Granular and Particle-Laden Flows

1 September to 19 December 2003

Report from the Organisers: JMNT Gray (Manchester), K Hutter (Darmstadt), JT Jenkins (Cornell), T Mullin (Manchester)

Scientific Background

This research programme at the Newton Institute was devoted to improving the understanding and mathematical modelling of granular and particle-laden fluid systems as they arise in nature, in the laboratory and in industry. Geophysical examples of granular and particle-laden flows include rock avalanches, debris flows, mud flows and turbulent boundary layers with suspended particles, such as volcanic ash flows, underwater turbidity currents and powder snow avalanches. Industrial examples include flows of cereals, pharmaceuticals, alumina, coal and concrete, in storage facilities, production lines, power stations and construction sites. Although these examples might seem to be of disparate nature, in fact, they have many common features. An objective of the programme was to develop synergies between the various mathematical approaches.

Mathematicians and physicists are fascinated by such flows because they involve a wide range of flow regimes and abrupt transitions between them. Granular and particle-laden flows often involve grain segregation by size, mass, or other properties. Such flows often exhibit interesting patterns, chaotic motion, and transitions between structure and chaos.
The diverse regimes of physical behaviour require different mathematical models: methods of both nonlinear continuum mechanics and non-equilibrium statistical mechanics are employed; computer simulations that are the analogue of molecular dynamic techniques are used to inform modelling of the solid, fluid and gaseous states, including the transition from one to the other; concepts from continuum mixture theory and turbulence are often required to describe the interaction between the fluid and the grains in both dense granular flows with little interstitial fluid as well as in flows of dilute suspensions; and scaling arguments are often helpful in guiding theoretical formulations.

Prior to the programme, the mathematical description of granular and particle-laden flows proceeded on a case-by-case basis and a unified description, developed from first principles, seemed out of reach. Because such a description was likely to involve a range of complex and sophisticated theoretical concepts and to require an interdisciplinary approach, the time was felt to be ripe for a concerted effort among mathematicians, physicists and engineers to focus on the fundamental mechanisms responsible for the observed phenomena and to attempt to unify the seemingly disparate models.

Organisation and Participation

This four-month programme involved 42 long-term visitors, 26 of them from the UK. There were also 55 short-term visitors, 28 from the UK. An additional 141 individuals, 76 of them from the UK, participated in the two workshops and the five focus days that augmented the research programme.

Upon arrival, visitors provided short, introductory overviews of their research that enabled rapid communication among participants. During the course of the programme, J Rice delivered the Rothschild Lecture on Granular Flows and Earthquake Faulting and A Acrivos provided the Distinguished Lecture of the Institute for Mathematics and its Applications (IMA) on Particle Segregation in Rimming Flows of Viscous Suspensions. In addition, tutorial reviews that focused on their research perspectives were provided by A Acrivos, RP Behringer, S Edwards and EJ Hinch.

Workshops and Focus Days

Flow and Failure of Dense Fluid- Infiltrated Granular Materials
Focus Day, 17 September 2003
Organiser: J Rice
In this one-day meeting, experts on dense, fluid saturated granular systems highlighted the role played by the pore fluid in mobilizing them. An emphasis was placed on the modelling of the saturated granular gouge in an earthquake fault. Here the pressure in the pore fluid and the mobility of the fault is found to be influenced in important ways by the heat generated by frictional sliding. Pore fluid pressure also was shown to be important to the fluidization of debris flow when incorporated into a depth-averaged theory for their flow.

Flow Regimes, Transitions and Segregation in Granular and Particle-Laden Flows
Workshop, 22-26 September 2003
Organisers: JT Jenkins and T Mullin
This workshop exposed beginners and experts to developments in modelling, experiments and computation over the wide range of behaviour exhibited by granular and particle-laden flows. During the course of the workshop, it became clear that non-local constitutive relations were necessary to describe dense, dry, frictional flows and that pore water pressure played an important role in the liquefaction and mobility in many dense, particle-laden flows. Other presentations emphasized the similarities and differences of particle segregation in a variety of dry and fluid-saturated flows. Segregating flows are similar in that they all involve particle fluctuations; they are different in the forces that are exerted between the constituents. The workshop helped to reveal at least some common physical mechanisms present over a range of seemingly diverse phenomena.

Mathematical Issues in Granular Flows
Focus Day, 20 October 2003
Organisers: DG Schaeffer and M Shearer
Six problems of granular and particle-laden flow were phrased and viewed in a mathematical context. These included the formulation of boundary-value problems for steady, fully developed collisional flows, the posing and numerical solution of rate-independent flows in

hoppers, the foundations of a general theory for the rate-independent flow of frictional grains, the difficulties encountered in the solution of depth-averaged equations for avalanches, and the incorporation of fluctuations in theories for the deformation of elastic-frictional aggregates.

Geophysical Granular and Particle-Laden Flows, Satellite Workshop at the University of Bristol,

27-31 October 2003

Organisers: JMNT Gray, AJ Hogg and K Hutter

This multidisciplinary meeting proved to be highly successful in providing an opportunity for scientists from very different communities and backgrounds to communicate and exchange ideas with one another on the topic of geophysical granular and particle-laden flows. A broad selection of disciplines was represented, from geologists, Earth scientists, geographers and oceanographers to mathematicians, applied mechanicians and engineers. For many it was the first time that they had met with scientists from other disciplines working on similar and related problems in different contexts. Avalanches on massive scales in very different contexts were seen to involve similar physics. Here, again, the pressure of the interstitial gas or liquid emerged as an important feature of the description. The ability of mixture theories and/or depth-averaged equations to describe the flows was another issue that extended across discipline boundaries. A measure of the stimulating nature of the workshop was the proposal to hold a similar meeting in two years' time.

Computational Aspects of Granular and Particle-Laden Flows

Hewlett-Packard Day, 17 November 2003

Organiser: K Hutter

The four presentations dealt with a range of computational techniques that are relevant to granular and particle-laden flows. These are discrete-element simulations of aggregates of individual particles, the integration of the multidimensional equations of a continuum theory, the integration of the reduced equations of a depth-averaged theory, and numerical simulations of particle clustering in a turbulent gas.

A Young Person's Guide to Granular and Particle-Laden Flows

Focus Day, 8 December 2003

Organisers: JMNT Gray and T Mullin

A fresh perspective was placed on granular and particle-laden flows in seven talks given by young scientists. The talks included studies of spheres rolling on a viscous film, the splash of a water droplet on a particulate bed, colliding spheres flowing down a rough inclined slope, cellular foams that expand when stretched, transient segregation in a rotating drum, soil liquefaction, and segregation in vibrated beds.

Statics and Dynamics of Systems of Rigid Particles

Focus Day, 10 December 2003

Organiser: K Hutter

The four talks involved computer simulations, theoretical models and the mathematical analysis of rigid, or nearly rigid, systems. Computer simulations illuminated the role played by the simplest measure of the local geometry; simple theories for the yield of rigid disks and the double shearing of nearly rigid disks were outlined; and the mathematical properties of simple solutions for the steady flow over heaps were demonstrated.

Outcome and Achievements

Although the topics that were treated in the workshops and the focus days were diverse, the research programme evolved in a way that led it to concentrate onto a relatively small number of rather clearly defined areas. These were: development of theories for rigid and nearly rigid particles and for dense, dry frictional flows; behaviour of the depth-averaged equations for granular avalanches; description of particle size segregation; modelling of pore pressure in fluid-particle systems; mathematical aspects of soil plasticity theories; and the application of kinetic theory to particle-laden flows.

Efforts were made to understand the generality of theories for a marginally stable packing of rigid particles and their relation to theories for deformable particles. The development of theory for elastic, frictional particles focused on the prediction of the stress relation and the onset of failure (shear bands) from a consideration of the statistical geometry of the particle interactions. Experimental and numerical evidence for correlation lengths much larger than a particle diameter and the success of scalings based on these observations led to attempts to derive a more general theory for dense, dry, frictional flows. The consideration of depth-averaged equations for granular avalanches focused on the degree of complexity of the constitutive relations necessary for a faithful description of the physics. Also, attention was devoted to the construction of solutions to the hyperbolic system of conservation laws that describe avalanche break-up, entrainment and particle size segregation.

Particle size segregation emerged as one of the topics of broad interest in both granular and fluid-particle systems. Segregation in such systems is typically driven by shearing or vibrations. Particle velocity fluctuations play an important and, perhaps, similar role in segregation in all of these systems, but the fluctuations typically result from different mechanisms in different systems. The relationship between the local strength of the fluctuations and the forces driving the system led to an active area of research.
Models for the influence of pore pressure on the evolution of dense particle-fluid systems are required for granular soils, earthquake fault gouges, and dense avalanching flows of debris and mud. In these systems, increases in pore pressure can result in liquefaction and the catastrophic loss of bearing capacity. Related activity concentrated on the mathematical features of soil plasticity. Here the issue was the loss of well-posedness in several formulations and how one could regularize such theories to avoid it. Attempts were made to apply kinetic theory to solid-fluid mixtures as diverse as fluidized beds and viscous suspensions of buoyant particles. There is accumulating evidence that the kinetic theory may provide a paradigm for the understanding of systems of particles that interact within a fluid. Again, a description of the particle velocity fluctuations is seen to be the key to the appropriate description, but the transfer of energy between particles and between particles and the fluid remains to be characterized.

Collaboration between participants took place in small groups. It is anticipated that strong publications from these newly established collaborations will be a hallmark of the programme. This activity has advanced the current understanding of granular and particle-laden flows by providing a forum for interactions between experts with similar and different interests. It has also exposed a large number of researchers to a broad overview of granular and particle-laden flow processes, the results of experiments and numerical simulations, methods of modelling such as mixture theory, kinetic theory and depth-averaging, and the relevant mathematical analysis.

A total of 46 seminars at other UK universities contributed to the increase in interest in the UK in the subject of granular and particle-laden flow. The activities of the programme brought focus to a broad and diverse community of researchers interested in these burgeoning subjects. The programme was particularly successful in attracting young UK researchers to the workshops and focus days.

An agreement has been established with the Royal Society to publish a subset of the contributions to the workshop on Geophysical Granular and Particle-Laden Flows as a special Theme Issue of the Philosophical Transactions of the Royal Society of London. Also, the writing of several research monographs was advanced during the programme.

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