FINAL REPORT

Grothendieck-Teichmüller Groups, Deformation and Operads [GDO programme]

January-April 2013

Organisers: Herbert Gangl *(Durham)*, John Jones *(Warwick)*, Pierre Lochak *(Paris VI)*, Bruno Vallette *(Nice)* and Nick Woodhouse *(CMI)*.

Programme Partner: Clay Mathematics Institute.

State of the art

The topics of the present programme lie at the crossroad of current research in Algebra, Geometry, Topology and Mathematical Physics.

Grothendieck-Teichmüller theory goes back to A. Grothendieck's celebrated Esquisse d'un programme. In 1991, V. Drinfeld related it to number theory and to deformation theory. On the other hand, the notion of an operad, which was introduced in Algebraic Topology over half a century ago, has enjoyed a renaissance in the 90's notably in the works of M. Kontsevich in deformation theory.

Two proofs of the deformation quantization of Poisson manifolds, one by himself and one by Tamarkin, led Kontsevich to conjecture an action of a Grothendieck-Teichmüller group on such deformation quantizations, thereby delineating a precise relationship between these two topics, which form the core of the programme.

Programme goals and topics

In the past few years, seminal results have been proved which lie at the intersection of Grothendieck-Teichmüller theory, homotopical deformation theory, operads and multiple zeta values. These fields of research now clearly overlap and the goal of this programme was to use the cross-fertilization between them to push the intrication further into new explorations.

The main topics of the programme were:

- **Operads** (Koszul duality theory, homotopy algebras, moduli space of curves, Deligne conjecture),
- **Deformation quantization** (Lie bialgebras, Hopf algebras, Poisson manifolds, Kashiwara-Vergne conjecture, Duflo isomorphism),
- **Grothendieck-Teichmüller theory** (absolute Galois group, GT groups and GT Lie algebras, Drinfel'd associators),
- **Multiple zeta values** (mixed Tate motives, polylogarithms, shuffle algebra, Deligne-Ihara algebra).

Programme structure

The programme opened with an **introductory workshop** (8-10 January 2013) where the speakers were asked to present their topics of interest to a wide audience. This allowed the participants coming from seemingly different fields to begin to interact.

The backbone of the programme was made up of **four weekly courses** (January-March 2013), each of these stretching over one month and covering one of the aforementioned topics. Given the partly pedagogical intent, these courses included some exercise sessions and were mainly intended for students and `non-experts'; they played an important if not decisive role in `putting the pieces together', both from a scientific and social viewpoint. They were organised in such a way as to make them logistically accessible to the students in the nearby Universities and were made available online after just a few days.

More **advanced mini-courses** were organised, taking advantage of the presence of experts participants, thus enabling international `leaders' to present recent their results and theories in detail.

On top of it all, a **weekly seminar** was scheduled which made it possible to a very sizable number of participants to present their own work. We decided to let it fill basically the entire afternoon to recall the necessary definitions and background and to go into the details of the proofs whenever it seemed desirable or necessary.

By the end of March 2013, the many long-term participants of the programme were hopefully prepared to take part in **two international conferences** where `experts' (most of which actually staying already at the Newton Institute) were invited to present the recent advances in these fields.

- **Higher Structure 2013: Operad and Deformation theory** (2-5 April 2013)
- **Grothendieck-Teichmüller Theory and Multiple Zeta Values** (8-12 April 2013)

Finally a **satellite workshop** (15-16 April 2013) was organised by the Clay Mathematics Institute at the University of Oxford on the broader topic of **Higher Structures in Topology and Number Theory**. Two of the keynote speakers were the Rothschild (D. Zagier) and Microsoft (E. Getzler) Professors of the GDO programme.

Outcomes and achievements

In the end, the GDO programme hosted a total of **90 participants** from all over the world and altogether, the various workshops were attended by **over 200 people**. All the talks, including the lecture series, were recorded, which now provides the mathematical community with an extremely useful databasis on the subject. **A total of 108 talks are available**, representing nearly one terabit of data. So far, 27 000 views have been counted (July 2013).

These fields of research were somewhat underrepresented in the British mathematical community. So the GDO programme helped spread these ideas across the country, all the more because a good many participants gave invited talks in UK universities. Moreover a number of students were able to attend the activities of the programme, notably *thanks to the financial support of the Clay Mathematical Institute.*

The most obvious scientific outcome of the programme is the **creation of a new community of researchers** at straddling the boundaries of Algebra, Geometry, Topology and Mathematical Physics.

Thanks to the GDO programme and the interaction of experts coming from these various fields, the following **essential results** have been demonstrated.

- The notion of an operad was proved to organise various algebraic structures emerging from Multiple Zetas Values (e.g. shuffle and stuffle products), Grothendieck-Teichmüller theory, and modern homotopy theory.
- Conversely specific number theory oriented fields like Grothendieck-Teichmüller theory do provide researchers in operad theory with new seminal applications (interpretation of Drinfeld's associators as morphisms of operads).
- The deformation theory of morphisms of operads introduced a new and crucial way of describing the Grothendieck-Teichmüller Lie algebra.
- Last but not least, the deformation quantization of Poisson manifolds has been shown to admit a natural faithful action of the prounipotent Grothendieck-Teichmüller group.

Multimedia Activity

One should take note the crucial role played by the recorded lectures. Participants who arrived after one month or two, as well as those who had to leave earlier, could benefit from the entire programme through online access of the talks. Moreover students from outside the Cambridge-London area could attend the weekly courses in full.

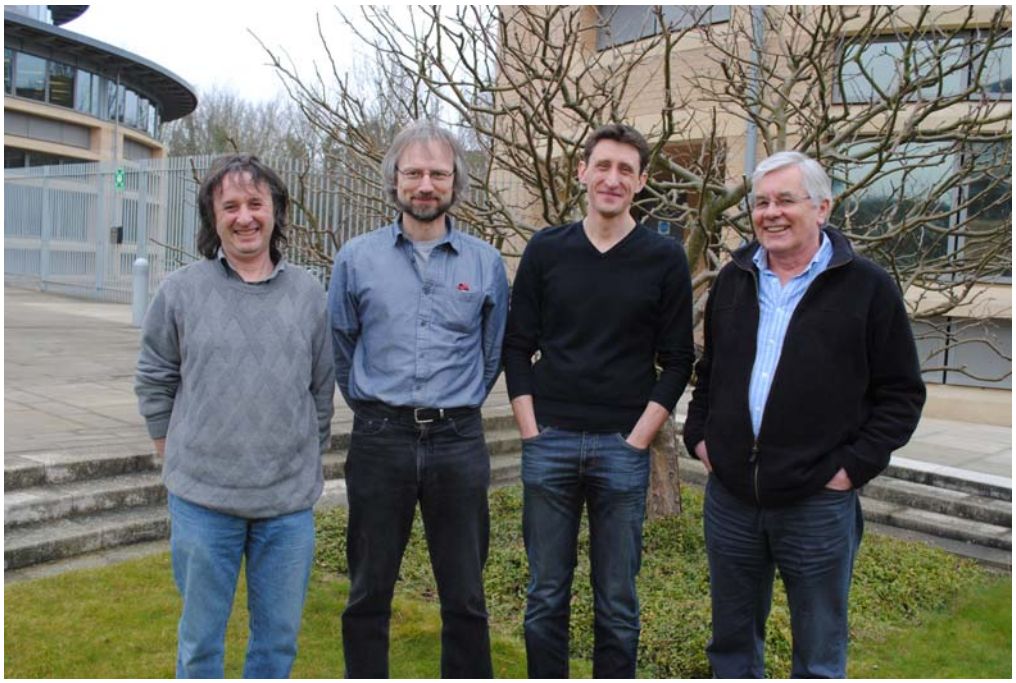
Publications

At least one book covering the entire activity of the programme is in preparation. It should provide a fairly complete and accessible view of the new research field of mathematics made up by these four topics. All the results readhed by the participants will naturally be published in high quality journals and since some of them feature genuine breakthroughs in their respective fields they are likely to appear in the highest quality journals in the world.

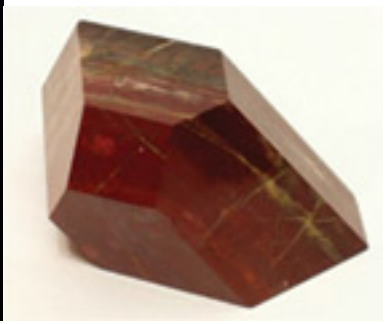
Illustrations

Participants of the introductory workshop
8-10 January 2013

Participants of the workshop
Higher Structure 2013 : Operads and Deformation Theory
2-5 April 2013



The organisers
Pierre Lochak, Herbert Gangl, Bruno Vallette and John Jones



The Borromean Rings and the Associahedron