Groups, Representations and Applications: New Perspectives
2020/2022

Group Theory is essentially the theory of symmetry for mathematical and physical systems, which underpins much of modern pure mathematics, with major connections to diverse areas of mathematics, as well as physics, chemistry, and information science. Born more than two centuries ago in the work of Évariste Galois, it achieved a major milestone when the Classification of Finite Simple Groups (CFSG) was completed. Since then, important and deep connections with areas as varied as topology, algebraic geometry, Lie theory, homological algebra, and mathematical physics, have been discovered and exploited. Still, the area abounds with basic problems and conjectures, including the conjectures of Brauer, McKay, Alperin, Broué and Dade in group representation theory, many of which have been open for decades.

The goal of the programme was to bring together leading experts in group theory and representation theory, on the one hand, and from several different areas of mathematics on the other, with two major focuses: first, to attack some of the representation-theoretic conjectures mentioned above; and second, to take the many connections between CFSG and other areas of mathematics to the next level. As it happened, the programme was divided into two halves by the Covid pandemic – the first January-March 2020, and the second May-July 2022. In fact, several of the most exciting achievements of the programme were announced during the second half, resulting from research undertaken during the first half and in between. Some of the main highlights relate to the conjectures mentioned before. These conjectures are all concerned with the representations of an arbitrary finite group with respect to a given prime number p. Britta Späth completed some final steps towards a proof of McKay's conjecture, and announced the full resolution for p=3. Lucas Ruhstorfer, one of the early career researchers on the programme, presented his proof of two of the main conjectures for the case p=2 – namely, Brauer's height zero conjecture and the Alperin-McKay conjecture. Major progress on one of the deepest of all the conjectures – Broué's abelian defect group conjecture – was announced by Sasha Kleshchev. There were also breakthroughs reported by Nick Katz and Pham Tiep on another big part of the programme, connecting CFSG with number theory and algebraic geometry via the theory of ℓ-adic local systems.

There were five workshops during the programme – an introductory one, three workshops on computational aspects, applications of groups to other areas, and representation-theoretic conjectures, and a final workshop bringing together all the main themes of the programme.

Overall, this was an exciting and successful programme, with great progress on its major goals, much of it achieved by the many early career researchers participating.