Black holes: bridges between number theory and holographic quantum information

Final Report

Organisers:

Alejandra Castro (University of Cambridge)
Robert de Mello Koch (Huzhou University)
Yang-Hui He (City University, London; University of Oxford)
Gabriel Lopes Cardoso (Instituto Superior Técnico, Lisboa)
Sameer Murthy (King's College London)
Suresh Nampuri (Instituto Superior Técnico, Lisboa)
Lárus Thorlacius (University of Iceland)

September 4 - December 20, 2023

July 19, 2024

Summary

Constructing a mathematical language and formalism that encodes the organising principles underlying black hole theory is one of the most challenging core research endeavours in the development of a mathematically consistent theory of quantum gravity. Recent advances in this vein in the context of black holes in superstring theory have revealed fascinating connections between the statistical physics of these black solutions, number theory and quantum information theory. This provided a conceptual genesis point for our four-month long programme which was a successful proof-of-concept demonstration of how bringing together mathematicians, mathematical physicists, string theorists and quantum information researchers to stimulate fertile interactions between them using black hole entropy as a central nexus of engagement, can result in unveiling new directions of research that could potentially advance our understanding of the holographic formulation of quantum information in quantum gravity.

Keywords: Black holes and number theory, number theory and machine learning, integrability and machine learning, black hole entropy and quantum information.

1 Background and context

Black hole thermodynamics and the underlying statistical mechanics provide a rigorous testing ground for any putative mathematically consistent theory of quantum gravity. In the 1970's, Bekenstein and Hawking showed that black holes possess thermodynamical properties such as thermodynamic entropy. The semi-classical description of a black hole yields its thermodynamic entropy as proportional to the area of a surface, called the event horizon, that separates the interior of the black hole from the outside region. When viewed from the point of view of quantum gravity, the entropy of a black hole should have a statistical mechanical interpretation, i.e. it should be given by the integer-valued dimension of a well-defined Hilbert space constituted of quantum black hole states (or black hole microstates). Statistical degeneracy being a quantum field theoretic or quantum mechanical number is an integer and further, the organising principle at work in the construction of this Hilbert space must be expressible in the language of quantum information theory. Following on from these arguments, one can posit two central questions in any mathematically consistent theory of quantum gravity:

1. What are the mathematical structures in number theory underlying the counting of black hole microstates?
2. How does quantum information theory encode the quantum statistics of black holes?

Both questions have generated two parallel streams of research in quantum gravity which, in the last decade, have experienced unprecedented conceptual and computational leaps, as follows:

1. The pioneering work of Strominger and Vafa [1] has shown that it is possible to identify and to count black hole microstates for special classes of black holes, called BPS black holes, in the context of string theory. This has given rise to a microstate enumeration programme for BPS black holes in superstring theories, which is at the heart of the first stream of research. This enumeration programme has revealed fascinating connections with number theory and has recently led to deep insights into novel modular forms [2, 3, 4].

2. The second stream of research into quantum gravity deals with the question of how quantum information is stored in black holes. In general, black holes carry temperature and emit thermal radiation, called Hawking radiation. If the black hole were to be initially in a pure quantum mechanical state, the complete evaporation of the black hole would result in a final state containing thermal radiation, i.e; the final state would be a mixed quantum mechanical state. Thus, the evaporation process of a black hole appears to imply a violation of the principle of unitary evolution in quantum mechanics, and hence of the conservation of quantum information. Since the principle of unitary evolution in quantum mechanics is a foundational physical principle, it is essential to understand how conservation of quantum information is ensured in the presence of black holes. This apparent loss of unitarity in black holes at the semi-classical level is referred to as the information paradox for black holes and constitutes a long standing fundamental problem in quantum gravity.

   A giant leap was made in addressing this problem when, inspired by black holes, Maldacena [5] made an astonishing conjecture, namely that quantum gravity in a certain class of spacetime backgrounds has a dual description in terms of a quantum field theory defined on the conformal boundary of the spacetime manifold. Maldacena's conjecture, often referred to as the AdS/CFT correspondence, provides a concrete realization of earlier proposals

> by 't Hooft [6] and Susskind [7] of a fundamentally holographic nature of gravity. The holographic view of quantum gravity has become a central research endeavour in string theory."
>
> A prominent question posited therein is the following: what is the nature of quantum information within holography? The structure of information in quantum field theories is characterized by states that are quantum entangled. Entanglement is quantified in terms of a computational quantity, namely the entanglement entropy. In the context of black holes, modelling them as ensembles in a holographically dual conformal field theory implies that the information entropy of black holes is related to the entropy of quantum entanglement in the dual ensemble. Hence, understanding the structure of holographic quantum entanglement will simultaneously resolve the information paradox for black holes.
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> Thus the critical question in a holographic description of quantum gravity can be posed as: How is the entropy of quantum entanglement in the holographically dual theory encoded in the gravity description? An incisive contribution to the answer was made by the Ryu-Takayanagi formula [8] and its generalization, the quantum extremal surface [9], relating holographic entanglement entropy computation to a geometrical surface minimization problem in gravity. Thus, 'extremal surfaces' as defined by a minimization process in a given spacetime metric, combined with a wide swathe of techniques drawn from the quantum information theory toolkit, including quantum error correcting codes [10], MERA tensor networks [11] and complexity theory [12], have become key probes for studying entanglement in quantum field theories. Quantum entanglement itself has proved to be a resourceful template for thinking about the mathematical foundations of quantum mechanics, formulating aspects of quantum gravity via holographic quantum field theories in terms of operator algebras and more recently, coupled with two of the fundamental concepts in computer science, Turing's universal computing machine, and complexity classes of problems, has been the basis of a groundbreaking result abbreviated as MPI* = RE [13] in both computer science, as well as operator algebra theory. The upshot of these developments in mathematical physics, quantum gravity and computer science has been to lay the groundwork for an exciting new avenue of research, namely setting up a new dictionary between quantum gravity and computer science, aided by recent advances in Machine Learning (ML) applied to string theory and its mathematics [14, 15]. The primary research goal here is to map concepts and proofs in computer science/quantum information theory to geometrical structures and operations in gravity such as extremal surface minimization – a new interface with the potential to revolutionize both fields.

These two parallel research programs (one based on number theory and the other based on holographic quantum information) have operated in their own independent domains, and interface between them has been minimal. They have, however, one common focal point, namely understanding quantum information encoded in black hole entropy.

This programme successfully established bridges between these two parallel streams of research in quantum gravity, by bringing together experts and young researchers from various research areas in mathematics, mathematical physics, string theory and quantum information, and kickstarting sustained synergistic interactions and engagement between them, resulting in cementing new and exciting connections between black holes, number theory, machine learning and quantum information theory.

2 Programme timeliness, scope and outline

The programme was based at the Moeller Institute, on the Churchill College grounds, and its top-of-the-line on-site facilities as well as spatial topographies designed for interactive co-working and board work along with the highly supportive staff contributed in no small measure to the considerable satisfaction of all programme participants.

The programme ran from 4 September to 20 December 2023. It was divided into three modules, each of which was designed as a customised interface between a subset of the 5 main research areas (black holes, number theory, integrability, machine learning and quantum information theory) that the programme delved into. Each module culminated in a one-week workshop dedicated to the specialised topics encompassed by the module.

Each module featured a series of preparatory review lectures by experts introducing the state of the art in terms of achievements and challenges in the designated interface topics of the module, setting the scene for open and vibrant exchanges, both in-field and across, in a collegiate and collaborative environment between specialists.

The programme also featured:

1. A machine learning bootcamp, whose goal was to expose the participants to this powerful computational tool with the aim of using it to help uncover unexpected connections between these research fields;

2. A one-day Gateway event, `Machine Learning: Portents and Possibilities`, which brought together thinkers and practitioners across a wide range of thought streams spanning industry representatives, string theorists, mathematicians, biologists and ethicists to have free flowing conversations (structured as panel discussions and seminars) on the possibilities and burgeoning challenges of having machine learning reshape the landscape and methodology of human knowledge and the intentionalities and consequences thereof;

3. A Rothschild public lecture by the BLH programme Rothschild Distinguished Visiting Fellow, professor Robert de Mello Koch;

4. A special session on `Equity, Diversity and Inclusion (EDI)` in Mathematical Sciences.

2.1 Module 1: Number theory, machine learning and quantum black holes

The first module (September 4 - October 6) explored emergent synergies between number theory, machine learning and quantum black holes.

The first 3 weeks of this module featured 3 review lectures per week on these topics according to the table given below, aimed at introducing recent advances and outstanding issues in this area to the other participants. The fourth week, termed absorption week, provided ample room for discussions and (initiation of) collaborations on the topics covered in this module.

Module 1 culminated in a one-week workshop (October 2-6) entitled `Number theory, machine learning and quantum black holes` dedicated to the topics encompassed by the module and aimed at bringing together researchers from number theory, automorphic representation theory and string theory to explore how automorphic forms that encode degeneracies of BPS black holes could be related to and feed into information theoretic concepts such as quantum error corrections, applied to BPS and near-extremal black hole backgrounds, and to develop machine learning techniques to unveil some of the hitherto undefined automorphic structures underlying the generating functions for BPS black hole microstates in superstring models.

Workshop 1 featured 20 one-hour research talks as well as a workshop wrap up talk entitled `Calabi-Yau periods, Modularity and Arithmetic Geometry: New Results, Challenges and Possibilities`.

Module 1: review lectures	
September 4	Atish Dabholkar (ICTP Trieste) `Quantum Entropy, Entanglement and Black holes`
September 6	Kyu-Hwan Lee (University of Connecticut) `How to Apply Machine Learning to Number Theory`
September 7	Kevin Buzzard (Imperial College London) `Machine learning and formal verification in mathematics`
September 11	Jerome Gauntlett (Imperial College London) `Equivariant Localization in Supergravity`
September 13	Caner Nazaroglu (University of Cologne) `Rademacher Expansions for False and Mock Modular Forms`
September 14	Johanna Erdmenger (Julius-Maximilians-Universität Würzburg) `Insights from quantum field theory and AdS/CFT for machine learning`
September 15	Yang-Hui He (London Institute for Mathematical Sciences; Merton College) `The AI Mathematician, Part I`
September 18	Albrecht Klemm (Bonn University) `Non-perturbative Topological String on Compact Calabi-Yau 3-folds`
September 20	Larry Rolen (Vanderbilt University) `Indefinite theta functions-- Theory and Applications`
September 22	Yang-Hui He (London Institute for Mathematical Sciences; Merton College) `The AI Mathematician, Part II`

2.2 Module 2: Machine learning toolkits and integrability techniques in gravity

The second module (October 9 - November 10) probed the utility of machine learning toolkits to integrability techniques in gravity and featured an introductory hands on machine learning bootcamp.

Weeks 1, 2 and 4 of this module featured 3 review lectures per week according to the table given below, aimed at introducing recent advances and outstanding issues in this area to the other participants. Week 3, termed absorption week, provided ample room for discussions and hands on initiation to machine learning following the machine learning bootcamp lectures by Challenger Mishra (University of Cambridge) in weeks 1 and 2.

Module 2 culminated in a one-week workshop (November 6-10) entitled `Machine learning toolkits and integrability techniques in gravity`, aimed at not only bringing together researchers in solution-generating techniques and integrability, as well as Riemann-Hilbert factorization problems in gravity, but also bringing in expertise in AI and machine learning as a tool. The workshop featured 16 one-hour research talks.

One day of the workshop (November 8) was dedicated to a Gateway event, entitled `Machine Learning: Portents and Possibilities`, aimed at bringing together thinkers and practitioners in mathematical physics as well as industry to exchange ideas, opinions and philosophies relating to the possibilities and burgeoning challenges of having machine learning (ML) reshape their respective landscapes. This event featured 3 sessions consisting of a morning session of talks by two representatives of academia and two from industry, based on the theme of `ML apps and industrial objectives: two sides of one revolving door`, followed by two afternoon discussion panels on

1. `ML applications to problem solving: universal and specific features`
2. `Biases, ethics and predilections in the age of ML: perils and countermeasures`

Module 2: review lectures	
October 9	Manus Visser (University of Cambridge) `Quantum de Sitter space`
October 11	Axel Kleinschmidt (Max-Planck-Institut für Gravitationphysik Golm) `Consistent truncations and D=2 gravity`
October 13	Challenger Mishra (University of Cambridge) `Learning to work with Learning Machines`
October 16	Yolanda Lozano (Universidad de Oviedo) `Gravitational solutions, quivers and AdS/CFT`
October 18	Challenger Mishra (University of Cambridge) `Learning to work with Learning Machines`
October 20	Challenger Mishra (University of Cambridge) `Learning to work with Learning Machines`
October 25	Justin David (Indian Institute of Science Bangalore) `Modular forms and black holes in N=4 string theories`
October 30	Sven Krippendorf (Ludwig-Maximilians-Universität Munich) `How can we learn mathematical structures?`
November 1	Thomas Mohaupt (University of Liverpool) `Gravity and string theory in general spacetime signature`
November 3	Gabriel Lopes Cardoso (University of Lisbon) `Integrable systems in gravity: a Riemann-Hilbert approach`

2.3 Module 3: Bridges between holographic quantum information and quantum gravity

The third module (November 13 - December 1) was dedicated to the very recently surfaced exciting cutting edge possibilities of using quantum information theory as an ontological framework to build up a holographic quantum field theoretic description of quantum gravity.

This module featured 3 review lectures, as described in the table given below, and culminated in a one-week workshop (November 27 - December 1) entitled `Bridges between holographic quantum information and quantum gravity`, which drew maximal audiences and participants and was punctuated by robust and vigorous cross-questioning as well as the emergence of audacious approaches to quantum gravity. Workshop 3 featured 20 one-hour research talks, a workshop wrap up discussion session entitled `Holographic quantum information and quantum gravity - a discussion` and a poster session which offered ECRs a platform to showcase succintly and concisely, cutting edge results in the field which provoked active conversations and dialogue both in-session and after.

Module 3: review lectures	
November 20	Monica Kang (University of Pennsylvania) `Infinite-dimensional holography: bulk reconstruction, relative entropy, and operator algebra`
November 22	Roberto Emparan (ICREA and Universitat Autonoma de Barcelona) `The correspondence between black holes and fundamental strings`
November 24	Lárus Thorlacius (University of Iceland) `Black hole informatics`

2.4 Rothschild Distinguished Visiting Fellow

The BLH programme Rothschild Distinguished Visiting Fellow was professor Robert de Mello Koch from Huzhou University, China. He gave his Rothschild public lecture, entitled `From Black Holes to Holographic Spacetime`, on October 23.

Professor de Mello Koch is one of South Africa's most outstanding researchers with exceptional acuity in both scientific research as well as science communication, as evinced by a plethora of awards from three continents such as the National Research Foundation's President's Award (2001), the South African Institute of Physics Silver Jubilee Medal (2001), the Convocation Distinguished Teacher's Award, Faculty of Science (2001) as well as multiple awards from the Royal Society (UK) to establish research links between South Africa and the United Kingdom as well as the Thousand Talents Award (2021) by the Zhejiang province, China and the Changjiang Scholar Award (2022).

His research spans an impressively wide spectrum from topics in Holographic Gauge/Gravity duality specialised to AdS/CFT scenarios in string theory, Group Theoretic approaches to and Integrability aspects in Quantum Field Theory, advanced Signal Processing techniques as well as optimization algorithms for gravity gradiometry instrumentation and phased radar arrays.

2.5 Equity, Diversity and Inclusion

Demographically, the programme organisers, with the active encouragement of the Isaac Newton Institute, strove for an EDI-compliant policy. Consequently, we had a good mix of established and early career researchers from various parts of the world, including developing countries. This was, in part, made possible by the generous DAC funding scheme of the Isaac Newton Institute, for which we are very grateful.

In this context, the programme included a special session (December 5) on EDI entitled `Examining, discussing, and promoting EDI initiatives in the mathematical sciences`, with the aim of igniting discussions and sharing valuable insights as well as inspiring actionable steps toward creating an inclusive and equitable academic community.

The event started with an overview to some core aspects of EDI, followed by an interactive session aimed at engaging in dialogues covering different corners of EDI relevant to mathematical sciences, and ending with a round table discussion among all participants and keynote speakers.

3 Media

Almost all of the formal presentations given during the programme (review lectures, workshops, Gateway event) were live streamed and made publicly available permanently via the INI YouTube channel as well as by links shared on social media such as X, formerly Twitter.

4 Scientific Outcomes and Highlights

4.1 Scientific advancement

This programme produced a significant expansion of research possibilities.

Programme participants were invited from a diverse collection of interacting fields related to number theory, string theory and machine learning. To facilitate interactions and enable progress, a mixture of regular activities and research time was programmed, with researchers able to discuss their projects in the common areas and offices. Important stimulating activities were the three research workshops, regular review lecture seminars, which all provided opportunities for starting new collaborations and exchanging feedback from different points of view. Various existing collaborations were reinforced when old collaborators reunited during overlapping periods of the programme, apart from new first time collaborations that were kickstarted as a direct result of conversations, talks and discussions during the programme.

A partial list of ongoing intersectional first time collaborations between researchers from different sub-disciplines and their collaboration topic(s) as well as current state of the project is elucidated below for illustrative depth, with details provided by participants in response to a post event survey:

1. Joan Simon, Roberto Emparan, Marija Tomasevic and Alejandra Castro on quantum super-radiance.

2. Simon Ross, Vijay Balasubramanian and Monica Kang on multiparty entanglement in holography. The seminar by Netta Engelhardt on Algebraic ER=EPR was key in inspiring this project.

3. Marina David, Suzanne Bintaja and Alejandra Castro on logarithmic corrections to black hole entropy in AdS spacetimes, specifically related to subtleties in the order of regularization schemes that one must do when investigating these types of corrections.

4. Sumit Das and Robert de Mello Koch on target space entanglement in the Calogero-Sutherland model.

5. Dmitry Melnikov and Vijay Balasubramanian on classification of different types of quantum entanglement types using topological quantum field theories and possible applications in AdS/CFT.

6. Masahito Yamazaki and Suresh Nampuri on Counting N=8 black hole microstates via quiver models.

7. Kyu-Hwan Lee, Gabriel Cardoso, Suresh Nampuri and Shailesh Lal on machine learning modular forms relevant to black hole counting.

8. Gabriel Cardoso, Swapna Mahapatra and Silvia Nagy on the relation of classical integrability in gravity with integrability in Yang-Mills theory through the single copy, with a preprint in its final stages of preparation.

Most notably, the fecund and fruitful atmosphere of the programme motivated some of the participants, Vijay Balasubramanian, Johanna Erdmenger and Dmitry Melnikov, to plan a 2025 Southern Hemisphere programme along similar lines in Natal, Brazil.

4.2 Research Communities

One particularly exciting aspect of the programme was that it brought together different research communities, namely researchers working in number theory, mathematical physics, string theory and quantum information, machine learning.

Another important aspect of this programme was that it had a large percentage of early career researchers.

4.3 Overall impact

An emerging relationship:
machine learning applications to number theory and string theory, as evinced by some of the INI preprints that have arisen out of work either initiated during the programme, or of which a substantial part was carried out during the programme.

To sum up, this event was definitely successful in terms of kick-starting fertile cross-pollinations across classic and emergent streams of research. Some striking examples include recently published/forthcoming papers by groups of programme participants on the application of machine learning techniques to determine integrability in gravitational systems and very exciting reformulations of automorphic forms that act as generators of the statistical microstate degeneracy of a class of supersymmetric black holes in superstring theories, in terms of quantum information theory applied to the dual holographic QFT.

5 Publications and programme legacy

5.1 Publications

Below we list 27 INI preprints that were either seeded or worked on during the programme.

1. Robert Bourne, Alejandra Castro, Jackson R. Fliss

   `Spinning up the spool: Massive spinning fields in 3d quantum gravity`, arXiv:2407.09608

2. David A. Lowe, Lárus Thorlacius

   `Post AdS/CFT`, arXiv:2407.06860

3. Luis Apolo, Suzanne Bintanja, Alejandra Castro, Diego Liska

   `The light we can see: Extracting black holes from weak Jacobi forms`, arXiv:2407.06260

4. Gabriel Lopes Cardoso, Damián Mayorga Peña, Suresh Nampuri

   `Classical integrability in the presence of a cosmological constant: analytic and machine learning results`, arXiv:2404.18247

5. Nana Cabo Bizet

   `T-dualities in gauged linear sigma models`, arXiv:2404.15025

6. M. Cristina Câmara, Gabriel Lopes Cardoso
   `Riemann-Hilbert problems, Toeplitz operators and ergosurfaces`, arXiv:2404.03373
7. Alejandro Cabo-Bizet
   `The Schwarzian from gauge theories`, arXiv:2404.01540
8. Matthias Harksen, Diego Hidalgo, Watse Sybesma, Lárus Thorlacius
   `Carroll Strings with an Extended Symmetry Algebra`, arXiv:2403.01984
9. Nana Cabo Bizet
   `Energy scales and scalar evolutions on a string model`
10. Nana Cabo Bizet
    `New string symmetries`
11. Shinji Hirano, Masaki Shigemori
    Conformal field theory on $T\bar{T}$-deformed space and correlators from dynamical coordinate transformations, arXiv:2402.08278
12. Giorgi Butbaia, Damián Mayorga Peña, Justin Tan, Per Berglund, Tristan Hübsch, Vishnu Jejjala, Challenger Mishra
    `Physical Yukawa Couplings in Heterotic String Compactifications`, arXiv:2401.15078
13. M. Médevielle, T. Mohaupt
    `T-duality across non-extremal horizons`, arXiv:2401.00296
14. Pantelis Panopoulos , Ioannis Papadimitriou
    `Supersymmetric Casimir energy on N=1 conformal supergravity backgrounds`, arXiv:2312.17740
15. Alejandro Cabo-Bizet, Marina David, Alfredo González Lezcano
    `Thermodynamics of black holes with probe D-branes`, arXiv:2312.12533
16. Roberto Emparan, Javier M. Magán
    `Tearing down spacetime with quantum disentanglement`, arXiv:2312.06803
17. Arjun Bagchi, Cynthia Keeler, Victoria Martin, Rahul Poddar
    `A generalized Selberg zeta function for flat space cosmologies`, arXiv:2312.06770
18. Yang-Hui He, Vishnu Jejjala, Challenger Mishra, Max Sharnoff
    `Learning to be Simple`, arXiv:2312.05299
19. Kaiwen Sun, Haowu Wand, Brandon Williams
    `Hyperbolization of affine Lie algebras`, arXiv:2312.03234
20. Panos Betzios, Nava Gaddam, Olga Papadoulaki
    `Black hole|wormhole transitions in two dimensional string theory`, arXiv:2312.02257
21. Wei Li
    `Chiral algebra from worldsheet`, arXiv:2312.02138

22. Dmitry Galakhov, Wei Li

    `Charging solid partitions`, arXiv:2311.02751

23. Meer Ashwinkumar, Abhiram Kidambi, Jacob M. Leedom, Masahito Yamazaki

    `Generalized Narain Theories Decoded: Discussions on Eisenstein series, Characteristics, Orbifolds, Discriminants and Ensembles in any Dimension`, arXiv:2311.00699

24. Kyu-Hwan Lee

    `Data-scientific study of Kronecker coefficients`, arXiv:2310.17906

25. Nana Geraldine Cabo Bizet, Josué Díaz-Correa, Hugo García-Compeán

    `Non-Abelian T-dualities in Two Dimensional (0,2) Gauged Linear Sigma Models`, arXiv:2310.09456

26. Sergei Alexandrov, Khalil Bendriss

    `Hypermultiplet metric and NS5-instantons`, arXiv:2309.14440

27. Robert de Mello Koch

    `Gravitational dynamics from collective field theory`, arXiv:2309.11116

5.2 Public engagement

Robert de Mello Koch, Rothschild public lecture `From Black Holes to Holographic Spacetime`, October 23.

Video interview with the organizers Robert de Mello Koch, Yang-Hui He, Gabriel Lopes Cardoso, Suresh Nampuri, Lárus Thorlacius.

Participants in the programme were encouraged to give seminars at universities in the UK.

5.3 Future activities

Building on this experience, the organisers will apply for a follow-up workshop at the Isaac Newton Institute, to ride and sustain the momentum buildup of research at the ever deepening intersection of number theory, superstring theory and quantum information theory, with the nascent but fast evolving tool of machine learning.

6 Survey

23 participants responded to an Isaac Newton Institute survey about the programme. 18 (5) indicated that the scientific quality of the programme was excellent (good), and 13 (7) indicated that the programme had a significant (partial) effect in opening up research directions for them, with 16 indicating that their participation in the programme led to new collaborations. In addition, 13 of respondents indicated that they had discovered new applications of work in this area of which they were previously unaware.

Furthermore, 5 of the respondents have indicated that they will make applications for funding for projects kickstarted or evolved during this programme, and 7 have indicated that they had given a talk/seminar at another UK institution, presenting their work, as a result of the programme. 2 participants indicated that they had begun collaborations with colleagues at the Centre for Mathematical Sciences (University of Cambridge) as a result of the programme.

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