Contents

APPENDICES

Please note that the following statistical information may be obtained from the Institute on request, or from **http://www.newton.cam.ac.uk/reports/0203/appendices.html**

Director's Foreword

The year under review began auspiciously with the celebration, on 2 July 2002, of the tenth anniversary of the Institute. A rich programme of mathematics, and a meeting of our correspondents from many universities, was followed by my inaugural lecture as NM Rothschild & Sons Professor. This was chaired by Sir Alec Broers, our ever supportive Vice-Chancellor, and we were delighted to welcome Sir Evelyn and Lady de Rothschild and our Honorary Fellow Dr Dill Faulkes. The generosity of our several benefactors, individual and corporate, has been essential to the establishment of the Isaac Newton Institute as one of the foremost centres of excellence in the mathematical world.

That excellence depends of course on the quality of proposals, and on the willingness of good mathematicians to define and organise effective programmes. It is good to report that we have an excellent pipeline of forthcoming programmes, running now into 2006. I have every confidence that we shall maintain the very high scientific standard in our second decade, and I pay tribute to programme organisers who put so much into their demanding role.

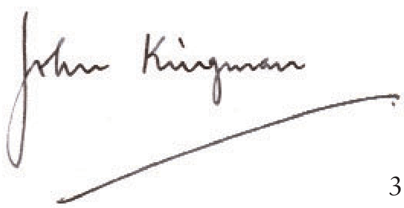
Only by keeping up the excitement and quality of our science can we expect the support of funding bodies that have to be highly selective in the allocation of their funds. During the year we faced a stringent review by the Engineering and Physical Sciences Research Council of the quality of research and the cost effectiveness of the Institute. We emerged from this review with flying colours, and with an assurance of continued funding at least until 2008.

This is crucial for our planning, because EPSRC and its sister research councils are almost the only sources of funding for the continuing operation of our programmes. Support for long term participants, and for essential support staff, comes almost entirely from them. Shorter workshops usually require separate funding, and this is increasingly difficult to achieve. We need to develop new funding streams to give confidence to programme organisers to arrange such events in the knowledge that they can meet the expenses of their visitors.

The Institute exists to advance mathematics and its many applications, to the benefit of the whole UK scientific community. I have continued to tour the country, seeking ideas and criticism from universities in all parts, and have been encouraged by the constructive approach of those with whom I have discussed our work. One particularly encouraging development has been a closer relationship with the International Centre for Mathematical Sciences in Edinburgh, under its new Director Professor John Toland. The ICMS and the INI are complementary in their missions, the former concentrating on much shorter events, and together they add great strength to UK mathematics.

Among the unsung heroes are those who referee programme proposals, and the members of the Scientific Steering Committee who maintain rigorous standards in judging which will result in successful meetings. This is a major undertaking, and I am grateful to those who take it on, as I am grateful too for the work of members of the National Advisory Board and of the Management Committee.

What impresses our visitors, however, is the warm welcome and friendly efficiency of the Institute staff. I am proud of the way we are able to look after the finance, accommodation, secretarial, computing and library requirements of participants, and the way in which inevitable problems are cheerfully tackled. The pleasant and constructive atmosphere of the Newton Institute, which is part of our reputation, owes everything to the work of all the staff, to whom I record my deep gratitude.



31 July 2003

Brief Scientific Report on Programmes

For full scientific reports see pages 19 to 44.

Programme 47: Foams and Minimal Surfaces

Foams are familiar gas–liquid systems of wide importance to industry, exhibiting many properties of interest to physicists, mathematicians, engineers and other applied scientists. In recent years computation has given fresh impetus to the subject, since it enables us to explore the consequences of idealised models for complex disordered foams by accurately simulating them. Some of the problems have direct bearing on potentially significant areas of application, such as in determining the properties of metallic foams.

This programme was conceived as an opportunity to maximise the cross-fertilisation of ideas between mathematicians, physicists and allied disciplines such as chemical engineering. It was successful in attracting world-leading figures from all these communities. The topics considered ranged from refined minimisation problems to physical experiments and their simulation, including drainage and rheology.

The communities which were brought together interacted well, and many future collaborations should result which would not have emerged without such a stimulus.

Programme 48: Computation, Combinatorics and Probability

The relationships between computer science and mathematics have become more wide ranging and profound, and this has been particularly evident in quantitative theoretical computer science, especially in the areas known as computational complexity and the analysis of algorithms. The aim of this programme was to explore two particularly fruitful interfaces, namely those involving combinatorics and probability theory. Within the broad area delineated by the title, four themes received special emphasis: randomised algorithms, random graphs and structures, phase transitions and probabilistic analysis of distributed systems. A series of three very well attended workshops provided rather hectic periods of concentrated interaction, interspersed with quieter interludes for reflection.

As a result of this broad scope, the research achievements of the programme were varied in topic. The work was highly collaborative, and often bridged two or more areas. The overall success of the programme can be judged by the volume and quality of the output.

Programme 49: New Contexts for Stable Homotopy Theory

The versatility of stable homotopy theory and its associated range of cohomological techniques has made it an important branch of mathematics. Recently there have been several fundamental developments which have been used to solve a number of long-standing questions in areas such as complex algebraic geometry, number theory and elliptic cohomology.

This programme was structured around three workshops (one being a pedagogical NATO Advanced Study Institute) and a number of related lecture series. It started and finished with intensive workshop activity, separated by a period for sustained research and collaboration amongst the longer term participants.

The programme was the foremost international event in the subject in 2002, and one of special significance as it brought together several strands of work at seminal stages of their development. A remarkable collection of researchers from a wide range of areas took part, and many new collaborations were born. The programme made significant progress in a number of areas, including:

- Algebra of rings up to homotopy
- Elliptic and chromatic phenomena
- Equivariant cohomology theories
- Algebraic groups and cohomology theories
- *K*-theory
- Algebraic and motivic homotopy theory
- Abstract homotopy theory

The programme organised, disseminated and consolidated the existing state of the subjects.

Programme 50: Computational Challenges in Partial Differential Equations

The study of partial differential equations (PDEs) is a fundamental area of mathematics which links important strands of pure mathematics to applied and computational mathematics, with relevance to the physical, natural and social sciences. Scientists and mathematicians have naturally been led to seek techniques for the approximation of solutions. High speed computation has led to consideration of very large systems of PDEs and the development of appropriate numerical algorithms and their analysis, both theoretical and applied.

This programme focussed on some of the most exciting and promising mathematical ideas, and on those branches of PDE theory that provide a source of physically relevant and mathematically hard problems to stimulate future developments. The three main themes were:

- Cognitive algorithms: adaptivity, feed-back and *a posteriori* error control
- Hierarchical modelling: multi-scale mathematical models and variational multi-scale algorithms
- Nonlinear degenerate PDEs and problems with interfaces

The programme attracted a large UK participation owing to the current strength of the UK in this field. A total of nine events, from one-day meetings to week-long workshops, drove the programme forward. Significant collaborative work was done in several areas, including nonlinear PDEs, free boundary problems, superconductivity, evolving surfaces, microstructure, adaptivity, multiscale algorithms and wave propagation. Applications in engineering and in industry received special attention.

Programme 51: Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics

Quasilinear hyperbolic systems in divergence form, commonly called “hyperbolic conservation laws”, govern a broad spectrum of physical phenomena in compressible fluid dynamics, nonlinear material science, general relativity, etc. Such equations admit solutions that may exhibit various kinds of shocks and other linear and nonlinear waves (propagating phase boundaries, fluid interfaces, gravitational waves, etc.) which play a dominant role in multiple areas of physics: astrophysics, cosmology, dynamics of (solid–solid) material interfaces, multiphase (liquid–vapour) flows, combustion theory, etc.

In recent years, major progress has been made in both theoretical and the numerical aspects of the field, while the number of applications has skyrocketed. Hyperbolic models arising in applications often face serious mathematical difficulties related to the occurrence of discontinuities, coordinate singularities, resonance between two or more wave speeds, elliptic regions in phase space, etc. This programme set out to build new bridges between the most recent developments in the general mathematical theory and the areas of applications that are currently most active. In particular, five main themes were addressed:

- Nonclassical shock waves and propagating phase boundaries
- Well-posedness theory of systems of conservation laws
- Multidimensional hyperbolic problems
- Computational methods for complex fluid flows
- Hyperbolic models in general relativity

Weekly seminars, short courses, workshops and conferences provided valuable opportunities for reviewing the state of the art for physical models, techniques of mathematical analysis and numerical analysis. Many challenging problems in the field were discussed.

One of the most unexpected and lasting outcomes of the programme was the launching of a new mathematical research journal, the *Journal of Hyperbolic Differential Equations*, devoted precisely to the topics treated during this programme.

Programme Participation

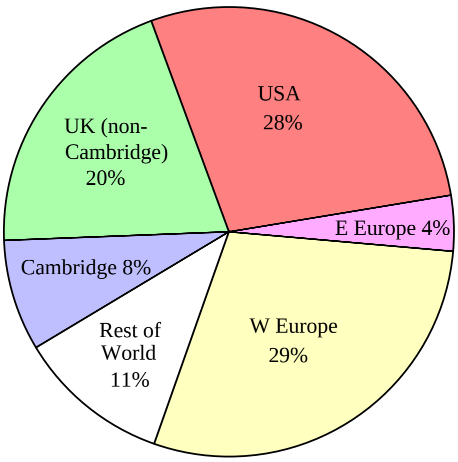
A total of 1114 visitors was recorded for 2002/2003. This includes 240 long-stay participants, each staying between two weeks and six months (6 weeks on average), and 351 short-stay participants who stayed for two weeks or less. Within the five completed programmes there was a total of 27 workshops (periods of intense activity on specialised topics) which attracted a further 297 visitors to the Institute. There were many others who attended informally at lectures, workshops, Institute Seminars or other events. Within all the programmes, workshops and other activities, around 988 seminars were given in total at the Institute during the year.

In addition to workshops, which serve to widen UK participation in programmes, programme organisers are encouraged to organise more informal special days, short meetings or intensive lecture series which can attract daily or short-term visitors, so further opening the activities of the Institute to the UK mathematical community.

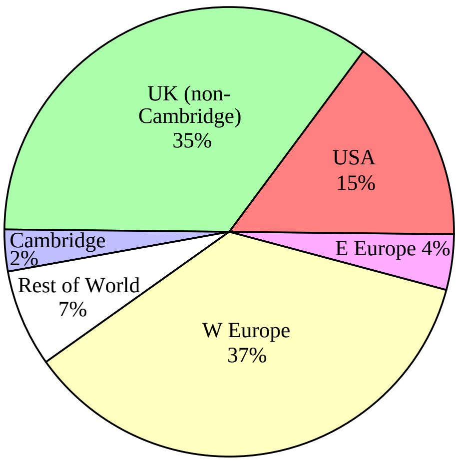
The Institute also funds visits by programme participants to other UK institutions to give seminars, and 123 such seminars took place last year.

Programme	Long-stay participants	Mean stay (days)	Short-stay participants	Mean stay (days)
Foams and Minimal Surfaces	27	23	37	7
Computation, Combinatorics and Probability	46	72	23	9
New Contexts for Stable Homotopy Theory	38	58	41	10
Computational Challenges in Partial Differential Equations	57	41	125	5
Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics	72	32	125	7
Totals 2002/2003	**240**	45	351	7

The pie charts below show the percentages of long-stay and short-stay participants broken down by country of residence:

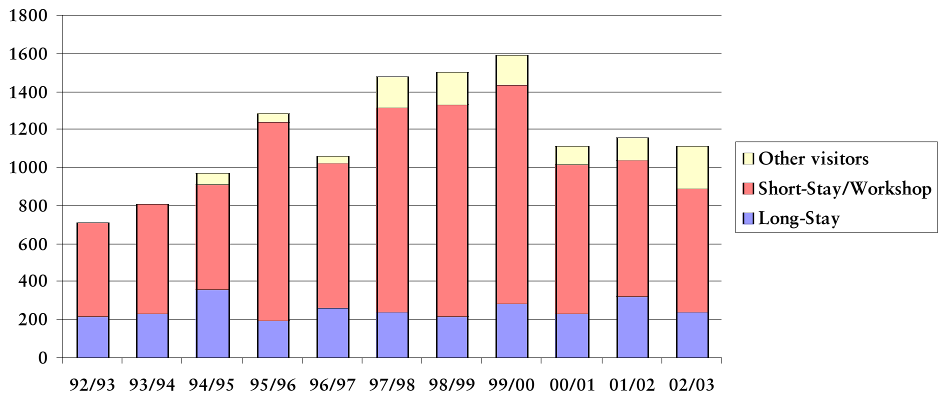


Long-stay participants

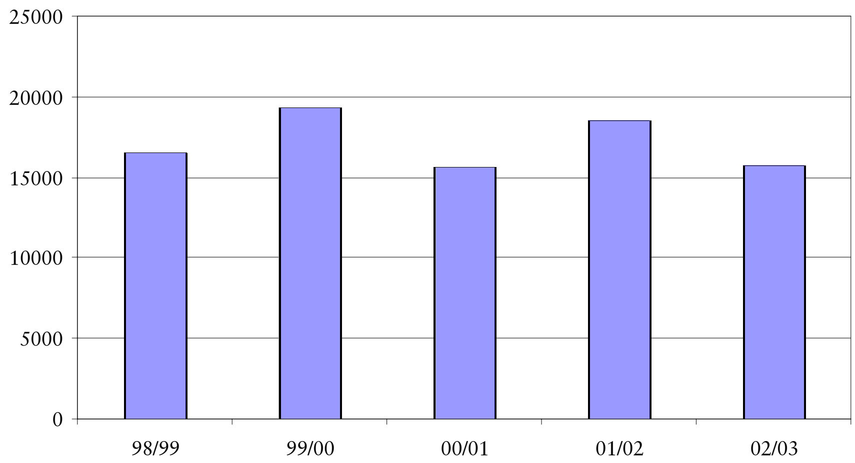


Short-stay and workshop participants

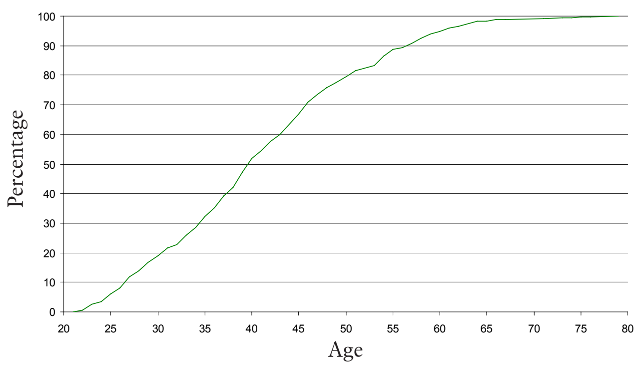
The following chart summarises the total figures for long- and short-stay participation since the Institute began:



The following chart summarises the total number of person-days for long- and short-stay participants combined:



The median age for long- and short-stay participants combined in 2002/2003 is 40 years, with an interquartile range of 33–48 years. The following chart shows the cumulative frequency of participant ages:



More detailed statistics, including visit dates and home institutions of participants, and a complete list of seminars and papers are given in the Appendices, available separately from the Institute or at

http://www.newton.cam.ac.uk/reports/0203/appendices.html

National Advisory Board and UK Mathematics

Membership of the National Advisory Board as at 31 July 2003 was as follows:

Professor Sir Michael Berry FRS	University of Bristol
Professor J Brindley	University of Leeds
Professor KA Brown FRSE	University of Glasgow
Professor J Howie FRSE	Heriot-Watt University
Dr RE Hunt	Deputy Director, Newton Institute
Professor Sir John Kingman FRS	Director, Newton Institute
Professor MA Moore FRS	University of Manchester
Dr H Ockenden	University of Oxford
Professor EG Rees FRSE	University of Edinburgh
Dr M Sheppard	Schlumberger Cambridge Research Ltd
Professor AFM Smith FRS (Chairman	Queen Mary, University of London
Professor DM Titterington FRSE	University of Glasgow

National Advisory Board

Following discussions with EPSRC, a National Advisory Board (NAB) for the Institute was established during 1999. Its remit is "To advise the Director in all matters relating to the role of the Newton Institute as a National Institute for the Mathematical Sciences."

The membership, as at 31 July 2003, is given in the table above. The overlap with the Scientific Steering Committee and Management Committee is deliberate and intended to ensure good communication with the Board. Some of the issues addressed by the NAB have been:

- The attendance of young UK scientists
- The Institute's strategy vis-à-vis its national role, interdisciplinarity and outreach
- Flexibility in the scientific programming to respond to new developments
- The interests of the EPSRC programmes which contribute to the Institute
- A focus on the Institute's databases to enable it to produce information on subject coverage, the geographical distribution of participants and the status of participants
- The Institute's mode of interface with industry

Anyone with views about the national role of the Institute is invited to make these known to any member of the NAB.

Symposia Activities

The Institute continues to maintain a list of forthcoming UK symposia, workshops, etc., in the Mathematical Sciences. This list is maintained in consultation with representatives of LMS, IMA, RSS, ICMS (Edinburgh) and the Warwick Mathematics Research Centre. For details, see

http://www.newton.cam.ac.uk/symposia.html

UK correspondents

During 2000/01, at the suggestion of the National Advisory Board, the Newton Institute established a list of correspondents in UK Universities to act as a channel of communication between the Institute and the mathematical sciences community in the UK. During 2002/03, following further suggestions, the Director wrote to a number of relevant *non*-University institutions and learned societies to invite them also to nominate correspondents. All correspondents are regularly informed about activities of the Institute, and it is their responsibility to ensure that the information is disseminated to relevant groups and individuals within their institution, in both mathematics and other appropriate departments. Correspondents also provide feedback to the Institute. The names of all the correspondents so far established can be found on the Institute website at

http://www.newton.cam.ac.uk/correspondents.html

Universities and other relevant bodies not yet represented on this list are encouraged to provide a suitable nominee.

Seminars

Long-term participants in Newton Institute programmes are strongly encouraged to visit other UK institutions during their stay at the Institute, and many did so during 2002/2003 (see p 4). To promote this activity, the Institute covers on request the travel costs within the UK for any overseas participant.

The Institute has recently set up a register of overseas participants who are willing to travel to other UK institutions to give seminars. It is hoped that organisers of seminar series will consult this register when planning their schedule of speakers and will contact potential speakers directly. The register can be found at

http://www.newton.cam.ac.uk/programmes/Speakers.html

Satellite Workshops

The Institute encourages organisers of longer (4- or 6-month) programmes to cooperate with local organisers in holding "satellite" workshops at UK Universities and institutions outside Cambridge.

Satellite workshops are on themes related to the Institute programmes, and involve a significant number of longer-term overseas participants from the Institute. They also, crucially, draw in and involve UK mathematicians and scientists who might not otherwise be able to participate substantially in the Institute programme.

Costs for satellite workshops are typically approximately £10,000 (excluding the overseas travel costs of Institute participants) and are shared approximately 50/50 between the Institute and the host institution. Both EPSRC and LMS welcome applications from host institutions for grants to cover their share of the costs (subject to the usual review procedures), and we are very grateful to both organisations for the fact that all such applications have so far been successful.

Institutions interested in holding satellite workshops should contact either the organisers of the relevant programme or the Deputy Director, Dr RE Hunt (R.E.Hunt@newton.cam.ac.uk).

Institute Seminars

The regular series of Institute Seminars, held on Mondays during term-time, is intended to be of general interest and to attract a wide range of mathematical scientists. Audio files of Institute Seminars, with accompanying transparencies and stills, are published on the web at

http://www.newton.cam.ac.uk/webseminars/

This year's seminars were:

- László Lovász (Microsoft Research), *Global information from local observation*
- Bill Dwyer (Notre Dame), *Lie groups and their classifying spaces*
- Jeffrey Steif (Chalmers), *An overview of certain phase transitions*
- Douglas Ravenel (Rochester), *What is elliptic cohomology?*
- Steve Lichtenbaum (Brown), *Special values of zeta-functions*
- Tom Pence (Michigan), *Multifield theory for materials*
- Charlie Elliott (Sussex), *Mathematical models and computation of flux penetration in type II superconductors*
- Carsten Carstensen (Vienna), *Mathematical and computational aspects of nonconvex minimisation problems allowing for microstructure*
- Constantine Dafermos (Brown), *Progress in hyperbolic conservation laws*
- Franco Brezzi (Pavia), *Theoretical and numerical questions related to thin elastic plates and shells*
- Joel Smoller (Michigan), *Cosmology of the large scale structure of the universe*

International Activity

EMS

The European Mathematical Society (EMS) was founded in 1990 in Madralin, near Warsaw (Poland). The meeting which created the EMS was held under the auspices of the European Mathematical Council, chaired by Sir Michael Atiyah before he came the first Director of the Newton Institute.

The purpose of the Society is "to further the development of all aspects of mathematics in the countries of Europe." In particular, the Society aims to promote research in mathematics and its applications, as well as concerning itself with the broader relation of mathematics to society. The EMS acts as an intermediary between mathematicians and those in charge of politics and funds in Brussels; the membership consists of about 50 mathematical societies throughout Europe and around 2000 individual members who have joined through their national societies. More information can be obtained from the Society's website at **http://www.emis.de/**

The current Director of the Newton Institute, Sir John Kingman, became President of the EMS in January 2003. His term of office runs until the end of 2006.

ERCOM

ERCOM ("European Research Centres On Mathematics") is a committee of the EMS consisting of the directors of all the research centres and institutes throughout Europe which have a substantial visitor research programme in the mathematical sciences. ERCOM was founded in 1997 and meets annually. The current chair is Professor M Castellet of the Centre de Recerca Matemàtica in Barcelona. The purposes of ERCOM are:

- to constitute a forum for communication and exchange of information between the centres themselves and with EMS
- to foster collaboration and coordination between the centres and with EMS
- to foster advanced research training on a European level
- to advise the Executive Committee of the EMS on matters relating to activities of the centres
- to contribute to the visibility of the EMS
- to cultivate contacts with similar research centres within and outside Europe

ERCOM is particularly concerned at present with the EC's Framework programmes, and is making representations to ensure that basic research receives sufficient support in both the current and future frameworks, and that the underpinning nature of mathematics in the sciences (and indeed more widely) is properly recognised.

More information about ERCOM can be obtained from its website at **http://www.crm.es/ERCOM/** or from the Newton Institute. European visitors are particularly encouraged to pick up a leaflet.

EPDI

The European Post-Doctoral Institute for Mathematical Sciences (EPDI) was founded in 1995 by the Newton Institute, the Institut des Hautes Études Scientifiques (Bures-sur-Yvette, France) and the Max-Planck-Institut für Mathematik (Bonn, Germany). Since then six more centres have joined the group, in Leipzig, Vienna, Djursholm, Warsaw, Barcelona and Zurich.

Each year, EPDI offers five two-year grants to young European scientists who have recently completed their PhDs, on condition that they spend at least 18 months in a foreign country and between 6 and 18 months at one of the EPDI institutes. Competition is strong and the scientific quality is high. British applicants, who are currently under-represented, are particularly encouraged.

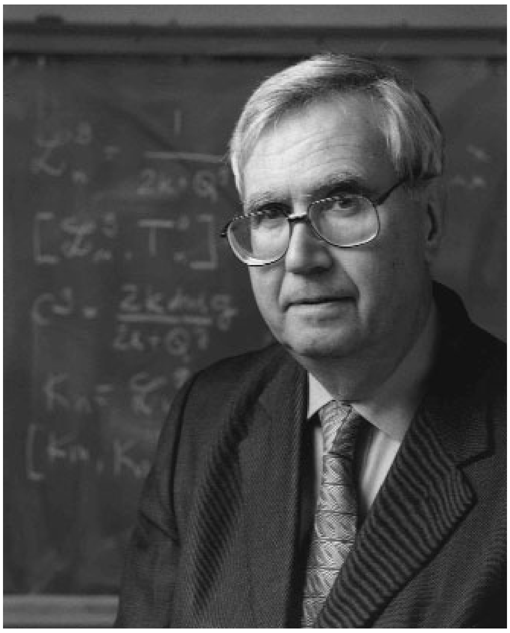
Further information, including full conditions for grant applications, can be found at the EPDI website, **http://www.ihes.fr/EPDI/**

Other Institute News

Peter Goddard

Professor Peter Goddard CBE ScD FRS, one of the prime movers in the establishment of the Isaac Newton Institute, is to become the eighth director of the Institute for Advanced Study, Princeton, on 1 January 2004.

J Hedgecoe



Professor Peter Goddard

Professor Goddard's research is in string theory and conformal field theory. He was elected a Fellow of the Royal Society in 1989, and received the Dirac Prize and Medal of the International Centre for Theoretical Physics in 1997 in recognition of his "farsighted and highly influential contributions to theoretical physics" over many years. He received his CBE in 2002, the same year in which he became President of the London Mathematical Society (as which he will now stand down in November 2003). He is also a Fellow of the Institute of Physics and of the Royal Society of Arts.

The Newton Institute was formally incorporated in 1990, two years ahead of its official opening. The first Director of the Institute, Sir Michael Atiyah, was also President of the Royal Society at the time and so the then Dr Goddard, who had been pivotal in the establishment and development of the Institute, was appointed as its first Deputy Director. In 1994 he moved to become Master of St John's College, Cambridge, and Professor in the Department of Applied Mathematics and Theoretical Physics.

The Institute for Advanced Study was founded in 1930 and is an independent, private institution with an academic membership of around 200 at any one time. The Institute's research fields include mathematics, social sciences, historical studies and natural sciences, and past faculty members include Albert Einstein, John von Neumann and Kurt Gödel.

Stokes Centenary Meeting

On Tuesday 18 March 2003 the Institute hosted a meeting to celebrate the centenary of Sir George Gabriel Stokes (1819–1903). The day-long event was devoted to his life, his scientific work and his influence on modern mathematics and science. The speakers were Professors Michael Hayes (University College Dublin), Keith Moffatt (Cambridge), Alastair Wood (Dublin City University), David Wilson (Iowa State), Sir Michael Berry (Bristol) and Sir Michael Atiyah (Edinburgh). Following the talks themselves, an exhibition of Stokes memorabilia (including original letters and other documents) was held at Pembroke College before dinner. The meeting was extremely well attended.

The talks can be heard online at

http://www.newton.cam.ac.uk/webseminars/stokes/

S Wygard

Professor Keith Moffatt delivering his lecture at the Stokes Centenary Meeting

National Science Week

Dr Robert Hunt, Deputy Director of the Institute, gave a public lecture on Saturday 22 March 2003 as part of National Science Week. In his lecture, entitled *Maths at work in the real world*, Dr Hunt explained how mathematics, despite sometimes appearing abstract and irrelevant to many people, is actually at work in our everyday lives. The talk focussed on examples of mathematics in nature and on how pieces of technology which we take for granted depend on mathematics for their very existence, and conveyed the excitement of applying mathematics. The lecture was well attended and the audience included many teenagers.

S Wygard

Dr Robert Hunt addresses the audience at his public lecture entitled "Maths at work in the real world"

Posters in the London Underground

The *Maths in the Underground* project, which ran during 2000 in the trains of the London Underground, was described in the Institute's *Annual Reports* for 1999–2000 and 2000–2001 but continues to generate considerable interest. A reprinting, kindly funded by EPSRC, resulted in free sets of the posters being distributed to every UK secondary school courtesy of the OCR Examinations Board in September 2001. Amongst other film and TV appearances, the posters were recently used as set dressing in the MGM film *Agent Cody Banks* released in 2003, and will be featured in the maths classroom in the next series of *Grange Hill* to be shown on BBC1 for 20 episodes starting in January 2004. They will also be shown in a video entitled *Journey through College Mathematics* being produced by Valencia Community College in Orlando.

All available copies of the posters have now been distributed or sold, despite several reprintings. However, the Institute still receives regular requests for copies, so an agreement has now been reached for the Mathematical Association to reprint the posters and sell them through its website. The Institute has also produced a pocket-sized booklet of the posters, which will be sold at the Institute and via its website for a nominal £1.00 (plus postage and packing) per booklet.

The posters can be seen on the web at

http://www.newton.cam.ac.uk/wmy2kposters/

where details of the booklet and the Mathematical Association reprinting can also be found.

Elements of Grace Exhibition

From 5 September to 31 October 2002 the Institute hosted an exhibition by Canadian artist Catherine M Stewart (see front and back covers), with the financial assistance of the Canada Council for the Arts. The exhibition consisted of two suites: *Elements of Grace* is a collection of photo-etchings combining diagrams from *Principia Mathematica* with photographic details of the human body, and *Copernican Notes* is a set of multiple plate etchings combining text and diagrams from *De Revolutionibus Orbium Cœlestium* with images of moving figures.

The artist explained that "The use of the human form in combination with these scientific references highlights the fact that it is through our bodies that we experience reality and acquire knowledge about the physical world. In both sets of prints, the human body is visually linked to the eternal forces of nature."

Catherine M Stewart kindly donated one of the works, *Deriving the Motions of Saturn and Jupiter II* from *Copernican Notes*, to the Institute, where it is now displayed on the first floor.

Public Understanding Events for 10th Anniversary Year

S Wilkinson

Rob Eastaway and his willing assistant during "How Long is a Piece of String?"

Last year the Newton Institute celebrated its tenth anniversary as a national resource for research in the mathematical sciences since it was opened in 1992 by HRH the Duke of Edinburgh.

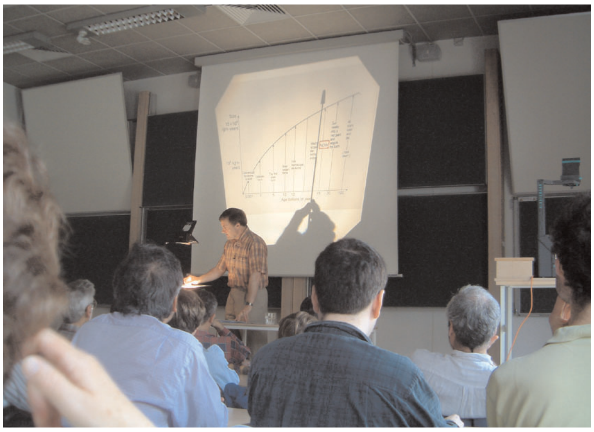
A variety of events took place in summer 2002 to mark the occasion, some academic and others aimed at the general public. The academic events have already been described in last year's *Annual Report*. The public events were aimed at three separate age groups and were extensively advertised within Cambridgeshire and the local area, especially to schools, and more widely via the web.

S Wilkinson

Kjartan Poskitt in full flow during "Newton's Apple (and other interesting maths)"

Each of the three public talks was well attended by around 100 people including many teenagers and younger children, and they were all enthusiastically received. The talks were:

- *How Long is a Piece of String?* for 8–11 year olds, by Rob Eastaway (author of *Why do Buses Come in Threes?* and *How Long is a Piece of String?*), Wednesday 3 July 2002
- *Newton's Apple (and other interesting maths)* for 11–16 year olds, by Kjartan Poskitt (author of the *Murderous Maths* series of books), Saturday 29 June 2002
- *Our Universe and Others* for ages 16+ and the general public, by John Barrow (author of 15 popular books including *Pi in the Sky*, *The Artful Universe* and *The Book of Nothing*), Saturday 6 July 2002



S Wilkinson

John Barrow presenting "Our Universe and Others"

More photographs from all three events together with audio recordings of the complete talks can be found at

http://www.newton.cam.ac.uk/webseminars/10th_anniversary

S Wilkinson

Visitors trying to solve puzzles during the interval

Newton Institute Publications

Papers and Preprints

Over 100 papers were produced or in preparation by participants at the Institute during 2002/2003 (a complete list is given in Appendix 7). Many of these are included in the Newton Institute's Preprint Series to which participants are encouraged to submit papers. A web page giving details of Newton Institute preprints is available at

http://www.newton.cam.ac.uk/preprints.html

Books arising from Newton Institute Programmes

The following titles were published during 2002/2003:

M Beck (Ed.)
Environmental Foresight and Models: A Manifesto
Elsevier, 2002
473pp, ISBN 008044086X, (Hbk) £75.00

M Burger and A Iozzi (Eds.)
Rigidity in Dynamics and Geometry
Springer, 2002
492pp, ISBN 3540432434, (Hbk) £56.00

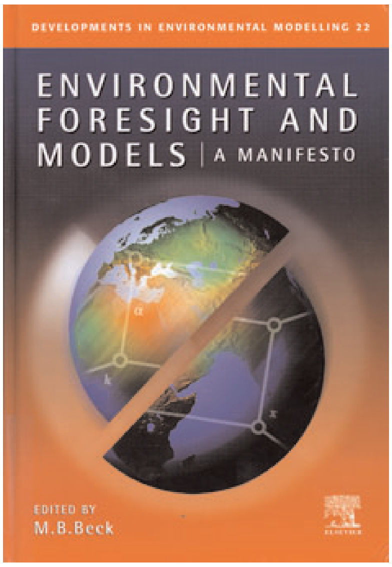
S Fomin (Ed.)
Symmetric Functions 2001: Surveys of Developments and Perspectives
Kluwer, 2002
288pp, ISBN 1402007736, (Hbk) £69.00

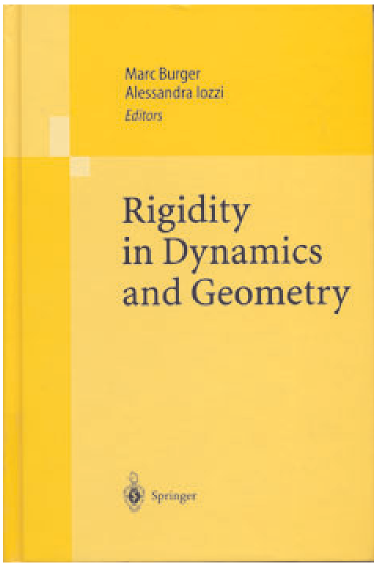
J Norbury and I Roulstone (Eds.)
Large-Scale Atmosphere–Ocean Dynamics I: Analytical Methods and Numerical Models
Cambridge University Press, 2002
400pp, ISBN 052180681X, (Hbk) £50.00

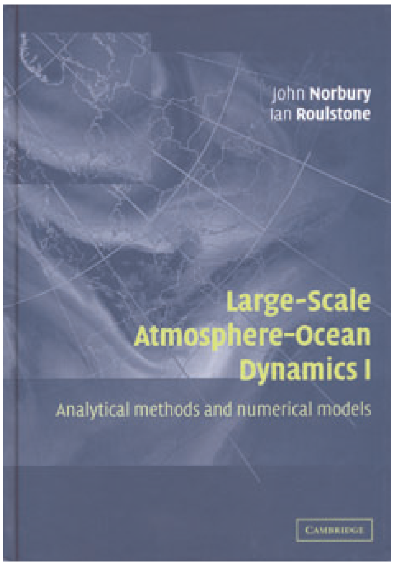
J Norbury and I Roulstone (Eds.)
Large-Scale Atmosphere–Ocean Dynamics II: Geometric Methods and Models
Cambridge University Press, 2002
394pp, ISBN 0521807573, (Hbk) £50.00

A complete list of books published as a result of Newton Institute programmes is available at

http://www.newton.cam.ac.uk/inibooks.html







Young Scientists

The Institute holds a number of events each year which are specifically targeted at young scientists. In 2002/2003 these events included:

- Euroworkshop on *Randomised Algorithms*
- Euroworkshop on *Elliptic Cohomology and Chromatic Phenomena*
- Euroconference on *Multiscale Modelling, Multiresolution and Adaptivity*
- Euroconference on *Hyperbolic Models in Astrophysics and Cosmology*
- Workshop on *Very High-Order Numerical Schemes for Conservation Laws*
- Series of lectures entitled *An Introduction to Numerical Methods*

The following young scientists were recipients of bursaries from the Cambridge Philosophical Society in 2002/2003:

Computation, Combinatorics and Probability
- A Hubenko (Memphis)
- N Fountoulakis (Oxford)

New Contexts for Stable Homotopy Theory
- N Ganter (MIT)
- S Wuetrich (Berne)

Computational Challenges in Partial Differential Equations
- O Lakkis (Sussex)
- V Styles (Milan)

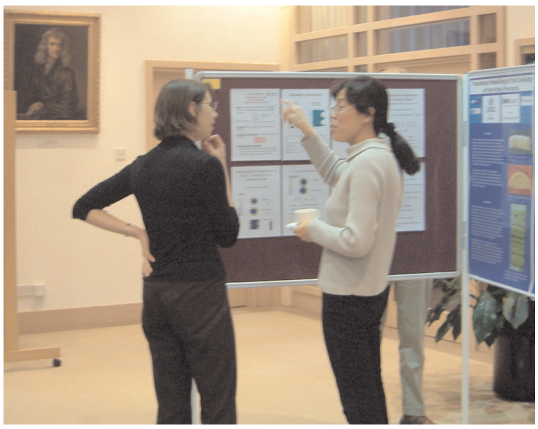
Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics
- AO Arancibia (Manchester Metropolitan)
- T Iguchi (Tokyo Institute of Technology)

The Institute recognises that junior researchers have much to contribute to and much to gain from Institute programmes and events. In order to maximise the information available to junior researchers, and to facilitate their involvement in Institute activities, we introduced in 1997 a category of Junior Membership of the Newton Institute. To be eligible for membership you must be a Research Student or within 5 years of having received a PhD (with appropriate allowance for career breaks) and you must work or study in a UK University or a related research institution.

Junior Members receive regular advance information about programmes, workshops, conferences and other Institute events via a Junior Members' Bulletin; detailed information about any workshops of an instructional or general nature likely to be of special interest to young researchers; and information about suitable sources of funding or support for visits to the Institute, when available.

The Institute makes available some of its general funds specifically to support junior researchers in Institute activities. Junior Members may apply for grants from these funds. The types of involvement supported include (but are not limited to) attendance at workshops, conferences, etc., and visits of up to two weeks to work or study with longer-term participants in the Institute's programmes. Those wishing to become Junior Members should consult the Institute's web site at

http://www.newton.cam.ac.uk/junior.html



S Wygard

Participants in the programme "Computational Challenges in Partial Differential Equations"

Scientific Steering Committee

Membership of the Scientific Steering Committee at 31 July 2003 was as follows:

Professor S Abramsky	University of Oxford
Professor JM Ball FRS FRSE	University of Oxford
Professor C Bernardi	University of Paris
Professor SK Donaldson FRS	Imperial College London
Professor Sir John Kingman FRS (Secretary)	Director, Newton Institute
Professor AJ Macintyre FRS FRSE	University of Edinburgh
Professor TCB McLeish	University of Leeds
Professor MA Moore FRS	University of Manchester
Professor EG Rees FRSE (Chairman)	University of Edinburgh
Professor G Ross FRS	University of Oxford
Professor BW Silverman FRS	University of Bristol
Professor J Stark	Imperial College London
Professor JR Whiteman	Brunel University

The Institute invites proposals for research programmes in any branch of mathematics or the mathematical sciences. The Scientific Steering Committee (SSC) meets in April and October each year to consider proposals for programmes (of 4-week, 4-month or 6-month duration) to run two or three years later. Proposals to be considered at these meetings should be submitted by 31 January or 31 July respectively. Successful proposals are usually developed in a process of discussion between the proposers and the SSC conducted through the Director, and may well be considered at more than one meeting of the SSC before selection is recommended. Proposers may wish to submit a shorter 'preliminary' proposal in the first instance with a view to obtaining feedback from the SSC prior to the submission of a full 'definitive' proposal.

Further details of the call for proposals, including guidelines for submission, can be found on the Institute's website at

http://www.newton.cam.ac.uk/callprop.html

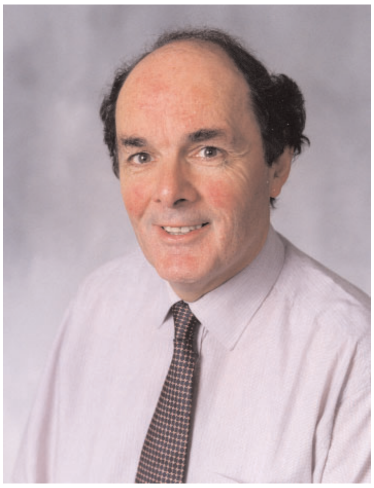
The scientific planning and organisation of each programme are the responsibility of a team of three or four Organisers (aided in some cases by an Advisory Committee). The Organisers recommend participants in the programme, of whom up to twenty can be accommodated at any one time; they also plan short-duration workshops and conferences within the programme, to which many more participants may be invited. Each programme is allocated a budget for salary support, subsistence allowances and travel expenses.

The following members of the Scientific Steering committee stepped down at the end of their term of service on 31 December 2002:

- Professor RH Dijkgraaf (Amsterdam)
- Professor CM Elliott (Sussex)
- Professor WT Gowers FRS (Cambridge)

The following new members were elected:

- Professor S Abramsky (Oxford)
- Professor SK Donaldson FRS (Imperial)
- Professor BW Silverman FRS (Bristol)



Professor EG Rees, Chairman of the Scientific Steering Committee

Scientific Policy Statement

From its inception, it has been intended that the Newton Institute should be devoted to the Mathematical Sciences in the broad sense. In this respect the Institute differs significantly from similar institutes in other countries. The range of sciences in which mathematics plays a significant role is enormous, too large for an Institute of modest size to cover adequately at any one time. In making the necessary choices, important principles are that no topic is excluded *a priori* and that scientific merit is to be the deciding factor.

One of the main purposes of the Newton Institute is to overcome the normal barriers presented by departmental structures in Universities. In consequence, an important, though not exclusive, criterion in judging the ‘scientific merit’ of a proposed research programme for the Institute is the extent to which it is ‘interdisciplinary’. Often this will involve bringing together research workers with very different backgrounds and expertise; sometimes a single mathematical topic may attract a wide entourage from other fields. The Institute’s Scientific Steering Committee therefore works within the following guidelines:

(a) the mixing together of scientists with different backgrounds does not *per se* produce a successful meeting: there has to be clear common ground on which to focus;

(b) each programme should have a substantial and significant mathematical content;

(c) each programme should have a broad base in the mathematical sciences.

Research in mathematics, as in many other sciences, tends to consist of major breakthroughs, with rapid exploitation of new ideas, followed by long periods of consolidation. For the Newton Institute to be an exciting and important world centre, it has to be involved with the breakthroughs rather than the consolidation. This means that, in selecting programmes, a main criterion should be that the relevant area is in the forefront of current development. Since the Institute’s programmes are chosen two to three years in advance, it is not easy to predict where the front line will be at that time. The best one can do is to choose fields whose importance and diversity are likely to persist and to choose world leaders in research who are likely to be able to respond quickly as ideas change.

Although the novelty and the interdisciplinary nature of a proposed programme provide important criteria for selection, these must be subject to the overriding criterion of quality. With such a wide range of possibilities to choose from, the aim must be to select programmes which represent serious and important mathematical science and which will attract the very best mathematicians and scientists from all over the world. However, the Institute is receptive also to proposals of an unorthodox nature if a strong scientific case is made.

Although the Institute operates on a world-wide basis and contributes thereby to the general advancement of mathematical science, it must also be considered in the context of UK mathematics. A natural expectation of all those concerned is that each programme will be of benefit to the UK mathematical community in a variety of ways. If the UK is strong in the field, UK scientists will play a major part in the programme; if the UK is comparatively weak in the field, the programme should help to raise UK standards. Instructional courses, aimed primarily at younger researchers and research students, will play a vital role here.

Because of the wide base of support for the Newton Institute in the EPSRC and elsewhere, the Institute’s programmes shall as far as possible represent an appropriate balance between the various mathematical fields. In order to retain the backing of the mathematical and scientific community, the Institute will run programmes over a wide range of fields and, over the years, achieve this balance. Such considerations, however, are secondary to the prime objective of having high quality programmes.

Future Programmes

The diagram below shows the forthcoming programmes which have been selected by the Scientific Steering Committee. To participate in a workshop, registration is required. For longer-term participation in a programme, an invitation is usually required, and applications are best made to the programme organisers in the first instance. Further details of each of these programmes, including

- the scientific content and background
- the names of the organisers
- the names of those who have so far been invited to take part in the programme
- contact details
- dates, topics and information about workshops which will take place during the programme

can be found on the Newton Institute website at **http://www.newton.cam.ac.uk/programmes/**

Further information on how to participate in Newton Institute programmes can be found at **http://www.newton.cam.ac.uk/participation.html**

JAN JUL SEP DEC

2003

Computational Challenges in Partial Differential Equations
Spaces of Kleinian Groups and Hyperbolic 3-Manifolds
Granular and Particle-Laden Flows
Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics
Interaction and Growth in Complex Stochastic Systems

2004

Statistical Mechanics of Molecular and Cellular Biological Systems
Magnetic Reconnection Theory
Magnetohydrodynamics of Stellar Interiors
Random Matrix Approaches in Number Theory
Quantum Information Science

2005

Model Theory and Applications to Algebra and Analysis
Pattern Formation in Large Domains
Developments in Quantitative Finance
Global Problems in Mathematical Relativity

2006

Principles of the Dynamics of Non-Equilibrium Systems
Spectral Theory and Partial Differential Equations
TBA
Logic and Algorithms
Noncommutative Geometry

Management Committee

Membership of the Management Committee at 31 July 2003 was as follows:

Dr A Bramley	EPSRC
Professor J Brindley	Co-opted at the discretion of the Committee
Professor WJ Fitzgerald	Council of the School of Technology
Professor GR Grimmett	Head of Department, DPMMS
Professor EJ Hinch FRS	Trinity College
Professor J Howie FRSE	LMS
Dr RE Hunt (Secretary)	Deputy Director, Newton Institute
Professor PT Johnstone	St John's College
Professor Sir John Kingman FRS	Director, Newton Institute
Professor PV Landshoff (Chairman)	General Board
Professor TJ Pedley FRS	Head of Department, DAMTP
Professor EG Rees FRSE	Chairman of Scientific Steering Committee
Professor Sir Martin Rees FRS	Council of the School of Physical Sciences
Dr C Teleman	Faculty of Mathematics

The Management Committee is responsible for overall control of the budget of the Institute, and for both its short-term and long-term financial planning. The Director is responsible to the Management Committee, which provides essential advice and support in relation to fund-raising activity, employment of staff at the Institute, appointment of organisers of programmes, housing, library and computing facilities, publicity, and general oversight of Institute activities.

Its aim is to facilitate to the fullest possible extent the smooth and effective running of the visitor research programmes of the Institute and all related activities. The Committee is especially concerned with the interactions between the Institute and its funding bodies, particularly the UK Research Councils, Cambridge University, the Cambridge Colleges, the London Mathematical Society, the Leverhulme Trust and others. It generally meets three times a year.

Staff of the Institute

The staff (full-time and part-time) of the Institute at 31 July 2003 was as follows:

- Wendy Abbott, Director's Administrative Assistant
- Amy Abram, Conference and Programme Assistant
- Dr Mustapha Amrani, Computer Systems Manager
- Tracey Andrew, Conference and Programme Secretary
- Elsie Batcheler, Assistant Librarian
- Lynn Berry, Catering Assistant
- Jonathan Chin, Deputy Computer Systems Manager
- Esperanza de Felipe, Housing Officer
- Louise Grainger, Receptionist
- Matt Hodson, Technical Assistant
- Dr Robert Hunt, Deputy Director
- Professor Konstantin Khanin, Hewlett-Packard Senior Research Fellow
- Professor Sir John Kingman FRS, Director
- Doreen Rook, Clerk
- Christine West, Institute Administrator
- Sara Wilkinson, Librarian and Information Officer (on leave)
- Stephen Williams, Senior Accounts Clerk
- Sarah Wygard, Acting Librarian and Information Officer

Hewlett-Packard Senior Research Fellow

Report from Professor Kostya Khanin

In the past year I have been working on three different problems: Burgers turbulence in unbounded domains and KPZ scalings, rigidity theory in one-dimensional dynamics and KAM theory via renormalisations.

In a joint paper with J Bec (Nice) we started a study of Burgers turbulence in unbounded domains more than a year ago. The statistical properties of shocks and the velocity field are determined by the structure of minimisers for a random time-dependent Lagrangian system with a potential of the form $F(x)W(t)$, where $W(t)$ is the white noise. It is interesting that fluctuations of minimisers for potentials of this type are the same as for KPZ potentials $F(x,t)$ which are white both in x and t. In both cases the scaling exponent for the spatial fluctuations of minimisers is equal to $-2/3$. However, the structure of minimisers seems to be much simpler in the first model which perhaps makes it more tractable rigorously. Notice that the analysis of the statistics of minimisers is closely related to the problems of first-passage percolation and directed polymers. Potentials of the type $F(x)W(t)$ are very interesting on their own, and it seems that they have not been studied before. We call such potentials quasi-stationary. Indeed in many cases they behave like stationary time-independent random potentials. At the same time, in the case when $F(x)$ is bounded, many properties of directed polymers and minimisers in quasi-stationary potentials are similar to the properties of the corresponding objects for completely independent non-stationary random potentials $F(x,t)$. The main aim of this ongoing research project is to study directed polymers in quasi-stationary potentials in different regimes and settings.

In the area of rigidity theory the main efforts of our joint research project with A Teplinsky (Ukraine) were concentrated on analysis of smoothness for conjugacies between two circle diffeomorphisms with singularities. We consider both critical circle maps and diffeomorphisms with breaks. It is assumed that both mappings have the same irrational rotation number and the same local structure of singular points, that is the same order of critical point or the same size of break. The main result in the case of critical circle maps can be formulated in the following way: two critical maps with the same but arbitrary irrational rotation number are C^1-smoothly conjugate to each other. In other words, the asymptotic metric properties of their trajectories are the same. It is quite remarkable that the rigidity properties are stronger in the presence of singularities. Indeed, in the case of smooth diffeomorphisms a similar result is known to be false. The result will be proven in a paper which is still in progress but we hope to finish in 2004. To the best of my knowledge this is the first rigidity-type result which holds without any Diophantine restrictions on a rotation number.

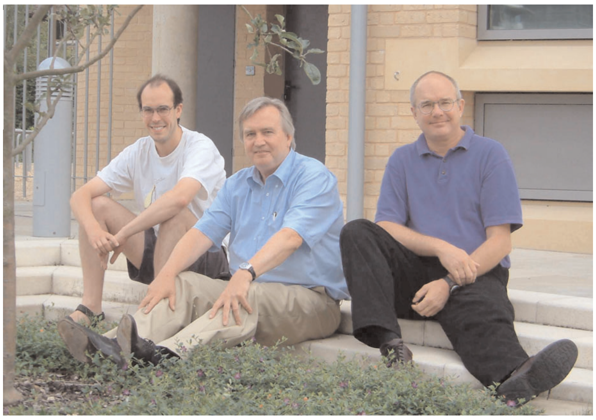
The third topic, KAM theory via renormalisations, is not new. Renormalisations in this context were used previously by R MacKay, K Khanin & Y Sinai, H Koch and others. Most of the results were obtained in the two-dimensional case which corresponds to Hamiltonian systems with two degrees of freedom. In this case the aim is to construct two-dimensional invariant tori with quasi-periodic trajectories on them. Using Poincaré first return maps the problem can be reduced to construction of invariant curves for two-dimensional area-preserving maps. In this case one can use continued fractions to define a natural renormalisation scheme. However, in the multi-dimensional case the situation is more complicated. The new idea which we use in a joint work with J Marklof (Bristol) and J Lopes-Dias (Instituto Superior Tecnico, Lisboa) is to use a multi-dimensional continued fraction algorithm based on the geodesic flow in the space of lattices. This algorithm was proposed by J Lagarias about 10 years ago but was not used in dynamical systems applications. In my view renormalisation based on Lagarias' algorithm is a very powerful tool which will be used intensively in dynamics in the future.

Foams and Minimal Surfaces

29 July to 23 August 2002

Report from the Organisers: A Kraynik (Sandia), H Stone (Harvard), E Terentjev (Cambridge), DL Weaire (Dublin)

S Wilkinson



Left to right: S Cox, Secretary to the organising committee, with organisers DL Weaire and A Kraynik

Background

Foams are familiar gas–liquid systems of wide importance to industry, exhibiting many properties of interest to physicists, mathematicians, engineers and other applied scientists. In mathematical terms, their governing principle (at least in equilibrium) is the minimisation of surface area. The study of idealised models of foam therefore throws up a long list of significant mathematical problems, some of them associated with the name of Joseph Plateau and his classic 1873 book. In recent years computation has given fresh impetus to the subject, since it enables us to explore the consequences of idealised models for complex disordered foams by accurately simulating them. Some of the problems have direct bearing on potentially significant areas of application, such as in determining the properties of metallic foams.

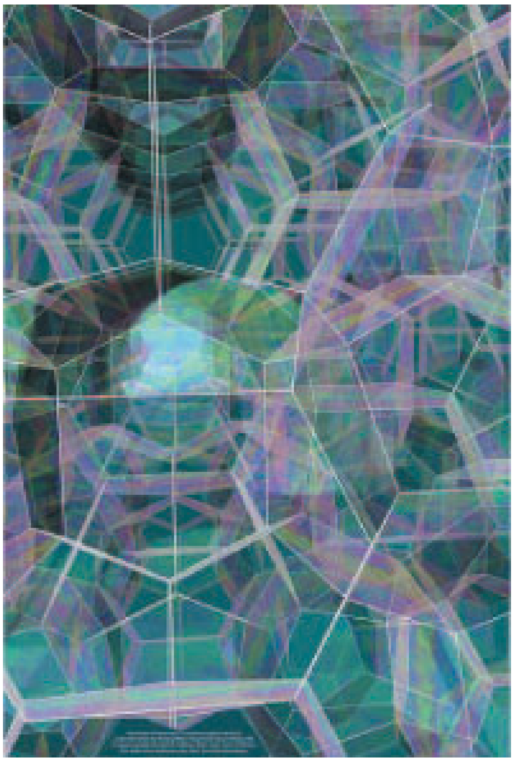
Programme Outline

This programme was conceived as an opportunity to maximise the cross-fertilisation of ideas between mathematicians, physicists and allied disciplines such as chemical engineering. It was successful in attracting world-leading figures from all these communities. Particularly notable are Thomas Hales, who in recent years has produced proofs of classic minimal conjectures (the Kepler problem and the honeycomb problem mentioned below), and Ken Brakke, originator and developer of the Surface Evolver software which is widely used in problems of surface energy minimisation.

The schedule of topics moved progressively from refined minimal problems to physical experiments and their simulation. At the outset, Hales outlined his strategy for the honeycomb and other problems. That the 2D honeycomb minimises line length for cells of equal size and any shape has been recognised for centuries, but proving this has been intractable until now unless restrictions such as straight edges are imposed. The general proof of Hales is what he calls an 'engineering' solution, patched together out of various inequalities which rigorously entrap every possible case.

What about the 3D case, the so-called Kelvin Problem? In 1887 Kelvin conjectured that surface area was minimised by a regular stacking of his 'tetrakaidecahedron'. His remarkable notebooks, in which some of his sudden insights into this problem are recorded, are stored in the Cambridge University Library, and participants were kindly given permission to scrutinise them during the programme itself. His conjecture was overthrown by the computational discovery of the Weaire–Phelan structure in 1994, and a proof by Sullivan and Kusner that this indeed has a lower surface area. But the proof that it is an absolute minimum remains to be found, and Hales commented (not very optimistically) on this difficult challenge.

Another challenge was thrown up by Andrew Kraynik, in the course of describing extensive simulations of large samples of disordered foam. It appeared that in the case of bubbles of equal volume, the lowest possible energy in such a bulk structure is that of a regular dodecahedron, obeying Plateau's laws for angles and curvature. However, Ken Brakke was able to show, by conformal transformation, that the regular dodecahedron is a saddle point, and that the energy *can* be lower for asymmetric bubbles. Yet another challenge concerns an upper bound on the shear elastic modulus of a 2D disordered foam. In addition, John Sullivan adduced a long list of rather general conjectures about periodicity, pressures and other properties of equilibrium foams. In some cases the physicists protested that the answers were 'obvious' and indeed were able to show the value of physical insights.



The Weaire–Phelan structure, as visualised by John Sullivan

Careful analysis of the statistics of the cellular patterns that represent foams in two dimensions have thrown up several intriguing correlations; these were reviewed by Schliecker, Delannay and others. The case of three dimensions has proved more difficult. For example, what is the 3D equivalent of von Neumann's law in 2D, which relates the growth rate of a cell to its number of sides? Sascha Hilgenfeldt explained the relevance of theorems of Minkowski which concern integrals of mean curvature of a cell. These lead to a 3D growth law which plays the role of that of von Neumann, if only in an average sense.

The phenomena of drainage (the motion of liquid through a foam) and rheology (which deals with the flow of the foam) lie at the frontier of current research. Howard Stone and others debated current ideas on drainage, for which an adequate phenomenology now distinguishes between foams with effectively rigid or free surfaces. Rheology is a notoriously difficult subject to get to grips with, and the description of foam flow at finite shear rates remains far from settled. Reinhard Höhler represented the current efforts to measure rheological properties reliably. While at the Institute, Masao Doi was able to derive and present a constitutive model for viscous foams that is based on affine film deformation. Another difficult question remaining to be resolved is concerned with normal stress differences.

It is much too soon to say that a truly comprehensive practical theory of foam behaviour is available. Many of its individual ingredients, mentioned above, remain poorly defined, and their couplings (e.g., drainage and rheology, drainage and coarsening) are still to be properly understood. Often the results of experiments are rationalised afterwards, based on the influences of the different surfactants used. This made for lively discussion.

Participants were joined throughout the programme by colleagues from different departments in Cambridge. Several participants also made visits to these departments to discuss 'foam-inspired' topics of mutual interest. On several occasions we were joined by researchers from Schlumberger Cambridge Research. Also, Mike Ashby from the Department of Engineering made a special presentation on the mechanical properties of metal foams.

The programme was enlivened by occasional glimpses of the history of the subject in the form of biographical sketches. Some of the personalities included Kelvin, Riemann, Minkowski and Cyril Stanley Smith.

The typical schedule for a given day was for no more than four half-hour talks or focussed one-hour discussions. Throughout the meeting the discussion was lively and interactive and it was particularly valuable that participants from different backgrounds generally attended all of the talks. To allow participants to solicit feedback on problems of current interest, we also held small sets of short five-minute talks with five further minutes allotted for discussion. This proved particularly successful.

Special Events

Surface Evolver Extravaganza

The workshop started with a Hewlett-Packard day, the 'Surface Evolver Extravaganza', in which several of the world's leading authorities on the use of Brakke's Surface Evolver entertained us with various applications from triply periodic crystal structures to knots. The day finished with a presentation by Brakke on the stability of soap film junctions, after which we studied protein foams from a local brewery.

Soap Bubble Geometry Contest

The *Soap Bubble Geometry Contest* was an opportunity to educate the general public. Frank Morgan performed demonstrations and set questions to test the audience's knowledge of how soap films interact. The contest was well attended, and the youngest participants, less than ten years old, showed great enthusiasm.

T Andrew

Frank Morgan presenting the "Soap Bubble Geometry Contest"

Presentation of Sculpture

Prior to the contest, John Sullivan presented the Institute with a minimal surface sculpture (below) as a token of gratitude on behalf of the participants. The sculpture will be displayed in the Institute building.

John Sullivan's computer-designed 'Minimal Flower 3' surface sculpture

Outcome and Achievements

The communities which were brought together interacted well, and many future collaborations should result which would not have emerged without such a stimulus. Some such collaborations were already active during the programme. Ken Brakke kindly volunteered to conduct practical sessions in the use of the Surface Evolver. These tutorials introduced several new users to the program and its applications.

It was particularly pleasing to observe the fruitful discussions between mathematical scientists from many different intellectual backgrounds. The Newton Institute is ideally suited and designed to facilitate such interactions. The general conclusion was that two-phase mixtures of academics can be as interesting, profitable and unpredictable as the mixture of gas and liquid that constitutes a foam.

Computation, Combinatorics and Probability

29 July to 20 December 2002

Report from the Organisers: M Dyer (Leeds), M Jerrum (Edinburgh), P Winkler (Bell Labs)

S Wygard

Left to right: M Dyer and M Jerrum

Scientific Background

Over time, the relationships between computer science and mathematics have become more wide ranging and profound. The traffic in ideas has been in both directions. On the one hand, computer science has opened up new areas of study to mathematics; on the other, increasingly sophisticated mathematical methods have been brought to bear on problems in computer science. These developments have been particularly evident in quantitative theoretical computer science, especially in the areas known as computational complexity and the analysis of algorithms. Both of these areas are concerned with the resources (often time or space) required to achieve specified computational tasks; the rough distinction is that the former concerns itself with general phenomena surrounding resource-bounded computation, while the latter probes the inherent difficulty of specific computational tasks.

The aim of this programme was to explore two particularly fruitful interfaces between computer science and mathematics, namely those involving combinatorics and probability theory. Since computers perform manipulations of finite structures (bit patterns or data structures of varying complexity, depending on the level at which we view the computation) it is not at all surprising that combinatorics is pervasive in the design and analysis of algorithms. However, the importance of probability may come as a surprise, given that determinacy and predictability would seem to be desirable features of a real computer! Nevertheless the notion of randomised computation has assumed an increasingly important role, both in theory and practice.

Within the broad area delineated by the title, a number of themes were to receive special emphasis.

- Randomised algorithms. This theme addressed the design and analysis of algorithms that make random choices, and also of deterministic algorithms when run on random instances. The theoretical basis of this study includes, for example, the study of parameters connected with random walks on graphs – coupling time, expansion, multicommodity flow, mixing time, etc. – and the relationships between them.

- Phase transitions in statistical physics and computer science. This heading covers not only finite random systems, but also (infinite sequences of) finite systems that exhibit phenomena akin to phase transitions. An example of the latter is the satisfiability threshold for random instances of CNF Boolean formulas with k literals per clause. Particular attention was paid to the apparent link between phase transitions and computational tractability.

- Random graphs and structures. We were concerned here with finite random structures, of which the Erdös–Rényi random graph model is the primal example. Algorithmic aspects received special attention.

- Probabilistic analysis of distributed systems. The central concern was to study the behaviour of distributed systems in the absence of global control. The prime example of such a system is the Internet, though one could also think of traffic on road networks, for example. This is a very new interdisciplinary area bringing together economics, game theory (Nash equilibria of agents in networks) and computer science.

Structure of the Programme

The programme attracted a strong contingent of nearly fifty long-term visitors. It began and ended with very well attended periods of heightened activity built around the workshops. Between these rather hectic periods of concentrated interaction, there was a quieter interlude for collaborative research projects. Momentum was maintained through a programme of regular weekly seminars. We were fortunate in having J Steif (Chalmers) and L Lovász (Microsoft Research) on hand to depict some of our interests to the wider world in the Institute's Monday Seminar series.

Workshops

Randomised Algorithms

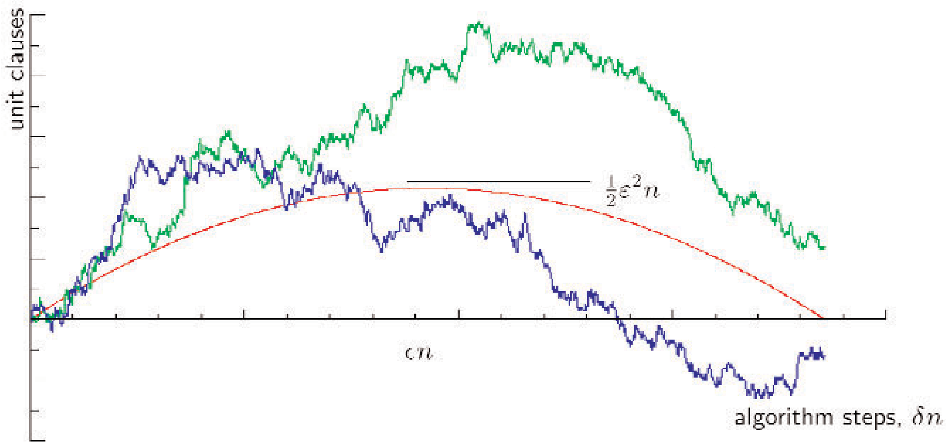
Euroworkshop, 5–16 August 2002

Organisers: M Dyer, M Jerrum and P Winkler

One of the appeals of randomised algorithms for the mathematician is that whereas the algorithms themselves are often quite simple, their analysis can be very challenging. Because of this, the study of randomised algorithms has prompted one of the most profound and fruitful interactions between computer science and mathematics. This workshop aimed to stimulate progress in this interdisciplinary area, by bringing together computer scientists with probability theorists, combinatorialists and others working in relevant areas of mathematics.

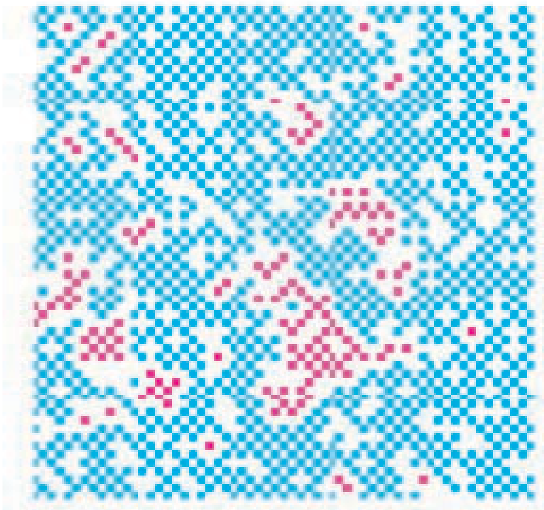
The plan was that the first of the two weeks would be concerned with mathematical foundations, for example, random walks on graphs, concentration of measure, etc. The second would deal with the design and analysis of algorithms which make random choices, including Monte Carlo and Markov Chain Monte Carlo (MCMC) algorithms and deterministic algorithms when run on random instances. This plan was roughly followed, though availability of speakers meant in practice that there was some blurring of the theory/application distinction.

A typical day was built around four talks of one-hour duration, a timetable that left plenty of time for informal discussions. Amongst the speakers giving talks of a more expository nature were: C Greenhill (Melbourne) who gave a tutorial on the "Differential equations method", an analytical tool whose value was amply illustrated in a number of other talks during the course of the workshop; M Jerrum (Edinburgh) who surveyed



Two realisations and the nominal behaviour of a stochastic process arising in the analysis of a MAX-2 SAT heuristic the varied methods, many of them quite recent, for the non-asymptotic analysis of rates of convergence to stationarity of Markov chains; A Sokal (New York) who demonstrated how methods from statistical physics can answer long-standing open questions from combinatorics (and how combinatorics can in the agenda of statistical physics); and P Winkler who untangled for us the web of mixing times (measures of convergence of Markov Chains).

Many of the "leading lights" of the field were at the workshop: M Dyer (Leeds) introduced his surprising discovery that dynamic programming is useful for counting as well as optimisation; R Kannan (CMU) described "blocking conductance", a new and more precise tool for bounding mixing times of Markov chains; and A Sinclair (UC, Berkeley) explained how Clifford algebras might one day provide a practical approach to estimating the permanent of a non-negative matrix.



A configuration of the "hard core lattice gas" model – a finite square fragment of a conceptually infinite random configuration

Random Structures

Workshop, 26 August–6 September 2002

Organisers: M Dyer, M Jerrum and P Winkler

This two-week workshop was structured into two complementary parts of equal duration: *Combinatorial and Computational Aspects of Statistical Physics*, and *Random Graphs and Structures*. The two parts were to be united by the notion of phase transition, broadly interpreted. Roughly speaking, Part 1 was to concentrate on phase transitions in infinite systems (e.g., the Ising model on $\mathbb{Z}^2$), and Part 2 on "phase transitions" in finite structures (e.g., random graphs or random partial orders). The interdisciplinary aspect was evident, with computer scientists meeting with probabilists, mathematical physicists (particularly in Part 1) and combinatorialists (particularly in Part 2).

In the event there were about sixty officially registered participants, and a significant additional number (particularly from in or around Cambridge) attended on an informal basis. Any selection of talks from the workshop will necessarily be somewhat arbitrary, but here is a brief summary of a few notable contributions of more general interest. G Grimmett (Cambridge) surveyed some open problems in statistical mechanics, reported on recent progress on their solution and expressed optimism that they could be completely resolved in the next few years, in the light of recent work on "stochastic Löwner evolutions". J Kahn (Rutgers) presented an important new result (with D Galvin) that the hard-core lattice gas model in $\mathbb{Z}^d$ exhibits a phase transition at an activity that decays to 0 with d. (This has long been expected, but not proved.) M Krivelevich (Tel Aviv) gave a personal account of the future of colouring problems on random graphs. S Janson (Uppsala) and P Tetali (Georgia Tech.) described progress on concentration inequalities for combinatorial applications; such inequalities are an essential tool for research in this area. Finally, A Scott (UCL) drew a surprising connection between two seemingly unrelated topics: a theorem of Dobrushin regarding uniqueness of Gibbs measures, and the Lovász Local Lemma. (This was joint work with A Sokal.)

The meeting was supported by the MathFIT initiative of EPSRC and the London Mathematical Society.

Topics in Computer Communication and Networks

Workshop, 16–20 December

Organisers: M Dyer, L Goldberg, M Jerrum, F Kelly and P Winkler

This one-week meeting, partly supported by Cisco Systems Inc., attracted a greater than expected number of participants, suggesting that the topic has captured the imagination of the community.

A number of themes could be identified. One of these was models for the web graph. The World Wide Web can be viewed as an ever-evolving graph, with web pages as nodes and links as directed edges. This graph, though random in some sense, has very different properties to the usual Erdös–Rényi random graph; for example, the vertex degrees obey a power law. A number of participants, among them J Chayes (Microsoft Research) and C Cooper (King's College, London), proposed and analysed graph models that could potentially explain these observed features.

A second theme was the "cost of dispensing with global control". In other words, how far can a Nash equilibrium (obtained when agents optimise their own utility) be from a globally optimal solution imposed by centralised control? This question was taken up by T Roughgarden (Cornell) in the context of routing in networks.

The final theme in this non-exhaustive list was the complexity of computing Nash equilibria. Problems involving these equilibria are interesting and challenging from a computational viewpoint, because they do not seem to fall naturally into the usual complexity-theoretic classes such as "NP-complete". V Vazirani (Georgia Tech.) presented one of the first moves in the direction of positive results.

Outcomes and Achievements

The programme was broad in scope, covering work in computer science, probability, statistical physics, graph theory and combinatorics. Consequently the research achievements of the programme were varied in topic. The work was highly collaborative, and often bridged two or more areas. We summarise the outputs below, alphabetically by first collaborator. Most have resulted in one or more publications, and several formed the topic of a seminar during the programme. The overall success of the research programme can be judged by the volume and quality of the output. Further details of many of the items mentioned below may be found in the growing list of Newton Institute preprints, and in due course in a special issue of *Random Structures and Algorithms*.

Bollobás collaborated with Brightwell on the paper *How Many Graphs are Unions of k-cliques?*, and with Simonovits on extending their theorem, with Balogh, concerning the number of graphs that exclude a certain set $\mathcal{L}$ of subgraphs. (When $\mathcal{L}$ is the singleton set containing the complete graph on k vertices, this is the Erdös–Kleitman–Rothschild theorem.) Brightwell worked with Winkler on problems at the interface between graph theory and statistical physics. Asymptotics were obtained for the following interesting problem: When does the unique simple invariant Gibbs measure for the hard-core gas model on the Bethe lattice (i.e., infinite regular tree) cease to be extremal?

Cryan worked with Dyer and Randall on the counting problem for a cell-bounded version of the contingency table problem, and with Dyer, Jerrum and Karpinski on polynomial time generation of independent sets in hypergraphs. Contingency tables are matrices of non-negative integers with given row- and column-sums, which arise in statistics; in determining whether an apparent correlation is statistically significant it is necessary to be able to count and/or sample such tables.

Dyer completed work on a new approach using dynamic programming to polynomial time approximate counting for the knapsack and related problems. This gives simpler, and more efficient, algorithms than were known previously. He worked with Frieze and Vigoda on sampling from the Ising and hard-core models on "large girth" graphs. Related work, undertaken with Goldberg and Greenhill, attempts generalisations to (other) H-colouring (also known as graph homomorphism) problems. Also, with Goldberg and Jerrum, he investigated alternative dynamics for sampling particle systems on "simple" graphs, in particular colourings. With Sinclair and Vigoda, he completed some work on the relationship between spatial and temporal mixing in physical systems.

Fill continued collaboration with Janson on a series of papers giving refined analysis of the Quicksort algorithm. Quicksort is a classical randomised algorithm for sorting items using pairwise comparisons. Describing the distribution of the number of comparisons used in sorting n items is a challenging problem.

Goldberg, Martin and Paterson completed work on Markov chains on three-colourings of the rectangular grid in $\mathbb{Z}^2$, and showed rapid mixing of the Glauber dynamics. Somewhat surprisingly, the question of rapid mixing for four to six colours remains open, even though movements within the state space appear then to be less constrained.

Greenhill worked with Rucinski (and Wormald) on applying the "differential equation approach" to random hypergraph processes. The idea, made rigorous by Kurtz, and introduced to the algorithms community by Karp and Sipser, is to approximate the evolution of a Markov chain on $\mathbb{Z}^d$ by the solution to a differential equation. As the size of the state space increases, the approximation becomes more exact.

Hubenko worked (with Gyárfás) on certain factors of the n-cube. The particular feature of these factors is that they arise as induced subgraphs of the cube. The prototype is the "Snake in a box" (largest induced cycle in the n-cube) problem that arises in coding theory.

Janson collaborated with Rucinski (and Oleskiewicz) on large deviation inequalities for counts of (not necessarily induced) subgraphs in a random graph. He also worked with Chassaing on the Brownian snake. Both of these investigations have led to subsequent publications.

Járai worked with van den Berg on a "forest fire" model, and (with Athreya) on the abelian sandpile model in $\mathbb{Z}^d$; both of these are models of self-organised criticality.

Jerrum (with Son) continued investigations of the spectral gap and log-Sobolev constants of Markov chains. One outcome, with Tetali and Vigoda, is a remarkable decomposition theorem for the log-Sobolev constant which closely mirrors that known for spectral gap. He worked with Sinclair and Vigoda on (non-trivially) extending their celebrated solution of the permanent problem to the case of non-bipartite matchings. This work remains in progress. He also completed a monograph, *Counting, Sampling and Integration*, which has since been published by Birkhaüser.

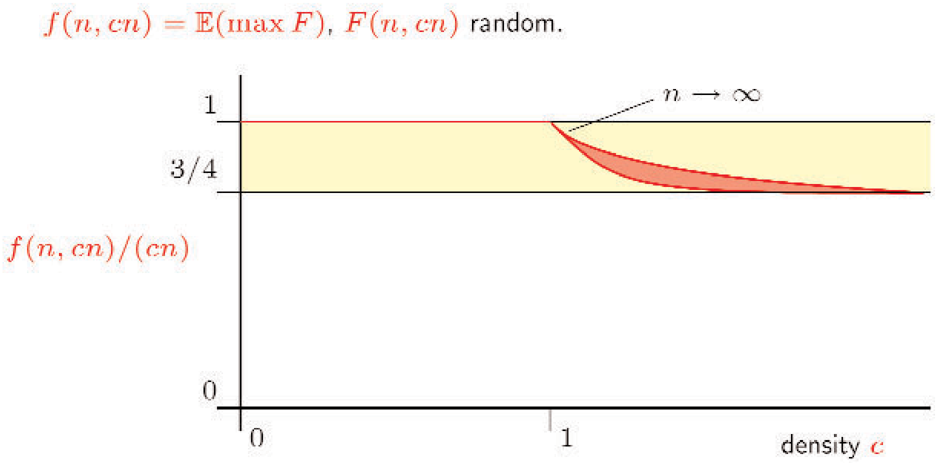
Kahn, during a short visit, worked with Randall and Sorkin on phase transitions for three-colourings of the lattice $\mathbb{Z}^d$ in high dimension d. The expectation, which is consistent with what has been discovered so far, is that there should be six Gibbs states; a typical such state has the even sublattice coloured largely "red" and the odd sublattice largely "blue" and "green". He also proved a conjecture of van den Berg.

Karonski worked with T Luczak and Thomason on local irregularity strength of graphs, with Pittel on perfect matchings in a class of sparse random graphs, and with Krivelevich and Vazirani on information hiding in graphs and its potential applications to software watermarking.

Kendall studied problems arising in image analysis concerning phase transitions in the Ising model defined on "augmented quad trees", and the form of the resulting interfaces.

Lovász wrote a paper (with Vempala) on the geometry of log-concave distributions: these occur widely in practice, the most familiar being the Gaussian distribution. A concrete outcome of their study is a particularly efficient algorithm for sampling a point from a log-concave distribution. Lovász also collaborated on other work in discrete probability and combinatorics.

M Luczak worked with T Luczak on phase transitions in the cluster-scaled model of a random



Phase diagram for the maximum number of satisfiable clauses in a random 2-CNF formula

graph, and with Winkler on showing that a random walk on the non-negative integers produces a uniform random subtree of the Cayley tree.

T Luczak and Simonovits collaborated on generalising results of Andrásfai–Erdös–Sós concerning the minimum degree of a graph of chromatic number *k* that contains no clique on *k* vertices. Pikhurko and Simonovits (with Füredi) investigated extremal hypergraph problems, and proved a conjecture of Mubayi and Rödl as to the maximum size of a 3-graph on *n* vertices that excludes a specific 3-graph on five vertices.

Randall continued her work on simulated tempering, a general method that has been proposed for speeding up simulations of "low temperature" systems. The idea is to introduce a dynamics that is able to move between low temperature states and high temperature states, where mixing occurs more freely. Preliminary results, both positive and negative, have been obtained.

Scott worked with Wilmer on a wide range of problems (extremal, algorithmic and complexity-theoretic) concerning classes of permutations which occur in large-scale genome changes.

Stougie proved a polynomial upper bound on the edge-diameter of transportation polyhedra. His initial results were improved during the programme in collaboration with van den Heuvel, and subsequently further improved in collaboration with van den Heuvel and Brightwell.

Van den Berg (with Brouwer) studied "self-destructive" percolation in which all sites in an infinite cluster become vacant through some catastrophe. With what probability do vacant sites need to be re-populated before an infinite cluster re-appears? They conclude the answer may be larger than one might suspect.

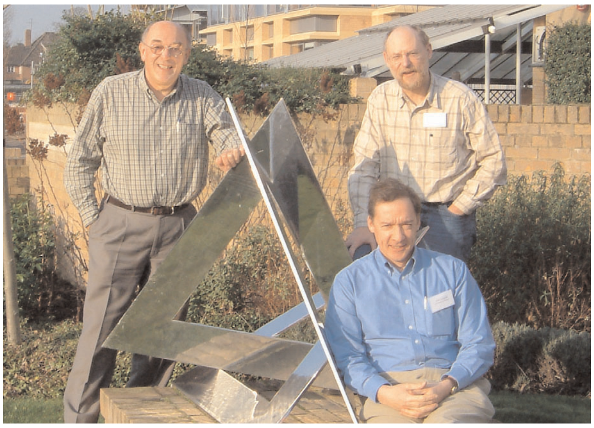
Van den Heuvel extended some of his own results on cyclic and distant 2-colourings of planar graphs. Vigoda (with Hayes) developed a new tool for analysing non-Markovian couplings, giving an application to sampling graph colourings. The idea should have many other applications.

New Contexts for Stable Homotopy Theory

2 September to 20 December 2002

Report from the Organisers:
JPC Greenlees (Sheffield), HR Miller (MIT), F Morel (Jussieu), VP Snaith (Southampton)

S Wygard



Back, left to right: VP Snaith, HR Miller
Front: JPC Greenlees

Scientific Background

Algebraic topology started in the late 19th century with the work of Henri Poincaré. In the beginning its objective was to study geometric objects, such as smooth manifolds, which arise in connection with the differential equations of physics, by converting the high-dimensional geometry into more accessible algebra using various cohomology theories.

Stable homotopy theory is the ultimate context in which to perform the type of conversion from geometrical to algebraic data which Poincaré began. The cohomology theories themselves are represented by geometric objects; only very recently have really satisfactory models been found, opening a wide range of new possibilities for exploiting algebraic ideas in topology, geometry, number theory and physics. The programme explored several of these new avenues.

Algebraic topology was applied with astounding success by Lefschetz, Hodge and others to tackle problems in complex algebraic geometry. The development of cohomology theories which behaved in positive characteristic like the classical cohomology used by Lefschetz was a major achievement of the 20th century. The Grothendieck school flourished in the 1960s and 1970s and left a legacy of powerful new techniques, famous successes (such as Deligne's proof of the Weil Conjectures) and a series of mysterious cohomological problems concerning Grothendieck's nebulous notion of a 'motive' (or motif).

In the 1990s, techniques of homotopy theory were adapted to the algebraic geometry context to form motivic homotopy theory by Morel, Voevodsky and others. Results in algebraic geometry can be exploited through structures familiar from algebraic topology. These ideas led to the construction of motivic cohomology and to Voevodsky's proof of the Milnor conjecture. This exploitation of motivic homotopy theory in number theory and algebraic topology was spurred on by the programme, which also played an important role in spreading the developing body of knowledge. The motivic world is only one of several new manifestations of stable homotopy theory which made important progress during the programme.

Another subject benefitting from the new structures of stable homotopy theory is elliptic cohomology. This covers a large and rapidly growing field, involving a conjunction of algebra, geometry, analysis and topology, of a depth and power unseen since the development of *K*-theory. The first elliptic genus to be widely studied was introduced by E Witten in connection with conformal field theory, as the equivariant index of a putative signature operator on the free loop space of a manifold. He provided a physical proof of its rigidity, establishing at the outset the connection with differential geometry.

A number of attempts to find geometric cycles for this theory were discussed and developed during the programme. So far the greatest success has been through homotopy theory. Quillen's discovery that the formal group law of complex cobordism is universal opened a new era for algebraic topology, because the theory of formal groups provides a remarkably accurate image of the stable homotopy category. It is natural to ask whether there is a functorial way to associate a complex orientable cohomology theory to a formal group, lifting parts of the category of formal groups to the category of spectra itself, not merely to the homotopy category. New techniques for doing this using the new models for stable homotopy theory were discussed during the programme. One notable success was the construction by Hopkins and Miller of a lift of the moduli stack of elliptic curves to the stable category. This is an elliptic analogue of Atiyah's *K*-theory with reality, and by taking the homotopy inverse limit of this diagram one obtains the spectrum **tmf** of topological modular forms. New calculations, variations and consequences of this remarkable object were central to several parts of the programme.

This programme at the Newton Institute furthered this synergistic development by bringing together the practitioners of stable homotopy in all its current diverse disguises – from arithmetic to physics to topological modular forms – to investigate possible new applications of their current techniques.

Programme Structure

The programme was structured around three workshops and a number of related lecture series: it started and finished with intensive workshop activity, separated by a period for sustained research and collaboration amongst the longer term participants. The programme opened in earnest with the very well attended two week NATO Advanced Study Institute *Axiomatic, enriched and motivic homotopy theory*, which provided introductions to several of the programme themes. Many participants stayed on for the workshop *K-theory and arithmetic*. During October and November, regular seminar series and three Newton Institute Seminars continued the themes these opened up. The final two weeks were taken up with an EU workshop, *Elliptic and chromatic phenomena*, giving a suitably climactic finish before Christmas.

Meetings and Workshops

Axiomatic, Enriched and Motivic Homotopy Theory

NATO Advanced Study Institute, 9–20 September 2002

Directors: JPC Greenlees and IB Zhukov
Organisers: PG Goerss, JF Jardine, F Morel and VP Snaith

This first workshop was a two week NATO Advanced Study Institute attended by 90 participants from 17 countries. It was arranged around a series of 15 introductory minicourses by experts, designed to make the remainder of the programme accessible to newcomers. These were supplemented by a handful of stand-alone lectures, a problem session and a poster session. Several new collaborations were born during the ASI.

The ASI came about because of a remarkable conjunction in two related subjects. The subject of stable homotopy theory has been transformed in the last ten years by key technical advances making distant dreams into reality, but the fact that its methods have also been used in recent spectacular progress in motivic homotopy theory was a quite separate development. The ASI brought together principal exponents of both themes, and it was gratifying to see how effective it was at encouraging substantial two-way interaction between them. This mathematical cohesion is illustrated by the manner in which workshops in the programme often referred directly to one another. For example, (i) the talks of Hornbostel, Jardine, Hesselholt, Kahn, Levine, Morel, Rognes, Snaith, Toen, Totaro, Weibel and Vishik from the motivic homotopy week of the NATO workshop were all cited during the *K*-theory workshop; and (ii) a similar relation is true of lectures of Goerss, Madsen and Strickland which surfaced again in the EU workshop on elliptic cohomology and chromatic phenomena. Particularly notable and stimulating was the number of originators of results who were present, many of whom had never given lectures in the UK before.

The objective of the ASI was to survey recent developments and outline research perspectives in stable homotopy theory, the homotopy theory of structured ring spectra and motivic homotopy theory. A major effort was made to encourage communication between those with experience from the various different themes.

The lectures at the ASI highlighted the directions of research in which major breakthroughs have been achieved in recent years. These can be broadly grouped into six areas:

- Abstract homotopy theory and models (Dwyer, Jeff Smith, Schwede, Strickland)
- Operads and structured ring spectra (McClure, May, Richter, Smirnov)
- Chromatic and geometric applications of structured ring spectra (Goerss, Madsen, Rognes)
- Foundations of motivic homotopy theory (Jardine, Weibel, Toen)
- Motivic homotopy theory (Hornbostel, Levin, Morel, Totaro, Vishik)
- *K*-theory (Bloch, Carlsson, Hesselholt, Snaith)

We are confident that the positive impact of this ASI will be felt in the mathematical community for many years to come.

K–Theory and Arithmetic

Workshop, 30 September – 4 October 2002

Organisers: S Lichtenbaum and VP Snaith

This workshop attracted over 80 participants and concentrated on aspects of interplay between algebraic *K*-theory, arithmetic and algebraic geometry. Particular emphasis was placed upon applications of the recently developed homotopy theory of geometric and motivic categories. In addition to lectures on current results, a number of expository lectures were scheduled to provide researchers and graduate students in related areas with an opportunity to learn about these new techniques. Topics of current interest in this area include: Beilinson–Soulé conjectures, Bloch–Kato conjecture, Beilinson–Borel regulators, Kato–Parshin–Saito higher class field theory, Lichtenbaum–Quillen conjecture, Milnor *K*-theory, motivic cohomology, Brumer–Coates–Sinnott conjectures, polylogarithms, modularity and special values of L-functions.

Elliptic Cohomology and Chromatic Phenomena

Euroworkshop, 9–20 December 2002

Organisers: HR Miller and DC Ravenel

Elliptic cohomology is the third of a sequence of natural probes into the nature of high dimensional objects bringing together ideas from geometry, arithmetic and analysis. Ordinary cohomology dominated the first half of the twentieth century, and *K*-theory dominated the second half. As Ed Witten has said, elliptic cohomology is a piece of twenty-first century mathematics that happened to commence in the twentieth. It has been most highly developed within the homotopy theory, but early discoveries were made by mathematical physicists studying conformal field theory. The formal structure of elliptic cohomology is becoming well understood, but its geometric foundations are still mysterious. Hints have come from physics (conformal field theory), algebraic geometry (stacks and 2-vector spaces) and representation theory (moonshine and vertex operator algebras), as well as traditional algebraic topology (structured ring spectra, equivariant cohomology).

This workshop attracted over 75 participants from 13 nations, bringing together top researchers in mathematical physics, homotopy theory, algebraic geometry and representation theory to investigate this mysterious object. Significant progress was reported, and discussions at the conference resulted in further progress.

The first week was focussed sharply on bringing together a wide range of perspectives on elliptic genera and elliptic cohomology. There were 19 talks during this first week. The quality of

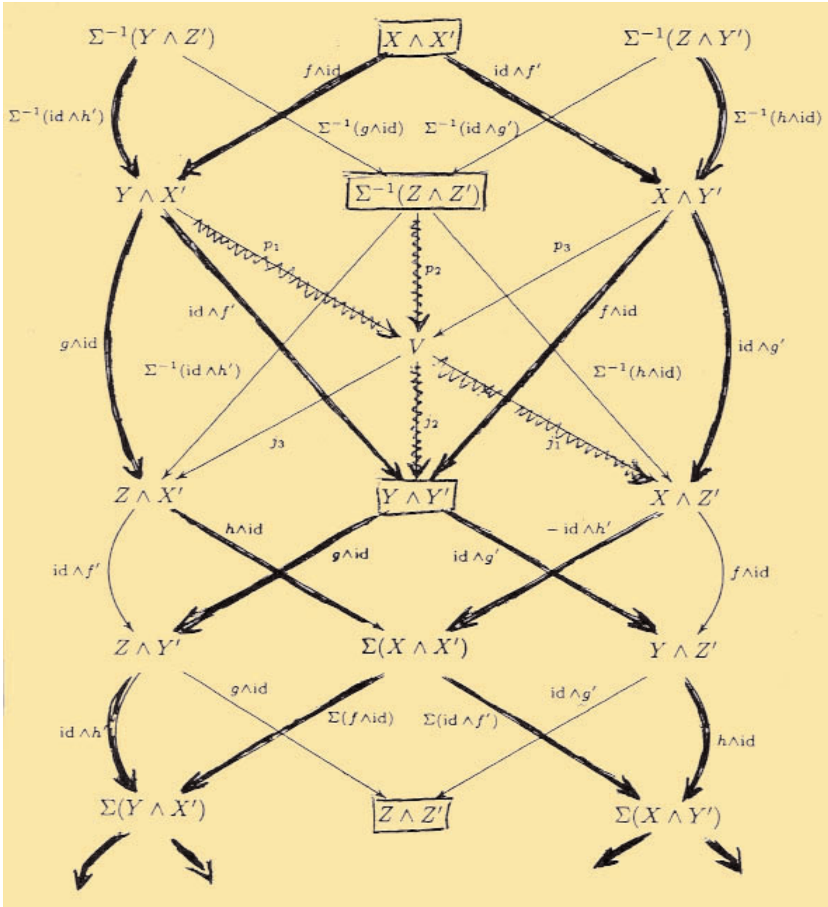
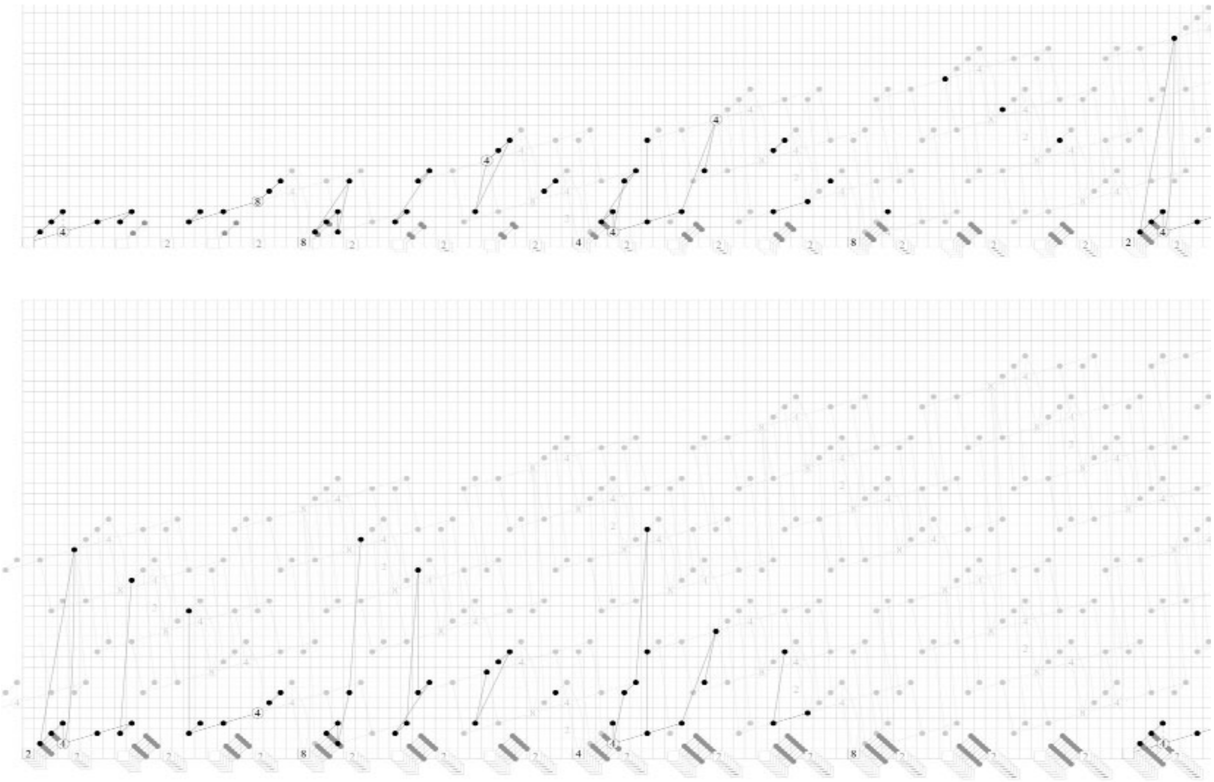


Diagram relating the smash product and triangulation in a stable homotopy category

exposition was simply excellent. The high degree of audience participation and the animated conversation, often between new acquaintances, made it clear that the hoped for cross-fertilisation was occurring. While each talk represented a different and important perspective, we might pick out the following sessions as shedding the most new light specifically on the meaning of elliptic cohomology.

- Mike Hopkins described the homotopy theoretical construction of elliptic cohomology, along with the ensuing theory of topological modular forms and the generalized Witten genus from string manifolds.
- John Rognes described joint work with Nils Baas and others giving a 2-vector space model for the Waldhausen K-theory of topological K-theory, and gave evidence of its relationship with elliptic cohomology.
- Stephan Stolz described his work with Peter Teichner, providing an operator algebra approach to the notion of string structure, and a corresponding candidate for elliptic objects.
- Charles Rezk described explicit formulae for homotopy theoretic logarithms in terms of power operations, resulting in a homotopy theoretic extension of the theory of Hecke operators.
- Vassily Gorbunov described the construction of a geometric object, a sheaf of vertex operator algebras, associated with a string structure.



*Summary of the Hopkins–Mahowald 2-local calculation of the coefficient ring of the topological modular form spectrum **tmf**, with the dark part giving the final answer.*

*The part which does not come from connective real K-theory is 192-periodic, so the picture (which stops at the 197th homotopy group) gives the complete answer. The spectrum **tmf** is central to the study of elliptic cohomology (final workshop) and its construction uses enriched homotopy theory (first workshop).*
(With thanks to MJ Hopkins)

- Burt Totaro described the universal character of elliptic cohomology in relation to complex projective varieties.
- Matthew Ando described how to explain the phenomenon of Witten rigidity from the perspective of elliptic cohomology.

The second week of this workshop was more narrowly homotopy theoretic, but the focus was broadened to encompass the next steps, beyond elliptic cohomology. From a homotopy theoretic perspective, ordinary cohomology, *K*-theory and elliptic cohomology form the first three of an infinite sequence of probes into the nature of high dimensional geometry.

The following were among the highlights of the ten talks. Hans-Werner Henn described the status of joint work with Paul Goerss, Mark Mahowald and Charles Rezk on the structure of the sphere spectrum from the elliptic perspective. Jorge Devoto described a speculative approach to a geometric theory of K3-cohomology. Andy Baker described the obstruction theory associated to the construction of ring structures on certain higher analogues of *K*-theory. Doug Ravenel described part of the theory of abelian varieties which seems related to these higher *K*-theories.

Outcomes, Achievements and Publications

The programme was the foremost international event in the subject in 2002, and one of special significance as it brought together several strands at seminal stages of their development. Some of this will be reflected in the proceedings volume for the NATO ASI *Axiomatic, Enriched and Motivic Homotopy Theory*, to be published by Kluwer.

Here we summarise some of the work that took place during the programme and was directly related to one of the main themes of the programme. Although there have been many new collaborations, it is especially notable how many participants benefitted from less formal interaction with those of more distant expertise. Which of these contacts will lead to new collaborations, and what the fruit of these will be, we will only know in a few years' time.

Some of the areas where significant work was done during the programme are as follows. Inevitably, there are many omissions. There are already preprints reflecting this work, several of them in the Newton Institute preprint series.

- Algebra of rings up to homotopy (Galois and étale maps, duality and finiteness conditions, centres)
- Elliptic and chromatic phenomena (localisation and comodules, small resolutions of local spheres, 2-vector bundles, generalised Mumford conjecture)
- Equivariant cohomology theories (elliptic cohomology, sigma orientation, rational torus-equivariant cohomology)
- Algebraic groups and cohomology theories (arithmetic Jacobians in the non-equivariant case and rational Jacobians in the equivariant case)
- *K*-theory (Weil-étale topology for number fields and zeta functions, additive dilogarithm, de Rham–Witt complex, Galois modules, the Vandiver conjecture)
- Algebraic and motivic homotopy theory (transfer functors, Balmer–Witt groups of classifying spaces, motivic stable homotopy groups, higher stacks)
- Abstract homotopy theory (operads and higher categories, sequence operads and stable *p*-adic homotopy theory, Franke equivalence, cubical sets, algebraic cobordism)

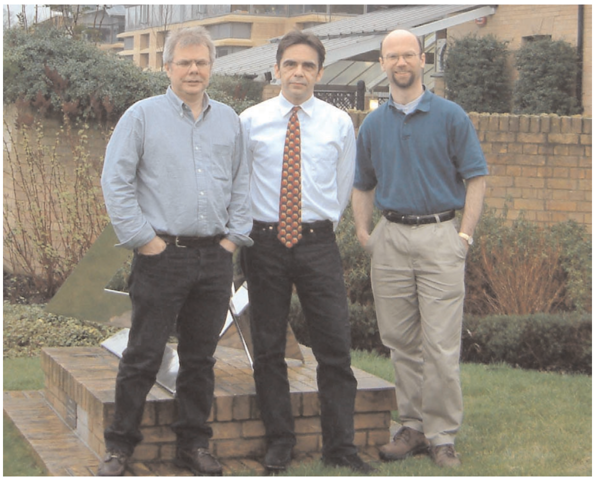
Conclusion

This programme brought together a remarkable collection of researchers from a wide range of areas including parts of homotopy theory, *K*-theory and elliptic cohomology. It organised, disseminated and consolidated the existing state of the subjects. Many new collaborations were formed, new dreams born, and new agenda set. The beneficial effects of this programme will appear and ripen over the coming years.

Computational Challenges in Partial Differential Equations

20 January to 4 July 2003

Report from the Organisers: M Ainsworth (Strathclyde), CM Elliott (Sussex), E Süli (Oxford)



Left to right: CM Elliott, E Süli and M Ainsworth

Scientific Background

The study of partial differential equations (PDEs) is a fundamental area of mathematics which links important strands of pure mathematics to applied and computational mathematics. Indeed PDEs are ubiquitous in almost all of the applications of mathematics where they provide a natural mathematical description of phenomena in the physical, natural and social sciences.

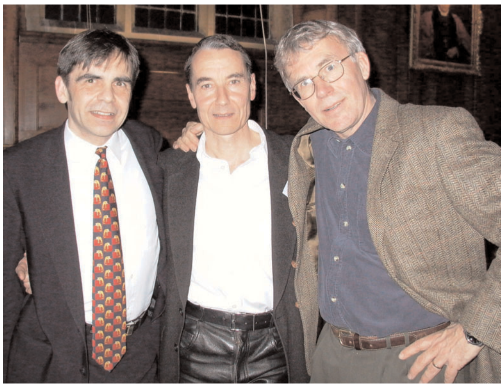
Partial differential equations and their solutions exhibit rich and complex structures. Unfortunately, closed analytical expressions for their solutions can be found only in very special circumstances, and these are mostly of limited theoretical and practical interest. Thus, scientists and mathematicians have naturally been led to seek techniques for the approximation of solutions. Indeed, the advent of digital computers has stimulated the incarnation of Computational Mathematics, much of which is concerned with the construction and mathematical analysis of numerical algorithms for the approximate solution of PDEs.

The most powerful and generally applicable algorithms for the approximate solution of partial differential equations rely on the concept of *discretisation*, whereby the PDE under consideration is replaced by a finite-dimensional problem. The transition from the partial differential equation to the discrete model is a non-trivial mathematical problem, and the selection of an appropriate finite-dimensional representation is rarely a matter of arbitrary choice: physical properties behind the mathematical model (such as energy and mass conservation, positivity, total-variation-boundedness, dispersion and dissipation) have to be borne in mind, as well as issues of resolution of relevant scales, complete and guaranteed control of the discretisation error, in addition to concerns about the efficiency and reliability of the resulting algorithm. The study of these mathematical questions represents the focus of the field of Numerical Analysis of Partial Differential Equations.

Simultaneously, the development of high speed digital computers has led to consideration of very large systems of PDEs in science and engineering and to scientific demands for highly accurate approximations. In addition, in recent years the applied analysis of nonlinear partial differential equations has progressed in parallel with their theoretical numerical analysis and the development of appropriate numerical algorithms, leading to a very important and intellectually rich area of applied mathematics referred to as Computational Partial Differential Equations (CPDEs).

This programme focussed on some of the most exciting and promising mathematical ideas in these fields, and those branches of PDE theory that provide a source of physically relevant and mathematically hard problems to stimulate future developments. The three main themes were:

- Cognitive algorithms: adaptivity, feed-back and *a posteriori* error control
- Hierarchical modelling: multi-scale mathematical models and variational multi-scale algorithms



E Süli, W Dahmen and C Johnson

- Nonlinear degenerate PDEs and problems with interfaces

Structure of the Programme

Apart from the nine events described below we had a seminar programme. Because of the large number of talks during the events and also in the parallel programme, we restricted the number of seminars to about one per week. Most of the long-term participants delivered a lecture or seminar during the programme.

There were 57 long-term (21 UK) as well as 125 short-term (approximately 56 UK) participants. The meetings also attracted others who were not formally participants in the programme. The large UK participation is a reflection of the current strength of and interest in the field in the UK. The three organisers M Ainsworth, CM Elliott and E Süli together with J Barrett were in residence throughout the programme. Other participants staying for substantial periods, partially supported by EPSRC fellowships, were C Carstensen, PB Monk, C Johnson, R Nochetto and C Schwab. The Rothschild Visiting Professor was F Brezzi. These contributions added greatly to the cohesion of the programme throughout the six months.

The Newton Institute's Junior Membership scheme supported the participation of 11 young researchers. Other sources of funding for young researchers were the EU (for attendance at the Euroworkshop) and the EPSRC Computational Engineering Mathematics programme.

The parallel programme at the Institute, *Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics*, had a number of activities concerning computational aspects of hyperbolic and related systems. This led to several discussions between participants of the two programmes and to mutual attendance of seminars and lectures.

Workshops and Conferences

Mathematical Challenges in Scientific and Engineering Computation

Workshop, 20–24 January 2003

Organisers: M Ainsworth (Strathclyde), CM Elliott (Sussex) and E Süli (Oxford)

In the opening lecture R Rannacher (Heidelberg) discussed the issues of error control and adaptivity in numerical computation, and presented applications to numerical approximation of chemically reacting flows. These issues were taken up in the context of free boundary problems by R Nochetto (Maryland), who observed that many of the difficulties revolve around identification of an appropriate notion of residual for such problems, and showed how error control is accomplished for a range of examples including nonlinear degenerate parabolic equations, mean curvature problems and surface diffusion. The day concluded with an open discussion on the role of computational mathematics in complex industrial simulations chaired by J Ockendon (Oxford).

Problems involving interfaces formed the theme of the second day opened by O Pironneau (Paris VI), who described his recent work on the sensitivity of shocks in solutions of Euler equations which are important in computations of flutter. A variety of physical problems where curvature-dependent evolution of interfaces plays a lead role, including diffusion-induced grain boundary motion, along with an overview of the principal mathematical models was presented by CM Elliott (Sussex). Diffuse interface, or phase field, models and their application to solidification were discussed by A Wheeler (Southampton), while the use of simulation at the atomistic level was advocated by A Sutton (Oxford).

Atomistic and multi-scale modelling of materials were the topic on the following day, where E Tadmor (Technion) discussed the problems of dealing with widely varying length-scales found in materials science. He described the "quasi-continuum" approach whereby different local levels of resolution are identified using an adaptive approach. B Engquist (Princeton) continued the theme in his talk on the Heterogeneous Multiscale Method, in joint work with Weinan E (Princeton) who later described some of the new mathematics underlying the method. The importance of using constitutive laws at the molecular level was highlighted in the talk of T McLeish (Leeds), who gave two examples in polymer processing where difference in the structure of the polymer at the molecular level is responsible for completely different macroscopic behaviour.

Thursday was devoted to interaction with the EPSRC Computational Engineering Mathematics (CEM) programme (which provided an additional £10,000 of financial support for the meeting) and consisted of two lectures by overseas experts: R Glowinski (Houston) and T Chan (UCLA). These were followed by a number of short talks based on projects supported through the CEM initiative presented by the principal investigators, and a poster session comprised of other projects supported under CEM. The day concluded with presentations by EPSRC personnel from the engineering and high-performance computing programmes, paving the way for an open forum discussion on the topics raised.

The final day witnessed a change of pace as, firstly, A Quarteroni (EPFL, Lausanne) discussed the applications and advantages of numerical modelling of the cardiovascular system, a problem requiring coupled, heterogeneous PDE models of the entire system. PB Monk (Delaware) gave an overview of numerical techniques used in solving inverse problems and described a new technique, the linear sampling method, developed in collaboration with D Colton. The meeting closed with talks by R Davies (Aberystwyth) on PDEs in the food industry and issues of mesh generation in computational modelling by N Weatherill (Swansea).

The organisers prepared an article which appeared in the June 2003 issue of *SIAM News* that described the meeting and its context in the six-month programme at the Newton Institute.

Computational Challenges in Micromagnetics and Superconductivity

Hewlett-Packard Event, 13–14 February 2003

Organisers: M Ainsworth (Strathclyde), CM Elliott (Sussex) and PB Monk (Delaware)

One of the aims of this two-day meeting was to identify the major applications in each area where progress in computational techniques for PDEs is needed and will have a significant impact. Leading experts, including physicists, engineers, numerical analysts and theoretical PDE analysts delivered survey lectures on their area of expertise.

The speakers were selected with the aim to provide a comprehensive overview of the current state of the field, with particular emphasis on the interactions between theory, applications and practice.

J Chapman (Glasgow) described experimental results in micromagnetism obtained using Lorentz microscopy showing the existence of free vortices in magnetic permalloy films and reversal magnetisation effects in multi-layer structures. T Schrefl (Vienna) discussed the different length- and time-scales that arise in micromagnetic simulations, and demonstrated the performance of adaptive finite element schemes for some three-dimensional applications. A Prohl (Zürich) discussed the numerical analysis of microstructure with applications to micromagnetism. G Carbou (Bordeaux) presented results on the existence, uniqueness and regularity of solutions of the Landau–Lifschitz equations. PB Monk (Delaware) concluded the day with a presentation on his recent collaborative work on the numerical modelling using an eddy current micromagnetic model.

The second day focussed on mathematical models for superconductivity. M McCulloch (Oxford), in his talk *Using PDEs to develop the application of superconductors*, described the importance of computational methods for PDEs in engineering applications and in particular the use of superconductors in electric motors. J Chapman (Oxford) discussed in his talk *Macroscopic models of superconductivity* a hierarchy of models from the Ginzburg–Landau equations to Bean's critical state model. Q Du (HKUST) described numerical computations associated with *Quantized vortices: from Ginzburg–Landau to Gross–Pitaevskii*. V Styles (Sussex) discussed critical state computations in her talk *A finite element approximation of a variational formulation of Bean's model for superconductivity*. Finally, L Prigozhin (Ben Gurion) gave an overview of his research on the *Solution of critical-state problems in superconductivity*.

Computational PDEs: Giving Industry the Edge

Joint Workshop with the Smith Institute, 4–5 March 2003

Organisers: CM Elliott (Sussex), RE Hunt (Newton Institute), R Leese (Smith Institute), J Ockendon (Oxford) and E Süli (Oxford)

The challenges of industry call for new mathematics of the highest scientific quality. The Isaac Newton Institute and the Smith Institute for Industrial Mathematics and System Engineering held this joint workshop in order to bring together world-class researchers and industrialists to work on real problems. The meeting presented six current Smith Institute projects involving collaboration between industry and University mathematicians in the areas of food, electro-magnetism and violent mechanics. The industrial collaborators described the background and the problems being addressed. The academic researchers discussed the development of mathematical models together with the resulting PDE problems. The Computational Overview sessions illustrated some of the current numerical approaches to the areas of interest, and there were chaired discussions on the critical computational issues in the projects. Topics considered included bread crusting, scraped surface heat exchangers, microwaving, inertia-dominated free surfaces, droplet impact on water layers, electromagnetic compatibility and shaped charge mechanics.

Multiscale Modelling, Multiresolution and Adaptivity

Workshop, 7–11 April 2003

Organisers: M Ainsworth (Strathclyde), W Dahmen (Aachen), C Schwab (Zürich), and E Süli (Oxford)

This meeting was devoted to identifying new algorithmic paradigms which will lead to computationally affordable numerical algorithms for the approximate solution of multiscale problems, with focus on recent developments. This was achieved by bringing together leading experts from applied mathematics, scientific applications and various branches of scientific computation, who work on different aspects of multiscale modelling. The meeting stimulated interactions and cross-fertilisation between the various subject areas involved, and resulted in an assessment by leading researchers of the state-of-the-art in the field, the identification of key problems and obstacles to progress, and the indication of promising directions for future research.

Fifteen leading scientists working in the field were invited to present plenary lectures.

S Wysgard

Participants at the workshop "Multiscale Modelling, Multiresolution and Adaptivity"

M Ainsworth (Strathclyde) discussed the use of high-order finite element methods for the approximation of Maxwell's equations. He presented families of hierarchic basis functions for the Galerkin discretisation of the space $H(\mathrm{curl}; \Omega)$. The conditioning and dispersive behaviour of the elements was discussed along with approximation theory.

F Brezzi (Pavia) highlighted the fact that residual-free bubbles proved to be a powerful technique to deal with subscale phenomena and to have optimal stabilising effects. He also discussed, through several examples of PDEs, the amount of computational effort required to solve the subgrid problem.

C Canuto (Torino) used the properties of wavelet bases to design an adaptive descent algorithm for solving a convex minimisation problem in a function space of Hilbert type. He proved the convergence of the algorithm and discussed its optimality in the framework of best N-term approximation results due to A Cohen, W Dahmen and R DeVore.

C Carstensen (Vienna) considered three applications for a nonconvex minimisation problem model: the examples included phase transitions, optimal design tasks and micromagnetics. Through the three examples at hand, he showed that relaxation theory provides auxiliary problems (relaxed models) which are convex but not uniformly or even strictly convex. Their numerical simulation in the quasiconvexified approach to computational microstructures is relatively easy; he recommended the use of this approach, since even the Young measures generated in the nonconvex model can be recovered.

A Cohen (Paris) discussed the convergence analysis of a class of adaptive schemes for evolution equations. These schemes combined adaptive mesh refinement with wavelets based on a framework introduced by A Harten. The analysis mainly focussed on the difficult case of nonlinear hyperbolic conservation laws.

W Dahmen (Aachen) spoke on the adaptive application of (linear and nonlinear) operators in wavelet coordinates. He showed that there are two principal steps: first, the reliable prediction of significant wavelet coefficients of the output of such adaptive applications from those of the input, and second, the accurate and efficient computation of the significant output coefficients. Some of the main conceptual ingredients of such schemes were discussed. Moreover, it was also highlighted how this approach leads to adaptive solution schemes for variational problems that can be shown to have asymptotically optimal complexity.

At the conference dinner for "Multiscale Modelling, Multiresolution and Adaptivity"

R DeVore (South Carolina) discussed two results that have proven to be important in the analysis of convergence rates for adaptive finite element methods. The first is how to bound the number of additional subdivisions needed to remove hanging nodes in newest vertex bisection. Such a result is necessary for deriving computational complexity estimates in adaptive methods. The second result is how to generate a near-best adaptive approximation to a given function in linear time. This result was used in coarsening routines to keep control of the number of cells in adaptively generated triangular partitions.

Weinan E (Princeton) gave an extensive survey of various multiscale methods for a range of physical applications that exhibit multiscale behaviour, and then discussed the numerical analysis of multiscale methods for several classes of multi-physics problems.

B Engquist (Princeton and Stockholm) talked about the heterogeneous multiscale method and discussed its application to the numerical solution of stiff ordinary differential equations. He argued that the computational complexity of these techniques is, in many cases, lower than that of traditional methods. He illustrated this point through analysis and numerical examples.

T Hou (Caltech) showed that many problems of fundamental and practical importance contain multiple-scale solutions. He introduced a dynamic multiscale method for approximating nonlinear PDEs with multiscale solutions. The main idea is to construct semi-analytic multiscale solutions local in space and time, and use them to construct the coarse grid approximation to the global multiscale solution. He showed that the method provides an effective multiscale numerical method for computing two-phase and incompressible Euler and Navier–Stokes equations with multiscale solutions. Finally, he introduced a new class of numerical methods to solve the stochastically forced Navier–Stokes equations, and demonstrated that the method can be used to compute accurately high order statistical quantities more efficiently than the traditional Monte Carlo method.

C Johnson (Chalmers, Göteborg) presented a framework for adaptive computational modelling with applications to reaction–diffusion in laminar and incompressible flow. He estimated, using a duality argument, the total computational error in different outputs *a posteriori*, with contributions from both discretisation in space/time and subgrid modelling of unresolved scales. He considered subgrid models based on extrapolation or local resolution of subgrid scales, and presented computational results for laminar and turbulent Couette flow.

R Nochetto (Maryland) proposed an algorithm for saddle point problems consisting of two nested iterations, the outer iteration being an Uzawa algorithm to update the scalar variable and the inner iteration being an elliptic AFEM for the vector variable. He showed linear convergence in terms of the outer iteration-counter provided the elliptic AFEM guarantees an error reduction rate together with a reduction rate of data-oscillation (information missed by the underlying averaging process), and applied this idea to the Stokes system without relying on the discrete inf-sup condition. He finally assessed the complexity of the elliptic AFEM, and provided consistent computational evidence that the resulting meshes were quasi-optimal.

R Rannacher (Heidelberg) presented a systematic approach to error control and mesh adaptivity in the numerical solution of optimisation problems with PDE constraints. By the Lagrangian formalism the optimisation problem was reformulated as a saddle-point boundary value problem which was discretised by a Galerkin finite element method. The accuracy of this discretisation was controlled by residual-based *a posteriori* error estimates. The main features of this method were illustrated by examples from optimal control of fluids and parameter estimation.

C Schwab (ETH Zürich) showed that elliptic problems with multiple scales in bounded domains in $\mathbb{R}^d$ can be reformulated as elliptic problems with a single scale in high-dimensional domains. Standard finite element methods for such problems exhibit poor accuracy due to lack of scale-resolution. Schwab showed how a two-scale finite element method in a higher dimensional space can be developed to overcome this problem. He further showed how sparsification of the finite element space and hierarchical finite element spaces can be used to reduce computational complexity.

J Xu (Penn State) reported on some recent studies on using multigrid ideas in grid adaptation. The results presented included gradient and Hessian recovery schemes by using averaging and smoothing (as in multigrid), interpolation error estimates for both isotropic and anisotropic grids and multilevel techniques for global grid movement and local grid refinement.

The meeting included a further 12 (contributed) talks, mostly by young researchers (under the age of 35) working in the field, and four one-hour coordinated discussion sessions (on Adaptivity, Wavelets and multiresolution for PDEs, Multiscale methods for PDEs, and on Multigrid and multilevel methods); the meeting was attended by 79 participants from academia and industry. The discussion sessions were productive, lively, and, on a number of occasions, quite controversial. The workshop was a very successful event in bringing together leading researchers working in the field.

S Wysgard

Participants at the workshop "INTERPHASE 2003"

INTERPHASE 2003: Numerical Methods for Free Boundary Problems

Conference, 14–18 April 2003

Organisers: CM Elliott (Sussex), J Barrett (Imperial), G Dziuk (Freiburg) and R Nochetto (Maryland)

INTERPHASE 2003 was the latest in a series of meetings on numerical methods for free boundary problems. The title is a word-play on the ideas that free boundary problems may be interfaces, that many applications involve multiple phases and that the interdisciplinary nature of the workshop brings together computational mathematicians, analysts, modellers and scientists. The meeting was held over four days in which there were 21 talks and much time for discussion. There were 57 participants including well known experts in the field and many young researchers. The topics ranged from image processing, differential geometry, phase transitions and material science, cosmology, mesh generation and flow in porous media, to fluid drops.

Numerical Analysis of Maxwell Equations on Non-Smooth Domains

Workshop, 30 April 2003

Organisers: M Ainsworth (Strathclyde) and C Schwab (Zürich)

The numerical analysis of Maxwell equations on non-smooth domains poses a number of computational challenges due to the variational setting of the problem and the presence of edge and corner singularities in the solutions. M Dauge (Rennes) gave two survey talks on the structure of the singularities in three dimensions. M Costabel (Rennes) compared various alternative variational settings for the time-harmonic Maxwell equations and outlined recent theoretical results on the convergence of Maxwell eigenvalues using a regularised variational formulation. D Boffi began by surveying the theory for the numerical approximation of eigenvalues for non-coercive operators, and then showed how the general theory applies to the Maxwell equations and highlighted the importance of the discrete compactness property.

The First European Finite Element Fair

Workshop, 8–9 May 2003

Organisers: M Ainsworth (Strathclyde), C Carstensen (Vienna), CM Elliott (Sussex), C Schwab (Zürich) and E Süli (Oxford)

The European Finite Element Fair (EFEF) was the inaugural meeting of what is planned to become an annual series of completely informal small workshops throughout Europe with equal initial conditions for each speaker. The idea of EFEF is to provide a platform for high-level discussions on current research on finite element approximation, in the broadest sense, of PDEs. The format is based on a long series of such meetings held in the USA. A few, but strict, rules were applied in order to distinguish it from existing workshops and minisymposia in the field.

- Provided that he or she was present throughout the meeting, each participant was invited to talk.

- Based on the number of speakers the time available was divided into slots for the talks. The order of the speakers was determined through random choice, by drawing names out of a hat. Speakers could not request a specific time to talk.
- Each speaker had to introduce himself or herself, the title and topic, and was expected to leave sufficient time, within the allocated time-slot, for discussion. Speakers had to prepare a talk that could be trimmed to various lengths.

There were 34 participants at the meeting.

Partial Differential Equations and Computational Material Science

Spitalfields Day, 13 May 2003
Organisers: CM Elliott (Sussex) and M Luskin (Minneapolis)

Three lectures of a general nature were presented and were well attended by Institute participants from both parallel programmes and academics from within and without Cambridge. The aim was to present exciting new areas of research in materials science requiring significant computational mathematics based on PDEs.

A follow-up day on 14 May 2003, entitled *Partial Differential Equations and Computational Material Science and Solid Mechanics*, was more specialised and focussed on the interaction between the micro and macro scales.

Mathematics of Finite Elements and Applications

Satellite Conference at Brunel University, 21–24 June 2003
Organiser: JR Whiteman (Brunel)

The Newton Institute played a prominent role in the latest of this long-standing series of conferences. The Rothschild Visiting Professor, F Brezzi, gave the Isaac Newton Lecture: *Stabilising sub-grids and their construction*. The Director of the Institute, Sir John Kingman, gave the after-dinner speech and there was a Newton minisymposium running throughout the conference. In addition the following long term participants in the programme gave invited lectures: M Ainsworth, C Carstensen, C Schwab and E Süli.

Outcome and Achievements

At the date of printing there are over 35 preprints associated with the programme in the Newton Institute Preprint Series. The slides, overheads and computer presentations of a large number of lectures in the programme may be found on the web pages of the Institute. There was a great deal of interaction and collaborative research during the programme and we discuss this below under non-mutually exclusive headings.

Nonlinear PDEs and Free Boundary Problems

Diffuse Interface and Phase Field Methods

Z Chen, A Schmidt and R Nochetto made substantial progress in completing a project on *Adaptive FEM for diffuse interface models*. D Kessler, R Nochetto and A Schmidt derived *a posteriori* error bounds to use in *Error control for the Allen–Cahn equation*. J Barrett and CM Elliott began a project to derive fully discrete stable schemes for Cahn–Hilliard fluids entitled *Finite element approximation of Cahn–Hilliard, Navier–Stokes and Hele–Shaw systems*. J Barrett, H Garcke and R Nürnberg developed an approach to surface diffusion with elasticity via the coupling of elasticity equations to a degenerate Cahn–Hilliard equation.

Geometric Flows, Surface Diffusion and Level Set Methods

K Deckelnick and CM Elliott completed a project on *Uniqueness and error analysis for Hamilton–Jacobi equations with discontinuities*. This involves viscosity solutions and has applications to solving the level set eikonal equations in discontinuous media. There was significant activity associated with surface diffusion: K Deckelnick, G Dziuk and CM Elliott derived an algorithm and proved optimal order error bounds for *Fully discrete second order splitting finite element* *methods for anisotropic surface diffusion for graphs*; G Dziuk and CM Elliott developed level set methods; and P Morin and R Nochetto improved their algorithm for parametric evolution of surfaces by introducing volume preserving mesh regularisation. R Davies and CM Elliott derived a level set model for the proving of shaped bread rolls. O Lakkis and R Nochetto completed their work *A posteriori error analysis for the mean curvature of graphs*. K Mikula completed a long manuscript, *Computational solution, applications and analysis of some geometrical evolution equations,* with a major application oriented towards image processing.

Nonlinear PDEs

R Nochetto, A Schmidt, K Siebert and A Veeser collaborated on *a posteriori* error analysis for *Semilinear elliptic PDEs with free boundaries*. J Barrett, C Schwab and E Süli initiated a major project on the analysis and numerical analysis of kinetic models for polymeric fluids. J Barrett, X Feng and A Prohl collaborated on the numerical analysis of Erickson's model for nematic liquid crystals.

Pani collaborated with Süli on the error analysis of discontinuous Galerkin finite element methods and derived optimal error bounds in the L^2 norm. He also contributed to the Newton Institute workshop at the MAFELAP conference.

Superconductivity

L Prigozhin made substantial progress on his work *Penetration of nonuniform magnetic field fluctuations into type II superconductors and the AC losses*. CM Elliott, D Kay and V Styles developed an analysis of a degenerate Stefan formulation of a critical state model in *Finite element analysis of a current density–electric field formulation of Bean's model for superconductivity*.

Evolving Surfaces

G Dziuk and CM Elliott developed an elegant compact formulation for transport and diffusion on evolving surfaces. This has applications to, for example, the transport of surfactants on fluid surfaces. A finite element algorithm based on evolving triangulations was developed together with stability and error analysis.

Microstructure

C Carstensen developed an approach to the convergence of adaptive finite elements for degenerate convex problems.

Adaptivity and *A Posteriori* Error Control

R Nochetto, E Bäensch, O Lakkis, P Morin, K Siebert, A Schmidt and A Veeser continued their very active collaboration on the theory and implementation of adaptive finite element methods. Areas where significant progress has been made include error control for the Allen–Cahn equation, adaptive finite element methods for diffuse interface models, *a posteriori* error analysis for mean curvature on graphs and convergence of adaptive Raviart–Thomas finite element methods.

W Dörfler and M Ainsworth developed a new *a posteriori* error estimator based on the nonconforming Crouzeix–Raviart element for the Stokes equations.

C Schwab, P Houston and E Süli devised a new regularity estimation strategy for *hp*-adaptive finite element methods, E Süli and J Robson developed the *a posteriori* error analysis of finite element approximation of non-Newtonian flows of power-law type, while B Guo and M Ainsworth collaborated on the *a posteriori* error analysis of *h*- and *hp*-version finite element methods.

B Guo completed a paper on the *hp*-version of the boundary element method.

F Brezzi, LD Marini and E Süli developed an error analysis of the discontinuous Galerkin finite element method for hyperbolic problems which sheds new insight on the dissipation properties of these methods.

P Houston and TJ Barth developed a new post-processing technique for adaptive finite element approximations of hyperbolic problems.

C Carstensen and A Veeser collaborated on the convergence of adaptive finite element methods and developed a general technique of proof of convergence.

ADC Hill worked on the *a posteriori* error analysis of finite element methods for nonlinear parabolic PDEs.

Multiscale and Multilevel Algorithms

W Dahmen, A Cohen and R DeVore continued their collaboration on multiscale methods and the stability of solutions to conservation laws in metrics other than the usual ones. W Dahmen also started a new collaboration with Kunoth during their stay at the Institute on the *a posteriori* error analysis of wavelet-based methods in the framework of dual-weighted residual type estimators.

F Brezzi made considerable advances on the error analysis of a class of multiscale finite element methods based on residual-free bubbles. W McLean and M Ainsworth continued their collaboration on multilevel preconditioners for first-kind boundary integral equations with weakly singular kernels.

R Duran and R Rodriguez collaborated on the development and analysis of numerical methods for folded elastic plates.

P Plechac and MAK Katsoulakis collaborated on coarse-graining of multiscale discretisation algorithms, while R Scheichl worked on the numerical linear algebra of PDEs which arise from fluid flows in porous media.

Computational Wave Propagation

M Costabel and M Dauge completed a manuscript on the regularity of solutions of Maxwell's equations in non-homogeneous media, and, in collaboration with D Boffi, made significant progress on proving the discrete compactness property for higher order edge elements.

IG Graham completed a paper on the computation of diffraction coefficients in high frequency acoustic scattering. PB Monk completed a manuscript on the ultra-weak variational formulation for wave propagation in elastic media and another on computational micromagnetics, and initiated a project on discontinuous Galerkin methods for Maxwell's equations with P Houston. M Ainsworth completed analyses of the dispersive and dissipative properties of higher order elements for standard and discontinuous Galerkin methods and initiated a new collaboration on this topic with PB Monk. D Silvester developed software that will form part of his forthcoming book with A Wathen and HC Elman, and initiated a new collaboration with M Ainsworth and PB Monk on preconditioners for the Helmholtz equation.

Published Books

During the course of the programme, C Johnson completed a major book on mathematical analysis (for Springer-Verlag) in which constructive and computational/algorithmic aspects play a vital role, and E Süli completed the undergraduate textbook *An Introduction to Numerical Analysis* (Cambridge University Press, 2003).

Conclusion

Based on the reaction of the participants, the organisers feel that this has been an extremely exciting and fruitful programme in which a great deal has been achieved and in which a substantial number of new projects and collaborations has been initiated. We wish to express our sincere gratitude to all members of staff of the Isaac Newton Institute for their help and support throughout the programme, and for providing such a friendly, pleasant and stimulating working environment.

Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics

27 January to 11 July 2003

Report from the Organisers:
CM Dafermos (Brown), PG LeFloch (CNRS), EF Toro (Trento)

S Wysgard

Left to right: PG LeFloch, EF Toro, CM Dafermos

Background

Quasilinear hyperbolic systems in divergence form, commonly called "hyperbolic conservation laws", govern a broad spectrum of physical phenomena in compressible fluid dynamics, nonlinear material science, general relativity, etc. Such equations admit solutions that may exhibit various kinds of shocks and other linear and nonlinear waves (propagating phase boundaries, fluid interfaces, gravitational waves, etc.) which play a dominant role in multiple areas of physics: astrophysics, cosmology, dynamics of (solid–solid) material interfaces, multiphase (liquid–vapour) flows, combustion theory, etc. In recent years, major progress has been made in both the theoretical and numerical aspects of the field, while the number of applications has skyrocketed. Hyperbolic models arising in applications often face serious mathematical difficulties related to the occurrence of discontinuities, coordinate singularities, resonance between two or more wave speeds, elliptic regions in phase space, etc. The challenge for mathematicians is to comprehend the properties of nonlinear waves and their relationships with the dynamics of many physical phenomena.

The activities during the programme were organised around five main themes described below.

Nonclassical Shock Waves and Propagating Phase Boundaries

The first period of concentration (January–February) was centred around the concept of a kinetic relation, which determines the propagation of undercompressive waves such as those arising in phase transition dynamics. This idea was put forward by material scientists and applied mathematicians in recent years. The mathematical modeling of such problems and the theoretical and numerical aspects of the kinetic functions were both discussed. Several series of lectures were given by long-term participants, on continuum mechanical models of evolutionary structural transformations (Knowles) and on diffusive–dispersive singular limits for systems of conservation laws when both viscosity and capillarity effects play a role (Bedjaoui).

Mathematical Aspects of the Dynamics of Phase Transitions

Spitalfields Day, 10 February 2003

Organisers: CM Dafermos and PG LeFloch

This event was supported by the London Mathematical Society, and was an opportunity to review the state-of-the-art in the field and to initiate several collaborations between long-term participants. The meeting covered a broad range of topics, including the mathematical analysis of the kinetic relation (Asakura, Corli, Sablé-Tougeron), the dynamics of phase boundaries in solids (Berezovski, Knowles, Pence, Thanh) and the discretisation of continuous models (Friesecke, Shearer, Truskinovsky).

Well-Posedness Theory of Systems of Conservation Laws

Recent developments on the general theory of entropy solutions to one-dimensional hyperbolic systems of conservation laws were covered during this second concentration period. Over the two-week period 10–21 March, a total of six series of lectures was offered on:

- Fundamental concepts of the theory of shock waves and their connection with thermodynamics and continuum mechanics (Dafermos)
- Conservation laws with diffusion or relaxation (Marcati, Serre)
- Vanishing viscosity solutions (Bressan)
- Well-posedness theory for general hyperbolic systems (LeFloch)
- Boltzmann, Euler and Navier–Stokes equations of gas dynamics (Liu)

Mathematical Theory of Hyperbolic Systems of Conservation Laws

Workshop, 24–28 March 2003

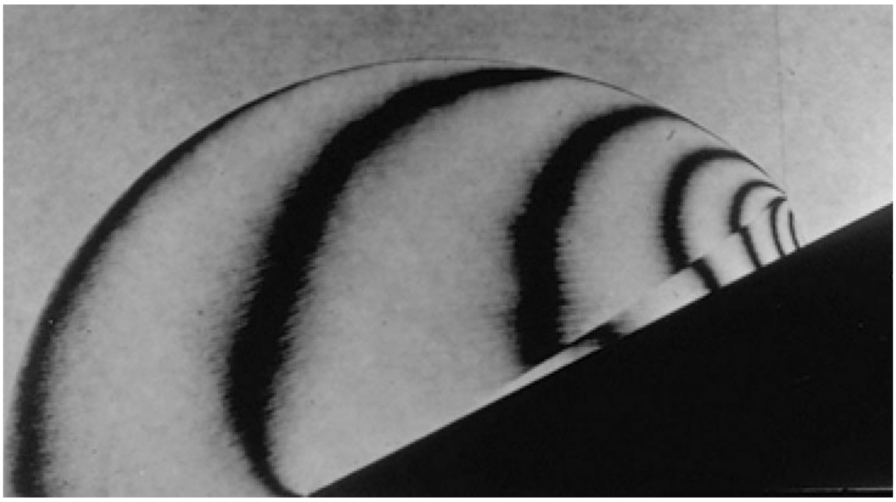
Organisers: CM Dafermos and PG LeFloch

This workshop was very successful. It attracted a large and enthusiastic participation from many of the worldwide experts in the field and provided a special opportunity to investigate current trends on the mathematical theory of shock waves and conservation laws. All contributions consisted of one-hour presentations, allowing plenty of time for questions and discussions during the breaks. Two poster sessions were also organised. The main themes treated during the week were:

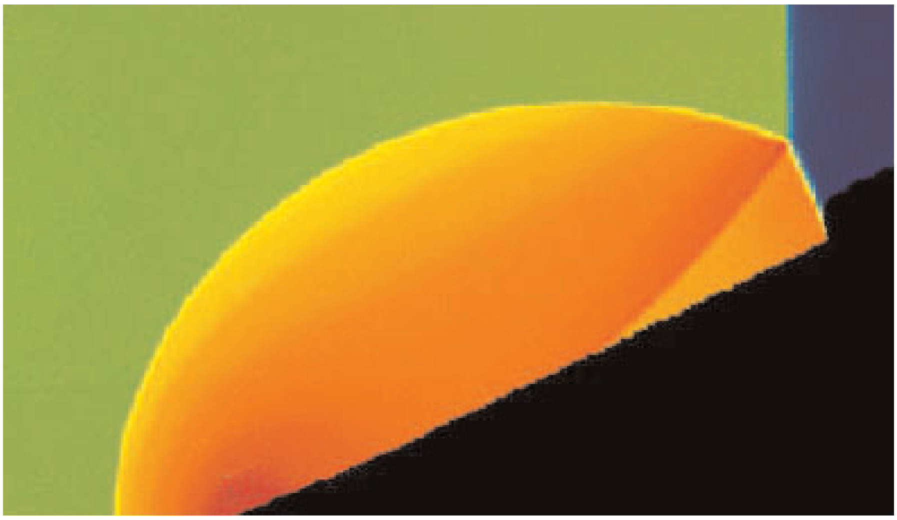
- General properties of hyperbolic systems arising in continuum physics (Godunov, Brenier, Ferapontov)
- Kinetic models from mathematical biology and discrete velocity Boltzmann equations (Perthame, Tzavaras)
- Well-posedness theory of systems of conservation laws, especially existence theory for general flux-functions and classical and nonclassical entropy solutions (Iguchi, LeFloch, Trivisa, Liu)
- Models of diffusive relaxation approximations (Marcati, Serre)
- Vanishing viscosity approximations (Bressan)
- Euler and Navier–Stokes equations including results on critical thresholds in Eulerian dynamics (Tadmor)
- Asymptotic behaviour (Nishibata)

S Wygard

Participants at the workshop on "Mathematical Theory of Hyperbolic Systems of Conservation Laws"



Shock wave on a wedge: experimental results



Shock wave on a wedge: numerical results

Multidimensional Hyperbolic Problems

In April, the programme concentrated on multi-dimensional aspects of hyperbolic conservation laws, including computational methods with applications to multiphase flows. Several series of three lectures each were given on multidimensional Riemann problems and related issues (Chen, Keyfitz) and on mixed equations of steady transonic flow dynamics (Morawetz). The equations of transonic flow are particularly challenging: change of type of the equations, degeneracy at the sonic line and the appearance of free boundary problems which describe shock interfaces. Building on her earlier results on mixed equations for transonic flow, CS Morawetz presented new existence theorems for viscous flows, including an application of Noether's theorem to the Tricomi equation. PD Lax's inspiring lectures presented an illuminating overview on the zero-dispersion limit for the Korteweg–de Vries equation.

Multiphase Fluid Flows and Multi-Dimensional Hyperbolic Problems

Workshop, 31 March–4 April 2003

Organisers: J Ballmann, PG LeFloch, R LeVeque and EF Toro

This workshop attracted a large international audience of about 80 participants from all over the world. The talks fell within three categories: first, multidimensional hyperbolic problems including mixed models for transonic flows (Morawetz, Chen), self-similar solutions to the Riemann problem in two space dimensions (Keyfitz) and the asymptotic stability of non-planar Riemann solutions (Frid); second, numerical methods, especially the evolution Galerkin schemes (Noelle) and the positive schemes for systems of conservation laws (Lax); third, a large variety of applications, especially the simulation of cavitation processes (Ballmann), the propagation of interfaces (Nikiforakis, Greenberg, Marquina), wave structure for elastic–plastic flow (Menikoff) and various problems from fluid dynamics (Toro, Drikakis, Falle, Peregrine).

Computational Methods for Complex Fluid Flows

During May, there were two main concentrated periods of activity. A short course (LeVeque, Shu, Toro) was organised over four days on numerical methods for hyperbolic conservation laws. The course was aimed primarily at young engineers and scientists within various disciplines in which modern numerical methods for hyperbolic systems are used. Support for young UK scientists was provided via the Institute's Junior Membership scheme. There were about 45 participants, 30 of whom were junior scientists and the remainder senior participants. The contents of the course included both basic theoretical aspects of conservation laws and notions on numerical methods for hyperbolic equations (Riemann solvers, high-order schemes). To illustrate the potential applicability of the methods three lectures were offered on traffic flow modelling (Shu), environmental fluid dynamics (Toro) and astrophysics (LeVeque).

S Wygard

One of PD Lax's lectures in progress

Very High-Order Numerical Methods for Hyperbolic Conservation Laws

Workshop, 27–30 May 2003

Organisers: N Nikiforakis and EF Toro

This four-day workshop was about current research on approaches for constructing numerical methods of very high-order accuracy for solving hyperbolic conservation laws. The emphasis was on algorithms that allow high accuracy (third order and above) in both space and time, for one- and multi-dimensional problems and for problems including additional algebraic source terms (balance laws) or higher order derivatives (viscosity, capillarity). The talks in the workshop fell within three categories: algorithm design, numerical analysis and applications. A central objective of the workshop was to include all major current approaches for hyperbolic equations, aiming at bringing together various schools of thought. Topics studied included: finite volume methods, total variation diminishing, essentially non-oscillatory schemes and variants, adaptive finite-element methods, discontinuous Galerkin methods, multi-dimensional upwinding, adaptive meshes and applications. The workshop enjoyed the participation of about 40 scientists from several countries. All contributions consisted of one-hour informal presentations, allowing plenty of time for questions and lively discussions.

S Wygard

Participants at the workshop on "Multiphase Fluid Flows and Multi-Dimensional Hyperbolic Problems"

Hyperbolic Models in General Relativity

The last period of the programme (June–July) was devoted to hyperbolic aspects of Einstein's field equations of General Relativity. Several major aspects of the mathematical theory were covered. This period started with short advanced courses on the Cauchy problem for the Einstein vacuum equations (Klainerman), shock wave solutions of the Einstein equations (Temple), linear hyperbolic equations in a black hole geometry (Finster) and the Dirac sea and the principle of the fermionic projector (Finster again). S Klainerman and I Rodianski presented a new conjecture on the optimal regularity of the solutions to the Einstein equations. The programme included also some activity on the interaction of gravity with other force fields, described by Einstein's equations coupled to the Yang–Mills, Maxwell, and Dirac equations (Smoller, Finster). These coupled, highly nonlinear equations have been shown to admit solutions with surprising properties.

Recent Activity on Numerical General Relativity

Hewlett-Packard Day, 2 June 2003

Organisers: PG LeFloch, and JM Stewart

The speakers discussed recent numerical work in General Relativity, addressing central issues related to cosmology, the collision of two black holes, the generation of gravitational waves and shock waves, etc. The themes included the most recent progress in numerical relativity concerning the binary black hole inspiral and merger (Hawke, Seidel), recent advances in numerical relativistic magnetohydrodynamics motivated by applications in astrophysics (Marti) and the Geroch reduction and the issue of axisymmetry in numerical relativity (Stewart).

Hyperbolic Models in Astrophysics and Cosmology

Euroconference, 23–27 June 2003

Organisers: CM Dafermos, PG LeFloch, JA Smoller and JM Stewart

This meeting was devoted to the Einstein field equations of General Relativity and covered both the general mathematical theory and various applications in astrophysics and cosmology. It brought to Cambridge the front-line specialists in mathematical general relativity. A large number of young researchers from the European Community participated in this event and the conference fostered exchanges between several research centres of excellence working on general relativity, including Cambridge, Golm, Oxford, Princeton, Southampton and Trieste. There were about 20 lectures of one hour each plus two poster sessions, leaving plenty of time for lively discussions between participants.

A presentation of the most recent research on existence theory for the Einstein equations was provided by Friedrich, Klainerman, Reula and

S Wygard

Lecture by CS Morawetz

Rodianski. The other themes were:

- Late-time behaviour of solutions (Rendall)
- Gravitational waves and shock waves (Griffiths, Stewart, Temple)
- Interaction of gravity with other force fields (Smoller, Linden)
- Recent progress on numerical methods in general and special relativity (Bona, Font)
- Geometric aspects of Lorentzian manifolds (Zeghib)
- The interior of charged black holes and the problem of uniqueness in general relativity (Dafermos)
- Many other topics such as black hole spacetimes and scattering problems (Komissarov, Lindblad, MacCallum, Mason, Melnick)

Outcome and Achievements

The programme proved very popular and brought to Cambridge a large number of short- and long-term participants, including both young and senior researchers. All participants were very favourably impressed by the working conditions offered by the Newton Institute and many would have liked to come for a longer period.

Weekly seminars, short courses, workshops and conferences provided valuable opportunities for reviewing the state of the art for physical models, techniques of mathematical analysis and numerical analysis. Many challenging problems in the field were discussed. Significant work was carried out by the participants and new bridges arose between the most recent developments in the general mathematical theory of shock waves and the areas of applications that are currently most active.

For instance, the programme helped to increase the interest of the applied mathematics community in relativistic fluid models. The lectures by H Friedrichs and JM Stewart were very helpful to clarify the most important issues in General Relativity and initiate exchanges between the long-term participants.

Progress was also made on nonclassical travelling waves with viscosity and capillarity terms and the concept of nucleation and its connection with the stability or instability of interfaces in phase transition dynamics.

One of the most unexpected and lasting outcomes of the programme was the launching of a new mathematical research journal, the *Journal of Hyperbolic Differential Equations*, devoted precisely to the topics treated during this programme. Almost all of the members of the editorial board were long-term participants, and many participants have already submitted papers written at the Institute for publication in the *Journal*.

Finances

Accounts for July 2002 to July 2003 (Institute Year 11)

	2001/2002 Year 10	2002/2003 Year 11 (13 months)
Income		
Grant Income – Revenue	1,247,710	1,230,928
Grant Income – Workshop	377,796	226,421
Trust Fund Income	78,975	183,904
Donations – Revenue	604	512
Interest on Deposits	31,901	31,525
General Income	5,066	7,661
Housing	10,938	7,239
Total Income	**1,752,990**	**1,688,190**
Expenditure		
Scientific Salaries	331,912	362,158
Scientific Travel and Subsistence	410,712	395,477
Scientific Workshop Expenditure	288,281	181,483
Other Scientific Costs	16,701	22,113
Staff Costs (Payroll)	290,285	348,294
Staff Costs (Contract and Agency)	18,789	21,738
Computing Costs	57,118	58,689
Library Costs	11,577	11,459
Building – Rent	221,258	227,896
Building – Repair and Maintenance	13,042	23,494
University Overheads	30,597	15,650
Consumables	28,113	27,370
Equipment – Capital	16,819	41,996
Equipment – Repair and Maintenance	4,819	5,000
Publicity	12,253	6,540
Recruitment Costs	3,761	2,519
Total Expenditure	**1,756,037**	**1,751,876**
Operating Surplus / (Deficit)	**(3,047)**	**(63,686)**
Transfer (to) / from Building Capital Fund	(4,964)	0
Transfer (to) / from Reprovision	21,829	49,291
Total Surplus / (Deficit)	**13,818**	**(14,395)**

Notes to Accounts

1. Accounting Period
The Institute's financial year has been amended from 1 July–30 June in previous years to 1 August–31 July in future, to be in line with the University Finance System. This has resulted in the current year 2002/2003 being of 13 months' duration.

2. Grant Income – Revenue
This breaks down as follows:

	2001/2002 Year 10	2002/2003 Year 11
EPSRC/PPARC Salaries	344,992	314,598
EPSRC/PPARC Travel and Subsistence	298,112	297,743
Trinity College (Isaac Newton Trust)	100,000	100,000
Hewlett-Packard	115,000	115,000
PF Charitable Trust	0	19,700
CNRS	25,668	0
Leverhulme	79,513	79,287
LMS	20,000	20,000
Cambridge Philosophical Society	2,250	2,000
Daiwa Anglo-Japanese Foundation	3,074	0
Royal Society	0	2,314
University of Cambridge (Staff)	37,843	22,390
University of Cambridge (Rent)	221,258	227,896
University of Cambridge (Equipment)	0	30,000
Total	**1,247,710**	**1,230,928**

3. Grant Income – Workshop
See note 8 below.

4. Trust Fund Income
This breaks down as follows:

	2001/2002	2002/2003
Rothschild – Visiting Professors	25,655	28,207
Rothschild – Director	53,320	155,697
Total	**78,975**	**183,904**

An additional late stipend payment of £31,793 for 2001/2002 has been included in the reported figures for 2002/2003.

5. Housing
This figure represents the net balance of income and expenditure, and is lower than in 2001/2002 mainly owing to reduced income from rental properties in the second half of 2002.

6. Scientific Salaries
This includes stipends paid to EPSRC/PPARC Fellows, Rothschild Visiting Professors, the Hewlett-Packard Senior Fellow, the Director and the Deputy Director.

7. Scientific Travel and Subsistence
This includes expenditure incurred by programme participants, Junior Members and the Hewlett-Packard Senior Fellow.

8. Scientific Workshop Expenditure
This figure is lower than in 2001/2002 owing to the reduced level of workshop activity planned by programme organisers. The grant income for workshops is also, therefore, correspondingly lower.

9. Other Scientific Costs
This includes costs relating to meetings of Institute committees, Institute Correspondents' expenses and the travel expenses of overseas participants who visit other UK institutions to give seminars during their stay (see p 7).

10. Building – Rent
This is the rental for both the main and Gatehouse buildings, and is covered by a grant from the University of Cambridge. The University also pays for all gas, electricity and rates, which have not been included.

11. Building – Repair and Maintenance
This includes rewiring costs related to the Library refit, provision of additional computer storage space and work on the car park.

12. Equipment – Capital
Purchase of security access and monitoring equipment, new Library furniture, kitchen equipment and shelving for the Gatehouse storeroom. Much of this is covered by a one-off Equipment grant from the University of Cambridge and by a transfer from the Reprovision fund.

13. Publicity
This includes costs for the production of the *Annual Report*, Institute brochures and any other outsourced printing. This budget has fallen since 2001/2002, as more of the Institute's publicity is now performed electronically when appropriate.

14. Donations in Kind
COMSOL Ltd donated their FEMLAB software for a six-month period for use by participants of the *Computational Challenges in Partial Differential Equations* and *Nonlinear Hyperbolic Waves in Phase Dynamics and Astrophysics* programmes.

Over 4,100 books and journals have been donated to date by a large number of publishers and individual members of the mathematical community.

Cumulative Financial Grants and Donations

SERC/EPSRC/PPARC	£10,571k	over 16 years
Trinity College (Isaac Newton Trust)	£2,510k	over 12 years
NM Rothschild and Sons	£2,083k	over 10 years
Anonymous Donation	£1,065k	
Hewlett-Packard	£1,065k	over 10 years
Dill Faulkes Foundation	£1,000k	
Leverhulme Trust	£855k	over 12 years
St John's College	£750k	over 5 years
NATO	£728k	over 10 years
European Union	£736k	over 11 years
University of Cambridge (excluding rent grant)	£573k	over 11 years
Le Centre Nationale de la Recherche Scientifique	£435k	over 10 years
Rosenbaum Foundation	£330k	over 7 years
PF Charitable Trust	£240k	over 3 years
London Mathematical Society	£225k	over 12 years
Clay Mathematics Institute	£152k	
Gonville and Caius College	£100k	
Prudential Corporation plc	£100k	over 4 years
Institute of Physics	£68k	over 7 years
British Meteorological Office	£64k	
Nuffield Foundation	£57k	
TSUNAMI	£40k	
Daiwa Anglo-Japanese Foundation	£36k	over 4 years
AFCU (Hamish Maxwell): $50k	£32k	
AFCU (Anonymous Donation): $50k	£32k	
Emmanuel College	£30k	
Jesus College	£30k	over 6 years
British Aerospace	£25k	
Rolls Royce	£25k	
Cambridge Philosophical Society	£25k	over 10 years
Corporate Members (FIN programme)	£22k	
British Gas	£20k	
DERA	£20k	
Magnox Electric	£20k	
Paul Zucherman Trust	£20k	
Thriplow Trust	£18k	
Schlumberger	£17k	
Bank of England	£15k	
Wellcome Trust	£15k	
Benfield Greig	£10k	
NERC	£10k	
Unilever	£10k	
Applied Probability Trust	£10k	over 3 years

This table includes sums agreed but yet to be received.