and more systems have been discovered or constructed which have unusual properties due to strong correlations between the electrons.

These include unusual alloys, such as those studied in Cambridge that display both ferromagnetism and superconductivity, but also fabricated mesoscopic systems such as quantum dots and nanotubes. These have not only presented new problems but also different realizations of old ones, ones in which the parameters are easier to control and hence offer more scope for detailed comparison of theory and experiment. This conference was organised to focus on these developments and the future potential directions of strong correlation research.

Amongst the many stimulating and exciting talks were those by D Cobden, S De Franceschi and K Matveev on experiment and theory on quantum dots and nanotubes, P Johnson, H Mook, M Randeira, P Horsch and S Kivelson on recent experiment and theory for cuprate superconductors, and HR Ott, M Aronson and TM Rice on the problems posed by the unusual behaviour observed in hexaborides.



Steve Kivelson at the final conference.

Achievements

During the programme VI Falko announced the solution of an outstanding problem in the theory of quantum Hall ferromagnets. The gap in the excitation spectrum is generally believed to be the activation energy of a skyrmion-antiskyrmion pair, but this is larger than the gap which is observed. Dropping the assumption of particle hole symmetry implicit in previous work Falko and Iodanski show that a much smaller activation energy exists due to the creation of a bare electron and an antiskyrmion.

Tai-Kai Ng finished a paper on topological effects in short antiferromagnetic Heisenberg spin chains. He gave numerical evidence, using the density matrix renormalization group technique, that the topological effect which is responsible for the difference between integer and half-integer spin infinite chains (Haldane conjecture) also manifests itself as end-states which exist even for short chains where the Haldane gap is unobservable.

JJ Betouras and JT Chalker completed a paper on the effects of interactions and disorder on mesoscopic conductance fluctuations in the two-

John Samson and Feodor Kusmartsev at the poster session of the final conference.

dimensional chiral metal.

Yong Baek Kim wrote a paper investigating the limitations of the Chern-Simons composite fermion theory of the fractional quantum Hall effect for the important case of a half-filled Landau level. He did this by considering the frictional drag between two two-dimensional electron gases in a double-well potential structure, and recently followed this up with a further paper using a more microscopic dipolar composite fermion approach.

KA Matveev completed work with AV Adreev on the thermoelectric properties of quantum dots in the Coulomb blockade regime. A particular interest of this work is the possible application of such small devices as micro-refrigerators.

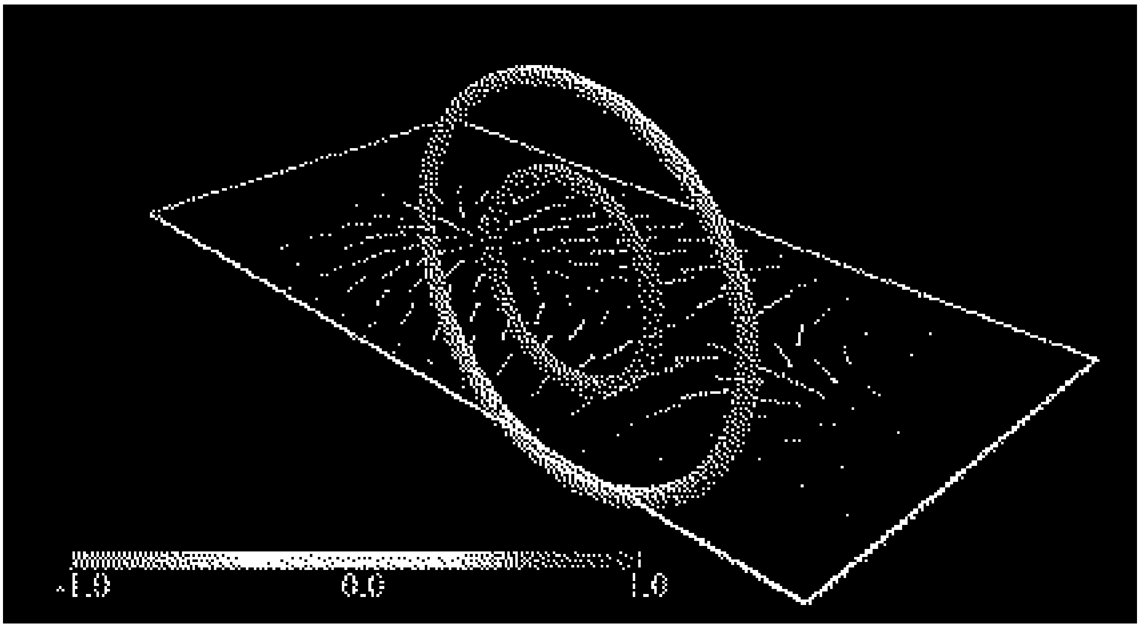
T Xiang completed two papers, one on spin chains and the other on superconducting phase fluctuations probed by tunneling. In the first, he and co-authors used the transfer matrix renormalization group method to calculate numerically the magnetic susceptibility and specific heat of the spin 1 Heisenberg chain with linear and biquadratic exchange. In the second, with Tai-kai Ng as a co-author, they showed how a current noise spectra of a tunnel junction between an underdoped and optimally doped cuprate superconductor can probe pair fluctuations in the underdoped material above its transition temperature. It is concluded that a suitable experimental study might settle the question of whether or not the famous observed pseudo-gap is an incipient pairing gap or not.

T Giamarchi completed a long paper on a general renormalization group analysis of creep and pinning in disordered media. In the context of the present programme this relates to charge density waves, Wigner crystals and vortex lattices.

C Nayak and P Fendley initiated a collaboration during the NATO ASI which has resulted in a paper on tunnelling between Luttinger liquids. They map the problem of coupled Luttinger liquids on to the 4-state chiral clock model and draw conclusions which challenge some of PW Anderson's ideas on high T_c cuprates.

Dynamic mean field theory has provided a way of studying the Mott metal-insulator transition in strong correlation models, such as the Hubbard model. The numerical calculations are however, difficult to carry out, particularly in the immediate region of the transition. Y Ono, R Bulla and AC Hewson applied a linearized form of this theory to the two-band Hubbard model. Their approach gives analytic results which generate the complete metal insulator phase diagram for this model, and they have shown that it is in excellent agreement with numerical results. This approach should be useful for studying the metal-insulator transition in other models.

Renormalization group techniques have proved to be very effective in tackling strong correlation impurity problems and have recently been combined with dynamic mean field theory for calculations for Hubbard and periodic Anderson models. S Bradley, AC Hewson, R Bulla and Y Ono combined resources to apply this approach to study the interplay of phonons and strong electron correlation. The first stage, the calculations for impurity models, has been completed and the work is being extended to include the dynamic mean field theory to study



The orientations of the magnetic moments within a propagating 'magnetic smoke ring' within a ferro-magnet, taken from the calculations of Nigel Cooper (Birmingham), which he presented in his talk at the final conference. The Hopf invariant is zero and the direction of propagation is perpendicular to the plane containing the red circle.

the behaviour of lattice models.

It was shown by AC Hewson, a few years ago, that another renormalization group technique, renormalized perturbation theory, is a very effective way of calculating the energy behaviour of a number of strong correlation impurity and lattice models. In particular, he demonstrated that, when carried out to second order for the impurity Anderson model, it gives asymptotically exact results for the leading low temperature conductivity and thermodynamic behaviour. Using an entirely different approach based on conformal field theory, H Saleur has derived exact expressions for the next leading order low temperature terms in the conductivity, using boundary conformal field theory. Renormalized perturbation approach taken up to fourth order would generate similar terms. It is of some interest, therefore, as the renormalized perturbation theory is a more generally applicable technique, to see whether it give these next leading correction terms exactly if taken to fourth order. These calculations are well under way and Hewson expects to be able to answer this question in the near future.

This programme brought together a number of experts in integrable models with different backgrounds. Those more concerned with the application to models describing experimental systems, such as FHL Essler, N Andrei and AM Tsvelik, had important discussions with S Lukyanov whose approach is more formal. Lukyanov and Essler collaborated on a renormalization group treatment of the short-distance behaviour of correlation functions in the integrable Sine-Gordon model. This complemented the form-factor approach to the long-distance behaviour used by Essler and Tsvelik in their paper, completed during the programme on optical conductivity of one-dimensional Mott insulators. Such systems, which may apply to some quasi-one-dimensional organic conductors, can be described by the integrable Sine-Gordon model within the framework of a low energy effective field theory. Tsvelik also collaborated with A Schofield and TM Rice on non-linear optical properties of Mott insulators.

The exact calculation of correlation functions in an integrable model is a formidable problem. As a possible first step FDM Haldane worked on the identification of the generators of the Yangian SO(4) quantum symmetry of the integrable single-impurity Anderson Model.

YY Lobanov and GS Tian were amongst the most mathematical participants and each produced a paper. Lobanov developed a new technique for numerical function integration which can be applied to open quantum systems. Tian proved two theorems stating inequalities between charge and spin excitiation gaps for some models of strongly correlated systems.

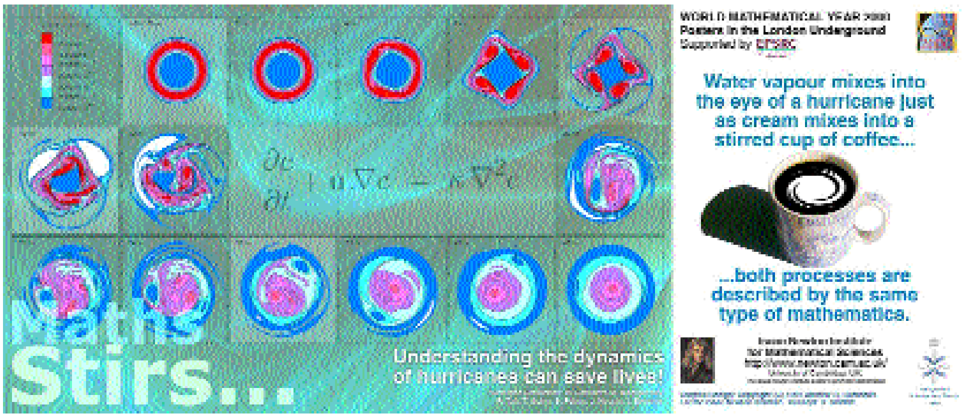
Rather than working on generic models, such as the Hubbard model, MW Long developed models for some specific materials, oxides of vanadium. The symmetry of different vanadium d-orbitals is regarded of prime importance and a new theory of the metal insulator transition in V_2O_3, in terms of orbital ordering, was written up. Long's incisive questioning during seminars was a notable and welcome feature of his two-month stay.

Amongst many excellent seminars a highlight was that of TM Rice on an 'N-patch' renormalization group analysis of the two-dimensional Hubbard model. Very recent work in his group shows the viability of a new scenario for the breakdown of a Fermi liquid which may relate to the high-T_c cuprates. On the path from a Fermi liquid to a Mott insulator, with increasing electron density, a precursor of the Mott gap opens on parts of the Fermi surface susceptible to umklapp scattering.

Posters on the Underground

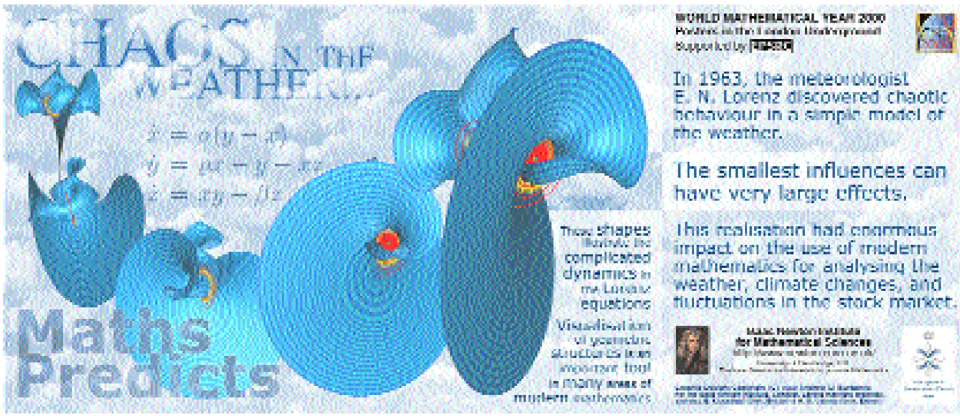
The Institute is participating in World Mathematical Year 2000 by designing twelve posters for display month by month in trains on the London Underground. These posters are part of an international initiative and similar posters have been placed in transport systems in over thirty cities worldwide. The Newton Institute's posters, which focus on the wide range of applications of mathematics, have attracted the attention of the national press (The *Guardian*) and radio stations (LBC and Radio 5 Live).

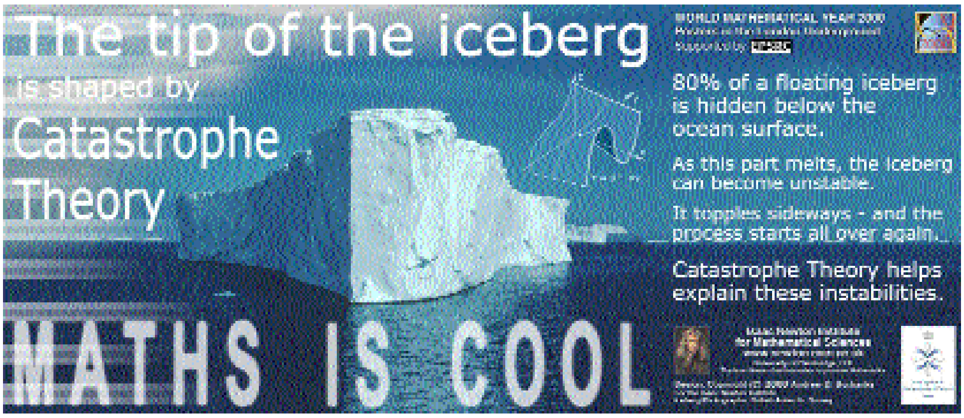
More detailed information about the mathematics behind the posters is available on the Institute's web site, and many schools and university departments have requested copies of the posters. The campaign was supported by funding from EPSRC's Partnership for Public Awareness Initiative and by the Victor Rothschild Memorial Fund. The first six posters displayed from January - June 2000 are shown below. Reprints of the posters will be published with the assistance of the World Scientific Publishing company.

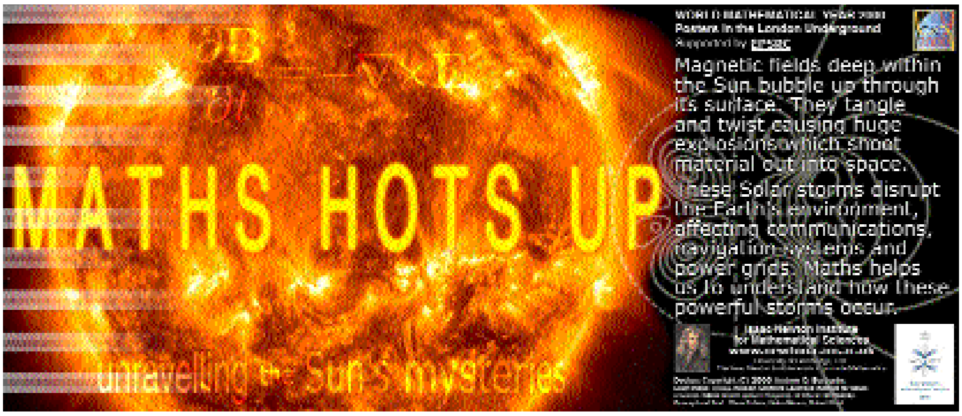


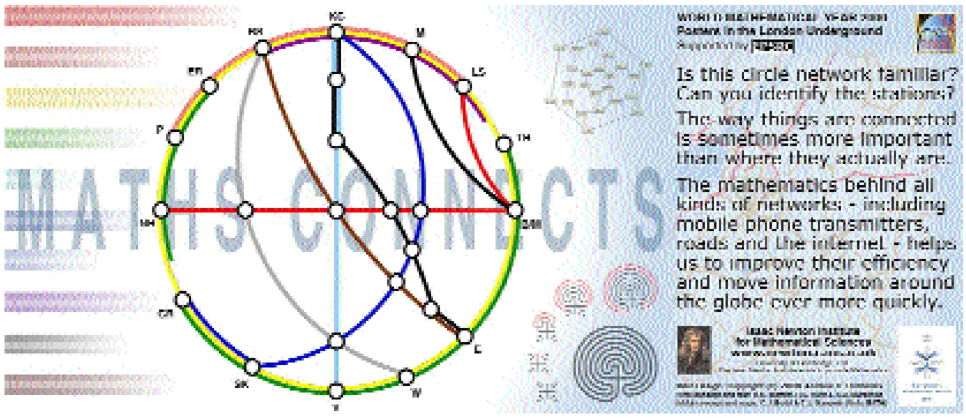
Above, *Maths Counts* (January) and *Maths Stirs* (February).

Opposite, *Maths Predicts*, *Maths is Cool*, *Maths Hots Up* and *Maths Connects* (March to June respectively).


CHAOS IN THE WEATHER
WORLD MATHEMATICAL YEAR 2000
Posters in the London Underground
Supported by EPSRC
In 1963, the meteorologist E. N. Lorenz discovered chaotic behaviour in a simple model of the weather.
The smallest influences can have very large effects.
This realisation had enormous impact on the use of modern mathematics for analysing the weather, climate changes, and fluctuations in the stock market.
Maths Predicts
Isaac Newton Institute for Mathematical Sciences


The tip of the iceberg
is shaped by
Catastrophe Theory
WORLD MATHEMATICAL YEAR 2000
Posters in the London Underground
Supported by EPSRC
80% of a floating iceberg is hidden below the ocean surface.
As this part melts, the iceberg can become unstable.
It topples sideways - and the process starts all over again.
Catastrophe Theory helps explain these instabilities.
MATHS IS COOL
Isaac Newton Institute for Mathematical Sciences


WORLD MATHEMATICAL YEAR 2000
Posters in the London Underground
Supported by EPSRC
Magnetic fields deep within the Sun bubble up through its surface. They tangle and twist causing huge explosions which shoot material out into space.
These Solar storms disrupt the Earth's environment, affecting communications, navigation systems and power grids. Maths helps us to understand how these powerful storms occur.
MATHS HOTS UP
unravelling the Sun's mysteries
Isaac Newton Institute for Mathematical Sciences


WORLD MATHEMATICAL YEAR 2000
Posters in the London Underground
Supported by EPSRC
Is this circle network familiar? Can you identify the stations?
The way things are connected is sometimes more important than where they actually are.
The mathematics behind all kinds of networks - including mobile phone transmitters, roads and the internet - helps us to improve their efficiency and move information around the globe ever more quickly.
MATHS CONNECTS
KC
M
LS
TH
BM
E
W
V
SK
GR
NH
P
BR
RR
Isaac Newton Institute for Mathematical Sciences

Finances

Accounts for July 1999 to June 2000 (Institute Year 8)

Income	**1998/1999 Year 7**	**1999/2000 Year 8**
Grant Income - Revenue	1,325,782	1,206,758
Grant Income - Workshop	179,280	290,908
Endowment Interest	0	50,000
Donations - Revenue	3,526	532
Donations - Capital	340	0
Interest on Deposits	-	92,944
General Income	46,999	2,617
Housing	6,231	20,212
Transfer from Reprovision	-	37,937
Transfer from Building Capital Fund	-	62,751
Total	1,562,158	1,764,659

Expenditure		
Scientific Salaries	279,388	310,046
Scientific Travel and Subsistence	436,938	389,121
Scientific Workshop Expenditure	139,533	239,878
Other Scientific Costs	20,610	7,634
Staff Costs	293,296	305,486
Computing Costs	36,548	76,228
Library Costs	28,326	15,338
Audio-Visual	1,566	1,340
Building - Capital	1,906	69,526
Building - Rent	190,000	195,700
Building - Repair and Maintenance	12,879	7,804
University Overheads	27,826	31,335
Consumables	49,989	32,834
Equipment - Capital	29,139	4,475
Equipment - Repair and Maintenance	5,360	1,883
Publicity	9,091	26,351
Recruitment Costs	11,125	4,569
Miscellaneous	(113)	356
Transfer to Future Reprovision	85,000	80,000
Transfer to Building Capital Fund	0	45,779
Total	1,658,407	1,845,681
Income Less Expenditure	(96,249)	(81,516)

Notes to Accounts

1. Grant income - Revenue

This breaks down as follows:

	1998/1999 **Year 7**	1999/2000 **Year 8**
EPSRC/PPARC Salaries	276,656	310,090
EPSRC/PPARC Travel and Subsistence	246,665	268,451
EPSRC PPA	0	11,430
Newton Trust	62,886	50,000
Hewlett-Packard	98,000	115,000
Rothschild	20,714	31,964
CNRS	40,549	36,960
Rosenbaum	27,504	21,951
Leverhulme	56,899	76,870
Royal Society	6,339	2,986
LMS	15,000	20,000
Institute of Physics	7,500	0
CPS	2,000	2,000
UofC Director's Fund	40,000	0
UofC/DAMPT (Staff)	57,272	60,734
UofC Rent	190,000	195,700
BP	5,000	0
Wellcome Trust	5,000	0
British Antarctic Survey	40,000	0
Bank of England	2,500	0
Barclaycard	1,500	0
Schlumberger	5,000	0
US Navy	2,960	1,847
Compugen	4,000	0
Smith Kline Beecham	2,000	0
RAE	95,999	0
CMG	10,040	0
ERCOFTAC	3,800	0
NSF	0	775
Total	**1,325,782**	**1,206,758**

2. Endowment

Endowment income for 1999/2000 comes from funds provided by the Isaac Newton Trust.

3. Grant Income and Expenditure - Workshops

Both income and expenditure on workshops are higher than expected because of high attendance rates.

4. Publicity Costs

These are high due to the Posters on the London Underground campaign which was the Institute's contribution to World Mathematical Year 2000.

5. Transfers from Reprovision

Transfers from reprovision have been made to cover computing equipment purchased during this financial year, and also to cover minor building works and reprovision of items of equipment.