Approximation, Sampling and compression in data science

Isaac Newton Institute Cambridge

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Short Report

Recently, driven by applications in engineering, biology, medicine and other areas of science, new challenging problems have appeared. The common feature of these problems is high, really high, dimensions. These problems are often challenging, due to the curse of dimensionality. Classical methods developed in multivariate approximation theory may work for moderate dimensions, say, up to dimension 40. The focus of this 6-months programme was to develop approximation machinery in understanding and solving challenging problems in high dimensions.

Activity was structured around four workshops, each of them with different focus. Three of them: "Challenges in optimal recovery and hyperbolic cross approximation" (February), "The Mathematics of Deep Learning, and Data Science" (May), and "Approximation, sampling, and compression in high dimensional problems" (June) took place in the Institute. Another workshop "Mathematics of data: structured representations for sensing, approximation and learning" (May) was organized in the collaboration of the Alan Turing Institute in London. This was fruitful and productive teamwork that can be used in the future programmes.

The workshops attracted well over 300 participants, coming from diverse areas of pure and applied mathematics such as approximation theory, harmonic analysis, functional analysis, signal processing, machine learning, optimization, etc. Outside workshops weeks, seminars and open problem sessions took place. Moreover, two introductory courses were given for earlier career participants. The first one was a course on information based complexity delivered by Erich Novak and the second on compressed sensing and sparse approximation by Ben Adcock. This programme hosted two special lectures by Ronald DeVore and Svitlana Mayboroda, which were aimed at a wide audience comprising experts and the general public. These lectures provided an overview of the state of art in numerical computation with special emphasis on high dimensional problems (R. DeVore) and localization phenomena under irregularities of systems (S. Mayboroda). We would also like to mention the talk given by Martin Buhmann at the Isaac Newton Institute Seminar Series about the Mathematikum in Giessen/Germany -- a mathematical centre for the public.

Scientific achievements of the programme can be seen as follows. On the one hand, the participants have achieved a breakthrough in solving several old problems from classical approximation theory, specifically, sampling problems, discretization, polynomial inequalities, function spaces. On the other hand, several interesting new directions were developed in multivariate approximation problems, where classical multivariate approximation approach does not work (problems in nonlinear approximation, in particular, m-term approximation, greedy approximations, etc).

The programme attracted much attention of many young researchers in early stages of their careers, who were actively participating in the scientific life. During the programme, the Institute also welcomed several mathematicians from such countries as India, Kazakhstan, Russia, Ukraine, therefore reaching another programme´s goal - to foster international collaboration among leading mathematical schools in the world.