Applicable Resurgent Asymptotics – short report

Asymptotic analysis has been a vital technique within mathematics for over a century, but recent breakthroughs have led to radical changes in the mathematical understanding of these methods. By exploiting the idea of "resurgence", researchers have been able to extend asymptotic analysis beyond the boundaries of the traditional asymptotic analysis of Poincaré. The new asymptotic machinery was used to finally establish a rigorous foundation for many long-standing ideas from perturbation theory. These newly developed methods have been applied to resolve open questions in areas such as fluid dynamics, aero-acoustics, pattern formation, dynamical systems, optics and biomathematics. Parallel developments were being pursued in theoretical physics, in the study of the non-perturbative structure and dynamics of quantum field theories, string theory, random matrix models and integrable models, with potential real life applications in the realm of condensed matter physics and other strongly correlated systems governed by relativistic hydrodynamics. Resurgent asymptotics was explored essentially independently in these two fields, who established different perspectives and tools for the same mathematical concepts. The main purpose of this programme was to bring together these perspectives to build a shared universal theory of resurgent asymptotics. In doing so, the programme aimed to establish the direction of future work in this field.

Following the outbreak of COVID-19, the programme was split into two parts. The first part was a four-month long online programme, taking place from March to June 2021, while the second part was a four-month in-person programme, taking place from September to December 2022.

The first part of the programme featured weekly online seminars and discussion rooms, with a high participation from across the globe. Participants were able to interact through an online platform, and there were several "theme weeks", arranged to focus more deeply on topics of cross-disciplinary interest, including relativistic hydrodynamics and Painlevé equations. The online platform and theme weeks encouraged more informal interactions between participants, and replicated the thought-inspiring discussions that would be had during an in-person programme. A highlight of the online portion of the programme was the Spring School, a targeted one-week series of lectures aimed at graduate students and early-career researchers held at the start of the programme. This Spring School drew on perspectives from applied mathematicians and theoretical physicists in order to provide a comprehensive introduction to the field.

The online portion of the programme, which ended with a summary workshop, also laid a strong foundation that allowed participants to begin strongly when the in-person component of the programme began in 2022 with the first of three workshops taking place just one week after the programme began. These three workshops were focal points of the programme, attracting participants internationally, including many early-career researchers, and allowing for a mix of presentations and discussions. In addition to the workshops, there were two weekly seminars and numerous smaller events aimed at generating discussion and academic connections between participants, including several events with a focus on early-career researchers. The final summary workshop provided an outline of the state of asymptotic analysis within both theoretical physics and applied mathematics, presenting key open questions from for researchers to study and resolve in the coming years. This particular workshop played an important role in focusing research efforts, and will shape the direction of future progress in resurgent asymptotics and its applications.

One important objective of the programme was to merge the two largely-independent research streams in resurgent asymptotics that had developed in recent years. In this capacity, the programme was a great success, with many new collaborative links formed between physicists and mathematicians. Bringing together researchers from such diverse academic backgrounds led to both entirely new ideas and genuine progress on existing open problems. This programme led to mathematical advances including the development of q-Borel techniques, the application of transasymptotic analysis to PDEs, and the development of asymptotic methods that can incorporate numerical or experimental data. Additionally, programme participants made significant advances in theoretical physics, notably the analysis of the types of branes being expressed in the transseries of matrix models, interpolation between different regimes in parameter space of observables, and transseries approaches to integrable field theories.

In summary, the major goals of the ARA programme were to establish connections between the asymptotic perspectives from two different fields (applied mathematics and theoretical physics), to use these connections to produce important results for both fields, and to shape the future direction of research in resurgent asymptotics. These goals were accomplished by participants in the programme, and give many reasons to be optimistic about both the immediate and long-term future of the field.