Cluster Algebra and Representation Theory Programme

Isaac Newton Institute, 6 September - 17 December 2021

Final Report, January 2022

Summary: The 3-month programme on Cluster Algebras and Representation Theory at the Isaac Newton Institute in Cambridge has been a very stimulating research experience for the physical and virtual participants involved. It has also brought on major advances in the field and uncovered many new and sometimes unexpected connections with other areas such as mathematical physics and geometry. The programme has fostered and produced many new collaborations and it was particularly successful in the integration of early career researchers into major research collaborations. As we are expecting many exciting new developments initiated during the programme to come to fruition over the coming years, we have already planned a series of follow up workshops and events to capitalise on the work and keep the momentum going. Besides the follow up workshop at the Isaac Newton Institute which we are planning for 2023, these include an application for an Oberwolfach meeting in 2024 and the possibility of a further semester programme in 2026 or 2027.

Organizers: Karin Baur (Leeds, UK), Bethany Marsh (Leeds, UK), Ralf Schiffler (Connecticut, US), Sibylle Schroll (Cologne, Germany)

I. Background and context

The theory of cluster algebras is without doubt one of the most active research areas in Mathematics over the last 20 years. Introduced by Fomin and Zelevinsky in 2002 in the context of Lie theory and total positivity, cluster algebras quickly developed deep connections to many different disciplines including quiver representations, tilting theory, Poisson geometry, Teichmüller theory, combinatorics, Coxeter groups, algebraic geometry and string theory. Among the major applications of cluster algebras one finds the construction of large parts of Lusztig's dual semicanonical basis in Lie theory, the discovery of generalised associahedra in combinatorics, the development of cluster-tilting theory in representation theory of finite dimensional algebras, the proof of Zamolodchikov's periodicity conjecture in conformal field theory, the development of a new approach to Teichmüller theory and the discovery of a fundamental relation to Donaldson-Thomas invariants and wall-crossing in algebraic geometry. The idea of the programme was on the one side to take stock of the developments and on the other hand to explore many of the emerging new connections and drive the field forward.

The primary focus of the programme was on the structural theory of cluster algebras, representation theoretic cluster theory, and connections to other areas of mathematics and physics. The main focal points were open problems in frieze patterns in cluster algebras and representation theory, categorification of cluster algebras with frozen variables, picture groups, stability conditions and scattering diagrams, cluster complexes, tau-tilting theory, cluster structures on Grassmannians, canonical bases for cluster algebras, mutation types of quivers with frozen vertices, Poisson cluster structures, categorification of cluster structures on orbifolds, connections between cluster algebras, representation theory and mathematical physics, categorification of surface phenomena such as cuts, contractions and boundaries in terms of associated categories related to cluster algebras.

Our aim was to create a strong community of participants. To achieve this, the programme consisted of several regular activities, interspersed with a research school at the beginning of the programme, in September, followed immediately by a scientific workshop, and a second scientific workshop in November (see below for a description). The activities included two weekly research seminars where participants (online and in person) presented their work. The weekly open problem session was a place for participants to present work in progress and find new perspectives from the broad group of participants. Both the Kirk and Rothschild fellows gave public talks and complemented these with a series of lectures. This set-up provided a fruitful mixture between regular activities, input through lecture series and research talks and space and time for discussion. This made it possible for the participating experts to make progress on several of the open problems. It also resulted in work on new questions and topics, often by new teams of collaborators: the research talks brought new development in the field to the table and the participants (online and in person) took these up.

In terms of organisation of the daily programme, the decision to involve early career researchers in running the regular activities gave them the opportunity to play an important role in the programme and created a very lively atmosphere. The weekly dinner after the seminar and the social hour were additional highlights.

II. Timeliness and scope

We addressed topics in and connections with representation theory, geometry, combinatorics and mathematical physics. Several of these topics lie in two or more research areas. Examples of topics we have addressed are: scattering diagrams, bases, g-vectors, Poisson-cluster structures, picture groups, frieze patterns, knots and cluster algebras, surfaces and cluster structures, surfaces and categories, perfect matchings, reduction techniques on cluster categories, Grassmannian cluster structures, oriented networks, orbifolds and generalized cluster structures, orbifolds and skew-group categories tau-tilting theory, cluster complexes and deformation, lattices of t-structures, gentle algebras, recollements, categorification of generalised cluster structures, BPS quivers, extriangulated categories, exceptional sequences, higher homological algebra, associahedra, toric and cluster varieties, syzygy categories, and super cluster algebras.

II.1 Research school and scientific workshops

The programme included a research school and two workshops. Because of the ongoing pandemic all three events took place in a hybrid format with both online and physical participants.

The research school, “New developments in representation theory arising from cluster algebras research school,” at the start of the programme introduced some of the central topics of the field and closely adjacent fields, and acted as an introduction to the later workshops and the programme in general. The following lecture courses were given:

- Stability conditions and silting (D. Pauksztello);
- Dimer models (C. Beil);
- Matrix factorisations (E. Faber);
- Extriangulated categories (Y. Palu);
- Grassmannians and amplituhedra (L. Williams)

A scientific workshop, “New developments in representation theory arising from cluster algebras”, directly followed on from the research school. The idea of the workshop was based on the fact that it was noticed early on in the development of the field that cluster algebras could be successfully categorified using the representation theory of finite dimensional algebras. Furthermore, the categorification of cluster algebras had introduced many innovative ideas to representation theory, giving rise to new areas of study that today are central to the theory, such as higher homological algebra, Calabi-Yau categories and τ-tilting theory. The aim of the workshop was to share the latest developments in these cluster-inspired areas of research and discuss their future development.

Talks covered themes from the module categories of finite dimensional algebras, such as gentle algebras, skew gentle algebras and torsion theory, as well as homological algebra more generally (often originally motivated by cluster theory): higher homological algebra, derived categories, extriangulated categories, tilting theory and the new silting theory. This included talks on the categorification of cluster algebras and cluster characters, which remains a big theme in the subject (and, in particular, was also the topic of Bernard Keller’s series of talks as Rothschild Distinguished Visiting Fellow), and the study of algebras arising from this theory, and their properties.

Other talks focussed on the theory of cluster algebras per se (although often influenced by representation-theoretic methods), including generating sets (motivated by representation theory) and cluster combinatorics and generalised cluster algebras. Talks also covered geometric models (often involving surface combinatorics), which play an important role in studying and visualising a cluster algebra and understanding its associated representation theory.

There were talks on interactions with and applications to other areas such as invariants in knot theory and noncommutative singularity theory.

The theme of the second scientific workshop, “Interdisciplinary applications of cluster algebras”, was based on the deep connections and applications of cluster algebras to a wide variety of research areas, including representation theory, combinatorics, algebraic, hyperbolic and symplectic geometry, dynamical systems, number theory, topology and mathematical physics. The aim of the workshop was to draw out these deep connections and applications, both within mathematics and in an interdisciplinary context. It brought together mathematicians and physicists in a variety of different areas relating to cluster algebras, such as those above and other fields, to provide an opportunity for exchanging ideas and stimulating further progress.

A collection of talks in the second workshop brought across strong connections between mathematical physics, cluster algebras and representation theory and combinatorial geometry arising from the theory of scattering amplitudes. These connections are driving developments both in mathematical physics and in pure mathematics and this area shows strong potential for future work.

Another strong theme in this workshop was that of geometric applications and interactions, which arose in various ways, including non-Archimedean geometry, dimer models on surfaces and orbifolds, and related representation theory and combinatorics. The combinatorial aspects included a key notion in the theory of cluster algebras: perfect matchings. Cluster algebras can be regarded as coordinate rings of Teichmüller spaces in a certain sense, and this theme was well-represented in the workshop, including talks on the Fock-Goncharov cluster structure and on harmonic maps.

Fans, which are collections of cones covering Euclidean space with good properties, and related polytopes, arise both in cluster theory and related representation theory (as well as the mathematical physics connections mentioned above), for example via the notion of g-vectors (which can be regarded as a certain choice of coordinates in both cases), and this theme was represented in the workshop. Geometric methods play a role in the study of cluster algebras per se, particularly in the theory of bases, and this topic was also represented in the workshop, as was the classification of cluster algebras of finite mutation type (developments in the case with frozen variables, which is an important area for future development generally).

Other geometric themes were mirror symmetry, including theta functions, an area where the role of cluster algebras is developing quickly, and Lie theory, one of the originating areas of cluster algebras and key source of examples. Applications to knot theory also appeared in this workshop, with work presented on braid groups and Legendrian knots and on the relationship between representation theory and knot theory via cluster algebras. One talk introduced the notion of q-analogues of real numbers and matrices, with a strong relationship to cluster theory and knot theory.

Representation theory was a topic in many talks; in some cases it was the main focus, such as in a talk on the notion of Bongartz completion, which plays an important role in tau-tilting reduction.

Frieze patterns have become an important part of cluster theory in recent years, and this theme was represented in a talk on Heronian friezes, showing how distance measurements in Euclidean space could be used to define more general kinds of friezes. A talk on picture groups acted as a good introduction to the series of talks from Gordana Todorov (Kirk Distinguished Visiting Fellow) on the topic, bringing together ideas from combinatorial geometry, representation theory and topology.

III. Scientific outcomes

III.1 Scientific advancement

This programme advanced research in the field significantly. Programme participants were invited from a diverse collection of interacting fields related to cluster theory. To facilitate interactions and enable progress, a mixture of regular activities and research time was programmed, with researchers able to discuss their projects in the common areas and offices. Important stimulating activities were the three research workshops, regular research seminars, and the open problem sessions, which all provided opportunities for starting new collaborations and exchanging feedback

from different points of view. Early career researchers were strongly involved and were given roles to encourage their participation. We give details of scientific advancement in the following list.

1. Frieze patterns of integers, which are arrays of integers in an infinite strip in the plane satisfying a combinatorial rule, were introduce d by Coxeter and Conway in the 1970s, and have become the subject of intense study after a close connection with cluster algebras was discovered by Caldero and Chapoton. Classically, friezes are associated to disks. However, a disk can be regarded as a surface with boundary, so a natural question is how to generalise the construction of friezes to more general surfaces. Ana Garcia Elsener, Anna Felikson, Ilke Canakci and Pavel Tumarkin started a new collaboration during the programme associating friezes to fundamental geometric spaces known as 'pairs of pants' and using techniques from hyperbolic geometry to study them. They completed this work during the programme.

2. Anna Felikson and Pavel Tumarkin completed their classification of cluster algebras of finite mutation type with coefficients. The case without coefficients had already been classified in their work with Michael Shapiro in 2012.

3. Oleg Karpenkov, Khrystyna Serhiyenko and Pavel Tumarkin started a new collaboration on SL_n tilings, which are friezes distributed across the whole plane, and on the three-dimensional Farey graph, which is a three-dimensional version of a beautiful classical combinatorial construction reflecting the properties of and relationships between rational numbers.

4. The associahedron is a geometric object, or polytope, which reflects aspects of the associativity property. For example, any bracketing of a sum of real numbers gives the same answer. There are two possible bracketings of three real numbers, and the fact that they are the same can be represented by a line. There are five bracketings of four real numbers, and the fact that they are the same can be represented by a pentagon. For five real numbers, the corresponding associahedron is a three-dimensional polyhedron with 14 vertices corresponding to the 14 bracketings. The theory of cluster algebras led to the introduction of generalised associahedra which can be associated to crystallographic reflection groups, classical groups which categorise fundamental geometric symmetries. Ana Garcia Elsener, Pavel Tumarkin and Emine Yildirim started a new collaboration on polytopal realisations of generalised associahedra for non-crystallographic reflection groups.

5. Noncrossing is a fundamental notion that plays a key role in cluster algebra theory. At heart, it just expresses the property that two geometric objects do not intersect. In the definition of a cluster algebra, the generators of the algebras are organised into equal-sized overlapping subsets which can be regarded as 'noncrossing' in a generalised sense. A new collaboration was started up between Ana Garcia Elsener and Sira Gratz on noncrossing theory for arbitrary surfaces.

6. Emily Gunawan and Raquel Coelho Simoes started a new collaboration with Emily Barnard and Ralf Schiffler combining different approaches in the representation theory of so-called gentle algebras. This work aims at understanding a specific type of representation known as maximal almost rigid representations, as a triangulation of an associated surface with boundary. In a related project, Gunawan and Schiffler have started working with Thomas

Brüstle on an interpretation of these representations in a category with a weaker exact structure.

7. One way of measuring a category appearing in representation theory is to describe its so-called exceptional sequences, certain sequences of objects in the category satisfying certain properties in terms of vanishing of homological spaces between them. However, the classical theory only worked well for module categories of hereditary algebras. Signed tau-exceptional sequences were recently introduced in work of Igusa-Todorov and Buan-Marsh to remedy this problem. Bethany Marsh and Aslak Bakke Buan developed an existing collaboration on the properties of tau-exceptional sequences during the programme.

8. A fundamental link between representation theory and cluster algebras is the use of categories in representation theory to model cluster algebras. This link is fruitful in both directions, giving an extra dimension to cluster theory while at the same time giving rise to new representation-theoretic ideas. Classically, categorifications were constructed using triangulated categories, but cluster algebras with frozen variables, which cannot be changed, arise in many applications, and cannot be categorified in this way. The solution is to use Frobenius categories (either exact or ex-triangulated). Bernard Keller gave an extensive series of talks on recent developments in this area, including work of his PhD student Yilin Wu, who also gave a seminar towards the end of the Programme, showing that such categorifications exist in a very general context. Eleonore Faber, Bethany Marsh and Matthew Pressland developed an existing collaboration during the programme on the relationships between different Frobenius categorifications.

9. Karin Baur and Bethany Marsh made good developments in their continuing project on the geometric and combinatorial properties of Grassmannians and their cluster algebra structure.

10. While preparing her talk on the Homology of Picture Groups, Gordana Todorov developed some surprising new ideas with Kyoshi Igusa and Orr on Massey products for quivers of type A. Massey products appear in algebraic topology, so this work builds stronger bridges between representation theory and algebraic topology. Gordana Todorov also reported on comments received during her series of talks being helpful for her future work, particularly on modulated quivers and pro-p-groups and pro-pictures. These are geometric representations of module categories which arise when limits are considered.

11. An important class of algebras in this field are the cluster-tilted algebras, or, more generally, the 2-Calabi-Yau tilted algebras, whose representation theory captures the combinatorial structure of the cluster algebra. For many of these algebras the representation theory is well understood. However, in general, it is very hard to get a complete grasp on the category of representations. It is then useful to restrict to a certain type of representation. One such type is given by the syzygies of the algebra, which, by definition, are built from subrepresentations of projective representations. These syzygies are the subject of an already active collaboration between Khrystyna Serhiyenko and Ralf Schiffler. Their results were considerably enhanced during the programme due to new developments following discussions in the first scientific workshop. The authors revamped their first paper and completed a second on the subject during the programme.

12. A novel relationship between cluster algebras and the deformation theory of Stanley-Reisner rings was established in a preprint by Nathan Ilten, Alfredo Nájera Chávez, and Hipolito Treffinger. One of the highlights is that the cluster complexes of finite (skew-symmetric) type are unobstructed. The authors consider this work a first step towards a systematic study of this relation, in particular, first order deformations, obstructions, and semi-universal deformations.

13. A recent development in cluster algebras and representation theory is a connection to knot theory developed by Véronique Bazier-Matte and Ralf Schiffler. To every knot diagram, they associate a quiver whose vertices correspond to the segments of the diagram. For each such vertex, they construct a representation of the quiver such that the Alexander polynomial of the knot is recovered from the associated element of the cluster algebra. The authors completed their first paper on the subject during the programme and made good progress on a follow-up paper.

14. Karin Baur and Ralf Schiffler made good progress in their ongoing project on developing combinatorial formulae for the cluster variables in cluster algebras of finite exceptional types.

15. Misha Gekhtman, Michael Shapiro and Alek Vainshtein completed an important component of their long-term research programme on the classification of cluster algebra structures in the rings of regular functions on the general linear group that are compatible with Poisson-Lie and Poisson-homogeneous structures. The Poisson structures were completely classified by Belavin and Drinfeld in 1982 and Gekhtman, Shapiro and Vainshtein conjectured in 2012 that each Belavin-Drinfeld Poisson structure admits a (generalised) cluster structure. During the programme, they proved that their conjecture holds for a large class of Poisson structures, the so-called oriented aperiodic structures, which are honest (non-generalized) cluster structures.

16. Fan Qin proved the equality of two important bases for cluster algebras, the bracelet basis introduced by Musiker, Schiffler and Williams in 2013 and the theta function bases of Gross, Hacking, Keel and Kontsevich in 2018. He presented his results in the second workshop of the programme.

17. Karin Baur and Misha Gekhtman made progress in their project on oriented networks on annuli, establishing explicit formulae for a new operation on them and initiated discussions with Misha Shapiro on related networks for asymptotic triangulations of annuli.

18. Karin Baur, Léa Bittmann, Emily Gunawan, Maria Ross, Gordana Todorov and Emine Yildirim started a collaboration on tame infinite friezes and established initial results pointing to a connection between the different affine types.

19. Karin Baur, Chris Fraser and Misha Gekhtman initiated a project on linking generalised cluster algebras to the skew cluster categories arising from the orbifold diagrams of the work of Baur-Pasquali-Velasco.

20. Mandy Cheung, Elizabeth Kelley and Gregg Musiker completed their work on cluster scattering diagrams and theta functions for reciprocal generalised cluster algebras.

21. Wen Chang, Haibo Jin and Sibylle Schroll made good progress on making a connection between recollements of derived categories of graded gentle algebras and cut and contract actions of the surfaces in the associated surface models.

22. Johanne Haugland, Karin Jacobsen and Sibylle Schroll completed their work on the role of gentle algebras in higher homological algebra. Higher homological algebra is born out of the representation theory of finite dimensional algebras arising from cluster theory.

23. Lang Mou and Sibylle Schroll initiated a collaboration on a representation theoretic approach to certain subcategories of the derived category of the Ginzburg differential-graded algebra associated to a quiver with potential coming from a surface triangulation arising in 4d N=2 gauge theories.

24. Lang Mou created scattering diagrams for generalised cluster algebras, relating these to theta functions. This generalised the existing cluster scattering diagrams of Gross, Hacking, Keel and Kontsevich.

III.2 Research Communities

One particularly exciting aspect of the programme was that it brought together different research communities. In particular, the participation of physicists was very valuable and allowed a unique opportunity for deep discussions over several weeks enabling a better understanding of the different perspectives and also the different languages in the different areas. The close connection of cluster algebras and ideas from mathematical physics, such as elementary particle scattering processes and brane webs in gauge theories, as well as connections of cluster algebras and other areas of mathematics such as K-theory (via picture groups) was also highlighted in the second workshop.

Another important aspect of this programme was that it had a large percentage of early career researchers. Almost all of the early career researchers were involved in the organisational aspects during the programme such as organising the local seminars, the problem session, the social events and also in the co-organisation of the workshops. This seems to have resulted in a very inclusive atmosphere with exciting new collaborations between the different participants in the programme. Furthermore, an open policy on ad hoc invitations during the programme brought new aspects to the programme. This was further driven by the agenda to have everybody who was visiting the institute give a talk on their research.

A strength of the programme was the way it brought together early career researchers with established researchers. New collaborations (from the list above) of this type were initiated during the programme as follows:
1. Ana Garcia Elsener, Anna Felikson, Ilke Canakci and Pavel Tumarkin;
2. Oleg Karpenkov, Khrystyna Serhiyenko and Pavel Tumarkin;
3. Ana Garcia Elsener, Pavel Tumarkin and Emine Yildirim;
4. Ana Garcia Elsener and Sira Gratz;
5. Emily Barnard, Raquel Coelho Simoes, Emily Gunawan, and Ralf Schiffler; Also Thomas Brüstle, Emily Gunawan and Ralf Schiffler;
17. Karin Baur, Léa Bittmann, Emily Gunawan, Maria Ross, Gordana Todorov and Emine Yildirim.

III.3 Impact on other research disciplines

As discussed above, there is a strong relationship between cluster algebras, algebraic geometry, representation theory and scattering amplitudes in mathematical physics which was well-represented in the programme, including talks in the November research workshop. There is a good potential for impact on the theory of scattering amplitudes originating from developments in and interactions and perspectives from cluster theory. On the physics side, the work is motivated by understanding and predicting the scattering amplitudes in collisions of elementary particles. It presents a new model that simplifies computations considerably and thereby takes the feasibility of such calculations to a new level. On the mathematics side, this new model corresponds to polytopes associated to cluster algebras of a certain type. Generalising to other types of cluster algebras may correspond to solving more complicated scattering models in physics.

III.4 Future investigations

Some areas where we see potential development in the future include:

The connection between cluster algebras and representation theory has a strong momentum and there are new ideas and insights on both sides, as has been seen during the programme. We expect this relationship to continue to be very fruitful in the future, in terms of representation-theoretic and homological models of cluster algebras and new ideas in cluster theory and on the representation-theoretic side. N-angulated, extriangulated, and n-exangulated categories are also a new area of development, bringing together the theories of exact categories and triangulated categories, and bringing in new higher homological theories, and strong development is expected here.

The relationship between cluster algebras, algebraic geometry, representation theory and scattering amplitudes in mathematical physics. These ideas manifest on the mathematical side in different ways, including the geometric structure of the amplituhedron, surfacehedra and new ideas in representation theory, and there is the potential for applications in either direction.

Another strong potential area for development is the theory of friezes and related surface tilings, where there are strong interactions between cluster theory, Grassmannian varieties, representation theory, and the combinatorial geometry of surfaces. This is part of a more general strong connection between cluster algebras and surfaces which is not yet well understood, which also relates strongly to representation theory.

Picture groups bring together ideas from algebraic topology, group presentations, cluster algebras, combinatorial geometry and representation theory. In particular, they provide a new way of looking at classical representation-theoretic objects, and open up new potential areas of investigation.

Another potential area for application and future development is the relationship between knot theory and cluster algebras. There is a successful history of the use of algebraic and combinatorial methods in the study of knots (eg knot invariants, Jones polynomial, Alexander polynomial) and the potential for the use of cluster algebras in this area has only just started being realised.

Tau-tilting theory, which was originally motivated by cluster theory, is a thriving area of representation theory and allows for ideas which were traditionally focussed on hereditary algebras only to be

generalised to arbitrary finite dimensional algebras. This, together with the strong interactions with cluster algebra theory, indicates strong future development.

There is a good potential for the development of interactions between ring theory and cluster algebras, for example via Stanley-Reisner rings, the theories of Gorenstein rings and Cohen-Macauley modules and the ring-theoretic properties of cluster algebras themselves.

Cluster algebras are closely related to integrable systems (and especially discrete integrable systems), with a strong relationship with Poisson structures, periodic properties of cluster algebras and the Laurent phenomenon. This is an area with a good potential for future development and application, with interactions feeding back into cluster theory insights.

In addition, each item in the list in Section III.1, Scientific advancement, indicates a direction for potential development in the future.

IV. Equality, diversity and inclusion

The programme was organised with equality, diversity and inclusion at the forefront, with particular emphasis on ensuring a diverse collection of participants and seminar speakers. The aim was, in addition, to ensure that every physical participant gave a seminar talk (either in the regular seminar or in the workshops) during their visit.

A panel session on Life decisions in mathematics was held, organised by members of the Cluster Algebras and Representation Theory programme and the Mathematics of Deep Learning programme and Christie Marr, the deputy director of the Isaac Newton Institute. Christie chaired the session, which had a diverse panel consisting of three members from each programme and covered the difficulties of making decisions during a mathematical career, taking into account personal circumstances, as well as aspects of creating a strong community of researchers in mathematics. The panel session was aimed at everyone, but with the aim of being particularly useful for Early Career Researchers, and attracted around 25 in-person participants and 40 online participants.

One of the organisers, a trans woman, was interviewed by staff at the Isaac Newton Institute about her experiences as a trans mathematician and her mathematical career (see IV.3, Public engagement, below).

V. Publications, public engagement and future plans

V.1 Planned books

Gordana Todorov (Kirk fellow) has used input and feedback during her lecture series to finalise plans for a book with co-authors Kiyoshi Igusa, Kent Orr and Jerzy Weyman on *Modulated quivers, semi invariant pictures and picture groups* which is accepted for publication by Cambridge University Press.

V.2 Publications and preprints

Ilke Canakci, Anna Felikson, Ana Garcia Elsener, Pavel Tumarkin, Friezes of a pair of pants, arXiv:2111.13135.

Nathan Ilten, Alfredo Nájera Chávez, Hipolito Treffinger, Deformation Theory for Finite Cluster Complexes, arXiv:2111.02566.

Véronique Bazier-Matte, Ralf Schiffler, Knot theory and cluster algebras, arXiv:2110.14740.

Anna Felikson, Pavel Tumarkin, Cluster algebras of finite mutation type with coefficients, arXiv:2110.12917.

Lauren Williams, The positive Grassmannian, the amplituhedron, and cluster algebras, (survey) arXiv:2110.10856.

Ralf Schiffler, Khrystyna Serhiyenko, A geometric model for syzygies over 2-Calabi-Yau tilted algebras II, arXiv:2110.09976.

Sira Gratz, Alexandra Zvonareva, Lattices of t-structures and thick subcategories for discrete cluster categories, arXiv:2110.08606.

Man-Wai Mandy Cheung, Elizabeth Kelley, Gregg Musiker, Cluster scattering diagrams and theta functions for reciprocal generalized cluster algebras, arXiv:2110.08157.

Lang Mou, Scattering diagrams for generalized cluster algebras, arXiv:2110.02416.

Yilin Wu, Categorification of ice quiver mutation, arXiv:2109.04503.

Yilin Wu, Relative cluster categories, arXiv:2109.03707.

Costa da Rocha, Joel, Bicolored tilings and the Scott map, arXiv:2112.08007.

Johanne Haugland, Karin M. Jacobsen, Sibylle Schroll, The role of gentle algebras in higher homological algebra, arXiv:2107.01045.

Emily Gunawan, Ben Drucker, Eli Garcia, Aubrey Rumbolt, Rose Silver, RSK tableaux and box-ball systems, arXiv:2112.03780.

V.3 Public engagement

Bernard Keller, Rothschild public lecture, ‘Cluster Algebras and Representation Theory’.

Gordana Todorov, Kirk public lecture, ‘Cluster algebras and many related notions on one picture’.

Video interview with the organizers Karin Baur, Bethany Marsh, Sibylle Schroll, and Ralf Schiffler https://downloads.sms.cam.ac.uk/3653632/3653640.mp4

Podcast interview with Bethany Marsh on her life as a transgender mathematician: https://www.newton.ac.uk/media/podcasts/post/33-becoming-bethany-marsh-life-as-a-transgender-professor-of-mathematics/

Podcast interview with Emine Yildirim, Simons postdoctoral fellow:
https://www.newton.ac.uk/media/podcasts/post/meet-the-simons-postdoctoral-fellows-mini-series-1-dr-emine-yildirim/

Podcast interview with the organizers Karin Baur, Bethany Marsh, Sibylle Schroll, and Ralf Schiffler:
https://www.newton.ac.uk/media/podcasts/post/35-discussing-the-cluster-algebras-and-representation-theory-programme/

V.4 Plans for future activities and grant applications

The organisers will apply for a follow-up workshop at the Isaac Newton Institute. They are also planning to apply for a workshop at the Oberwolfach Mathematical Research Institute in Germany. In the long term, they plan to organise another semester-long programme.

VI. Survey

A total of 24 participants (out of 64 who participated in the programme, including virtual participants) responded to an Isaac Newton Institute survey at the end of the programme. All respondents indicated that the scientific quality of the programme was excellent, and 83% indicated that the programme had a significant effect in opening up research directions for them, with 65% indicating that their participation in the programme led to new collaborations. In addition, 54% of respondents indicated that they had discovered new applications of work in this area of which they were previously unaware (an area that was mentioned several times was theoretical physics). Furthermore, 30% of respondents indicated that they will make applications for funding as a result of research in this programme, and 31% indicated that they had given a talk at another UK institution as a result of the programme. Three participants indicated that they had begun collaborations with mathematicians at the Centre for Mathematical Sciences (University of Cambridge) as a result of the programme.