FINAL REPORT ON THE RESEARCH PROGRAMME

Free Boundary Problems and Related Topics

Isaac Newton Institute for Mathematical Sciences

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Organisers: Gui-Qiang G. Chen , Henrik Shahgholian, Juan Luis Vázquez

Free boundary problems (FBPs) are boundary-value problems for partial differential equations (PDEs) which are defined in a domain, a part of whose boundary is a priori unknown; this part is accordingly named a free boundary.They arise typically in phase separation problems, which can be either stationary or moving free boundaries. The mathematical problem is then to determine both the location of the free boundary and the solution of the PDE(s) in the resulting domain(s). FBPs are one of the most important directions in the analysis of PDEs, with an abundance of applications across the sciences and real world problems.

A mathematical community has been forming over the last five decades with tight connections, and a series of major events have kept the focus on this frontier theme which lies between the analysis of PDEs and the geometric study of evolving surfaces, which separate the phases where the state equations hold. In the last two decades, various new ideas, techniques, and methods have been developed, and new important, challenging problems in physics, engineering, industry, finance, biology, and other areas have arisen. The Isaac Newton Institute programme (in Spring 2014) was aimed at keeping the intellectual connection, fostering the innovation and serving as a training ground for the younger participants.

The topics of this programme are at the forefront of current exciting developments. We realised that there had been few activities on FBPs in recent decades in the UK. We strongly believe that the thematic programme at the Newton Institute has made a significant impact on the development of FBPs from all points of view. The programme was directed towards theory (mainly Analysis of PDEs, Calculus of Variations, Conservation Laws, and Differential Geometry), numerics (from industrial computation to the analysis of convergence and stability of numerical methods), and applications (a wide coverage of different areas from fluid mechanics to biology/medicine to finance). It was devised to further enhance the interaction between UK mathematicians and top international researchers in this field and to strengthen the UK expertise in FBPs.

The study of FBPs is an extremely broad topic due to the abundance of applications. This breadth presents challenges and opportunities! Many problems treated by applied scientists and numerical analysts are not well known amongst theoretical people, and vice versa. This has led to the development of the field in different directions, sometimes with no or few links between them. However, very often, the underlying ideas with respect to methodologies and techniques are similar or interrelated, which can be coherently unified. The aim of programme was to enhance those links and develop unifying techniques and methods, by bringing together the relevant specialists, most of whom spend extended periods at the Institute.

The programme activities included an introductory school, two satellite workshops, and an international conference. The Introductory School on FBPs and Related Topics held in January, which aimed at providing an introductory background on several current topics of research in FBPs and set the framework for much of what was to come. Five leading experts in the field offered short courses, with main target at young researchers. These short courses cover the topics on the regularity of free boundaries in obstacle-type problems, the derivation of FBM models for tumor growth, numerical methods for FBS, geometric approaches to water waves and free surface flows, and problems involving long-distance interactions modeled by fractional Laplacians.

The first satellite workshop on Free Boundaries and Moving Interfaces at the University of Oxford emphasizes the interaction between the FB community and the community of

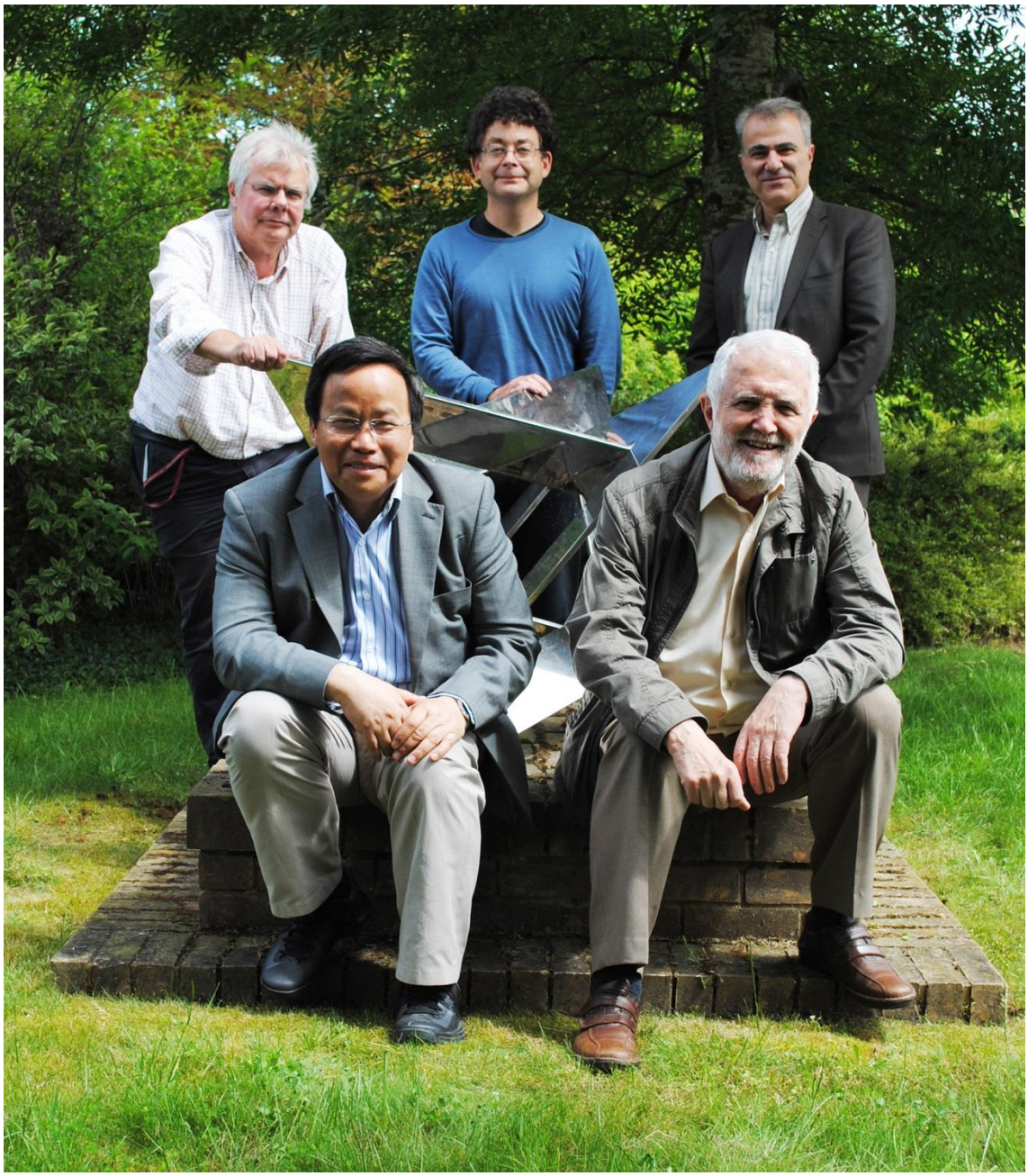
researchers working in fluid flow equations, such as the Euler, Stokes, and Navier-Stokes equations, as well as Hele-Shaw flows**,** quasi-geostrophic and Muskat problems. The second satellite workshop on Recent Developments and Challenges in Interface and Free Boundary Problems held at the University of Warwick with focus on some recent developments and challenges on new important problems arising in physics, engineering, industry, finance, biology, and other areas. In particular, numerical methods have to be specially developed to capture the evolution of interfaces, a major challenge in computational mathematics. These satellite workshops have allowed for a large body of updated information to be disseminated to scientists across the UK. They have also facilitated many new contacts and collaborations with researchers in related fields, a very valuable asset in our interdisciplinary work.

The International Conference on FBPs, Theory and Applications was held on 23–27 June 2014, organised by Gui-Qiang Chen, Charlie Elliott, and John King. It was a major event in the series that has been the central link of the mathematical FB community in the last decades, and was structured to assist in identifying the common challenges that have the greatest potential for research, knowledge transfer, public policy and commercial impact. In addition, the Turing Gateway Event brought together leading experts in multiple disciplines across biology and medicine which require mathematical modelling and computation involving FBs, interfaces and related concepts.

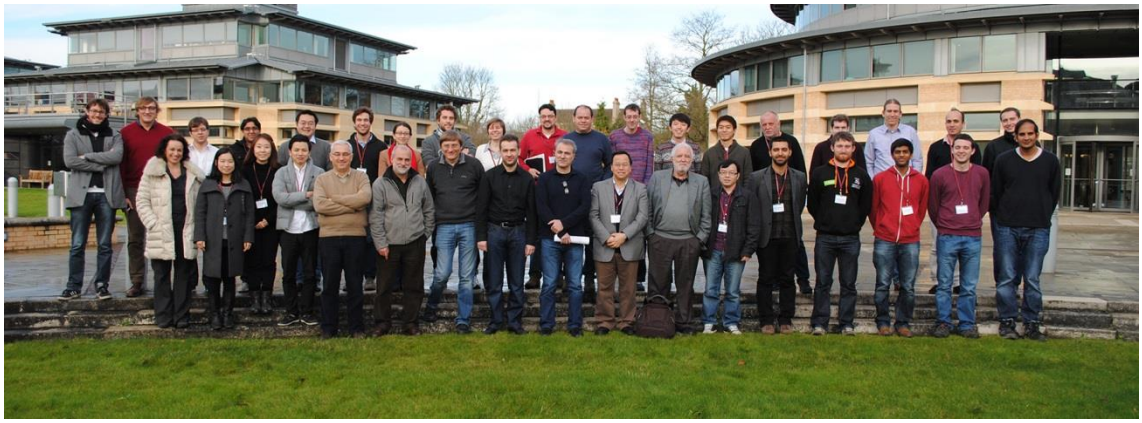
The programme's success was due to the presence at the Institute of a number of senior recognized researchers for long periods on one hand, while on the other hand, shorter visits by prominent figures allowed to enlarge the scope and quality of the information and expertise that were being shared. This also applies to the younger participants who had a combination of long and short stays. In that case, the usefulness of discussions and collaborations was greatly emphasized in the shorter stays. A Regular Weekly Seminar, led by Dorin Bucur and Donatella Danielli, was complemented by a Lunchtime Seminar led by Juan Luis Vázquez for the discussion of ongoing or recent work with a view to enhance free discussion and interaction, and by an Introductory Seminar led by Gui-Qiang Chen and Henrik Shahgholian for the introduction to some of the most central topics and new emerging directions in the field of FPBs. This novel organisational idea was initiated in a previous programme run at MSRI, Berkeley, in Spring 2011. The informal discussion sessions/seminars, discussion groups, and individual interactions are another important part to make the success of the programme.

A highlight point of the Programme was the presence of Rothschild Distinguished Visiting Fellow, Professor Luis Caffarelli (University of Texas-Austin), a major figure in the field of FBPs, more generally in PDEs and their geometrical analysis. In addition, the long list of participants has included many other prominent figures in pure and applied mathematics related to the field, who had a certain influence on the participants and the entire programme.

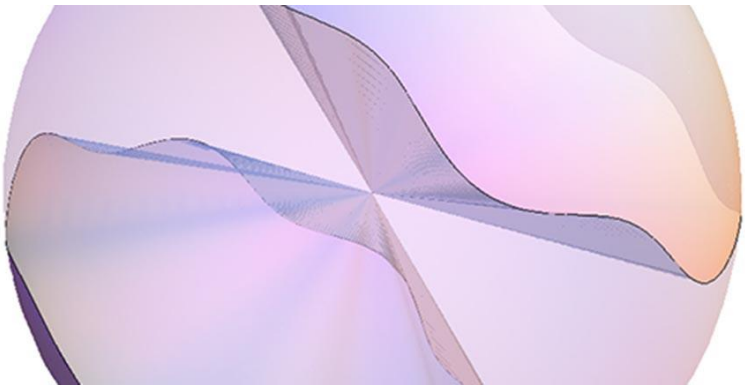
As a result of the programme, we have seen the emergence of significant results and new collaborations, and the development of new research projects that will shape the field in the near and far future. Together with the frequent viewing of streamed lectures, either in real time or later, these activities allow colleagues around the globe, unable to attend in person, were able both to contribute greatly to the scope of the Programme and to benefit from it. The archive of videos of lectures from the Programme will form an important source of material for researchers in the field which will extend the impact of the Programme into the future. However, it is hoped to achieve this in a more proactive way by editing a Special Theme Issue at the Philosophical Transactions of the Royal Society A, which consists of more than 10 survey articles, by leading experts, including open problems and recent developments/trends on FBPs and related topics, based initially on those identified during the Programme. In addition, associated with programme, there will be a Special Issue of the EMS Journal Interfaces and Free Boundaries, which consists of the papers for the new results obtained by the programme participants, besides those submitted to the INI Preprint Series.

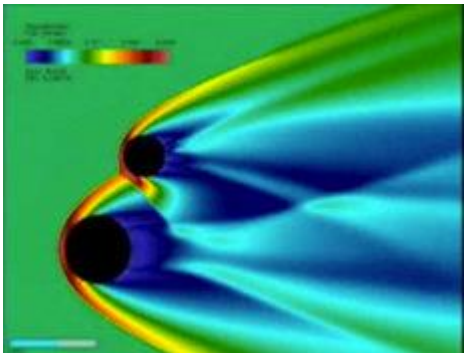


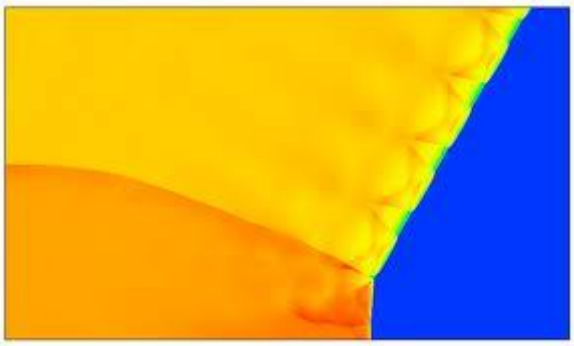
The Programme Organisers and Free Boundary Conference Organisers:
Professors Gui-Qiang G. Chen, Charlie Elliott, John King, Henrik Shahgholian, Juan Luis Vázquez

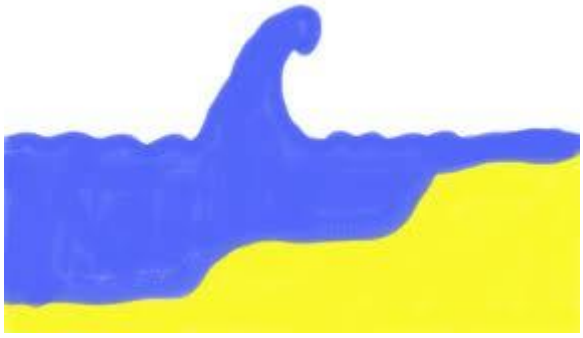


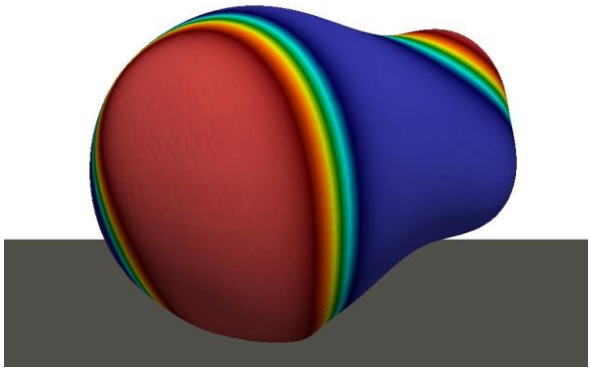
Participants in the Introductory School

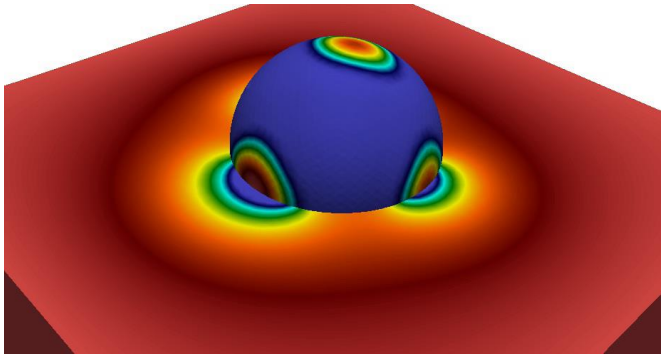


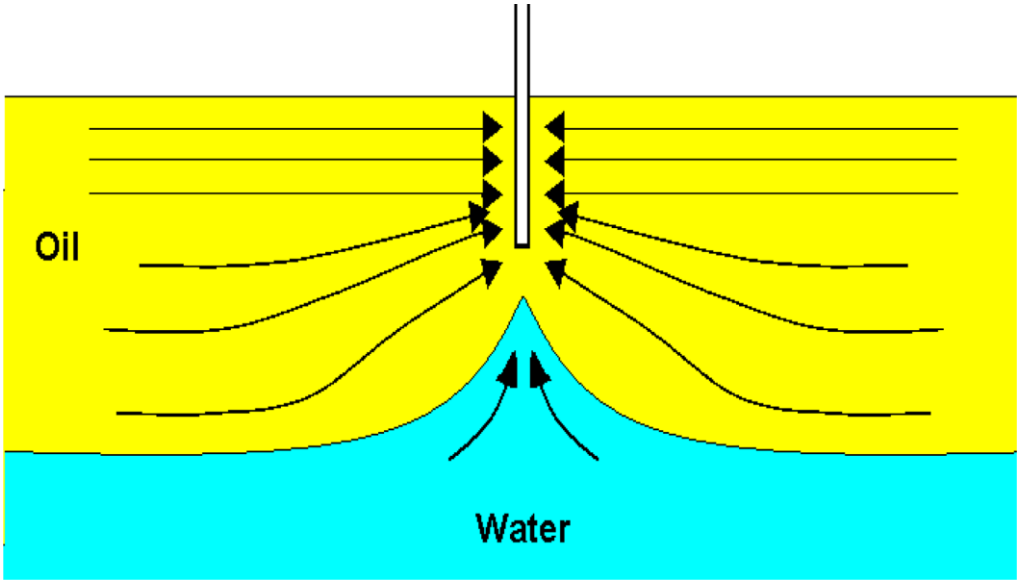











Oil
Water