# Contents

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## APPENDICES

Please note that the following statistical information may be obtained, on request, from the Institute or from  **http://www.newton.cam.ac.uk/reports/0001/appendices.html**

A   Long-Stay Participants
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# Director's Report

This is my fifth and final year as Director of the Newton Institute. It has been a challenging and fulfilling responsibility, and without a dull moment (sometimes I could have wished for one!). No less than 25 visitor research programmes will have been completed during my tenure as Director, and I warmly thank all in that wide national and international community of the mathematical sciences who have participated in these programmes and contributed in many ways to their success. I wish to record particular thanks to those whom we appoint programme organisers for their unstinting and committed involvement, without which no programme could succeed.

I pass on the responsibility of Directorship to Sir John Kingman, with effect from 1 October 2001. I believe that the Institute is in good heart, and well placed to respond to the challenges of the next decade. We already have a strong 'programme of programmes' up to December 2003 (see p14 of this report) covering a good mix of pure and applied mathematics. There has been no shortage of good proposals; the short summer programmes have been a popular innovation and are set to continue in future years in parallel with our normal programmes of four- and six-months duration.

On the fund-raising front, we have been extremely fortunate this year to have received two major donations: a bequest of about $1.38m from a donor in the USA who wished to remain anonymous; and a donation of £60k per annum for four years from the PF Trust. The latter activated a further donation of £50k per annum from the Isaac Newton Trust (the grant-giving arm of Trinity College, Cambridge). The Endowment Fund of the Institute now stands at just over £4m.

The 'Posters in the Underground' project, our contribution to WMY2000 and to the EPSRC Public Awareness Initiative, was completed in December 2000, and, judging by the demand from far and wide for copies of the posters, successful beyond all reasonable expectations. Again, I wish to thank all who contributed ideas, time and energy in seeing this project through to completion. Plans are now afoot within EPSRC to print 6000 additional sets of the twelve posters for distribution schools in the UK.

Construction of the Faulkes Gatehouse was completed during the month of June, and we expect to be making full use over the summer of the extra office and seminar room space that it provides. Our thanks are again due to Dill Faulkes for the munificent donation that made this possible. Our thanks also to Edward Cullinan for the exciting architectural design of the building, and to Sir Robert McAlpine for bringing the construction to successful completion despite the appalling weather during last winter and spring.

The Institute will soon bid farewell to Colin Sparrow, who has had a long association with the Institute, first as Hewlett-Packard Senior Research Fellow, then since 1996 as Liaison Officer with particular responsibility for liaison with the wider UK community. Colin will take up a Readership at the University of Warwick in October; he goes with our warmest good wishes and thanks for his dedicated service to the Institute.

We lost during the year our Institute Administrator, Ann Cartwright, who left to seek new challenges at the Wellcome/CRC Institute of Cancer and Developmental Biology in Cambridge. Ann gave unstinting service to the Institute over seven years, and we owe her a warm debt of gratitude. Ann was succeeded in December 2000 by Christine West, who came to us from her previous position with the Cambridge branch of the Further Education Funding Council, and who has ably entered upon the heavy responsibilities of her new post.

The task of Director of the Newton Institute is a difficult one; but it is greatly eased by the devotion of a wonderfully loyal and dedicated staff, who have responded to a succession of challenges with skill, commitment and good humour. Our participants pay regular tribute to the exceptional standard of service provided at the Newton Institute, and I am happy now to add my own tribute and thanks for the unwavering support that I have been privileged to enjoy these last five years.

*Keith Moffatt*

30 June 2001

# Introducing Sir John Kingman FRS

Sir John Kingman FRS takes over as the Director of the Isaac Newton Institute for Mathematical Sciences on 1 October 2001, on the retirement of Professor Keith Moffatt FRS, who has held the post since 1996. Sir John will also be the first N.M. Rothschild & Sons Professor of Mathematical Sciences. He is at present Vice-Chancellor of the University of Bristol and also Chairman of the Statistics Commission, which oversees the quality of official statistics in the United Kingdom.

Sir John works on the theory and applications of probability and stochastic analysis, including operational research and population genetics. He has written five books and about a hundred papers on these and related areas of mathematics, and has been President of both the Royal Statistical Society and the London Mathematical Society. He was elected to the Royal Society in 1971, and received its Royal Medal in 1983.

He gained his degrees at Cambridge, as a scholar and later Fellow and Honorary Fellow of Pembroke College.

After teaching in Cambridge he became a professor at Sussex in 1966 and then at Oxford in 1969. From 1981 he was Chairman of the Science and Engineering Research Council (SERC), before taking up his post at Bristol in 1985.

Sir John is married with two children, one a civil servant and the other an obstetrician and gynaecologist. His wife Valerie Cromwell is Director of the History of Parliament. He has served on a wide range of national and international bodies, including the British Council, the British Technology Group, the Parliamentary and Scientific Committee and the boards of IBM UK and SmithKlineBeecham. In 1988 he chaired the committee which produced the government report on the teaching of the English language.

His knighthood came in 1985 for his work with the SERC, and he holds honorary degrees from English, Russian and Canadian universities. He is an Officier dans l'Ordre des Palmes Académiques, and an Honorary Senator of the University of Hannover.



*Sir John Kingman FRS*

# Brief Scientific Report on Programmes

## Programme 35: Free Boundary Problems in Industry
### (17 July - 4 August 2000)

During this programme 70 researchers, all of an intensely collaborative nature, came together for each of three one-week sessions to discuss free boundary problems in the food, glass, and metals industries. Free boundary problems had been selected because of their proven ubiquity in industrial research, coupled with their mathematical fascination. The unstructured study group format attracted many representatives from UK industries including Unilever, Corus, Pilkington Glass and United Biscuits. What emerged were some dramatic revelations about the insights that all kinds of mathematics may soon be able to offer all branches of industry, ranging from high-tech to low-tech.

## Programme 36: Quantized Vortex Dynamics and Superfluid Turbulence
### (7 - 25 August 2000)

The subject of the programme was primarily superfluid turbulence and its explanation in terms of quantum vortex dynamics. As such it attracted researchers working in a number of fields: the superfluidity of helium-4 and of helium-3, superconductivity, equations of motion, quantized vortex dynamics and simulations of the dynamics, applications of the NLSE (Nonlinear Schrodinger Equation) to problems in helium-4 such as reconnection, and NLSE as a model of the Bose-Einstein Condensate (BEC) under various conditions. A number of new developments were reported and many unsolved problems were identified, making for three very exciting weeks.

## Programme 37: Singularity Theory
### (24 July - 22 December 2000)

Singularities arise naturally in a huge number of different areas of mathematics and science. As a consequence Singularity Theory lies at the crossroads of the paths connecting the most important areas of applications of mathematics with its most abstract parts.

The major idea of this programme was to bring together experts within the field and those from adjacent areas where singularity theory has existing or potential application, such as wave propagation, dynamical systems, quantum field theory, differential and algebraic geometry, and others. It was the programme's aim both to foster exciting new developments within singularity theory, and also to build bridges to other subjects where its tools and philosophy prove useful. To this end, the programme was attended by the majority world-wide of experts in the field together with a large number of leading experts in adjacent areas within mathematics and physics. It was an international event of extremely high significance for Singularity Theory, providing a valuable boost of new ideas and introducing many exciting problems to solve. Participants were uniformly positive in their comments and all reported productive discussions and new collaborations.

## Programme 38: Geometry and Topology of Fluid Flows
### (4 September - 17 December 2000)

The realization that the mathematical disciplines of topology and geometry are extremely valuable in furthering our understanding of fluid flows has been evolving steadily for many years within the fluid mechanics community. This programme was directed towards understanding phenomena in fluid mechanics and magnetohydrodynamics that appear to involve substantial questions in these disciplines. The programme brought together experts in widely varying fields to work on a set of research problems at the crossroads of fluid mechanics, modern topology, dynamical systems, analysis, and PDEs.

The areas of application fall roughly into four categories: the application of dynamical systems and topology to stirring and chaotic advection, the application of differential geometry to fluid mechanics, the application of topology to magnetic

and classical fluid flows, and the application of geometric and topological concepts to the problem of regularity/blowup in hydrodynamics.

The main goals were to inject ideas from modern differential geometry and topology into fluid mechanics and to inspire new directions in these mathematical fields from discussions on the major problems in fluid mechanics. There was a great deal of exposure to new disciplines and interaction between participants with a large range of backgrounds. Many new collaborations started during the programme. The large number of such interactions is the main measure of its success.

## Programme 39: Symmetric Functions and Macdonald Polynomials
### (8 January - 6 July 2001)

The overall programme was in the general area of symmetric functions and the representation theory of Hecke algebras and the symmetric groups and its generalizations, reflection groups and, in particular, Weyl groups. The importance of this subject area has been long recognised due to its applicability in a wide range of mathematical and scientific disciplines. The subject is also central to algebraic combinatorics and has a particularly distinguished history in the UK with such names as MacMahon, Young, Littlewood (DE) and Richardson regarded as the leading innovators in the field, culminating in more recent times with the breakthroughs by Ian Macdonald.

The programme was focused on Macdonald polynomials around which a great deal of recent developments have been centred. It was in the 1980's that a series of conjectures were formulated, now known as the Macdonald constant term conjectures. The goal of this programme was to unify and further develop the many disparate approaches which have since been taken with the ideal but not easily attainable outcome of a single theory which would encompass them all.

Many participants indicated that new ideas had arisen during formal and informal discussions during the programme which would be the basis

for future research, some of it in collaboration with other participants. The exceptionally stimulating environment of the Newton Institute was felt almost without exception to have been of immense benefit. Bringing together experts from many different backgrounds but who spoke a common language of symmetric functions and Macdonald polynomials produced a variety of new results.

## Programme 40: Nonlinear Partial Differential Equations
### (8 January - 6 July 2001)

This programme emphasised equations of elliptic and parabolic type, which traditionally model steady states and evolving processes. The programme was divided into four interrelated themes:

- Geometric evolution equations;
- Fully nonlinear equations;
- Variational problems with singularities;
- Reaction diffusion equations.

The first two themes were pursued mainly during the first three months of the programme and the last two themes during the last three months. The activities under each theme culminated in a workshop.

Many of the leading international researchers in these areas participated, typically as long term visitors or just for the workshops which, together with the weeks immediately preceding, were the most exciting and stimulating periods of the programme. Moreover the blend and standard of mathematics and ensuing synergies generated during those times must rank as the best ever in these themes.

The programme impacted on a huge range of scientific terrain, including general relativity, affine differential geometry, meteorology, complex flows, mass transport problems, vortices, transition layers and biological systems.

# Programme Participation

A total of 1085 visitors was recorded for 2000/2001. This includes 234 long-stay participants, each staying between two weeks and six months (7 weeks on average), and 389 short-stay participants who stayed for two weeks or less. Within the six completed programmes there was a total of 20 workshops (periods of intense activity on specialised topics) which attracted 390 visitors to the Institute. Finally, 100 visitors were registered as having taken part in the special events held outside the Institute programmes, and there were many others who attended occasionally for lectures, workshops or Institute Seminars. Withi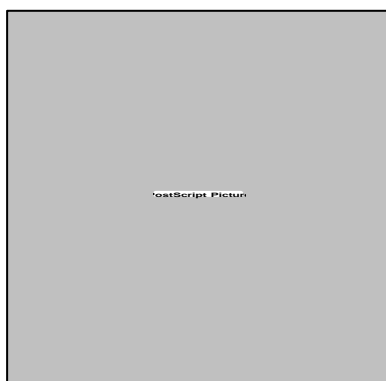n all the programmes, workshops and special events, around 1170 seminars were given in total at the Institute during the year.

In addition to workshops, which serve to widen UK participation in programmes, programme organisers are encouraged to organise more informal special days, short meetings or intensive lecture series which can attract daily or short-term visitors, so further opening the activities of the Institute to the UK mathematical community.

The Institute also funds visits by programme participants to other UK institutions to give seminars, and 204 such seminars took place last year.

| Programme | Long-stay participants | Mean stay (days) | Short-stay participants | Mean stay (days) |
|---|---|---|---|---|
| Free Boundary Problems in Industry | 19 | 18 | 56 | 6 |
| Quantized Vortex Dynamics and Superfluid Turbulence | 28 | 19 | 17 | 7 |
| Singularity Theory | 53 | 53 | 88 | 9 |
| Geometry and Topology of Fluid Flows | 42 | 49 | 54 | 7 |
| Symmetric Functions and Macdonald Polynomials | 52 | 68 | 56 | 8 |
| Nonlinear Partial Differential Equations | 40 | 48 | 118 | 8 |

The pie charts below show the percentages of long-stay and short-stay participants broken down by country of residence:
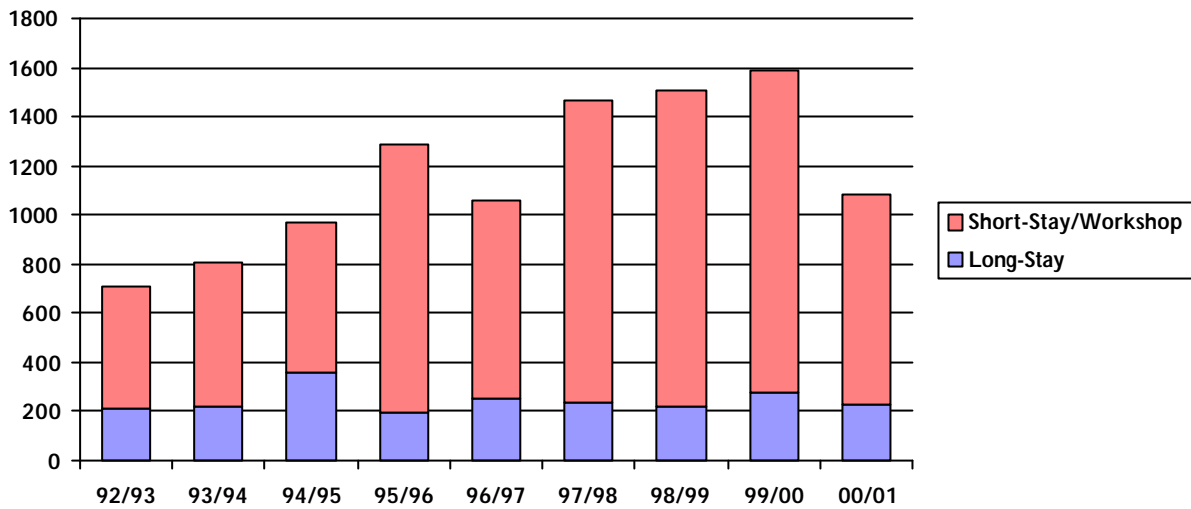


**Long-stay participants**
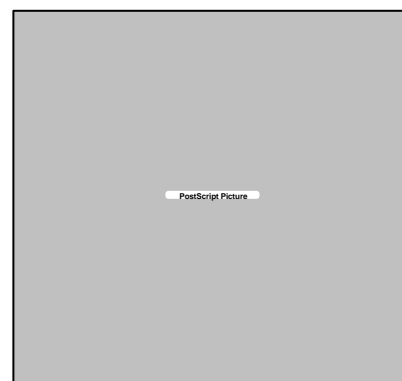


**Short-stay, workshop and special event participants**

The following chart summarises the total figures for long- and short-stay participation since the Institute began its programmes:



The median age for long- and short-stay participants combined is 41 years, with an interquartile range of 33-51 years. The following chart shows the cumulative frequency of long- and short-stay participant ages.
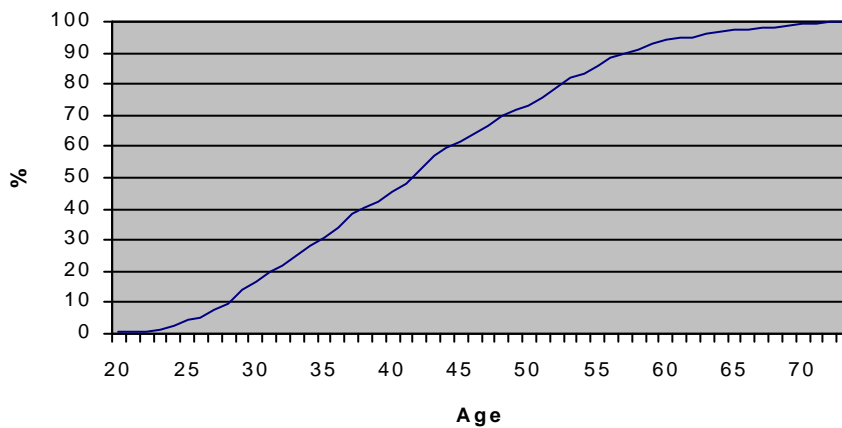


More detailed statistics, including visit dates, home institutions of participants and a complete list of seminars and papers, are given in the Appendices, available separately from the Institute or at

**http://www.newton.cam.ac.uk/reports/0001/appendices.html**

# Other Institute News

## *Fellowship of the Institute*

Dr Hans Rausing was elected an Honorary Fellow of the Newton Institute on 22 May 2001. The formal presentation, conducted by Professor Frank Kelly FRS, Chair of the Management Committee, was followed by a working lunch. Dr Rausing then took part in an afternoon meeting on recent developments in cosmology and astrophysics chaired by Professor Sir Martin Rees FRS, Astronomer Royal and a member of the Management Committee.



*Dr H Rausing (left) receiving his Scroll of Fellowship from Prof FP Kelly FRS*

## *Awards*

Professor AFM Smith, Chair of the National Advisory Board, was elected Fellow of the Royal Society on 14 May 2001.

Professor HK Moffatt, Director of the Institute, received the following awards and distinctions in the course of the year: elected President of the International Union of Theoretical and Applied Mechanics (IUTAM), 2000–2004; Hon DSc, University of Edinburgh; Foreign Member, Accademia Nazionale dei Lincei (Rome); Modesto Panetti and Carlo Ferrari International Prize and Gold Medal (2001) for distinguished research in applied mechanics; Blaise Pascal International Research Chair (Paris 2001).

## *National Science Week*

The Director gave a public lecture on Saturday 24 March 2001 as part of National Science Week. The lecture, entitled *Maths on the Undergound,* used the posters featured on pp46–47 as a basis for discussing the applications of mathematics to problems of everyday human concern. The lecture was well attended by over 100 visitors, many of whom took away a copy of their favourite poster as a souvenir.

## *Faulkes Gatehouse*

The Faulkes Gatehouse, shown on the front cover of this report, was completed in June, ready for use during the summer programmes. A topping-out ceremony was held on 19 February 2001 (following completion of the external building work) when Dr Dill Faulkes nailed a traditional evergreen branch to the roof timbers.

The Gatehouse comprises three generous offices at ground floor level and a semi-circular seminar room, accommodating up to 42, at first floor level. There is also ample basement storage. The Gatehouse will permit a considerable increase of flexibility in the planning of workshops in the future.



*Dr MC Faulkes (left) nailing the evergreen branch, assisted by Works Manager Mr L Bogic*

## Rethinking Cancer Research: 'Blue-Skying the Future'

A weekend workshop with this title was held on 1–3 December 2000 at the Newton Institute, under the auspices of CSFI (the Centre for the Study of Financial Innovation). The discussion, led by Dr M Baum and Dr A Hilton, involved oncologists on the one hand and a wide range of academics and lay participants on the other. The discussion was published by CSFI and the report can be seen, together with further details, at

**http://www.blueskyingcancer.net**

## Visit by Professor Donald Coxeter FRS

The Institute was honoured by the visit on 18 September 2000 of Donald Coxeter, Geometer extraordinary, who presented an Institute Seminar with the characteristically intriguing title *Five spheres in mutual contact.* Coxeter was a Fellow of Trinity College, Cambridge, and has this year been elected an Honorary Fellow of the College.



*Prof D Coxeter and Prof H Aref*

## JREI Grant

The Institute was awarded in February 2001 a grant from the Joint Research Equipment Initiative for its proposal titled *Computing and visualisation tools for interdisciplinary research programmes in the mathematical sciences.* The award of £92,225 from EPSRC, plus £10,000 from the Institute's own funds, was generously matched by a grant of £102,225 from Sun Microsystems in the form of a discount on their educational list prices.

This award has enabled the Institute to obtain a Sun Fire F3800 compute server, a Sun Enterprise 420R and two Sun Blade 1000 workstations. The high-performance server will allow participants to perform significantly more intense computational work than is presently feasible, while the workstations are equipped with Sun Expert3D graphics cards to enable participants to create super-high-resolution visualisations of complex 2D and 3D structures.

## Boules Court

The new landscaping around the Newton Institute allowed for the construction of a boules court on the western flank of the building. This has been available for staff and participants since early June: see the poem on p45, which seeks to capture the spirit of this quintessentially Newtonian game!



*A game of boules in progress*

## Burns Supper

A semi-traditional Burns Supper was held at the Institute on 25 January 2001; the *Address to the Haggis* was delivered by Dr Ron Anderson, *Tae a (Computer) Mouse* by the Director, and *John Anderson, my Jo* (in Russian) by Professor Pavel Plotnikov. The intricate patterns of a number of Scottish Country Dances were then subjected to geometrical and dynamical analysis by the participants, guided by the Deputy Director, Dr Robert Hunt.

# Newton Institute Publications

## *Papers and Preprints*

Over 100 papers were produced or in preparation at the Institute during 2000/2001 (a complete list is given in Appendix F). Many of these are included in the Newton Institute's Preprint Series to which participants are encouraged to submit papers. A web page giving details of Newton Institute preprints is available at

**http://www.newton.cam.ac.uk/preprints.html**

## *Books arising from Newton Institute Programmes*

The following titles were published during 2000/2001:

WJ Fitzgerald, RL Smith, AT Walden *et al* (Eds.)
*Nonlinear and Nonstationary Signal Processing*
Cambridge University Press, 22 February 2001, 484pp, ISBN: 0 521 80044 7, (hbk) £60.00

JCR Hunt and JC Vassilicos (Eds.)
*Turbulence Structure and Vortex Dynamics*
Cambridge University Press, 1 February 2001, 300pp, ISBN: 0 521 78131 0, (hbk) £55.00

AI Mees (Ed.)
*Nonlinear Dynamics and Statistics*
Birkhäuser, 29 September 2000, 473pp, ISBN: 0 8176 4163 7, (hbk) £77.00

JC Robinson and PA Glendinning (Eds.)
*From Finite to Infinite Dimensional Dynamical Systems*
Kluwer, April 2001, 215pp, ISBN: 0 7923 6976 9, (hbk) £61.00

JC Vassilicos (Ed.)
*Intermittency in Turbulent Flows*
Cambridge University Press, 30 November 2000, 276pp, ISBN: 0 521 79221 5, (hbk) £45.00

A complete list of books published as a result of Newton Institute Programmes is available at

**http://www.newton.cam.ac.uk/inibooks.html**
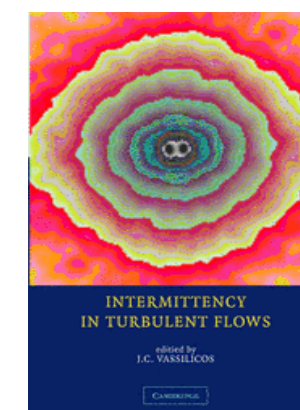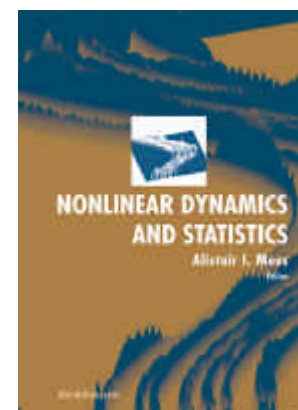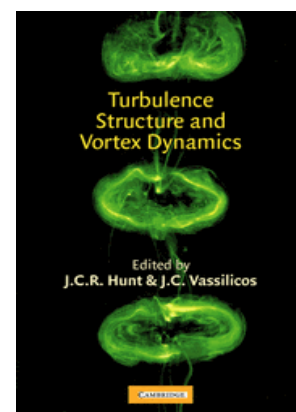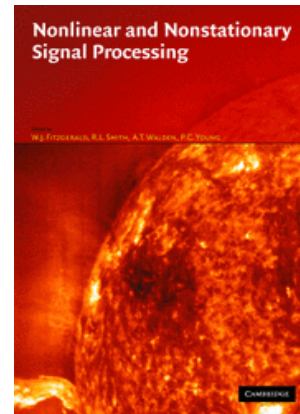
# UK and International Liaison

**Membership of the National Advisory Board as at 30 June 2001:**

| | |
|---|---|
| Professor AFM Smith FRS (Chair) | Queen Mary and Westfield College |
| Professor HK Moffatt FRS | Director, Newton Institute |
| Dr RE Hunt | Deputy Director, Newton Institute |
| Professor Sir Michael Berry FRS | University of Bristol |
| Professor J Brindley | University of Leeds |
| Professor KA Brown | University of Glasgow |
| Professor EB Davies FRS | Kings College London |
| Professor PJ Diggle | University of Lancaster |
| Professor CM Elliott | University of Sussex |
| Professor NJ Hitchin FRS | University of Oxford |
| Dr M Sheppard | Schlumberger Cambridge Research Ltd |
| Professor JR Whiteman | Brunel University |

## *National Advisory Board*

Following discussions with EPSRC, a National Advisory Board (NAB) for the Institute was established during 1999, and held its first meeting on 20 March 2000. Subsequent meetings have been held on 16 October 2000 and 14 May 2001.

The remit of the NAB is " To advise the Director in all matters relating to the role of the Newton Institute as a National Institute for the Mathematical Sciences."

The membership, as at 30 June 2001, is given in the table above. The overlap with the Scientific Steering Committee and Management Committee is deliberate and intended to ensure good communication with the Board.

Some of the issues addressed by the NAB have been:

- The attendance of young UK scientists
- The Institute's strategy vis-à-vis its national role, interdisciplinarity and outreach
- Flexibility in the scientific programming to respond to new developments
- The interests of the EPSRC programmes which contribute to the Institute
- A focus on the Institute's databases to enable it to produce information on subject coverage, the geographical distribution of participants and the status of participants

- The Institute's mode of interface with industry

Anyone with views about the national role of the Institute is invited to make these known to any member of the NAB.



*Prof AFM Smith FRS, Chair of the National Advisory Board*

# Symposia activities

The Institute continues to maintain a list of forthcoming UK symposia, workshops, etc., in the Mathematical Sciences. This list is maintained in consultation with representatives of LMS, IMA, RSS, ICMS (Edinburgh) and the Warwick Mathematics Research Centre. For details, see **http://www.newton.cam.ac.uk/symposia.html**

# UK correspondents

During 2000/01, at the suggestion of the National Advisory Board, the Newton Institute has established a list of UK University correspondents who act as a channel of communication between the Newton Institute and the mathematical sciences community of the Universities concerned. Correspondents are regularly informed about activities of the Institute, and it is their responsibility to ensure that the information is disseminated to the relevant University Departments, and also to provide any feedback to the Institute.

The names of the correspondents so far established can be found on the Institute website at

**http://www.newton.cam.ac.uk/correspondents.html**

Universities not yet represented on this list are encouraged to provide a suitable nominee.

# Satellite workshops

The Institute encourages organisers of longer (4- or 6-month) programmes to cooperate with local organisers in holding "satellite" workshops at UK Universities and institutions outside Cambridge.

Satellite workshops are on themes related to the Institute programmes, and involve a significant number of longer-term overseas participants from the Institute. They also, crucially, draw in and involve UK mathematicians and scientists who might not otherwise be able to participate substantially in the Institute programme.

Costs for satellite workshops are typically approximately £10,000 (excluding the overseas travel costs of Institute participants) and are shared approximately 50/50 between the Institute and the host institution. Both EPSRC and LMS welcome applications from host institutions for grants to cover their share of the costs (subject to the usual review procedures). We are grateful that LMS in particular has expressed a willingness to half-fund up to two such workshops a year.

Institutions interested in holding such workshops should contact either the Deputy Director, Dr RE Hunt (R.E.Hunt@newton.cam.ac.uk), or the organisers of the relevant programme.

# Seminars

Long-term participants in Newton Institute programmes are given every encouragement to visit other UK institutions during their time at the Institute, and many did so during 2000/2001 (see p5). Participants' travel costs within the UK were covered by the Institute on request.

# Institute Seminars

Audio files of selected Institute seminars with accompanying transparencies have been published on the web since 1999 at

**http://www.newton.cam.ac.uk/webseminars/**

This year's seminars included

- D Coxeter (Toronto), *Five spheres in mutual contact*
- EA Spiegel (Columbia), *From molecular chaos to deterministic chaos by dimensional reduction*
- L Caffarelli (Austin, Texas), *Plane-like minimal surfaces in periodic media*
- G Falkovich (Weizmann Institute), *Smog and rain on a windy day*
- J Palis (Brazil), *A global view of dynamics and recent related results*
- J Kingman (Bristol), *Some unsolved analytic problems in random process theory*
- L Nirenberg (New York), *Elliptic equations for composite materials*

# Young Scientists

The Institute holds a number of events each year which are specifically targeted at young scientists. In 2000/2001 these events included:

- NATO ASI / EC Summer School on *New Developments in Singularity Theory*
- Euroworkshop on *Applications to Quantum Field Theory*
- Workshop on *Applications to Geometry* (held at the University of Liverpool)
- NATO ASI *Pedagogical Workshop on Geometry and Topology of Fluid Flows*
- Workshop on *Singularities in Classical, Quantum and Magnetic Fluids* (held at the University of Warwick)
- Euroworkshop on *Conjectures, Recent Results and Open Problems Related to the Macdonald Polynomials*
- Euroconference on *Applications of the Macdonald Polynomials*
- Workshop on the *Heritage of I Schur's 1901 Dissertation*
- NATO ASI Workshop on *Symmetric Functions 2001: Surveys of Developments and Perspectives*
- Euroworkshop on *Geometric Evolutions and Nonlinear Elliptic Equations*
- Euroconference on *Nonlinear Elliptic Equations and Transition Phenomena*

The following young scientists were recipients of bursaries from the Cambridge Philosophical Society in 2000/2001:

*Singularity Theory*
- N Nekrasov (Princeton)
- P Pushkar (Moscow)
- F Napolitano (Paris 9)

*Geometry and Topology of Fluid Flows*
- R Ghrist (Georgia Tech.)

*Symmetric Functions and Macdonald Polynomials*
- M Roesler (Muenchen)
- M Rosas (Brandeis)

*Nonlinear Partial Differential Equations*
- J Wei (Hong Kong)
- D Labutin (Australian National)

The Institute recognises that junior researchers have much to contribute to and much to gain from Institute programmes and events. In order to maximise the information available to junior researchers, and to facilitate their involvement in Institute activities, we introduced in 1997 a category of Junior Membership of the Newton Institute. To be eligible for membership you must be a Research Student or within 5 years of having received a PhD (with appropriate allowance for career breaks) and you must work or study in a UK University or a related research institution.

Junior members receive regular advance information about programmes, workshops, conferences and other Institute events via a Junior Members' Bulletin, detailed information about any workshops of an instructional or general nature likely to be of special interest to young researchers, and information about suitable sources of funding or support for visits to the Institute, when available.

The Institute makes available some of its general funds specifically to support junior researchers in Institute activities. The types of involvement supported include (but are not limited to) attendance at workshops, conferences, etc., and visits of up to two weeks to work or study with longer-term participants in the Institute's programmes. Those wishing to become Junior Members should consult the Institute's web site at

**http://www.newton.cam.ac.uk/junior.html**



*An informal discussion at the NATO ASI workshop on Symmetric Functions*

# BRIMS

## *Report from the Hewlett-Packard Senior Research Fellow, Kostya Khanin*

During the past year I have been working on three different topics: Burgers turbulence, rigidity theory and multi-dimensional continued fractions. In the area of Burgers turbulence, Renato Iturriaga (CIMAT, Mexico and Newton Institute) and I have constructed a theory of stationary solutions and invariant measures for the $d$-dimensional random Burgers equation. This problem is closely related to the theory of minimising trajectories for random Lagrangian systems on compact Riemannian manifolds. We managed to prove that with probability 1 in a very general situation there exists a unique globally minimising trajectory. Using this uniqueness result we have introduced the new notion of topological shocks. The presence of the topological shocks is an unavoidable feature of stationary solutions to the random Burgers equation. From the physical point of view topological shocks correspond to the interaction between shock waves and the boundary conditions. The structure of shocks reflects the topology of the manifold. This structure is especially rigid in the 2-D case. For example, in the case of the two-dimensional torus which corresponds to periodic boundary conditions, topological shocks form a hexagonal tiling of the plane. If considered on the torus alone the topological shocks are formed by two triple points connected by three lines (see Fig). The statistics of the topological shocks were also studied numerically in a joint research project with Jeremie Bec (Observatoire de la Côte d'Azur, Nice).

My research in rigidity theory was concentrated around circle homeomorphisms with break singularity. In a joint paper with a PhD student, Dmitri Khmelev, we proved that in the case of rotation numbers with periodic continued fraction expansion, any two homeomorphisms with the same type of break singularity are smoothly conjugate to each other. The condition on rotation number is a bit restrictive and we hope to extend this result to a much wider class of irrational rotation numbers.

The third topic – multi-dimensional continued fractions – has become a very active research area in the last few years. This interest is mostly motivated by the applications of continued fraction algorithms in many fields varying from number theory to dynamical systems. Our main result is connected with a particularly symmetric algorithm which we call the $n$-dimensional Gauss transformation. In a joint article with David Hardcastle (Heriot-Watt University) we have proved that in the three-dimensional case the algorithm is strongly convergent almost everywhere.
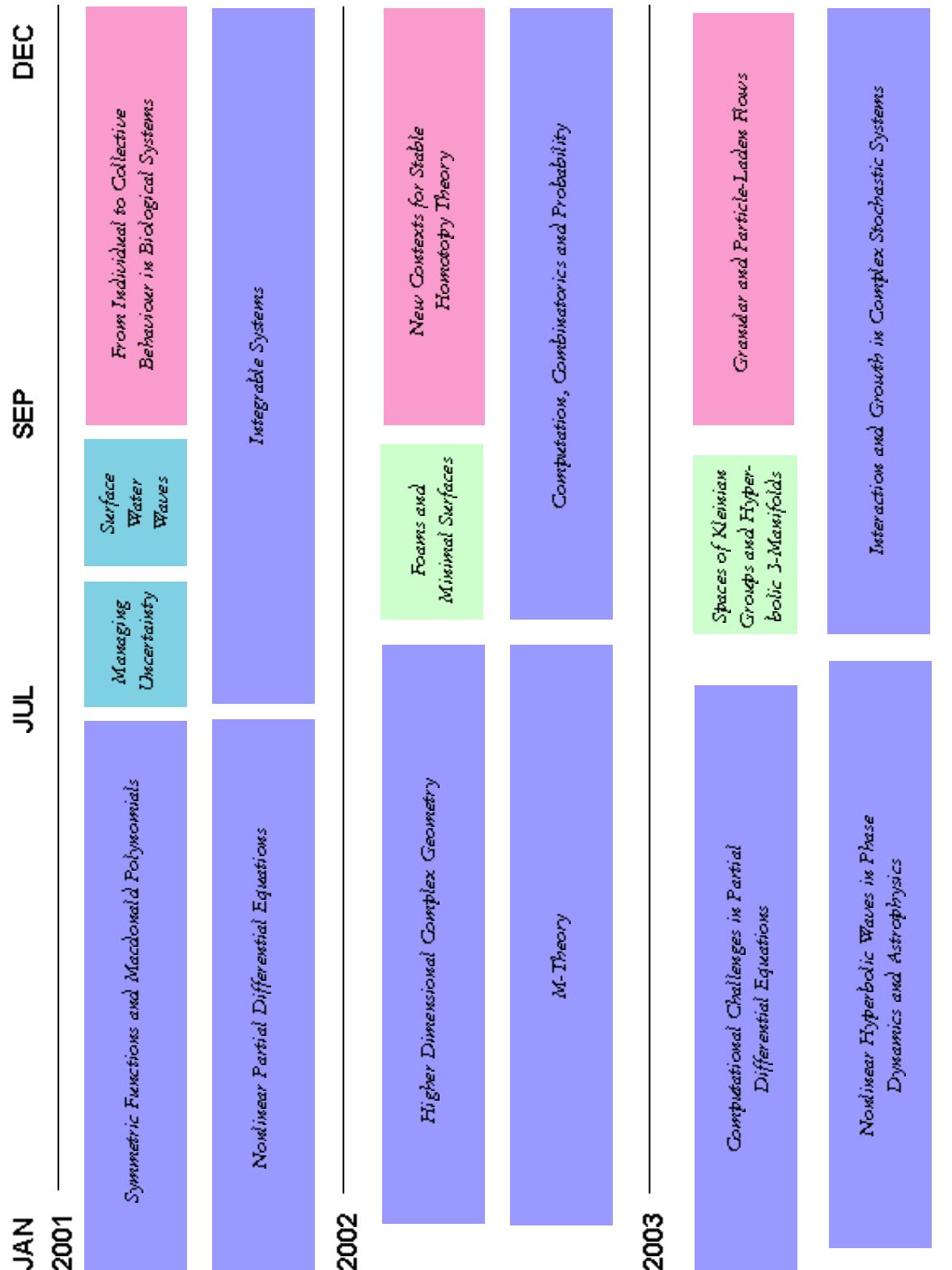
This is the first rigorous result on strong convergence in dimension greater than two. It is interesting to mention that a similar result in dimension two was proven by REAC Paley and HD Ursell in 1930 (Proc. Cambridge Philos. Soc. **26**, 127-144).
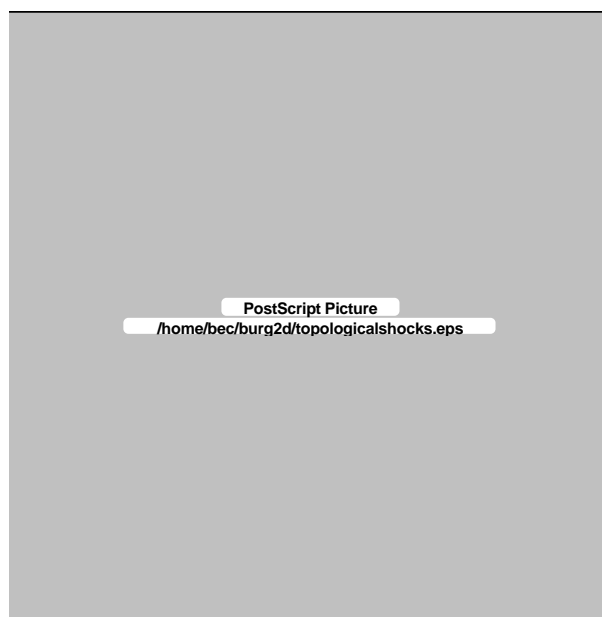


*Numerical simulations of topological shocks on a two-dimensional torus*

# Future Programmes

The diagram below shows the forthcoming programmes which have been selected by the Scientific Steering Committee. To participate in a workshop, registration is required. For longer-term participation in a programme, an invitation is usually required, and applications are best made to the programme organisers in the first instance. Full details of how to participate in programmes and workshops can be found on the Newton Institute website at **http://www.newton.cam.ac.uk/programs**



Timeline diagram of future programmes (2001–2003):

**2001**
- Symmetric Functions and Macdonald Polynomials (JAN–JUL)
- Managing Uncertainty
- Surface Water Waves
- From Individual to Collective Behaviour in Biological Systems (SEP–DEC)
- Nonlinear Partial Differential Equations
- Integrable Systems

**2002**
- Higher Dimensional Complex Geometry
- M-Theory
- Foams and Minimal Surfaces
- New Contexts for Stable Homotopy Theory
- Computation, Combinatorics and Probability

**2003**
- Computational Challenges in Partial Differential Equations
- Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics
- Spaces of Kleinian Groups and Hyperbolic 3-Manifolds
- Granular and Particle-Laden Flows
- Interactions and Growth in Complex Stochastic Systems

# Management Committee

The Management Committee is responsible for overall control of the budget of the Institute, and for its short-term and long-term financial planning. The Director is responsible to the Management Committee, which provides essential advice and support in relation to fund-raising activity, employment of staff at the Institute, appointment of organisers of programmes, housing, library and computing facilities, publicity, and general oversight of all activities of the Institute.

Its aim is to facilitate to the fullest possible extent the smooth and effective running of the visitor research programmes of the Institute and all related activities. The Committee is especially concerned with the interactions between the Institute and its funding bodies, particularly the UK Research Councils, Cambridge University, the Cambridge Colleges, the London Mathematical Society, the Leverhulme Trust, and others. It generally meets three times a year.



*Members of the Management Committee present on 18 June 2001. Left to right: FP Kelly, M Rees, RE Hunt, NJ Hitchin, WBR Lickorish, AEA Rose, HK Moffatt, PMH Wilson, WJ Fitzgerald, PT Johnstone, J Brindley, TJ Pedley, CT Sparrow*

# Scientific Policy Statement

From its inception, it has been intended that the Newton Institute should be devoted to the Mathematical Sciences in the broad sense. In this respect the Institute differs significantly from similar institutes in other countries. The range of sciences in which mathematics plays a significant role is enormous, too large for an Institute of modest size to cover adequately at any one time. In making the necessary choices, important principles are that no topic is excluded *a priori* and that scientific merit is to be the deciding factor.

One of the main purposes of the Newton Institute is to overcome the normal barriers presented by departmental structures in Universities. In consequence, an important, though not exclusive, criterion in judging the 'scientific merit' of a proposed research programme for the Institute is the extent to which it is 'interdisciplinary'. Often this will involve bringing together research workers with very different backgrounds and expertise; sometimes a single mathematical topic may attract a wide entourage from other fields. The Institute's Scientific Steering Committee therefore works within the following guidelines:

(a) the mixing together of scientists with different backgrounds does not *per se* produce a successful meeting: there has to be clear common ground on which to focus;

(b) each programme should have a substantial and significant mathematical content;

(c) each programme should have a broad base in the mathematical sciences.

Research in mathematics, as in many other sciences, tends to consist of major breakthroughs, with rapid exploitation of new ideas, followed by long periods of consolidation. For the Newton Institute to be an exciting and important world centre, it has to be involved with the breakthroughs rather than the consolidation. This means that, in selecting programmes, a main criterion should be that the relevant area is in the forefront of current development. Since the Institute's programmes are chosen two to three years in advance, it is not easy to predict where the front line will be at that time. The best one can do is to choose fields whose importance and diversity are likely to persist and to choose world leaders in research who are likely to be able to respond quickly as ideas change.

Although the novelty and the interdisciplinary nature of a proposed programme provide important criteria for selection, these must be subject to the overriding criterion of quality. With such a wide range of possibilities to choose from, the aim must be to select programmes which represent serious and important mathematical science and which will attract the very best mathematicians and scientists from all over the world. However, the Institute is receptive also to proposals of an unorthodox nature if a strong scientific case is made.

Although the Institute operates on a world-wide basis and contributes thereby to the general advancement of mathematical science, it must also be considered in the context of UK mathematics. A natural expectation of all those concerned is that each programme will be of benefit to the UK mathematical community in a variety of ways. If the UK is strong in the field, UK scientists will play a major part in the programme; if the UK is comparatively weak in the field, the programme should help to raise UK standards. Instructional courses, aimed primarily at younger researchers and research students, will play a vital role here.

Because of the wide base of support for the Newton Institute in the EPSRC and elsewhere, the Institute's programmes shall as far as possible represent an appropriate balance between the various mathematical fields. In order to retain the backing of the mathematical and scientific community, the Institute will run programmes over a wide range of fields and, over the years, achieve this balance. Such considerations, however, are secondary to the prime objective of having high quality programmes.

# Scientific Steering Committee

The Institute invites proposals for research programmes in any branch of mathematics or the mathematical sciences. The SSC meets in April and October each year to consider proposals for programmes (of 4-week, 4-month or 6-month duration) to run two or three years later. Proposals to be considered at these meetings are submitted by 31 January or 31 July respectively. Successful proposals are usually developed in a process of discussion between the proposers and the SSC conducted through the Director, and may well be considered at more than one meeting of the SSC before selection is recommended. Proposers may submit a 'preliminary' proposal in the first instance with a view to obtaining feedback from the SSC prior to the submission of a full 'definitive' proposal.

The scientific planning and organisation of each programme are the responsibility of a team of three or four Organisers (aided in some cases by an Advisory Committee). The Organisers recommend participants in the programme, of whom up to twenty can be accommodated at any one time; they also plan short-duration workshops and conferences within the programme, to which many more participants may be invited. Each programme is allocated a budget for salary support, subsistence allowances and travel expenses.

The following members of the Scientific Steering Committee stepped down at the end of their term of service on 31 December 2000:

- Professor AFM Smith FRS (Queen Mary and Westfield College)
- Professor A Newall (Warwick)
- Professor R Anderson FRS (Imperial)

The following new members were elected:

- Professor TCB McLeish (University of Leeds)
- Professor JR Whiteman (Brunel University)



*Prof NJ Hitchin,*
*Chair of the Scientific Steering Committee*

**Membership of the Scientific Steering Committee at 30 June 2001 was as follows:**

| | |
|---|---|
| Professor NJ Hitchin FRS (Chair) | University of Oxford |
| Professor HK Moffatt FRS (Secretary) | Director, Newton Institute |
| Professor RH Dijkgraaf | University of Amsterdam |
| Professor CM Elliott | University of Sussex |
| Professor WT Gowers FRS | University of Cambridge |
| Professor AJ Macintyre FRS | University of Edinburgh |
| Professor TCB McLeish | University of Leeds |
| Professor MA Moore FRS | University of Manchester |
| Professor EG Rees | University of Edinburgh |
| Professor J Stark | University College London |
| Professor S White FRS | Max-Planck-Institut für Astrophysik |
| Professor JR Whiteman | Brunel University |
| Professor D Zagier | Max-Planck-Institut für Mathematik |

# Free Boundary Problems in Industry

## 17 July to 4 August 2000

### *Report from the Organisers: N Barton (Sydney), EJ Hinch (Cambridge), JR Ockendon (Oxford)*

### Reprinted with kind permission from *SIAM News*

It was a bold step for one of the bastions of theoretical mathematical research – the Isaac Newton Institute, in Cambridge, England – to focus its first-ever short programme on mathematics-in-industry. The bare facts are that about 70 researchers, all of an intensely collaborative nature, came together for each of three one-week sessions to discuss free boundary problems in the food, glass, and metals industries. Free boundary problems had been selected because of their proven ubiquity in industrial research, coupled with their mathematical fascination. What emerged were some dramatic revelations about the insights that all kinds of mathematics may soon be able to offer all branches of industry, ranging from high-tech to low-tech.

### *New, Unstructured Study Group Format*

Many frameworks have been devised to bring mathematics to bear on industrial problems as effectively as possible. Well-established clinics, study groups, and workshops are held every year around the world, all geared to moving academic expertise across the chasm into industry in order to solve specific problems. The very act of posing an industrial problem in mathematical terms has been shown, time and again, to lead to new insight and new mathematics.

The Newton programme had quite a different format – each of the week-long sessions was an unstructured coming together, catalysed by a handful of overview presentations by industrialists on Monday, followed by three days of spontaneous open discussions. The output was an astonishing list of topics considered ripe for further specific studies, compiled on Friday (and available at

**http://www.newton.cam.ac.uk/programs/ old_progs/FBP/**).

Suffice it to say here that intense debates were generated on topics ranging from the modelling of mastication, especially so-called bolus formation, to the 'fictive temperature' of a solidifying glass, and from the extrusion of molten chocolate to the hot tearing of mushy alloys.

There were two particularly eye-catching examples from the food industry. The first, illustrated in Figures 1-4, concerns the manufacture of



*Figure 1. Top view of half-crumpet*



*Figure 2. Horizontal slice at one-third height from base*

crumpets. ('Crumpet' is one of those words that mean different things in different places, but it definitely does not mean 'English muffin'.) Like many breads, cakes, and biscuits, a crumpet is formed from dough that has been suitably kneaded and proved. (Why dough needs to be kneaded and proved, explained at the microstructural level, is a mathematical research topic in itself.)
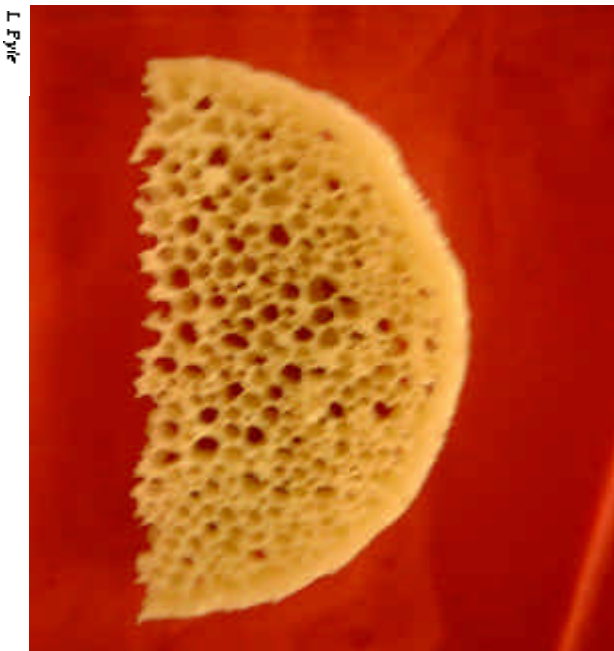


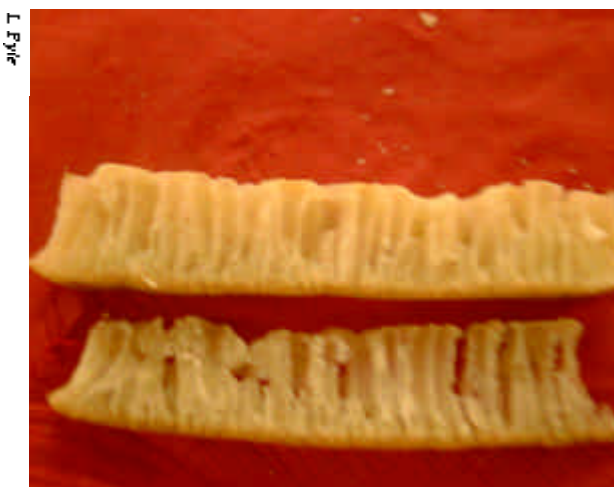*Figure 3. Horizontal slice at two-thirds height from base*



*Figure 4. Vertical slices of two crumpets: When the columns fail to reach the top the crumpet is said to be 'blind'*

In baking, most dough turns into a bubbly mixture, as in bread, with the solidified dough forming a more or less connected matrix. Crumpets, however, are heated in a special way, on a hot plate at 230°C. After a few seconds, crumpet dough takes on the 'columnar' morphology seen in Figure 4. To predict the conditions under which such fingering occurs – what a challenge for free boundary theorists! The difference in topology between a crumpet and bread is all important: Without its high genus, the crumpet would not be penetrated so easily by molten butter and jam, and it would lose all its distinctiveness.

A second, equally mouth-watering problem concerns the 'enrobing' of cake, biscuits, or ice cream with a thin layer of chocolate that is initially molten. Chocolate is a wonderful material that can solidify into at least six phases, and yet the only phase that is nice to eat is meta-stable! Fortunately, molten chocolate can be thought of as a Newtonian liquid, and we are led to the free boundary problem shown in Figure 5 for a biscuit.

We need to solve the slow flow equations

$$0 = -\nabla p + m\nabla^2 u + rg, \quad \nabla \cdot u = 0 \qquad (1)$$

for the pressure $p$ and velocity $u$ in the flow region $\Omega$, where $m$ is the chocolate viscosity, $r$ the density, and $g$ gravity, with no slip conditions on B; crucially, there is no traction on the free surface $\partial\Omega$, which is also a material surface. Discussions of similar problems with circular symmetry [1] have revealed an interesting nonuniqueness in the nearly unidirectional flow that occurs towards the base of B. But the most important prediction concerns the thickness of the chocolate at the 'shoulder' of B: No one will buy your biscuits if the coating at the shoulder is too thin.

The key to the diversity and novelty of the Newton programme was the ability of the academics and industrialists to confront each other on an equal footing, with everyone free to probe and enunciate questions of philosophy and strategy among themselves. At one time, for example, we had rigorous thermodynamicists arguing with expert chocolate manufacturers; such interactions created

as much astonishment as fervour. This eclectic approach had one dramatic consequence. Unlike settings in which specific problems are addressed, and in which it is mutually advantageous for industrial and academic researchers to work together, the Newton programme, lacking a common goal, allowed such frankness that wild disagreements were common. Even well-hardened study group veterans found themselves becoming intellectually overstressed!

The academics present had a rare glimpse into how industrialists select problems that they consider suitable for the attention of academics, and the industrialists were able to see the full scope and capabilities of modelling and analysis. It was these revelations to both groups that allowed the overwhelming plethora of opportunities to emerge as candidates for future targeted research.
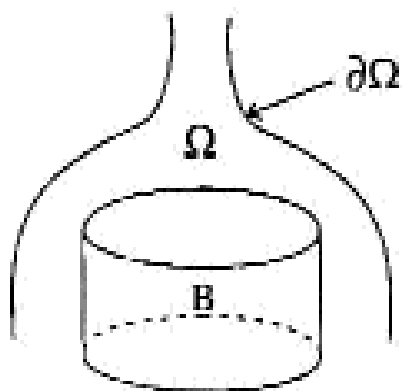


*Figure 5. Yet another example of a free boundary problem from the food industry: the coating of a biscuit with molten chocolate*

## Challenge to the Mathematics Community

One overriding implication surprised us all. If the mathematics community is ever to fulfil its potential as a source of new insights into industrial problems (rather than leaving industrial researchers to do their own mathematics), its members must learn to recognise that mathematics is everywhere, underlying activities that range from eating to flying. The task is so vast that mathematical scientists must learn to group their

activities in as manageable a way as possible if they are to avoid wasteful duplication of effort and present a coherent picture to students, politicians, and others. It is a tough job for a mathematician, let alone a politician, to overview an activity that encompasses so much of science – especially when it is so unstable to small modifications to well-established models.

Take traffic flow, for example. A crude way to model dense flows is to adopt the kinematic wave approach [2], where the vehicle velocity $u$ and flux $ru$ are related by the conservation law

$$\partial r/\partial t + \partial/\partial x\,(ru) = 0 \qquad (2)$$

It is common to model driver reaction with a decreasing function $u(r)$, say $u = 1 - r$, where $0 < r < 1$. At a recent Study Group in Shanghai, the traffic was bicycles (and thousand-foot traffic jams were commonplace at right turns). How do you control such traffic so as to minimise congestion? (When cars and bikes are interacting, the situation is even more of a nightmare to model.)

Colin Please and Alistair Fitt came up with the following neat idea for the case in which a fraction $l$ of the cyclists want to turn right. Write $u = u^*(1 - r)$, where the constant $u^* = 1$ on the highway and $u^* = a < 1$ at the turn. The maximum flux around the turn is then $a/4$ (the flow is 'choked' in gas dynamics parlance). Thence, a simple conservation-of-mass argument shows that jams will occur whenever $l > a/4$. There seem to be many generalizations of this kind of argument.

A few years ago, a Japanese Study Group in Kobe considered an equally fascinating transportation problem. Residents living near tunnel mouths on the Shinkahsen line experience house-rattling shock waves a few seconds before the emergence of each 200-mph train. This is another new problem for experts in hyperbolic PDEs – with the twist in this case coming from the effect of wall drag on tunnels longer than a mile or so.

One modelling approach is to modify the traditional inviscid gas dynamics equations, comprising (2) together with conservation of

momentum

$$\partial/\partial t\,(\boldsymbol{r}u) + \partial/\partial x\,(p + \boldsymbol{r}u^2) = 0 \qquad (3)$$

and energy

$$\partial(\boldsymbol{r}(e + \tfrac{1}{2}u^2))/\partial t + \partial(\boldsymbol{r}u(e + \tfrac{1}{2}u^2))/\partial x$$
$$+ \partial(\boldsymbol{r}u)/\partial x = 0 \quad (4)$$

by inserting a term proportional to $-\boldsymbol{r}u|u|$ on the right-hand side of (3). This term, a crude 'Fanno' approximation for the wall drag exerted on the air pushed ahead of the train, is important only over relatively long distances. The accurate incorporation of this term into asymptotic and numerical solutions continues to challenge applied mathematicians in the UK and Japan.

To get back to the Newton programme, and another class of problem altogether, the workshop revealed a beautiful inverse problem concerning the manufacture of windshields. In the 'sag' process, the wind-shield frame is positioned more or less horizontally, and an initially flat glass sheet is lowered onto the frame. Heaters are placed to make the sheet sag to a shape specified by the auto manufacturer.

In the simplest model, the windshield can be thought of as a viscous plate (or, better, as a viscoelastic shell) whose bending stiffness is a function of temperature, and hence of position, which is to be determined. Thus, we need to find the coefficients of an elliptic PDE with simply supported boundary conditions whose solution is as close as possible to a prescribed function. But this is not just a hyperbolic problem for the bending stiffness – near the corners of the frame, the windshield can easily 'lift off', thereby losing its convexity (even a 1-mm lift-off can be a disaster!). Hence, we have a novel contact problem embedded in the inverse problem. And what if the auto designer were to demand that the windshield be developable...?

With all these problems awaiting mathematical attention, how is our community to respond? In particular, how can we as mathematicians broaden our range of activities without diluting our knowledge so that we can make the contributions of which we are uniquely capable? This is a difficult question whose answer will depend on many factors. A crucial one will be the inevitable observation that mathematics-in-industry fares much better in some mathematics communities than others. The Newton meeting would not have revealed what it did had we not benefitted from a hard core of study group old hands from the UK and Europe, all of whom were used to entering into discussions on practical problems in an untrammeled way.

Of course, mathematicians all over the world engage in mathematics-in-industry on a more or less one-to-one level. Researchers involved in these time-consuming and academically low-profile activities often pay a heavy price – a lack of publications and a sense of ostracism – even though the ideas are every bit as exciting and important as those in traditional academic activity. But if these researchers remain voices in the applied mathematics wilderness, what hope is there for the wider realisation that the Newton programme revealed?

## References

[1] EO Tuck, M Bertwick, J Van der Hoek, *The Free Boundary Problem for Gravity-driven Unidirectional Viscous Flows*, IMA J. Appl. Math. **30** (1983), 191-208

[2] GB Whitham, *Linear and Nonlinear Waves*, Wiley, New York, 1974

# Quantized Vortex Dynamics and Superfluid Turbulence

## 7 August to 25 August 2000

### *Report from the Organisers: CF Barenghi (Newcastle), RJ Donnelly (Oregon), WF Vinen (Birmingham)*

The subject of the programme was primarily superfluid turbulence and its explanation in terms of quantum vortex dynamics. As such it attracted researchers working in a number of fields: the superfluidity of helium-4 and of helium-3, superconductivity, equations of motion, quantized vortex dynamics and simulations of the dynamics, applications of the NLSE (Nonlinear Schrodinger Equation) to problems in helium-4 such as reconnection, and NLSE as a model of the Bose-Einstein Condensate (BEC) under various conditions. We were encouraged to find that perhaps 25 of the attendees could be classified as 'young' compared to some of us who have been active for many years. Many professions were represented too: low temperature experimentalists and theorists, applied mathematicians and even one cryogenic engineer who undertook to let us know what is needed on the applied side. All of this worked to make a very exciting three weeks, and we trust that collaborations begun here will acknowledge the crucial role of the Institute in carrying all this out.

A number of exciting new developments were reported and they are listed below, in no particular order. Others would undoubtedly make a different list, for much new material was presented. At the same time many unsolved problems were identified, and progress towards their solution will have been greatly accelerated by discussions at the Institute.

1. Recognition that vortex reconnections are important for

- the dynamics of the quantum vortex tangle; and
- the emission of excitations (phonons and protons) from reconnecting vortices.

The emission of phonons has been observed in simulations (Tsubota, Adams) and may be relevant to dissipation of kinetic energy at very low temperature (still need to quantify it). Phonon emission may also come from Kelvin waves. The emission of excitations may be detected experimentally, ideally in a way that measures the excitation energy.

2. Helium II offers a huge dynamical range that challenges classical turbulence models and tests fundamental turbulence theories (Oregon swept grid, convection in helium gas).

3. The NLSE is an approximate model for helium II, but realistic for a trapped BEC. Current work with NLSE is focused on

- numerics: 3-D work is now possible in complex situations such as turbulence (Nore) and vortex-ion interaction;
- analysis: vortex nucleation, studied mathematically using bifurcation theory; and
- graphics: advanced 3-D time dependent visualisation.

4. Experiments on trapped BECs now address vortex systems. All issues studied in the past in helium II and some new ones, from nucleation to interaction to dissipation to perhaps turbulence (experiments at ENS), are now available in new BEC systems which have the advantage of more controlled experimental conditions (boundaries, etc.) and the great sophistication of atomic physics. In this context the NLSE is a realistic model. An entirely new area of quantized vortex dynamics is available now for investigation.

5. Helium II turbulence at low temperatures $T$: Svistunov and Vinen's conjectures of the importance of Kelvin waves have been demonstrated by Kivotides (evidence of a turbulent cascade which scales like $k^{-1}$). Classically this is a new interesting type of cascade. The role of small vortex rings in the decay of helium II turbulence at low temperatures needs to be clarified.

6. New tools for self-consistent calculation of flows in the normal and superfluid components, taking account of the mutual friction from vortices which couple them, are now available to study helium II turbulence at $T > 1$ K (Kivotides, Idowu).

7. Partly owing to the previous point, more attention is being paid to the normal fluid which has been a mystery in the past (due to lack of direct flow visualisation.) Now Particle Imaging Velocimetry (PIV) is being implemented in a cryostat designed by Vinen at Yale. Extension of PIV to helium II is an exciting possibility but presents many challenges.

8. The NLSE is being made more realistic for helium II.

- The nonlocal model discussed by Roberts includes rotons.
- Adams couples NLSE to hard sphere gas to model the normal fluid (ballistic).
- Griffin now has NLSE with hydrodynamic limit of normal fluid.

9. There is new interest in the relationship between the topology (broken by reconnections, hence release of energy) and the geometry of structure, which cannot be changed arbitrarily as done traditionally by topologists but changes according to the dynamics (Biot-Savart law, NLSE or Navier-Stokes).

10. Experiments suggest that for dense vortex line tangles the two fluids tend to move with the same velocity on length scales large compared to the vortex line spacing (that is, there are no signs of a temperature dependence, which characterises the Landau two-fluid model). Some theoretical justification for this idea exists (Vinen), but more rigorous arguments are required.

11. Helium II with vortices as a turbulent fluid: new derivations of the HVBK equations were given for the macroscopic hydrodynamic description of superfluid helium turbulence with quantum vortices (Holm). These derivations cast light on the energetics of these flows and the parallels of their mathematical structure with that of turbulence models for normal fluid – including 'eddy viscosity'. New applications of the HVBK equations in numerical simulations highlight the non-classical behaviour of their solutions in Couette flow.

12. Classical turbulence experts (Tsinober and others) provided a valuable comparison between the physics of Navier-Stokes and helium II turbulence. Many physical concepts and mathematical tools from classical turbulence are still underutilized in superfluid turbulence research. Two specific suggestions made were that higher order statistics be measured in superfluid turbulence, and that the characterisation of even the random vortex tangle by a single length scale may be inadequate, when one considers the vorticity amplification processes of Eulerian fluid mechanics.

13. Experiments being performed on the nucleation of individual vortices in helium-4 down to millikelvin temperatures suggest both thermally-assisted and quantum tunnelling nucleation mechanisms (Packard, Avenel & Varoquax).

14. Numerical simulations of vortex nucleation by spherical objects (ions) have been made in the NLSE framework by Berloff, Adams and co-workers. The calculated critical velocities take into account the structure of the relevant very small vortex configurations. But they relate to those at which the energy barrier (see the previous point) disappears, and they are therefore too high; calculations that identify the height and width of the barrier need to be carried out. The evolution of existing vortices has been simulated in 3-D using the NLSE by Brachet, Nore and their colleagues.

15. Koplik has discussed vortex reconnections in the framework of the NLSE, giving valuable insight into the way this process might proceed.

16. Sonin and Thouless discussed the forces that act on a moving quantized vortex; earlier controversies are now being resolved.

17. The experimental study of quantum turbulence in superfluid helium-3 is now appearing as a realistic possibility (Fisher), and it could contribute in an important way to our understanding of such turbulence.

18. Similarities between the behaviour of vortices in a Kosterlitz-Thouless transition and in two-dimensional turbulence were explored (Williams).

# Singularity Theory

## 24 July to 22 December 2000

*Report from the Organisers: VI Arnold (Moscow and Paris IX), JW Bruce (Liverpool), V Goryunov (Liverpool), D Siersma (Utrecht)*

## *Scientific Background and Objectives*

Singularities arise naturally in a huge number of different areas of mathematics and science. As a consequence Singularity Theory lies at the crossroads of the paths connecting the most important areas of applications of mathematics with its most abstract parts. For example, it connects the investigation of optical caustics with simple Lie algebras and regular polyhedra theory, while also relating hyperbolic PDE wave fronts to knot theory and the theory of the shape of solids to commutative algebra.

The main goal in most problems of singularity theory is to understand the dependence of some objects of analysis and geometry, or phenomena from physics or some other science, on parameters. For generic points in the parameter space their exact values influence only quantitative aspects of the phenomena, their qualitative, topological features remaining stable under small changes of parameter values.

However, for certain exceptional values of the parameters these qualitative features may suddenly change under a small variation of the parameter. This change is called a perestroika, bifurcation or catastrophe in different branches of the sciences. A typical example is that of Morse surgery, describing the perestroika of the level variety of a function as the function crosses through a critical value. (This has an important complex counterpart: the Picard-Lefschetz theory concerning the branching of integrals.) Other familiar examples include caustics and outlines or profiles of surfaces obtained from viewing or projecting from a point, or in a given direction.

In spite of its fundamental character and the central position it now occupies in mathematics, singularity theory is a surprisingly young subject. So, for example, one can consider the singularities arising from the orthogonal projections of a generic surface in 3-space, a problem of surely classical interest. Their classification was completed as recently as 1979. In one sense singularity theory can be viewed as the modern equivalent of the differential calculus, and this explains its central position and wide applicability. In its current form the subject started with the fundamental discoveries of Whitney (1955), Thom (1958), Mather (1970) and Brieskorn (1971). Substantial results and exciting new developments within the subject have continued to flow in the intervening years, while the theory has embodied more and more applications.
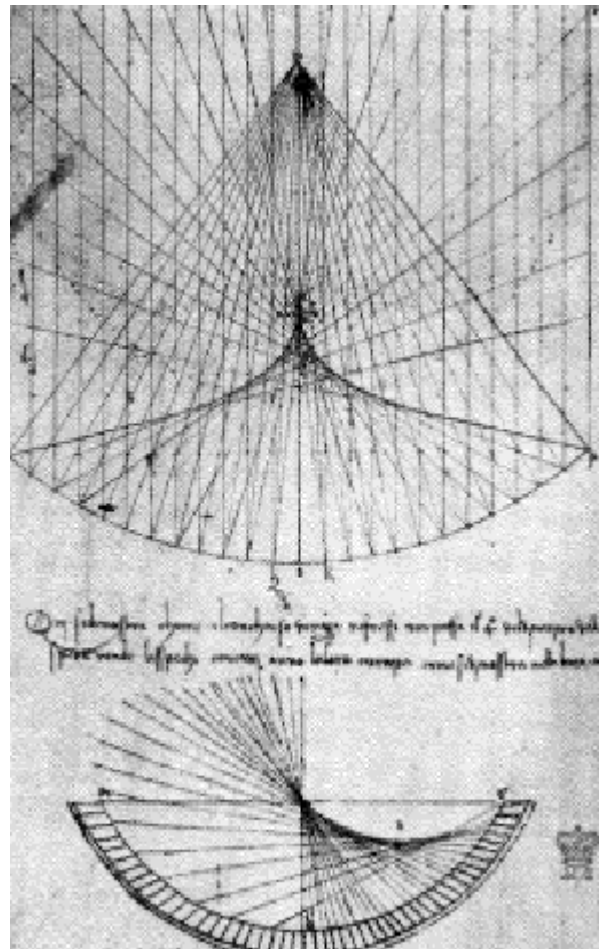


*The first known drawing of a caustic, by Leonardo da Vinci c. 1508*
*(British Library: codex Arundel 263, folio 87v)*

The major idea of this programme was to bring together experts within the field and those from adjacent areas where singularity theory has existing or potential application, such as wave propagation, dynamical systems, quantum field theory, differential and algebraic geometry, and others. It was the programme's aim both to foster exciting new developments within singularity theory, and also to build bridges to other subjects where its tools and philosophy prove useful.

## Organisation

The programme was planned by Arnold, Bruce and Siersma. Unfortunately, Arnold was not able to attend the programme at all due to his recent cycling accident. The day-to-day organisation was carried out by Goryunov and Siersma.

There was very valuable assistance from Zakalyukin, Wall, Giblin and Nikulin for specific workshops during the programme.

In addition to the main meetings, the programme ran seminars on a regular weekly basis on Tuesday and Thursday afternoons, with at least 2 talks in each session. There were also extra, occasional seminars arranged by the organisers and other participants.

Two of the participants gave lecture series on special subjects. From September to October, a series of 9 lectures on Frobenius manifolds was given by B Dubrovin. Frobenius manifolds are now finding numerous applications in physics. However, they are a rather new (but highly natural) topic for Singularity Theory. In November, another series of lectures, on ramified integrals, was given by VA Vassiliev, devoted to generalisations of works by Newton and Atiyah. Both lecture series attracted excellent audiences.

In December the frequency of talks increased and the penultimate week was run on a full conference scale, with up to 5 talks daily.

N Nekrasov, AN Varchenko, VA Vassiliev and VV Goryunov spoke in a two-day Discussion Meeting on *Topological Methods in the Physical Sciences*

held at the Royal Society in November. The meeting was organised together with the parallel programme on *Geometry and Topology of Fluid Flows*, and brought together leading pure and applied mathematicians from the UK and other countries. It was attended by many British specialists and was a great success.

## Participation

The programme was attended by 53 long-term and 87 short-term visitors. Some of them came to the Institute several times. In addition, there was very strong participation in the conferences, workshops and other events during the programme. Dubrovin, as Rothschild Visiting Professor, made a remarkable contribution to the programme during his stay.

The majority of experts in the field attended the programme, and a large number of leading experts in adjacent areas visited the Institute. There was a strong presence from Europe and Russia (the latter allowed by the generous support from the Leverhulme Trust), as well as from North America.

Many of the meetings and individual talks, plus special lecture courses, attracted mathematicians from both Cambridge and other British universities. The EU support for two of the meetings and the Junior Membership scheme were highly useful for attendance by PhD students and young postdocs. A number of participants gave talks in other UK departments, such as Edinburgh, Hull, Leeds, Liverpool and Warwick.

## Meetings and Workshops

### NATO Advanced Study Institute / EC Summer School: *New Developments in Singularity Theory,* 31 July - 11 August 2000

Organisers: D Siersma, VM Zakalyukin, JP Brasselet, VA Vassiliev and CTC Wall

This first meeting was designed to study developments in singularity theory, especially during the last 5 years. Its key topics included

- Singular complex varieties: global theory

- Singular complex varieties: local theory

- Singularities of holomorphic maps

- Singularities of real maps

- Study of discriminant spaces and Vassiliev-type invariants

The primary purpose of the ASI was the introduction of new developments in singularity theory to a broader audience, thus establishing new contacts and advancing a broad front of research. The importance of pedagogical skills had been borne in mind in choosing the ASI lecturers, as the instructional aspect was regarded as central to the success of the meeting.

GM Greuel, A Fruehbis-Krueger and C Lossen (all from Kaiserslautern) demonstrated the latest version of the computer package SINGULAR which the group in Kaiserslautern has been developing over the last decade for the specific needs of singularity theory. The meeting finished with a problem session. The forum was remarkably large and served as an excellent start to the entire programme: the total number of participants exceeded 95, many of them younger mathematicians. This was made possible thanks to the combined generous support from NATO and the EU. A volume of proceedings is now being published by Kluwer.
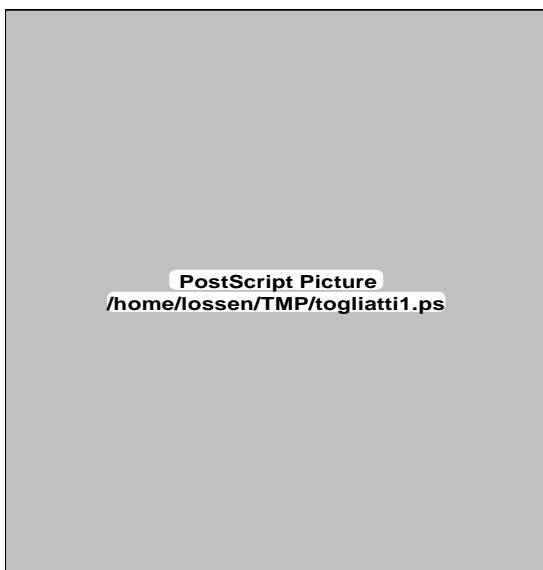
Some of the ASI participants stayed on at the Institute for one week to attend the mini-workshop on polynomial and meromorphic functions conducted by A Dimca.

## Workshop on *Applications to Wave Propagation Theory and Dynamical Systems*, 25 - 29 September 2000

Organisers: RM Roberts and VM Zakalyukin

The topics of the second meeting included:

- Singularities in symplectic and contact spaces: caustics and wave fronts, shock waves, Hamilton-Jacobi equation, Lagrangian intersections, Legendre knots.

- Applications to control theory and differential equations, game theory. Singularities in subriemannian geometry, optimization, Pfaffian systems.

- Singularities of momentum maps, energy-momentum maps and integrable Hamiltonian systems. Monodromy in



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*Togliatti's quintic and Barth's sextic: surfaces of degrees 5 and 6 with the maximal possible numbers 15 and 65, of isolated singularities. Images created using the SURF package (University of Mainz), part of the SINGULAR package (University of Kaiserslautern).*

Hamiltonian systems, applications to physics.

- Bifurcations of (relative) equilibria of Hamiltonian systems and time-reversible equivariant dynamical systems. Singularity theory and KAM theory.

The meeting consisted of 35 lectures, with 53 official participants. It attracted many specialists in Hamiltonian systems and this provided a considerable source of new questions (including those arising from physical experiments) to which Singularity Theory is very likely to find answers.

## Euroworkshop on *Applications to Quantum Field Theory*, 23 - 27 October 2000

Organisers: B Dubrovin, V Goryunov, N Nekrasov and V Nikulin

Over the last years, new and very interesting relations have been discovered between singularity theory and algebraic geometry on one hand, and quantum field theory and string theory on the other. For example, mirror symmetry for Calabi-Yau manifolds, Gromov-Witten invariants, Frobenius manifolds and integrable systems became important tools in string theory and quantum field theory. The October Euroworkshop was devoted to further interactions between all of these areas. Supported by the EU, it brought together leading specialists and young researchers in areas very rapidly developing nowadays. It was one of the most inspiring events in the entire programme bringing a large number of new problems into the area of singularity theory. There were 24 lectures with good spacing between them allowing numerous discussions.

## Informal Discussion Meeting on *Different Aspects of Singularity Theory*, 24 - 25 November 2000

Organiser: D Siersma

The meeting concentrated on deformation theory, analysis and topology of singularities. This was a rather small scale session (21 participants), but

intensive and productive. The discussions were very stimulating and useful for establishing new links with other branches of mathematics.

## Satellite Workshop at the University of Liverpool on *Applications of Singularity Theory to Geometry*, 16 - 21 December 2001

Organisers: JW Bruce, J Damon, PJ Giblin, VV Goryunov and CTC Wall

The final event of the programme was a satellite meeting in Liverpool. Its theme was one of the most traditional in the field: much of Singularity Theory was inspired by geometrical problems. Thom's early work on differential geometry via families of functions has borne enormous fruit in a richer understanding of the higher geometry of surfaces, and this in turn has found application in other fields such as computer vision. Projections of surfaces to planes, giving apparent contours, and the general theories of caustics and wave fronts are other examples where new techniques were motivated by geometrical problems. Remarkable duality connections have been found between some of these problems, and there are applications to algebraic geometry and other mathematical fields.

This workshop took as its theme interactions between singularity theory and geometry in its many modern guises. Besides the topics mentioned above one could mention Gauss mappings, the geometry of discriminants and bifurcation diagrams, coadjoint orbits, billiards, arrangements and the global geometry of singular waves and varieties.

The number of requests for talks in Liverpool was very high and it was decided to run a week-long full-scale pre-workshop at the INI, which included a most inspiring lecture series by A Losev exposing new ideas of applying singularity theory to the study of Frobenius structures.

There were 34 talks in Liverpool itself. The meeting was supported by the LMS conference grant and a special grant from the INI. It was attended by 63 participants.
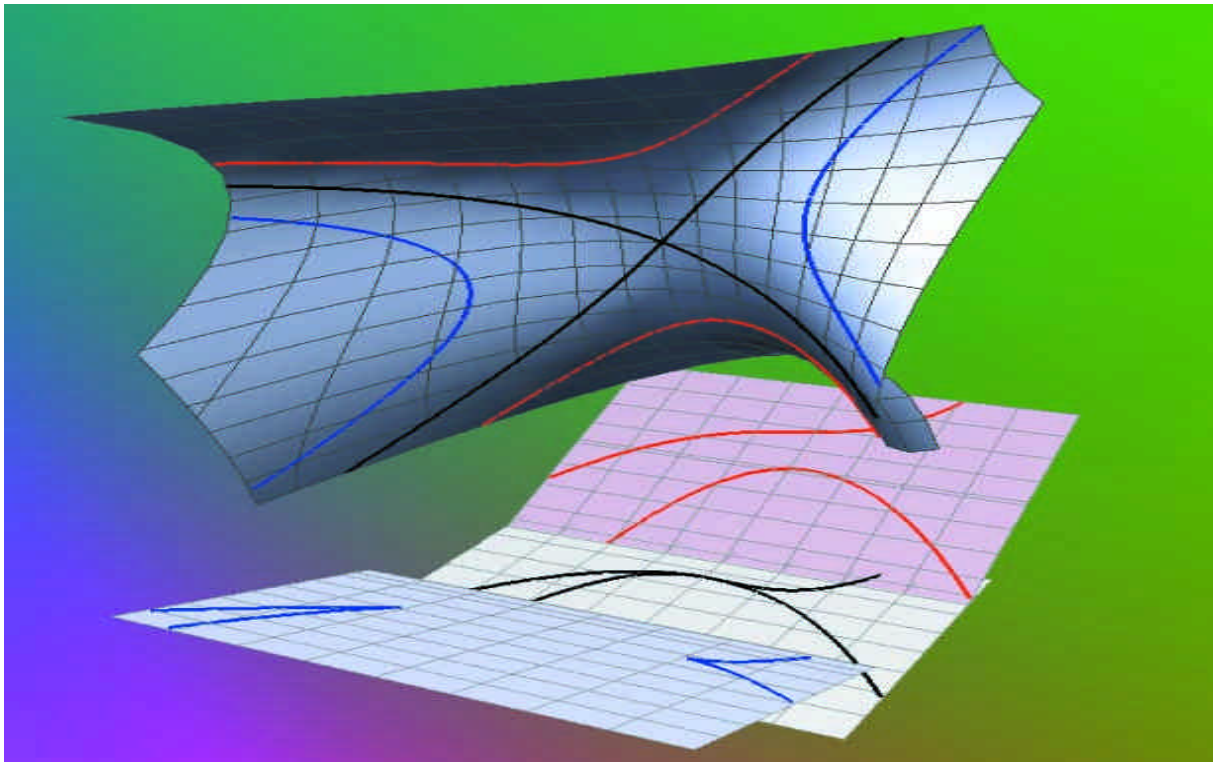
*A beaks singularity is observed when the surface is viewed from a succession of directions. The blue, black and red curves in the three planes are the 'apparent contours' of the surface viewed from three directions, which are at right angles to the respective planes.*

## *Outcome and Achievements*

The programme was highly successful. It did achieve its main goal to bring together the best researchers working on singularities and specialists in other branches of mathematics and physics where singularity theory either already is an efficient tool or can become such a tool in the future. The programme was an excellent school for its younger participants.

There was a certain amount of interaction with the participants of the parallel programme on *Geometry and Topology of Fluid Flows*. Contacts with Pelz, Ricca, Michor, Khesin, Shnirelman and others helped our participants to realise the current needs of this branch of applied mathematics.

Over the programme, its participants worked both on their long-term projects and on new problems that came to their attention during their stay. The comments of participants were uniformly positive, and all reported productive discussions and new collaborations. A high number of papers were reported as either submitted or being in preparation during the programme.

Anisov worked on Matveev's complexity of three-manifolds. He obtained an upper bound (which is very likely to be precise) for the complexity of torus fibrations over the circle.

Bogaevsky and Ishikawa studied singularities of Legendre mappings, in particular a generalisation of Mather's theory to the Legendrian case.

Buchstaber and Rees continued their joint work on symmetric products of linear spaces which is a multi-dimensional version of the Vieta theorem.

Campillo and Olivares made considerable progress in describing singular one-dimensional foliations in terms of the associated polar map.

A lot of attention during the programme was devoted to the famous Jacobian Conjecture requiring deep understanding of singularities of plane algebraic curves at infinity. Campillo, Piltant and Reguera studied singularities of the

characteristic cone of the surface obtained by blowing up the infinite points of such a curve.

Chekanov worked on the Arnold-Givental conjecture claiming that a Lagrangian submanifold, which is a fixed-point set of an anti-symplectic involution, must have sufficiently many points of intersection with its image under a Hamiltonian symplectomorphism. He succeeded in constructing a counter-example to the degenerate part of this conjecture. The result can be seen as the first step to disproving the degenerate part of Arnold's famous symplectic fixed-point conjecture, whose non-degenerate part was proven recently by Fukaya-Ono and others.

Davydov, with the involvement of Zhitomirskii, continued his study of singularities of implicit differential equations. He obtained new results in two directions: on generic singularities of higher order equations, and on normal forms of first order equations involving one dependent and many independent variables. The results are promising to be very important for applications.

Dubrovin continued his research on applications of Frobenius manifolds to integrable hierarchies and to Gromov-Witten invariants of higher genera. He also started new investigations, together with Goryunov and Kazarian, on applications of Frobenius manifolds to singularities of functions on complete intersections and on singular space curves.

Ebeling and Gusein-Zade made considerable progress in their work on an algebraic formula for the index of a differential form at an isolated complete intersection singularity.

Gaffney worked on his long-term project to give an integral closure formulation to all equisingularity conditions controlled by analytic inequalities. Together with Houston, he started a project on finding necessary and sufficient conditions for families of finitely determined map-germs from 3-space to 3-space to be Whitney equisingular. Damon, Gaffney and Mond worked on a conjecture claiming that the image of a map-germ from $n$-space to ($n$+1)-space is a free star divisor in a sense of Damon. This would yield a formula for the image Milnor number. Gaffney, Trotman and



*Participants at the NATO ASI*

Wilson finished their paper on equisingularity of sections.

Denef, Melle and Veys worked on the Monodromy Conjecture on the topological zeta-function of a holomorphic function-germ.

Kazarian obtained very impressive results on classifying spaces of singularities and Thom polynomials, on characteristic classes related to singularities of Lagrange and Legendre mappings.

Matveev obtained new strong results on the global theory of geodesically equivalent metrics.

Merkov, helped by Vassiliev and Kazarian, made considerable progress in the study of Vassiliev-type invariants of ornaments (collections of plane curves no three of which have common points) and doodles (triple-point-free collections of plane curves).

Polyak worked on Kontsevich's universal formula for a deformation quantisation of the algebra of functions on a real linear space. He succeeded in interpreting this formula as a degree of a map from a certain configuration to a multi-dimensional torus. Polyak also obtained a very elegant generalisation of the classical Crofton formula to the case of generic immersed plane curves instead of only convex ones.

Pushkar continued working on his long-term and extremely promising project on Legendrian K-theory and relative Morse theory.

Ruas, using the methods of Gaffney and Damon, made progress in the problem of finding sufficient conditions for topological triviality in families of function-germs on analytic varieties.

Scherbak and Varchenko worked on the resonance case of the problem of behaviour of critical points of products of powers of linear functions. This question turned out to be closely related to the representations of sl(2). Also Scherbak finished a paper on boundary singularities and non-crystallographic Coxeter groups.

Sedykh studied admissible homotopies of space curves, that is those not involving in particular curves with self-intersections and inflexion points. This allowed him to solve Arnold's problem on the impossibility of deforming certain curves to curves without flattening points via such homotopies.

Shapiro, Kazarian and Goryunov studied Hurwitz spaces developing an approach to calculate Hurwitz numbers in the general case of a meromorphic function with multiple complicated branch points. Shapiro also worked on understanding relations between various Poisson structures on the space of unipotent upper triangular matrices.

Tibar wrote a paper on a problem of describing vanishing cycles in non-generic Lefschetz pencils on complex and symplectic spaces. Together with Siersma, he worked on understanding deformations of polynomials at infinity.

Vassiliev finished the formulation of a purely combinatorial algorithm for calculating combinatorial formulae for knot invariants. This is an important part of his extensive programme on effective calculation of cohomology classes of spaces of non-singular geometrical objects. He wrote 4 papers and edited a translation of one of his books.

Wall and du Plessis worked on their papers on theorems of Cayley-Baeharach type and on generic projections.

Zakalyukin and Goryunov made considerable progress in their study of the monodromy in vanishing homology of families of $2 \times 2$ matrices. Together with Bortakovsky, Zakalyukin proved that local singularities in generic completely non-integrable logical-dynamical systems with a low number of switches correspond to generic corner singularities. Together with Giblin, Zakalyukin worked on classification of generic singularities of envelopes of families of chords in affine geometry.

The programme was an international event of extremely high significance for Singularity Theory and related subjects. It provided a very valuable boost of new ideas and introduced many exciting problems to solve.

# Geometry and Topology of Fluid Flows

## 4 September to 17 December 2000

### Report from the Organisers: H Aref (Urbana-Champaign), T Kambe (Tokyo), RB Pelz (Rutgers), RL Ricca (UCL)

The realization that the mathematical disciplines of topology and geometry are extremely valuable in furthering our understanding of fluid flows has been evolving steadily for many years within the fluid mechanics community. This realization was the impetus for many workshops and conferences: a 1989 IUTAM Symposium on *Topological Fluid Mechanics* held in Cambridge (UK), a 1990 NATO workshop on *The Global Geometry of Turbulence* held in Rota (Spain), a 1991 five-month programme on *Topological Fluid Mechanics* at the Institute for Theoretical Physics at the University of California, Santa Barbara, and a 1992 workshop on *Singularities in Fluids, Plasmas and Optics* in Crete.

With these collaborative precedents between pure and applied mathematicians, physicists and engineers, and the need to enhance the dialogue between these disciplines in mind, we planned and organised this programme at the Newton Institute directed towards understanding phenomena in fluid mechanics and magnetohydrodynamics (MHD) that appear to involve substantial questions of topology and geometry.

The programme brought together experts in widely varying fields to work on a set of research problems at the crossroads of fluid mechanics, modern topology, dynamical systems, analysis, and PDEs. The Newton Institute, with its first-class facilities and optimal working environment, provided an ideal location to encourage the fruitful collaboration between pure and applied sciences.

Since Poincaré's seminal work, it has been recognised that the global geometric point of view is essential for understanding Newtonian mechanics. Modern differential geometry and topology have brought new insight into mechanical and fluid-mechanical systems having symmetry and geometrical structures. Integrals of motion or conservation laws were found to be closely related to the structure of the topological space. The existence of certain integrals of the equations of MHD and inviscid hydrodynamics, for example, can be interpreted in terms of the linking of magnetic fields and vortex lines.

The initial success in using sophisticated methods from geometry and topology to solve problems in fluid mechanics suggests that at least some of these methods belong in the "tool kit" of the applied mathematician. Researchers in a number of areas of fluid mechanics and MHD are realising that an infusion of mathematical knowledge which is non-traditional in these subjects can greatly enhance their ability to understand and explain the phenomena that they observe.



*RL Ricca (right) and colleagues*

## Application Areas

A few important application areas ripe for mathematical input were addressed in the programme by bringing many experts together for collaboration over an extended period, having daily seminars and expository lectures and running numerous workshops.

The topics fall roughly into four categories: the application of dynamical systems and topology to stirring and chaotic advection, the application of differential geometry to fluid mechanics, the application of topology to magnetic and classical fluid flows, and the application of geometric and

topological concepts to the problem of regularity/blowup in hydrodynamics.

### Stirring and Chaotic Advection

The application of topological concepts to the kinematic problem of movement of a scalar field in a given flow has provided a mathematical theory for the process, created quantitative measures for mixing, and allowed a framework for design and optimization techniques for the enhancement or suppression of mixing. Numerous experts shared recent developments and developed collaborations with other participants.

### Geometry of Fluid Motion

Perhaps half of the lectures and discussions in the programme centred on the application of differential geometry to hydrodynamics and MHD. A dynamical system can be visualised as a field of vectors in a phase space, on which a solution is an integral curve. Geometrical theory then provides a means of describing the global properties of the family of solution curves, which fill up the entire phase space. The group theoretical approach to hydrodynamics considers that the fluid motion is a geodesic curve on a group of volume-preserving diffeomorphisms with right-invariant metric given by the kinetic energy. This provides a Lagrangian description of the particle motion of an ideal fluid, allowing the Lagrangian stability to be determined by the curvature of the geodesic, and unifies ideal fluid flow with other systems like Korteweg de Vries and MHD.

### Topological Ideas in Fluid Mechanics

Topological concepts have not only been applied successfully to kinematics, but have provided very physical constraints on the dynamics of ideal fluids and MHD. The knottedness of vortex and magnetic field lines is preserved in time allowing application of many concepts of knot theory to such flows. In particular, the complexity of a vortex tangle can be quantified using measures like the linking number. Energy relaxation methods based on various energy functionals have been applied to non-trivial topologies of various physical systems.

### The Hydrodynamic Regularity / Blowup Problem

One year ago, this problem was elevated to one of seven 21th century, Hilbert problems. The Clay Mathematics Institute has offered $1m for proving that the equations of hydrodynamics are regular or alternatively that spontaneous blowup can occur in finite time. Many discussions and seminars during the programme centred on this question. Numerical solutions have exposed a possible asymptotic behaviour with self-similar scaling in the inner layer. Concepts from both topology and geometry were applied. Work on the curvature and structure of the manifold was initiated.
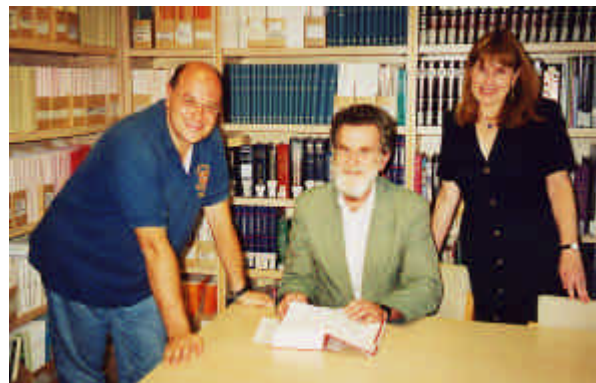


*Left to right: RL Ricca, P Drazin and S Freidlander during the NATO workshop*

## Meetings and Workshops

Four workshops were held during the programme and were specifically designed to provide either an introduction to topology and geometry and the application areas above, or a state-of-the-art assessment of recent advances in these areas.

### NATO ASI: Pedagogical Workshop on *Geometry and Topology of Fluid Flows*, 11 - 22 September 2000

Organisers: RL Ricca and A Shnirelman

This workshop was held at the beginning of the programme and, as its name suggests, was designed to introduce and provide a background in the subject areas of the programme. The topics and speakers included *Differential Geometry and Knot Theory* (Langevin, Kauffman, Weber),

*Topological Fluid Mechanics and MHD* (Berger, Khesin, Moffatt, Ricca, Spiegel), *Topological Kinematics and Advection* (Aref, Boyland), *Geometry of Fluid Flows* (Misiolek, Kambe, Shnirelman, Vladimirov) and *Finite-time Singularities* (Childress, Frisch, Hornig, Pelz). There was a poster session for all others. There were 85 researchers in attendance, many from NATO participant countries, and the proceedings will be published by Kluwer.


*Informal discussion over a glass of wine*

## Spitalfields Day: *In Search of the Ideal Knot*, 12 October 2000 with a follow-on day of talks

Organiser: RL Ricca

Six world specialists presented results and latest discoveries on mathematical and physical knots. Speakers included Stasiak (Lausanne), Niemi (Uppsala), Winfree (Arizona), Fernandez-Ranada (Complutense), Maddocks (EPFL), Sumners (Florida State) and O'Hara (Japan).

## Satellite Workshop at the University of Warwick (MIR@W) on *Singularities in Classical, Quantum and Magnetic Fluids*, 20 - 23 October 2000

Organisers: X He, R Mackay, S Nazarenko and R Pelz

Speakers included Zakharov, Kuznetsov, Pelz, Okamoto, Moffatt, Gibbon, He, Cowley, Greene, Rubenchick, Shnirelman, Stuart, Kerr, Turitsin,

Barenghi and Grauer. Funding was provided by LMS, the University of Warwick and INI.

The main aim of this workshop was to initiate an exchange of ideas and methods of studying singularities among researchers working in different fields such as classical Navier-Stokes and Euler fluids, superfluids, magnetohydrodynamics and fluid-like nonlinear properties of optical media.

## Royal Society Discussion Meeting on *Topological Methods in the Physical Sciences*, 15 - 16 November 2000

The organisers were Arnold, Bruce, Moffatt and Pelz. Speakers included Kontsevich, Nekrasov, Hannay, Varchenko, Khesin, Marsden, Misiolek, Moffatt and Holmes. This meeting was a joint effort with the concurrent INI programme, *Singularity Theory*.

The first day was primarily string theory, while the second was on dynamical systems and hence more relevant to our programme.

## BRIMS Day: *Differential Geometry in Fluid Dynamics and Dynamical Systems*, 20 November 2000 with a follow-on day of talks

The organiser was Tom Kambe. Speakers included Brenier (Paris), Fukumoto (Kyushu), Holm (Los Alamos), Kambe (Tokyo), Khesin (Toronto), Misiolek (Notre Dame), Ebin (SUNY) and Wocher (Schoedinger Inst).

# *Summary*

The main goals were to inject ideas from modern differential geometry and topology into fluid mechanics and to inspire new directions in these mathematical fields from discussions on the major problems in fluid mechanics. There was a great deal of exposure to new disciplines and interaction between participants with a large range of backgrounds. Many new collaborations started during the programme. The large number of such interactions is the main measure of its success.

# Symmetric Functions and Macdonald Polynomials

## 8 January to 6 July 2001

### *Report from the Organisers: P Hanlon (Michigan), IG Macdonald (QMW), AO Morris (Aberystwyth)*



*Left to right: IG Macdonald, S Fomin,
AO Morris, P Hanlon*

## *Scientific Background*

The overall programme was in the general area of symmetric functions and the representation theory of Hecke algebras and the symmetric groups and its generalisations, reflection groups and, in particular, Weyl groups. The importance of this subject area has been long recognised due to its applicability in a wide range of mathematical and scientific disciplines. The subject is also central to algebraic combinatorics and has a particularly distinguished history in the UK with such names as MacMahon, Young, Littlewood (DE) and Richardson regarded as the leading innovators in the field, culminating in more recent times with the breakthroughs by Ian Macdonald.

The programme was focused on Macdonald polynomials around which a great deal of these recent developments have been centred. It was in the 1980's that a series of conjectures were formulated, now known as the Macdonald constant term conjectures. These conjectures predicted the constant term of certain power series indexed by parameters related to semisimple Lie algebras. During the following two decades, these conjectures and related problems concerning Macdonald polynomials generated more activity amongst those working along the interface between algebra and combinatorics than any other specific problem.

The conjectures (and others like them) when they first appeared seemed to be isolated curiosities and it was not clear what (if anything) lay behind them. That became clear a few years later with the introduction of Macdonald polynomials. These are polynomials $P_{\lambda}(q, t)$ in several variables, depending on parameters $q$ and $t$, indexed by the dominant weights $\lambda$ for a root system $\Phi$ (partitions $\lambda$ of $n$ for the more classical case of symmetric groups). The Macdonald constant term conjectures, in their more general form, predict the value of $\langle P_{\lambda}, P_{\lambda} \rangle$. It was believed that there is some deep algebraic, topological or geometric phenomenon which has these constant term conjectures as a consequence.

The $P_{\lambda}(q, t)$ for the root system $A_{n-1}$ coincide with a number of well-studied symmetric functions under suitable evaluation or deformation of the parameters $q, t$, for example, the spherical functions for the classical symmetric spaces, the Hall–Littlewood symmetric functions and the Jack symmetric functions. Attempts to interpret the Macdonald polynomials come in different forms – for example there have been attempts to interpret the transition functions between the Macdonald polynomials and the Schur functions. These transition functions $K_{\lambda, \mu}(q, t)$ are polynomials in $q$ and $t$ : the $(q, t)$-Kostka–Foulkes polynomials.

In the intervening years, several different explanations have been discovered coming from different directions. The first method, conceived by the Dutch mathematicians Eric Opdam and Gert Heckman, proves the constant term identities inductively making use of special operators on polynomial operators on polynomial rings called *shift operators*. Ivan Cherednik showed that these shift operators are realised by the action of special elements in double affine Hecke algebras and proceeded to give the first proof. The second approach, pioneered by Phil Hanlon, interprets the two sides of the conjectures as two ways to

compute the Euler characteristic of the homology of a certain Lie superalgebra. According to this approach, the conjectures are trying to tell what the homology is and the approach leads to a series of new conjectures called the strong Macdonald conjectures.

The third approach to the Macdonald polynomials (for the root system $A_{n-1}$), developed by Adriano Garsia and Mark Haiman, involves the study of the diagonal action of the symmetric group $S_n$ on the ring of polynomials in two set of variables $\{x_1,\ldots,x_n,y_1,\ldots,y_n\}$. The Macdonald polynomials are interpreted in terms of analogues of classical results about the harmonic/invariant structure of the polynomial ring in one set of variables $\{x_1,\ldots,x_n\}$. The Garsia–Haiman conjectures concern the $S_n$-module $V_I$ obtained by applying all partial derivatives in the $x_i$ and $y_i$ to a certain polynomial $v_I$. A particularly intriguing form of their conjecture states that $V_I$, viewed as an ungraded $S_n$-module, is equal to the regular representation. In particular, this contains their celebrated $n!$ conjecture. The fourth approach seeks to interpret the Macdonald polynomials as spherical functions on an appropriately defined quantum group.



*Andrei Okounkov in front of the
Logan–Shepp–Vershik–Kerov limit shape*

Each of these approaches involves the development of a deep theory and collection of conjectures and results which provide an interesting interpretation of the Macdonald polynomials and have the original Macdonald constant term conjectures as a consequence. What is remarkable is that all these theories deal with quite different mathematical constructs so that on the face of it, these approaches are unrelated. The goal of the programme was to unify and further develop these disparate approaches to the Macdonald polynomials with the ideal but not easily attainable outcome of a single theory which would encompass all the above approaches. At the same time, the algebraic combinatorics and representation theory of Hecke algebras and quantum affine algebras and their relations with the Macdonald polynomials and Kostka–Foulkes polynomials have become increasingly significant in mathematical physics. For example, $q,t$ deformations of Virasoro algebras and $W$-algebras are related to Macdonald polynomials, and the combinatorics of crystals are related to Kostka–Foulkes polynomials. With other equally significant applications in mathematical physics and different approaches to the same subject, one of the main aims of the programme was to provide a unique opportunity for an interdisciplinary exchange of ideas.

## *Organisation*

The organisation was shared between the three organisers, with Phil Hanlon mainly responsible for the organisation ahead of the programme and Alun Morris for the day to day organisation during the programme. In addition, the programme benefited from the excellent workshop organised by Jean-Yves Thibon (Principal Organiser) in April and the stimulating NATO ASI organised by Sergey Fomin (NATO Countries Organiser) at the conclusion of the programme in late June to early July.

During the two workshops, satellite meeting and the final NATO meeting there was a full programme of lectures scheduled to leave time for participants to interact with each other. In the intervening weeks, seminars were on the whole restricted to Tuesday and Thursday afternoons with on average three to four one and half hour

sessions per week. This left participants with time to continue with their own research work and to carry on more informal discussions.

## Participation

The programme was fortunate to have around 55 long term participants staying for periods from three or so weeks to a full six months. There was a similar number who attended for a shorter term. Two of the organisers, IG Macdonald and AO Morris, were present for the whole programme as were S Kumar and A Ram whose active participation contributed strongly to the success of the programme. We were also fortunate that a number of other leading figures in the field spent long periods of three or four months at the Newton Institute. Participants were uniform in their praise for the unique opportunities offered at the Institute; both old and new collaborations flourished and all concerned benefited through interaction with different aspects of the subject.

Participants were in great demand as Visiting Lecturers at other UK Universities during their visit to the Newton Institute; the breadth of the field is illustrated by the different subject seminars which were visited.

## Meetings and Workshops

### Euroworkshop on *Conjectures, Recent Results and Open Problems Related to the Macdonald Polynomials,* 8 - 12 January 2001

Organisers: P Hanlon, IG Macdonald and AO Morris

The first meeting was designed to launch the programme with the aim of introducing the subject to those parts of the European Community, especially the UK, where the subject is not strongly represented at present. Its title indicated its importance in setting goals for the overall programme. A series of lectures was given by five lecturers who are acknowledged to be the leading

authorities and the main innovators in this field: IG Macdonald (QMW London), A Garsia (UC San Diego), P Hanlon (Michigan), E Opdam (Amsterdam) and JR Stembridge (Michigan).

There were a number of participants who were new to the subject and the responses, especially from the younger participants, indicated strongly that the programme had achieved its main aims.

### Euroconference on *Applications of the Macdonald Polynomials,* 17 - 20 April 2001

Organisers: J-Y Thibon, B Leclerc, ML Nazarov and M Noumi

The second workshop was devoted to applications of Macdonald polynomials. The programme was mainly, but not exclusively, devoted to applications to Harmonic Analysis and Integrable Quantum Systems. The main aim was to introduce mathematicians working in combinatorics, algebra or group theory to methods coming from physics or analysis. The speakers and participants included most of those who had historically been involved in the subject and the world leading specialists in this field.

The programme included two introductory lectures by P Forrester (Melbourne) on the applications of Jack polynomials in Quantum Statistical Mechanics and Random Matrix Theory and three lectures by members of the team which discovered the deformed Virasoro algebra: H Awata (Nagoya), S Odake (Shinshu) and J Shiraishi (Tokyo). The remaining talks were devoted to the announcement of new results. Apart from the themes mentioned above, several talks discussed interesting problems about quantum groups, affine Lie algebras, differential equations and algebraic combinatorics.

The lectures clearly demonstrated the wide range of applicability of the Macdonald polynomials in physics and harmonic analysis and gave a strong impulse for further research.

*C Dunkl and S Fomin during a seminar*

## Satellite Meeting at Gregynog Hall, University of Wales on *The Heritage of I Schur's 1901 Dissertation,* 2 - 5 June 2001

Organisers: AO Morris, C Bessenrodt, S Donkin, GD James, A Kerber and JB Olsson

The meeting celebrated the centenary of I Schur's Dissertation which has been so influential in the development of representation theory. Also, it was in this work that Schur functions first appeared especially in their relationship to the representation theory of general linear groups and symmetric groups. Schur functions were the precursors of the Macdonald polynomials. JA Green in his work on the characters of general linear groups, where what are now called Hall–Littlewood polynomials (a generalisation of Schur functions) first appeared, and also in his influential Springer Lecture Notes where Schur's work was presented in modern terminology has made an equally significant contribution. This meeting served to celebrate his contribution and also his 75th birthday. The main aim was to bring Newton Institute participants into contact with those who work in contemporary representation theory.

The meeting turned out to be especially successful not only on the research front but also because participants enjoyed the beautiful venue provided at Gregynog Hall. The meeting was funded jointly by a Conference grant from the London Mathematical Society and by the Newton Institute. A highlight of the meeting was the announcement that the 2001 De Morgan Medal will be awarded to JA Green.

## NATO Advanced Study Institute: *Symmetric Functions 2001: Surveys of Developments and Perspectives,* 25 June - 6 July 2001

Organisers: S Fomin, G Olshanski, S Kerov (deceased), P Hanlon, IG Macdonald and A Okounkov

The NATO ASI consisted of 13 mini-courses containing 38 one-hour lectures distributed over 10 working days. These mini-courses surveyed recent developments, including those that occurred during the rest of the programme, and indicated directions of future research in various fields for which fundamental questions can be stated in the language of symmetric functions. These courses brought insight from such areas as algebraic geometry, mathematical physics, invariant theory, combinatorics, statistical mechanics, classical representation theory, random matrices, special functions and orthogonal polynomials, Lie theory and quantum groups. On one day the work and contribution of Sergei Kerov, who was one of the two original co-directors of the ASI but who sadly died a year earlier, was commemorated and celebrated.

The ASI was also supported financially by a grant from the EC and from EPSRC funds. The meeting was regarded as a highly successful one by all concerned and in particular by the 102 participants. It was a most worthy event to celebrate the culmination of the Newton Institute programme on Symmetric Functions and Macdonald Polynomials.

## *Outcome and Achievements*

A month or so after the end of the programme is far too early to properly assess the achievements of the programme. Many participants in their

individual reports indicated that new ideas had arisen in the formal and informal discussions which would be the basis for their research in the future, some of it in collaboration with other participants. These reports commented almost without exception that the participants had benefited immensely from the exceptionally stimulating environment of the Newton Institute and from interaction with other participants. In comparing with other well-known mathematical institutes, one participant stated "never before have I been so thoroughly pleased with the facilities, staff, location, quality of the scientific program and general comfortable atmosphere."



*A Lascoux and S Sahi*

The outcomes and general proceedings may have been rather different from those outlined in the original proposal three years earlier. The subject has been extremely active in the intermediary period with some problems solved, but generally with the subject moving in new directions. However, as a perceptive referee of the original proposal had said, "Bringing together the experts listed as potential participants would likely result in a variety of new results, since they come from different backgrounds, but do speak a common language of symmetric functions and Macdonald polynomials." In the event, this proved to be exactly what happened.

Shortly in advance of the commencement of the programme, M Haiman had, amongst other things, announced the proof of his and A Garsia's celebrated $n!$ conjecture which also establishes the Macdonald positivity conjecture. During the programme he gave a number of lectures on this subject and the connections between symmetric functions and Macdonald polynomials, combinatorics and the geometry of Hilbert schemes. He completed the second in a series of four papers on this subject and did considerable preparatory work on the remaining two papers. Far more important however was that he, like others, took full advantage from the environment at the Newton Institute to take stock after his proofs of these major theorems.

The research activities of S Fomin and A Zelevinsky centred on their ongoing long-term project on Cluster Algebras. This project aims at constructing an explicit algebraic/combinatorial framework for the study of dual canonical basis and total positivity in semisimple groups. Two papers were completed and two more are forthcoming. In addition, both interacted in a positive way with a number of other participants; for example, a further paper in the series will involve R Marsh. Also, both were involved in discussions with M Noumi on the relationship between their approach to totally positive matrices and his approach to birational Weyl group actions.

During his two short visits, not only did M Noumi give a number of lectures to the April workshop and final ASI, but he also had discussions with G Heckman and M Olshanetsky on moduli spaces related singularity theory and nonlinear integrable systems, and with B Leclerc and J-Y Thibon on representation theoretic aspects of Macdonald polynomials and related combinatorics.

Although the French group based at Marne-le-Vallee already meet regularly, they were able to further develop their collaborative work at the Institute. A Lascoux, B Leclerc, J-Y Thibon together with G Duchamp and F Hivert co-operated on non-symmetric and multi-parameter Macdonald polynomials, R-matrices and Yang–Baxter equations. J-Y Thibon, F Hivert and A Lascoux were able to propose a definition of noncommutative and quasi-symmetric analogues of the classical Macdonald polynomials, which

followed earlier similar work on Hall–Littlewood polynomials. However, far more important possibly is the ambitious project about some new connections between affine Hecke algebras and Macdonald polynomials. The initial extensive computer calculations involving R-matrices which were completed are very promising of some significant new results.
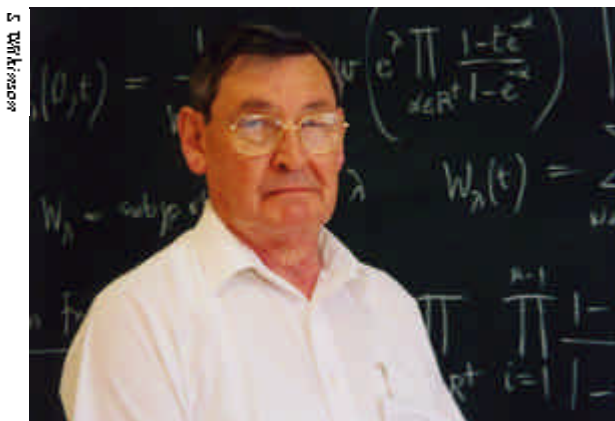


*IG Macdonald*

A Ram was present for the full programme and not only had a fruitful six months on his own account but also had a major impact on the work of other participants; equally though he took full advantage of the expertise of others readily at hand. He completed three papers during the programme, all of them in collaboration, with C Kirillov, A Shepler (who visited the Institute) and R Orellana. These involved the representations of graded Hecke algebras, the classification of graded Hecke algebras for complex reflection groups, and affine braids, Jantzen filtrations and the category *O* respectively. In addition considerable progress was made on three other projects: Kostka–Foulkes polynomials, category *O* for quantum groups at roots of 1 and branching rules for affine Hecke algebra representations. He also lectured at ten other venues in the UK and three on continental Europe.

Significant progress was made by J Stembridge during his four month visit in his research programme of understanding the combinatorial structure of root systems, Coxeter groups and

Weyl characters. In particular he was able to finish writing two papers, the first of which provides a minimal set of axioms for the most important features of Weyl characters, providing easy access to tensor product and branching rules. The second paper provides a interesting new insight into the combinatorial structure of the Bruhat ordering of any finite Weyl group. He also had fruitful discussions with A Schilling during her short visit on their long-term project on crystal bases and combinatorics.

RP Stanley, the programme's Rothschild Professor, was only able to be present for about five weeks but during that time was heavily involved in the programme. His main research activities were in three different areas: the enumeration of chains in the Bruhat order of the symmetric group, the Frobenius rank of a skew partition and Kerov's character polynomial of a *k*-cycle. The first of these involved a new approach via Schubert polynomials which he was able to discuss with the leading authorities who were present, J Stembridge, A Lascoux and IG Macdonald. The second was inspired by a lecture by M Nazarov and resulted in joint work with C Bessenrodt. The third took advantage of the presence of G Olshanski, A Okounkov and A Vershik.

C Bessenrodt also continued with her work on a number of problems concerning the representation theory of the symmetric groups and related groups and the associated combinatorics – including Schur functions and Schur *Q*-functions. For example, she together with JB Olsson worked on the problem of finding zeros on elements of prime order for the characters of symmetric and alternating groups which led to a purely combinatorial question of classifying partitions which are of maximal *p*-weight for all primes *p*. The question has been answered for *n* sufficiently large and $n \leq 9 \times 10^8$. She also obtained a similar answer for the covering groups. A number of questions involving Kronecker products of characters of these groups were also considered; in particular, she obtained a classification of multiplicity-free outer products of Schur *Q*-functions.

E Vallejo's research involved the difficult problem of the inner product of two Schur functions. He was able to show that $\langle s_{\boldsymbol{l}} * s_{\boldsymbol{m}} s_{\boldsymbol{n}} \rangle$ is equal to the number of 3-dimensional matrices of zeros and ones and plane sum vectors $\boldsymbol{l}$, $\boldsymbol{m}$ and $\boldsymbol{n}$ which satisfy some easily checked properties. He also collaborated with A Ram on the decomposition into irreducible modules of the restriction of an irreducible module from the affine Hecke algebra to the finite Hecke algebra.



*S Kumar and IG Macdonald*

M Nazarov and V Tarasov had already proved a conjecture of V Chari and A Pressley on the irreducibility of a tensor product of irreducible finite-dimensional modules over the Yangian $Y(gl_N)$ of the general linear Lie algebra $gl_N$ for a wide class of irreducible modules; while together at the Institute they worked on proving this in general taking advantage of discussions with many other participants.

In advance of the programme, B Sagan and M Rosas had worked on what turned out to be related problems. One was working in the context of symmetric functions in noncommuting variables and the other in the context of MacMahon symmetric functions; however, in both contexts a definition of an analogue of Schur functions was lacking. They were able to jointly obtain a combinatorial definition and explored further some of their properties; this will eventually result in a joint paper.

C Dunkl interacted strongly with many

participants during his four month visit. However, his greatest achievement was the successful extension jointly with E Opdam, who was also a participant, of the Dunkl operators to complex reflection groups. These are likely to be as influential as those earlier defined for real reflection groups. This work was a real blend of algebra and analysis, another example of the sort of interaction encouraged by the programme. AO Morris completed some work on root systems and Macdonald representations for complex reflection groups and benefited from discussions with them. C Dunkl also worked on a problem in generalised spherical harmonics with octahedral symmetry – the aim being to construct orthogonal bases for these objects. Typical of other interactions was the following: R Stanley conjectured a summation formula for a certain terminating hypergeometric series, C Dunkl provided a nice analytic proof which T Koornwinder was then able to connect with the problem of evaluating hypergeometric sums by computer algebra and which then led to more examples and generalisations.

M Rössler took full advantage of contact with C Dunkl and others to advance her work on special functions related with root systems and Dunkl operators. She studied in particular the integral kernel of the Dunkl transform and its asymptotic properties. Also, she commenced work on the theory of trigonometric Dunkl operators and Cherednik operators and the corresponding heat equation.

T Koornwinder also worked on the limit of Macdonald polynomials to the Heckman–Opdam Jacobi functions as $q$ tends to 1. He was able to prove this for the root system $A_n$ and for $t = q^k$ with $k$ a non-negative integer. Contact with T Koornwinder and C Dunkl turned out to be very valuable to P Forrester in his research into Jack and Macdonald polynomials during his short visit.

S Sahi during his visit wrote a joint paper with A Dvorsky describing a construction of a class of small unitary representations of semisimple Lie algebras which arise as conformal groups of real
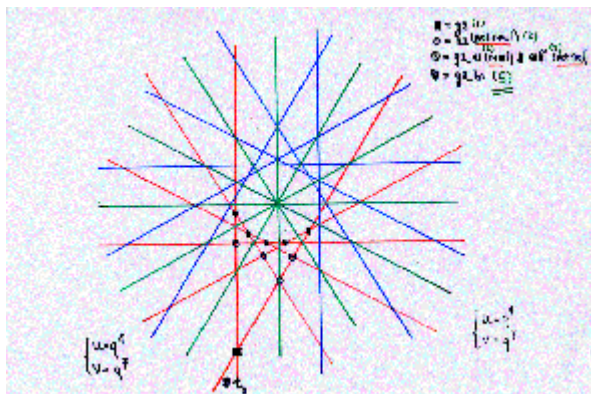
*The Opdam Fox – the weight spaces of the discrete series representations of an affine Hecke algebra of type $G_2$*

semisimple Jordan algebras. The intention is that this should lead to a generalisation of the theta-correspondence and of Howe's theory of dual reductive pairs. He also described his work on Jack polynomials – an interesting outcome is a new combinatorial formula for Jack polynomials which is new even for Schur functions.

E Sklyanin worked in collaboration with V Kuznetsov to construct a $Q$-operator for Jack polynomials – this is an operator whose eigenfunctions are Jack polynomials and whose eigenvalues are the separated polynomials arising from the separation of variables in Jack polynomials. Discussions with T Koornwinder in particular helped M Olshanetsky to further develop his work on unitary representations of the quantum Lorentz group and the theory of Bessel–Jackson functions.

M Wachs was able to complete work on a paper with P Hanlon on a conjecture closely related to the strong Macdonald conjectures – the property $M$ conjecture for the Heisenberg Lie algebra. The work resulted in a new conjecture. A number of lectures were given during the programme on the strong Macdonald conjecture; in particular C Teleman elucidated his recent proof in conjunction with I Grojnowski and S Fishel. A great deal of discussion of work in this area involved the above and S Kumar and J-L Loday. In fact, during his six months on the programme S Kumar made a major contribution to the discussions in general and especially so in the

weekly seminars.

Using the $q$-deformed algebra $U'_q(so_n)$, A Klimyk was able to derive some properties of the Macdonald polynomials $P_{\boldsymbol{\lambda}}(x;q,q^{1/2})$ and $P_{\boldsymbol{\lambda}}(x;q,q^2)$. He gave a group theoretical explanation of the relation $P_{\boldsymbol{\lambda}_1}P_{\boldsymbol{\lambda}_2} = \sum_{\boldsymbol{\lambda}} f^{\boldsymbol{\lambda}}_{\boldsymbol{\lambda}_1\boldsymbol{\lambda}_2} P_{\boldsymbol{\lambda}}$ for these Macdonald polynomials. He solved the problem of the positivity of these coefficients and also solved the problem relating the vanishing of these coefficients to the vanishing of the corresponding Littlewood–Richardson coefficients. B Srinivasan made further progress in her efforts to obtain a better understanding of the Green functions for Macdonald polynomials.

A Sergeev constructed a superanalogue $SL$ of the Calogero operator depending on a complex parameter $k$ which is related to the root system of the Lie superalgebra $gl(n|m)$. For generic $m$ and $n$, the superanalogues of the Jack polynomials constructed by A Kerov, A Okounkov and G Olshanskii are eigenfunctions of $SL$. He also showed that Schur $Q$-functions may be interpreted as bispherical functions of certain algebras involving the queer algebra $q(n)$, from which he characterised Schur $Q$-functions as common eigenfunctions of an algebra of differential operators.

IG Macdonald was able almost to complete his book on affine Hecke algebras and orthogonal polynomials during the six month programme. As this work was the inspiration for a great deal of what has occurred in the subject, the book will be eagerly awaited by all the participants.

Some other younger UK mathematicians such as A Cox, R Green, S Perkins, C Wensley and U Onn (from Israel) whose research interests may be regarded as peripheral to the main theme of the programme seemed to benefit from their short stays – all lectured and had valuable discussions with the other participants. Also, they made considerable progress in their research with publications resulting.

# Nonlinear Partial Differential Equations

## 8 January to 6 July 2001

### Report from the Organisers: H Brezis (Paris), EN Dancer (Sydney), JF Toland (Bath), NS Trudinger (Aust. Nat. Univ.)



*NS Trudinger and EN Dancer*

## Scientific Background

Nonlinear partial differential equations lie at the frontier of contemporary mathematics with deep theoretical challenges linked to diverse applications. This programme emphasised equations of elliptic and parabolic type, which traditionally model steady states and evolving processes. The programme was divided into four interrelated themes:

- Geometric evolution equations;

- Fully nonlinear equations;

- Variational problems with singularities;

- Reaction diffusion equations.

The first two themes were pursued mainly during the first three months of the programme and the last two themes during the last three months. The activities under each theme culminated in a workshop. The first two were integrated in a two-week workshop supported by the EC, as a Euroworkshop entitled *Geometric Evolutions and Nonlinear Elliptic Equations*, organised by B Andrews and NS Trudinger, from 26 March to 6 April 2001. A workshop on *Variational Problems*

*with Singularities*, organised by H Brezis and F Bethuel, was held from 25 to 29 June 2001, while the final activity of the programme, covering the last theme above, was a Euroconference entitled *Nonlinear Elliptic Equations and Transition Phenomena*, organised by EN Dancer and H Brezis, from 2 to 6 July 2001.

Many of the leading international researchers in these areas participated, typically as long-term visitors or just for the workshops which, together with the weeks immediately preceding, were the most exciting and stimulating periods of the programme. Moreover the blend and standard of mathematics and ensuing synergies generated during those times must rank as the best ever in these areas.

Apart from the main activities a shorter workshop on *Multiscaling*, organised by G Friesecke (Oxford), was held from 9 to 11 April 2001; and a Spitalfields Day, entitled *PDEs Today*, consisting of survey talks on a range of topics in partial differential equations, was held in London on 11 May. Outside of the workshop periods, there were regular seminars, normally two or three each week. There were a total of 156 participants in the programme (48 long stay, 118 short stay), drawn from 22 countries and including 33 from the United Kingdom.

The following is a selection of particular research activities embraced by the programme classified according to theme, which should also give some indication of the huge scope of scientific terrain impacted upon by the programme research.

## Outcome and Achievements

### Geometric Evolution Equations and Fully Nonlinear Equations

As already indicated the focus of the first half of the programme was fully nonlinear elliptic equations and geometric evolution equations. These are areas which have seen spectacular advances in recent years, including the resolution of long-standing open problems in general relativity, Riemannian geometry, affine differential

geometry and the calculus of variations, along with striking applications to areas such as optimal transportation, meteorology, image processing and crystal growth. Many of the leading researchers in these areas were assembled together for the first time. As well as hearing about the great breakthroughs in theory and applications by those who made them – such as Huisken (Tubingen) and Ilmanen (Zurich), who solved the Penrose conjecture using geometric evolutions, and Brenier (Nice), Caffarelli (Texas) and Cullen (Reading) who separately pioneered the application of the Monge–Ampère equation to transport problems and meteorology – the participants engaged in various joint research projects. Andrews combined with Guan (McMaster) and Ma (Shanghai) to apply geometric flow methods to the Christoffel–Minkowski problem. Trudinger and Wang (Canberra) completed their work on the solvability of the Plateau problem for Gauss curvature and answered another long-standing problem in affine geometry, namely that affine completeness of strongly convex hypersurfaces, of dimension larger than one, implies Euclidean completeness. Kuo (Taiwan) and Trudinger established Schauder estimates for fully nonlinear difference equations. There were also interactions with researchers in complex flows, Cao (Texas A&M) and Zhu (Princeton), and complex Monge–Ampère equations, Cegrell (Umea). One of the pioneers of the theory of fully nonlinear equations determined by elementary symmetric functions of curvatures, Ivochkina (St Petersburg), even liaised with the companion programme on Macdonald polynomials, presenting a seminar explaining the underlying arithmetic of symmetric polynomials used in nonlinear PDEs. All of the geometric flow researchers appreciated the opportunity to interact with Galaktionov (Bath), a leading researcher in (non-geometric) blow up for parabolic equations.

## *Variational Problems with Singularities*

The theory of variational problems with singularities has become a highly important area of nonlinear mathematics in recent years, which links deep theoretical studies with applications to areas
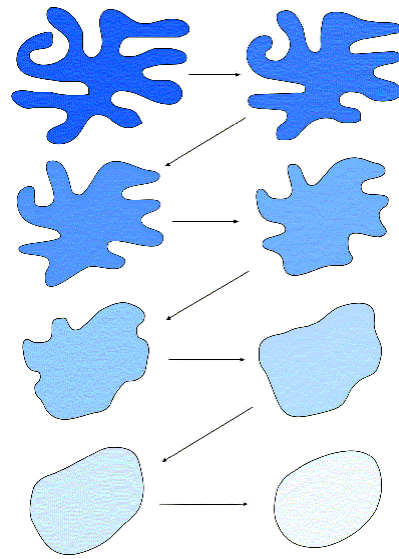


*Contraction of an embedded curve under curve-shortening flow.*

such as material science and superconductivity. The programme activities in this theme were largely concentrated in the June workshop, although there were extended visits by Sigal (Toronto) and Sternberg (Indiana). Of particular value were discussions between the Paris group, led by Brezis, and the North American group working on Ginzburg–Landau equations, and discussions between these two groups and those participants whose main interest was sharp transitions in reaction diffusion equations. The workshop itself was highly successful. Among many outstanding lectures were those of Ambrosio (Pisa) on optimal maps in mass transport problems, Sternberg and others on Ginzburg–Landau equations, including those by Sandier (Paris) and Serfaty (Cachan) on the distribution of vortices, as well as remarkable presentations by Soner, Shafrir, Jerrard, Alama, Bronsard, Sigal, Almog and Alberti. Various lectures described phenomena arising in physics, not related to type 2 superconductivity, but which now can be handled by the new machinery developed for the study of Ginzburg–Landau vortices (Aftalion and Mallick). Various aspects of minimal surfaces were covered in lectures by Chang (Beijing), Simon (Stanford) and Kirchheim (Leipzig).

*L Nirenberg*

### Reaction Diffusion Equations

The emphasis in this theme was on sharp transition layers for semilinear elliptic equations, with activity largely concentrated in the last month of the program. There were long-term visits by Gui (Vancouver), Sigal (Toronto), Sternberg (Indiana), Wei (Hong Kong) and Yan (Sydney), as well as numerous short-term visitors and conference attendees in the last week. A great deal of collaborative work occurred especially on sharp transitions and critical exponent problems, likely to result in many joint works over a longer timescale. In particular, there was much discussion on the use of the recent work on the DeGiorgi Conjecture in low dimensions (due to Gui-Ghousoub, Ambrosio, Cabre and others) and a better understanding of the profile of solutions obtained by Gamma convergence (due to Modica and Kohn-Sternberg among others). As already indicated, there was a great deal of fruitful discussion on the relationship between work done on the Ginzburg–Landau equations (by Brezis, Sigal, Sternberg and many others) and that on sharp transition layers for scalar equations (by Dancer, Gui, Wei, Yan and others), leading to much better understandings between the two groups.

The final Euroconference on *Nonlinear Elliptic Equations and Transition Phenomema* drew about 50 participants, with outstanding lectures on recent developments by Gui on the DeGiorgi problem, Lin and Chen (Taiwan) on critical exponent problems, Wei on applications to biological systems and Berestycki (Paris) on travelling waves. The presence of Nirenberg (New York) during this part of the program was greatly appreciated by all participants.

## Other activity

The participation of Toland (as an organiser) and Plotnikov (Novosibirsk) broadened the scope of the programme somewhat, by introducing concern for specific rigorous PDE problems arising in mechanics. Toland interacted substantially with another organiser, Dancer, and with Buffoni (Lausanne) with whom he was preparing a monograph on the application of analytic varieties to differential equations. Plotnikov, who was a long-term participant, spoke about his work on fully nonlinear equations and contributed to all aspects of the programme.



*H Brezis*

# TAM-day Musings by Keith Moffatt

*A poem to celebrate Theoretical and Applied Mechanics day, 27 August 2001.*
*After Tam O'Shanter, with apologies to Robert Burns*

When lecture rooms are growing stale,
And thoughts begin to turn to ale;
When blackboards thick in chalk are smothered,
With scarce a new truth there discovered;
When loud the arguments have railed,
But longed-for inspiration failed;
Why, that's the time to take a break,
To turn to coffee, tea and cake;
Or sip the vineyard's produce cool,
And contemplate the game of boules.

This truth was hap'ly brought to mind,
By letter sent and duly signed,
Conveying tidings of great joy
Hot from the heart of Illinois;
There it was formally declared
What ne'er had hitherto been dared:
Mechanics as a worldwide art
Should henceforth have a day apart,
A day when man may ruminate
Upon the subject's vibrant state;
A day when blackboard toil should cease,
And staff should have a bit of peace!
Straight from the land of Al Capone,
Instruction came by telephone;
It seemed there was no time to lose,
Such offers one cannot refuse!

And thus it was TAM-day was born,
An August Monday to adorn,
A day this year decreed by heaven
To fall on August twenty-seven;
So three times three, again times three,
A date on which TAM holds the key
To open Archimedes' door,
And celebrate on every shore;
What time of day?  you well may ask;
The answer's plain: from dawn till dusk.

In Cambridge town we heard the call,
And rallied to the central hall
Of mighty Newton's Institute,
Irreverently called by some the Newt!
He who bestrode the pebbled shore
Of Ocean's ever-mobile floor;
Here a pebble, there a pebble,
But was the system integrable?
Said Newton "If I shed a tear

Upon this ocean wide and clear,
Will this affect the rain in Laos?"
And thus he sowed the seeds of Chaos.

But to our tale: the day dawned bright,
The weather forecast had been right;
The warming sun on boules court shimmered,
The overarching crane fair glimmered;
That day a child might understand
An awesome drama was to hand.

Well practiced in the laws of motion,
And fortified by vintage potion,
The gifted savants slow foregathered
Hard by the shed where bikes are tethered;
From every land and clime they came,
Experts of legendary fame;
From Russia, It'ly and Japan,
And every country known to man;
From Poland, Greece and USA,
A clash of titans underway!

But here my Muse her wing maun cour,
This glorious game made such a stoor;
The boules of glittering steel were round,
And sped unerring on the ground;
They rolled, they arched, they spun, they clickit,
More action here than seen in cricket!
The game might well have run till dawn
There by the side of Newton's lawn.
The cochinet brent new frae France
Was kissed with steel, and touched perchance,
Balls tossed with Zakharovian skill;
Then Kruskal clad in T-shirt still,
He who could aim a ball and roll-it-on,
Invariant as any soliton,
With concentration ever keener
Entered upon that tense arena;
And cast one ball, a crafty throw
That scattered those of every foe.
The cochinet was split asunder,
Th'encircling crowd was mute with wonder.

Now, who this tale o' truth shall read,
Mechanics of whatever creed:
When overwork becomes a grind,
And problems tangle up the mind,
Fill well the cup and fill it full,
Take refuge in a game of boules!

# Posters on the Underground

The 'Maths in the Underground' project was completed with the month-by-month display of 6 more posters during July to December 2000. The December poster had a Christmas theme and a featured a festive puzzle. Correct answers were submitted by e-mail and prizes of Newton Institute T-shirts and mugs were sent to the winners. World Scientific kindly printed an additional 500 sets of posters, free of charge, for distribution on request to UK schools and universities. The Victor Rothschild Memorial Fund provided a grant of £5,000 towards costs of the project not covered by the initial grant of £20,000 from the EPSRC Public Awareness Initiative. EPSRC have kindly agreed to fund a reprinting of a further 6,000 sets of posters and these will be distributed to UK schools during September 2001 by UCLES (University of Cambridge Local Examinations Syndicate). The posters were used (with permission) by Channel 4 Television as a backdrop during classroom scenes in the programme *Dawson's Creek.* The posters also won a 'merit' level Heist award for excellence in an educational marketing campaign.

A booklet which features all 12 of the posters together with explanatory text is currently in preparation.

The posters are displayed on the web at

**http://www.newton.cam.ac.uk/wmy2kposters/**

and continue to attract much favourable comment.
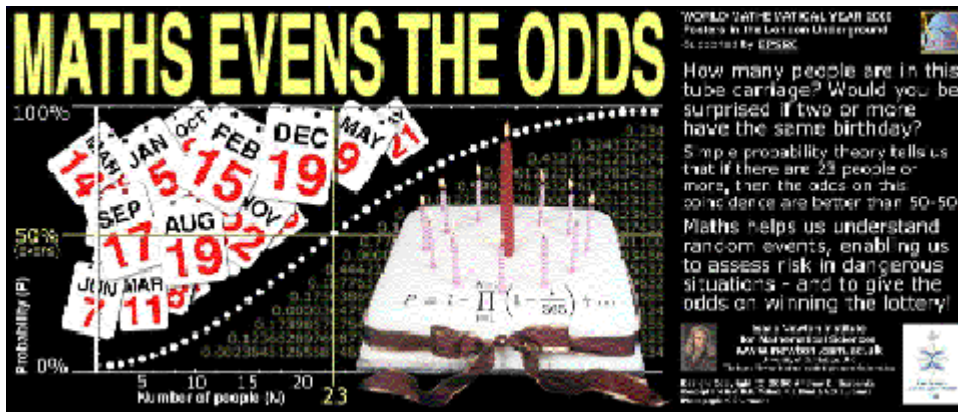




Above, *Maths Evens the Odds* (July) and *Maths Takes Off* (August).

Opposite, *Maths is Vital*, *Maths Breaks the Code*, *Maths Makes Waves* and *Maths is For Ever* (September to December respectively).

**MATHS IS VITAL**

DNA is the stuff of life.

The human genome project is generating vast sequences of DNA data. New mathematical techniques are being used to analyse this data.

This analysis is vital for the development of new medicines in the fight against viral disease.



**MATHS BREAKS THE CODE**

Many of today's secret codes rely on the difficulty of 'factorising' huge numbers which means solving problems like these

$5 \times 2 = 10$
$11 \times ? = 33$
$? \times ? = 91$

Every time you use a cash machine, your details must be checked with the bank's main computer. To protect your privacy, they are scrambled mathematically.

Number Theory is important not only for keeping data secret, e.g. when shopping by credit card on the net, but also for cracking codes like that of the wartime Enigma machine.

$? \times ? = 8577912293265445403162361462162997220043102876199$



Waves are a source of delight. They also cause enormous destruction.

We need to understand how they form and how they propagate, and find ways to harness their energy safely.

Maths holds the key to this understanding.

*maths makes waves*



On the first day of Christmas my true love sent to me a partridge in a pear tree

Taking the lyrics of the song literally, how many presents in total did my true love send to me over the twelve days of Christmas?

Answers and explanations to christmas@newton.cam.ac.uk Prizes will be awarded for the first few correct answers.

Once a mathematical result has been proved then, like true love, it lasts forever.

$1 + (2+1) + (3+2+1) + ...$

**Maths is For Ever**

# Finances

## *Accounts for July 2000 to June 2001 (Institute Year 9)*

| Income | 1999/2000 Year 8 | 2000/2001 Year 9 |
|---|---:|---:|
| Grant Income – Revenue | 1,206,758 | 1,371,368 |
| Grant Income – Workshop | 290,908 | 273,533 |
| Endowment Interest | 50,000 | 116,212 |
| Donations – Revenue | 532 | 9,091 |
| Donations – Capital | | 1,335,681 |
| Interest on Deposits | 92,944 | 86,801 |
| General Income | 2,617 | 5,435 |
| Housing | 20,212 | 18,199 |
| Transfer from Reprovision | 37,937 | 17,827 |
| Transfer from Building Capital Fund | 62,751 | 805,209 |
| **Total** | **1,764,659** | **4,039,356** |

| Expenditure | | |
|---|---:|---:|
| Scientific Salaries | 310,046 | 326,963 |
| Scientific Travel and Subsistence | 389,121 | 396,109 |
| Scientific Workshop Expenditure | 239,878 | 220,788 |
| Other Scientific Costs | 7,634 | 11,070 |
| Staff Costs | 305,486 | 297,732 |
| Computing Costs | 76,228 | 151,438 |
| Library Costs | 15,338 | 12,973 |
| Building – Capital | 69,526 | 805,209 |
| Building – Rent | 195,700 | 201,572 |
| Building – Repair and Maintenance | 7,804 | 7,447 |
| University Overheads | 31,335 | 30,752 |
| Consumables | 32,834 | 37,912 |
| Equipment – Capital | 5,277 | 4,508 |
| Equipment – Repair and Maintenance | 2,421 | 7,110 |
| Publicity | 26,351 | 16,660 |
| Recruitment Costs | 4,569 | 3,453 |
| Miscellaneous | 356 | 341 |
| Transfer to Future Reprovision | 80,000 | 80,000 |
| Transfer to Building Capital Fund | 45,779 | 41,697 |
| Transfer to Endowment Fund | | 1,382,674 |
| **Total** | **1,845,683** | **4,036,408** |
| **Income Less Expenditure** | **(81,024)** | **2,948** |

# Notes to Accounts

**1. Grant Income – Revenue.** This breaks down as follows:

|  | 1999/2000<br>Year 8 | 2000/2001<br>Year 9 |
|---|---|---|
| EPSRC/PPARC Salaries | 310,090 | 345,825 |
| EPSRC/PPARC Travel and Subsistenc | 268,451 | 268,108 |
| EPSRC PPA | 11,430 | 8,570 |
| EPSRC JREI | – | 92,225 |
| Newton Trust | 50,000 | 100,000 |
| Hewlett-Packard | 115,000 | 115,000 |
| Rothschild Visiting Professors | 31,964 | 20,097 |
| CNRS | 36,960 | 37,413 |
| Rosenbaum | 21,951 | – |
| Leverhulme | 76,870 | 78,867 |
| Royal Society | 2,986 | – |
| LMS | 20,000 | 20,000 |
| CPS | 2,000 | 2,000 |
| University of Cambridge (Staff) | 60,734 | 74,765 |
| University of Cambridge (Rent) | 195,700 | 201,572 |
| Daiwa Anglo-Japanese Foundation | – | 6,926 |
| US Navy | 1,847 | – |
| NSF | 775 | – |
| **Total** | **1,206,758** | **1,371,368** |

**2. Endowment Interest.** Endowment income for 2000/2001 arises from the Rothschild Professor Fund and from funds provided by the Isaac Newton Trust.

**3. Donations – Capital.** Donations were received from an anonymous donor and from the PF Charitable Trust, together with the fourth instalment of the Rothschild Professor Fund.

**4. Housing.** This figure represents the net position for the period.

**5. Transfer from Reprovision.** Transfers have been made to cover computing equipment and minor items of furniture purchased during the period.

**6. Transfer from Building Capital Fund.** This figure is matched precisely by expenditure on 'Building – Capital' for Gatehouse construction costs.

**7. Staff Costs.** The slight reduction for this period is the result of delays in filling vacant staff positions.

**8. Computing Costs.** This figure is considerably greater than in 1999/2000 because of expenditure on JREI equipment, covered entirely by the JREI grant from EPSRC.

**9. Publicity.** These costs are lower than in the previous financial year due to the completion of the *Posters on the Underground* campaign in 1999/2000.

**10. Transfer to Building Capital Fund.** This consists of interest accumulated from the Building Capital Fund (donated by Dr Dill Faulkes), part of the figure shown under 'Interest on Deposits'.

**11. Transfer to Endowment Fund.** This figure is the total of the anonymous donation, the fourth instalment of the Rothschild Professor Fund and endowment interest from the Isaac Newton Trust fund.

# *Cumulative Financial Donations*

| | | |
|---|---|---|
| SERC/EPSRC/PPARC | £9,033k | over 16 years |
| Isaac Newton Trust | £2,500k | over 7 years |
| NM Rothschild and Sons | £2,083k | over 10 years |
| Hewlett-Packard | £1,065k | over 5 years |
| Dill Faulkes Foundation | £1,000k | |
| Anonymous Donation (US$1.38m) | £924k | |
| St John's College | £750k | over 5 years |
| European Union | £708k | over 10 years |
| NATO | £696k | over 9 years |
| Leverhulme Trust | £615k | over 9 years |
| University of Cambridge | £512k | over 9 years |
| Le Centre Nationale de la Recherche Scientifique | £435k | over 10 years |
| Rosenbaum Foundation | £330k | over 7 years |
| PF Charitable Trust | £240k | over 3 years |
| London Mathematical Society | £219k | over 12 years |
| Gonville and Caius College | £100k | |
| Prudential Corporation plc | £100k | over 4 years |
| Institute of Physics | £68k | over 7 years |
| British Meteorological Office | £64k | |
| Nuffield Foundation | £57k | |
| TSUNAMI | £40k | |
| Daiwa Anglo-Japanese Foundation | £36k | over 4 years |
| AFCU (Hamish Maxwell): $50k | £32k | |
| AFCU (Anonymous Donation): $50k | £32k | |
| Emmanuel College | £30k | |
| Jesus College | £30k | over 6 years |
| British Aerospace | £25k | |
| Rolls Royce | £25k | |
| Corporate Members (FIN programme) | £22k | |
| British Gas | £20k | |
| DERA | £20k | |
| Magnox Electric | £20k | |
| Thriplow Trust | £18k | |
| Bank of England | £15k | |
| Cambridge Philosophical Society | £15k | over 10 years |
| Schlumberger | £12k | |
| Wellcome Trust | £12k | |
| NERC | £10k | |
| Trinity College | £10k | |
| Unilever | £10k | |
| Applied Probability Trust | £10k | over 3 years |
| DSM (Netherlands) | £6k | |
| GLOBEC | £6k | |

| | |
|---|---|
| BP | £5k |
| British Airways | £5k |
| Christ's College | £5k |
| Harlequin Software | £5k |
| Compugen | £4k |
| British Society of Rheology | £3k |
| JRC Ispra | £3k |
| Ove Arup | £3k |
| Agricultural and Food Research Council | £2k |
| Autonomy Systems Ltd | £2k |
| Smith Kline Beecham | £2k |

**Donations in Kind**

Computer equipment has in the past been donated by Hewlett-Packard, Apple UK and Silicon Graphics. Sun Microsystems has made many generous donations in recent years and has also sold equipment to the Institute at a very substantial discount. In particular, this year Sun has donated 14 flat panel Sun Ray 150 thin clients, provided processor and memory upgrades for our Sun Enterprise 450 server and, most notably, provided a handsome discount on high-performance equipment for the Institute's JREI grant (see p 8).

Software has been donated by NAG, Claris and Wolfram Research.

Over 4,000 books and journals have been donated by a large number of publishers and individual members of the mathematical community.