

# Mathematics of Atmosphere and Ocean Dynamics

## Review Meeting held 1-5 December, 1997

### *Report from the Organiser: Ian Roulstone (UK Meteorological Office)*

This meeting attracted about 30 participants, many of whom had taken part in the AOD programme (*Mathematics of Atmosphere and Ocean Dynamics*) from July to December, 1996. The original programme stimulated many new directions of research and triggered informal collaborative projects, and it was therefore timely to provide a forum for reviewing progress over the subsequent 12 months or so.

The scope of the presentations was very broad, ranging from hamiltonian fluid mechanics and balanced models to mesoscale meteorology and parameterisation schemes. During the first two days discussions on recent results on integrability, turbulence, and regularity of solutions of balanced models, predominated. The generation of small scales in the enstrophy in two-dimensional incompressible flow can be understood using the analytical tools which are used to prove existence and regularity of results for these equations. Many important issues relating to the enstrophy cascade in semi-geostrophic turbulence, and in the solutions of more general systems of equations, were discussed.

The theory of optimal mass transfer featured in several talks. The basic problem is as follows: given two sets of equal volume, find the optimal volume-preserving map between them, where optimality is measured against a cost function. Such problems arise when studying the semi-geostrophic equations. Recent research has focused on regularity theory, and existence of optimal mappings when the sets are replaced by manifolds. This is relevant to semi-geostrophic theory, and computations involving optimal transport were also presented.

Wednesday and Thursday had a slightly more geophysical theme, including some recent numerical results on predictability and growing normal modes, and numerical methods for the semi-geostrophic equations. Discussions also took place on sub-grid models and their interaction with resolved dynamics.

We were pleased to welcome several people who did not participate in the AOD programme in 1996, but have been involved in related work. Among those were Robert McCann (Brown), who spoke about free boundaries and discontinuities in optimal transport, Jean-David Benamou (INRIA), who spoke on the numerical resolution of the mass transfer problem based on an augmented lagrangian technique and also presented some numerical results, and Emmanuel Grenier (Paris VI), who gave a presentation on a new approach, from a pde point-of-view, to the derivation of the quasi-geostrophic and semi-geostrophic equations.

Overall, this meeting was a very worthwhile exercise which has, in turn, stimulated further collaboration and research. It was noted that a number of symposia at international conferences have been based around work emerging from the AOD programme.

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