

## REPORT FROM A NEWTON INSTITUTE PROGRAMME

### *Growth, Form, and Self-Organisation*

22 August – 20 December 2017

#### **Organisers:**

Arezki Boudaoud (École Normale Supérieure de Lyon)

Andrzej Herczyński (Department of Physics, Boston College)

Darryl Holm (Department of Mathematics, Imperial College)

Eric Lauga (DAMTP, University of Cambridge)

Jean-Luc Thiffeault (Department of Mathematics, University of Wisconsin)

### **1. Aims and Overview**

The programme was organized to coincide with the 100th anniversary of Sir D'Arcy Wentworth Thompson's *On Growth and Form*. Published in 1917, the book has had a lasting impact, effectively starting the field of biophysics and suggesting numerous ways in which mathematics, particularly as it relates to form, could be brought to understand living systems. The anniversary of its publication thus provided an opportunity to review progress, current research directions, and challenges in this interdisciplinary research area.

The *physics of living systems* can be considered a direct descendant of Thompson's ideas. This fast-developing field encompasses the search for universal scaling laws of biological systems, morphogenesis of single organisms at various scales, self-organization phenomena, and bio-fluid mechanics. Fruitful new mathematical approaches have also been deployed to a variety of complex *inanimate systems*, such as shape selection, shape deformation, and self-organization, and to a range of cross-disciplinary problems: from image analysis to medical diagnosis, from visual to performing arts, from dynamics of toys to physical aspects of sport.

The GFS programme aimed to bring these various contemporary strands of inquiry into focus. Its main goal was to stimulate new interactions and collaborations between applied mathematicians, biologists, and physicists – and also other experts interested in the mathematical approaches to form broadly understood. It was therefore conceived and organised to maximize opportunities for cross-fertilization of ideas within and across disciplines on various aspects of form and shape dynamics in diverse contexts and at diverse scales.

The broad range of themes of the programme necessitated a great diversity among the invited participants, and led to the extraordinarily rich and ever-changing community of researchers at the Institute, affording many serendipitous encounters and new research connections. Informal interactions, at the Institute and during many impromptu group lunches, dinners, were, arguably, the most rewarding and fruitful aspects of the programme. The structured activities consisted of two seminars per week, four workshops including a satellite at the University of Dundee, the Rothschild Lecture on 10<sup>th</sup> of October, Outreach Day (formerly Open for Business) sponsored by the Turing Gateway for Mathematics on 1<sup>st</sup> of December, and the Konrad Bajer Commemorative Symposium on 12<sup>th</sup> December.

Overall, there were close to 60 invited programme participants with visits at the Institute ranging from one week to four months. In addition, researchers from the University of Cambridge attended the GFS talks and events on an *ad hoc* basis. The four workshops attracted over 200 presenters, and were attended by most of the longer-term participants present. The total number of talks, seminars, and lectures given during the GFS programme was 135 – a remarkable number given that the programme duration was 121 days! Seminars and workshop lectures are discussed in the sections that follow.

## 2. Seminars and Lectures

The regular seminar series was inaugurated barely one week after the start of the programme, with a talk by Thomas Powers (Brown University) on 29<sup>th</sup> August. Seminars were held generally twice a week (Tuesday at 11 am and Thursday at 3 pm) except during the workshops. The last seminar was given by Mingming Wu (Cornell University) on 14<sup>th</sup> December, during the last week of the programme. Speakers were drawn mostly from among the GFS participants but included also a few researchers from the University of Cambridge – among them, Pierre Haas and Francis Woodhouse from DAMTP.

The range of seminar topics reflected the breadth of the programme itself: from fluid dynamical particle-wave duality (Yves Couder) to swimming and evolution of microorganisms (Nick Hill, Jay Tang, Pierre Haas), from cell migration (Mingming Wu) to the shape and function of the nasal cavity (Michael Brenner). A special seminar by Martins Bruveris, *Shape Analysis – An Introduction to Its Ideas, Methods, and Questions* – was offered on 7<sup>th</sup> November to provide an accessible primer on the subject of the upcoming Workshop III.

The keynote presentation and the highlight of the entire GFS programme was the Rothschild Lecture on the 10<sup>th</sup> of October. Alain Goriely (University of Oxford), the GFS Rothschild Fellow, gave a beautiful talk titled *On Growth and Form and Mathematics: Reading d'Arcy Thompson 100 Years On*. The lecture was advertised to the whole University community and beyond, and drew a record audience, filling the Seminar Room 1 and the Conference Room (where it was broadcast live). A wonderful reception was held in the Institute's lobby after the lecture and there was also a festive dinner for Professor Goriely and invited guests at Trinity College.

In addition to the Rothschild Fellowship held by Alain Goriely, the programme hosted eight Simons Fellows, all of whom gave lectures or seminars – some more than once. The GFS Simons Fellows were:

Arezki Boudaoud (École Normal Supérieure de Lyon); Martine Ben Amar (Université Pierre & Marie Curie-Paris); Pierre Degond (Imperial College London); Christophe Eloy (École Centrale de Marseille); Mimi Koehl (University of California, Berkeley); Jacques Prost (Institut Curie & CNRS); Neil Ribe (Université Paris-Sud); and Jane Wang (Cornell University).

### **3. Workshop I**

#### ***Form and deformation in solid and fluid mechanics***

The first workshop of the programme had a topic directly inspired by the legacy of D'Arcy Thompson. It was organized by Eric Lauga (University of Cambridge) and Jean-Luc Thiffeault (University Wisconsin). The workshop brought together a strikingly diverse group of scientists from all over the world unified by a common theme: the application of modern theoretical methods from the field of solid and fluid mechanics to a variety of problems in nature and bioengineering.

#### **3.1 Workshop in numbers**

The workshop featured 64 participants (11 women and 53 men) including 42 speakers (8 women and 34 men). About 30% of participants were based in the UK. Among the speakers there were 3 students and postdocs, 12 lecturers/assistant professors (6 short talks and 6 long talks), 11 mid-career scientists (6 long talks and 5 short talks) and 16 senior scientists (15 long talks and 1 short talk).

#### **3.2 Main topics**

Given the range of backgrounds of the participants, the topics of this workshop were thematically diverse.

*Pattern formation in physical mechanics.* This was a popular topic of the workshop. In many instances, a link was made between the “dry” system studied and a biological counterpart, in the spirit of D'Arcy Thompson, with a talk on soft robots and actuators joining the two. Topics included novel phenomena in solid mechanics and elasticity (planar elastica, shaping by swelling, indentation of plastic layers, granular media), fluid mechanics (magnetised plasmas, helicity and quantum vortices) and the interplay between a fluid and elastic medium (elasto-capillarity, a flexible Leidenfrost effect, flows in flexible tubes, bodies in anisotropic fluids).

*Solid and fluid mechanics of morphogenesis.* D'Arcy Thompson started the theoretical description of morphogenesis and the legacy of his work continues to loom large in today's work in the area. Reports on active research in the field ranged from detailed growth of plants to the development of embryonic lung. The link between the chemo-mechanical basis of morphogenesis at the cellular level and that of tissues was also addressed, culminating in a talk on the shape of animal skulls.

*Cellular fluid and solid mechanics.* A series of talks zoomed in on the physical mechanics at play at the cellular level and highlighted its consequence on biological phenomena. Here again, a wide variety of situations were considered, from packed cell tissue, microtubule rings, chromosomes and cilia, to biological interfaces, lipid tethers and bacterial colonies. Cell motility was also often addressed.

*Solid and fluid mechanics govern biological shapes and function.* Moving beyond morphogenesis and cellular physics, many talks highlighted the relationship between the mechanics of biological shapes and their functions. The biological systems included snake locomotion, fish slime, insects flight, the drinking strategies of mammals, locomotion of simple vertebrates, pulsating corals, and the fluid drag on plants. Some talks included impressive demonstrations, e.g. on the shape of the human foot or the ability of elephants to grab food with their trunk!

*Novel mathematical approaches.* As in many INI workshops, an important emphasis was placed on the development of novel mathematical models. After a review of classical work on the mathematics of phyllotaxis, new ideas presented included coarse-graining of collective models, Lagrangian multi-time dynamics, the dynamics of defects and new models for thin film of active liquid crystals.

### **3.3 Workshop highlights**

Some notable talks included Derek Moulton (Oxford) who discussed his research on the shapes of seashells. It was exciting to see that D'Arcy Thompson's most famous contribution has been refined to explain complex features of seashells as developmental features that involve solid mechanics. A related talk by Alain Goriely (also of Oxford) explored similar ideas for the development of folds in the brain. Neil Balmforth (University of British Columbia) discussed a fascinating problem: when we find dinosaur prints, what can we tell about the speed and gait of the creature by examining features of the indentation?

Practical applications of morphological analysis also abounded. Shreyas Mandre (Brown) talked about the importance of the transverse arch of the human foot, as opposed to the lengthwise arch which is more familiar. David Hu (Georgia Tech) showed the many methods that elephants use to grab objects with their trunk. Laura Miller (University of North Carolina) explained how the smallest insects manage to fly, and Sunny Jung (Virginia Tech) demonstrated how dogs drink.

### **3.4 Informal events**

There was time for many spirited discussions between the talks and at lunches at Wolfson Court, as well as during the banquet at Emmanuel College. In addition, a daily running group was formed and allowed discussions to continue at the end of the afternoon on the way to Grantchester or Fen Ditton.

## **4. Workshop II**

### ***Growth, form and self-organisation in living systems***

This satellite workshop, closely related to D'Arcy Thompson's legacy, was held at the University of Dundee in Scotland, a highly appropriate venue given that Thompson spent half of his career there. The principal organisers were Arezki Boudaoud (École Normal Supérieure de Lyon) and Gunnar Hornig (Dundee) who served as the local coordinator. Institutional support was provided by the University of Dundee and the International Centre for Mathematical Sciences in Edinburgh.

The focus was on recent advances, mathematical challenges, and promising new directions in research on form in living systems. Scientists from multiple disciplines – applied mathematics, physics, and biology – participated. An original feature of the workshop was that it bridged two usually disconnected topics: the generation of form – how organisms develop well-defined shapes – and the function of form – how the form enables organisms to function and survive. Accordingly, the main goal was to highlight various mathematical approaches to form and to provide a broad view while elucidating connections between topics.

#### 4.1 Workshop in numbers

There were 51 participants (17 women and 34 men) including 38 speakers (15 women and 23 men); about half of the participants were based in the UK. Among speakers: 26% students and postdocs (2 posters and 8 short talks), 31% lecturers/assistant professors (7 short, 5 longer talks), 42% senior scientists (1 short and 15 longer talks).

#### 4.2 Main topics

*Growth and form.* About half of the talks addressed the question of how plant and animal tissues or organs achieve their form, in normal or in pathological conditions (cancer). Experimental biologist presented their latest discoveries on underlying mechanisms and raised questions of how these mechanisms function together and whether they lead to observed forms. Mathematical models were either spatially discrete with cells represented as polygons in 2D, more faithful to cellular architecture but numerically challenging, or continuous and thus more amenable to mathematical analysis but inapplicable at the cell scale. New models now include stochasticity in order to link noise at the cellular level with variability at tissue level.

*Self-organisation and form.* Many talks considered morphogenesis from the point of view of complex systems, with a set of material points endowed with intrinsic dynamics and interacting, such as cells crawling in a dish and communicating through chemical signalling or hydrodynamic flows leading to specific collective behaviours. Similar phenomena occur for herds, flocks, or fish schools, as well as for the filaments and motors that make the cell cytoskeleton. This has led to the mathematical investigation of agent-based models and to the study of their continuous limits. Recently, continuous models demonstrated phase transitions between different behaviours in agreement with the numerical simulations of agent-based models.

*Form and function.* Many talks focussed on the function of form. How form enables locomotion (fish), self-cleaning (eyelash or tongue), chewing (jaws), or seed dispersal (dandelion)? Such questions can be phrased in terms of the optimization of cost functions that involve solving hydrodynamic or mechanical problems. This research appears to require a good knowledge of both the biological system of interest and of continuum mechanics, so it is critical to develop collaborations between the experimentalists and the theoreticians.

*Transverse themes.* A few talks bridged growth, form, and self-organisation by addressing how form adjusts to function. Kelp grows ruffled or not according to the strength of marine currents, which minimizes its hydrodynamic drag and maximises area for light harvesting. Plant stems grow in radius so as to optimize their self-support under gravity. The self-similar geometry of trees can be predicted by a model that incorporates competition for light and the mechanical resistance to wind.

*Mathematical questions and challenges.* A major challenge in building models is that living systems are active, which requires modification of classical frameworks in fluid or solid mechanics to incorporate non-conservative nature of living systems. Another challenge is building and analysing multi-scale models appropriate to the living systems which are organised into many levels. Many talks addressed these challenges via continuous limits of discrete models or of agent-based models. These models often involve a large number of degrees of freedom that are strongly coupled whereas continuum models pose difficulties when the underlying space is growing.

### **4.3 Workshop highlights**

The workshop featured three exceptional talks by members of national academies. Mimi Koehl (UC Berkley) demonstrated the links between kelp growth patterns, kelp form, and kelp hydrodynamic drag, by combining fieldwork, biomechanical measurements, and continuum models of growth. Herbert Levine (Rice University) made the point that variability in cancer is not only due to genetic mutations, but also to a phase transition in the collective behaviour of cells, by presenting mathematical models and corresponding experimental data. Jacques Prost (Institut Curie, France, and National University of Singapore) showed an elegant model for the dynamics of the cell cytoskeleton, that explained counter-intuitive experimental observations.

Among junior scientists, Hervé Turlier (Collège de France) presented a model for animal embryos similar to soap bubbles with varying surface tension, very much in the spirit of D’Arcy Thompson. Madeleine Seale (University of Edinburgh) showed how dandelion optimizes the dispersion of its seeds by coupling seed biomechanics with humidity, also in the spirit of D’Arcy Thomson. Marya Ptashnyk (University of Dundee) made a nice survey of multi-scale models for plant biomechanics and growth, with elegant applications of homogenisation.

### **4.4 Informal events**

In addition to self-organized dinner discussions, the main social event was an excursion to Blair Castle, followed by a dinner at Atholl Palace Hotel (Pitlochry) which started with a band playing bagpipes and reciting a classical “address to a haggis” referring to the Scottish traditional savoury sheep dish. Conversations at meals fostered camaraderie and helped establish links between participants.

An important event was a guided tour of the D’Arcy Thompson Zoology Museum at the University of Dundee. Although a large part of Thompson’s original collection was dispersed, participants were able to see some exquisite specimens still preserved and gain a new appreciation of the scope of his scientific contributions.

## **5. Workshop III**

### ***Shape Analysis and Computational Anatomy***

This workshop showcased a variety of recent advances in mathematical aspects of shape analysis (SA) and some of their implications for computational anatomy (CA), in the context of open problems relating to growth, self-organisation, and shape, both in living and inanimate systems. The workshop was organized by Darryl Holm (Imperial College) and Martins Bruveris (Brunel University).

#### **5.1 Workshop in numbers**

There were 46 participants (12 women and 34 men) including 28 speakers (6 women and 22 men); 18 participants were based in the UK. Among the speakers 9 were students and postdocs, 6 lecturers/assistant professors, 13 senior scientists.

## 5.2. Main topics

The two principal thematic strands of the workshop were shape analysis and computational anatomy. The term "shape analysis" is understood broadly, not only as a tool for analysing MRI or CT images of anatomy but also as a general approach to shape differentiation, variation, and development. SA is the study of the action of smooth invertible maps (diffeomorphisms) on embedded manifolds and immersions. The diffeomorphic methods of SA are important tools in CA; for example, image registration can be regarded as an optimal control problem for shapes.

But shape analysis is a rapidly developing branch of mathematics in its own right, linking the fields of geometric mechanics, Riemannian geometry, and control theory. The workshop lectures demonstrated the mathematical variety of shape analysis by drawing from functional analysis, variational principles, and the differential geometry of infinite-dimensional Lie groups. In particular, recently developed stochastic variational principles allow to account for naturally occurring shape variations and may also provide new tools for data assimilation and uncertainty quantification in fluid flows, e.g., in models for climate change.

Several of the participants and speakers of the workshop, including Ian Jermyn (Durham) and Karthik Bharath (Nottingham) were also participating in the programme "Variational Methods and Effective Algorithms for Imaging and Vision" which was taking place at the Newton Institute at the same time. Furthermore, Laurent Younes (Johns Hopkins, USA) was an organiser of the workshop "Flows, Mappings and Shapes" in that programme.

## 5.3. Workshop highlights

The workshop featured several inspiring talks. For example, Alain Goriely (Oxford) spoke about the challenge of modeling growth and the connections to isometric embeddings of Riemannian manifolds. Krastan Blagoev (NSF) spoke about the importance of shape in measuring tumor size. Marc Niethammer (North Carolina) showed how diffeomorphic image registration can be accelerated using neural networks and machine learning. Mads Nielsen (Copenhagen) talked about the challenges of achieving sufficiently high accuracy in image registration to be able to measure fine changes in brain anatomy.

A trio of talks by Sarang Joshi (Utah), Stefan Sommer (Copenhagen) and Alexis Arnaudon (Imperial) explored the effects of uncertainty and stochastic noise on shape deformations. Chris Klingenberg (Manchester) and Arezki Boudaoud (Lyon) applied ideas from shape analysis to biology, in particular the understanding of evolution and the study of tree leaves. Among junior scientists, Jean Feydy (ENS Cachan, France) talked about accelerating the computation of image registration using automatic differentiation libraries, Sophie Hecht (Imperial) presented a model for tissue growth, and Nina Miolane (INRIA, France) talked about the statistical pitfalls one can encounter when estimating mean shapes and how to overcome them.

## 5.4 Informal events

The formal dinner at Clare College was attended by most of the participants. Discussions at other meals and coffee breaks also helped build links between participants. At the end of the workshop a movie night was organised to see the comedy "The Death of Stalin" in the local cinema.

## **6. Workshop IV**

### ***Form in Art, Toys, and Games***

This was the last of the four GFS workshops with the theme that was at once more open and broader in scope than those of the earlier three. It was organised by Andrzej Herczyński (Boston College) and Roberto Zenit (Universidad Nacional Autónoma de México).

The goals were three-fold. The first was to extend the scope of the programme beyond the traditional areas where mathematical studies of form have proven relevant, such as topological fluid dynamics, physics of living systems, and computational anatomy, venturing beyond the biophysical context of Thompson's *On Growth and Form*. The second aim was to stimulate new insights, research, and ideas on applications of shape analysis, and other mathematical approaches, to the understanding and analysis of art processes and art objects, as well as to artful toys, broadly understood, and sport activities such as swimming or rowing. The third purpose was to establish new connections and collaborations between researchers pursuing such "diversions" as ancillary to their main research efforts, and thus begin forming a community of scientists—anchored by their dialogue with artists and art historians—who share scientific curiosity about these heretofore esoteric subjects.

The workshop was also distinct in a few other ways. It was shorter – three days long instead of five – but augmented by an Outreach Forum (formerly designated as "Open for Business" forum) organized in collaboration with Turing Gateway for Mathematics (TGM), and held on Friday, December 1. This was also the first workshop at INI, which was accompanied by an art exhibition, conceived and organized with the aim of provoking new ideas and "illustrating" some of the themes of the lectures. To make the connection stronger and to explore the possibilities of the dialogue with the artistic community, a special session dedicated solely to the presentations by the exhibiting artists was organized during the workshop (more details are given below).

#### **6.1 Workshop in numbers**

The workshop attracted 46 registered participants (of whom 10 women) with many additional attenders from the CMS and the university community, and a number of local artists as well (they were encouraged to attend and the registration fees were waived). There were 23 talks given by scientists, each 40-minute long, and 9 presentations by the artists (eight of whom contributed pieces to the exhibition), varying in length between 20 and 40 minutes.

The TGM outreach event attracted 76 delegates, the number that included most of the workshop participants; over  $\frac{1}{4}$  of those attending were women. There were 9 lecturers in total, three speakers were women and six were men.

The estimated total number of people who came to some of the workshop events over the four days is about 120. A few lectures, such as the opening lecture by Sir. Michael Berry or the closing presentation by Tadashi Tokieda, drew audiences that very nearly filled the INI Seminar Room 1 to capacity.



## **6.2 Artists' talks**

Presentations by the artists, during the session on Wednesday, November 30, in the afternoon, focused on their creative inspirations and details of their artistic processes, and included images of their recent work. Among notable highlights were a demonstration of modeling a reclining female figure in clay by the sculptor Emma Rodgers, and a multifaceted light-sculpting display by Paul Friedlander.

## **6.3 Themes and highlights**

The workshop was organised according to the three main themes. The first and second day (Tuesday and Wednesday) were devoted to form in art, and the intersection of the sciences and the arts. Thursday was devoted to toys and their forms and to the related theme of form in games. The outreach forum on the last day (Friday) featured talks intended for general audience touching on all three themes.

Among the highlights of the first day was a beautiful lecture by Sir Michael Berry on the surprising physics of the Japanese “magic mirrors” and its equally surprising possible applications. Aubrey Jaffers provided a fascinating account of the rarely appreciated marbling technique, while Tomohiro Tachi gave an engaging account of the mathematically aided origami constructions, with a nice collection of actual models on display. Jane Wang’s engaging talk on the physics of falling paper was followed by a 20-minute demonstration in the Institute’s foyer, accompanied by live music on cello. Among lecture on toys and games, Keith Moffatt’s lecture on the fascinating and chaotic “beaver ball” and Alain Goriely’s masterful mathematical analysis of chains of magnets were especially captivating.

The outreach forum included a lecture by Ray Goldstein, which connected the shape of stalactites, icicles, and ponytails (Ray’s approach “makes you first smile and then think” commented Mike Cates), and a talk by the composer Deidre Gribbin on her quartet inspired by “evolving genes.” The workshop concluded with a spectacular demonstration of the possibilities hidden in a simple sheet of paper presented by Tadashi Tokieda.

## **6.4 Informal events**

As usual, many of the questions and exchanges begun during the workshop lectures were continued later in informal discussions and impromptus over coffee breaks, lunches at Wolfson Court, and in the Institute’s common areas. There was a welcome wine reception after the first day of talks on Tuesday, and a similar reception following the artists’ session on Wednesday.

The workshop’s formal dinner was held on Thursday evening at Murray Edwards College. Lunches were provided in the Institute’s Discussion Room for all participants of the outreach forum organised by TGM. Since one of the talks on the previous day (by Patrick Weidman) concerned terminal motion of sliding and spinning discs, Professor Keith Moffatt provided a (Dutch-style) shuffleboard game and colleagues took their turn to try their skills in the game during the lunch break and at later times. The outreach day – and the workshop – concluded with a wonderful reception attended by all.

## 7. Art Exhibition

Accompanying workshop IV of the programme, and extending its interdisciplinary character, was an art exhibition *Form in Art: Art of Form*, open November 15 – December 15, 2018. The exhibition was organized at INI, with art objects displayed at all levels of the building. It was curated by Barry Phipps, an art historian interested in the intersection of art and science and Curator of the Works of Art at Churchill College, with assistance from Henry Jabbour, a local painter. An attractive brochure was prepared describing the event and listing all the exhibiting artists with their brief biographical sketches.

The exhibition encompassed 14 artworks by ten leading contemporary painters and sculptors whose work engage science in some manner – either by coopting physics in the creative process, or by drawing inspiration from nature, physical theories, or technological advances.

The exhibited works included five canvases, three prints, three bronze sculptures, a spectacular light sculpture by Paul Friedlander (London), and one video installation accompanied by a photograph by Nedyalka Panova who is based in Ireland. Among the contributors were two prominent British artists, Nigel Hall and Peter Randall-Page, who are Royal Academicians with works in the permanent collections of the Tate Gallery, the Victoria & Albert Museum, and other premier art institutions. Other participating artists were painters Mark Francis (London), Ulyana Gumeniuk (Cambridge), Henry Jabbour (Cambridge), and Manoel Veiga (Sao Paulo, Brazil), and sculptors Emma Rodgers and Mella Shaw (now in Edinburgh).

## 8. Commemorative Symposium

In addition to regular bi-weekly symposia, four workshops, the outreach day under the auspices of Turing Gateway for Mathematics, and Rothschild Lecture, the GFS programme included the one-day Konrad Bajer Commemorative Symposium.

Konrad Bajer (PhD from Cambridge University in 1989) was a prominent Polish physicist and fluid dynamists. He was an Associate Professor at the Institute of Geophysics of Warsaw University, organiser of numerous conferences, Board member of the Copernicus Science Center in Warsaw, and a well-known popularizer of science. In 2012, he was one of the leading organisers of the INI programme on Topological Dynamics in the Physical and Biological Systems. Konrad participated in early discussions of a proposed programme on the mathematics of form, and later served as a very active member of the Scientific Advisory Committee for the GFS. This work was cut short when in December 2013, Konrad was struck with a rare form of cancer. He died on August 29, 2014.

The Commutative Symposium, held on December 12, 2017, was a tribute to Konrad's service to the fluid dynamics community, and his critical contributions to the GFS programme, in which he was not able to participate. All speakers knew Konrad's well personally and spoke of his work at various stages of his career. The last speaker was his PhD advisor and long-time collaborator Keith Moffatt.

The event brought together a larger number of Konrad's colleagues from Cambridge, from elsewhere in England, Spain, Russia and the United States, collaborators and friends from Warsaw University and the Polish Academy of Sciences, and his family. Reception at INI, and a dinner at Trinity College, followed the Symposium.

## 9. New Collaborations

The lasting effects of a fruitful research programme are in new research links established, new collaborations begun, and new ideas that emerged. By all accounts, and based on comments from participants, GFS has been a successful programme in this regard. Below a few examples of new initiatives are briefly described.

A notable new initiative emerged serendipitously when Mimi Koehl (UC Berkeley) and Lisa Fauci (Tulane) met in their shared office at INI. Mimi and Lisa knew of each other's work from some years but discovered only during the program their common interest in the hydrodynamics of swimming and prey capture by choanoflagellate protozoans. They soon began to work together and developed a new mathematical model of the process. Mimi has also begun a collaboration with Mingming Wu on mechanisms used by the same organisms to locate dense patches of bacterial prey, and also with Shilpa Khatri (UC Merced) on analysis of errors in PIV data.

At the request of John Kessler (University of Arizona), we invited him and Nick Hill (Glasgow) to visit at the same time. Nick and John collaborated in the past but not in many years. At INI, they discovered new strands of shared interest and begun working out the mathematics. Another former collaborator, Tim Pedley, whose office is at DAMTP, soon joined them. Thus a dormant collaboration was rekindled – the three working researchers became something of a fixture in the second floor lobby!

During the GFS satellite Workshop II, Christophe Eloy began a conversation with Mimi Koehl on the motion of fish larvae in turbulent flow – this dialogue continued upon their return to INI. At the same workshop, Mariya Ptashnyk (Dundee) and Arezki Boudaoud met for the first time and started a brand new collaboration on the homogenization of discrete models of plant growth; Mariya later visited INI to continue the work. Separately, Mariya began a new research project with Naomi Nakayama (Edinburgh) on modelling stem biomechanical reinforcement.

Impromptu discussions begun during coffee breaks of workshop III between Alexis Arnaudon (Imperial College), Martins Bruveris (Brunell University), Stefan Sommer (University of Copenhagen) and Stephen Marsland (Massey University) led to new ideas and resulted in the preparation of a COST networking grant in applied geometry, which was submitted in April 2018.

As a result of an impromptu conversation one morning – when Darryl Holm (Imperial College) stopped by Andrzej Herczyński (Boston College) office to ask “what is growth” – Daryl, Martin Bruveris, and Andrzej began a conversation which resulted in a draft of a text which may be converted into a paper of the same title. Separately, Roberto Zenit (UNAM) and Andrzej begun conversations with the artist Manoel Veiga on the physics of his painting process, which involves diffusion of pigment and fingering.

## 10. Concluding Remarks

As this report demonstrates, the GFS programme was broad in scope, and in some ways – experimental demonstrations, musical accompaniment, displays of toys and Origami, participation of artists, the art exhibition – broke new ground. Arguably, the theme of *form in art, toys, and games* stretched the boundary of what had been attempted at the INI heretofore.

The four workshops, organised independently by teams of two researchers each, varied in style and appeared complementary. They were vigorous, informal, and most enjoyable as well. Based on the feedback received from the participants, they were in general judged highly. To quote a comment about Workshop I, “The greatest success of this first workshop was in its ability to gather in the same room researchers from the whole spectrum of science, from mathematics to experimental biology, with common interests in the relationship between form and function. Some of the most exciting questions asked during the talks came from people on the opposite side of the scientific spectrum, and they galvanised spirited discussions!”

The four workshops, the well-attended regular seminars, the outreach event, the Commemorative Symposium, and, above all else, the intense debates and the pressing questions raised, contributed to a fertile, enriching, and invigorating programme.

Perhaps the most gratifying comment came from Professor Mike Cates, as he closed the outreach event on the last day of Workshop IV: “I have to say, I have been watching these proceedings with ever diminishing scepticism!”