Mathematical, Foundational and Computational Aspects of the Higher Infinite (HIF)

In the late 19th century, Georg Cantor observed that there are different sizes of infinity when he proved that the set of real numbers cannot be brought in one-to-one correspondence with the set of natural numbers. Cantor's \emph{set theory} gave rise to a rich and interesting structure of infinities and is nowadays a mature research field within mathematics using sophisticated techniques such as G\"odel's constructibility and Cohen's forcing in studying the Higher Infinite and its properties. In addition to being a research field in mathematics, set theory also provides a unified foundations for mathematics as a whole and has many applications in other fields of mathematics, theoretical computer science, and beyond.

After the Second World War, set theory had largely moved to America, and until a few decades ago, research in set theory was dominated by North American researchers. This has recently changed and a number of European initiatives witness the current strength of set theory in Europe, most prominently the formation of the \emph{European Set Theory Society} (ESTS; 2007), its new conference series of \emph{European Set Theory Conferences}, and the Research Networking Programme INFTY funded by the European Science Foundation (2009--2014).

The programme HIF developed as a natural next step after the network INFTY to bring together the leading researchers in set theory from North America, Europe and other parts of the world and stimulate the exchange of ideas among researchers pursuing mathematical, foundational, and computational approaches to infinity, linking set theory to other parts of mathematical logic, mathematics, computer science, and other disciplines.

The programme was very successful in attracting the most prominent set theorists from around the world: almost all of the leading thinkers of our field came to Cambridge during HIF, enjoyed the hospitality of the Newton Institute, and contributed to the scientific programme, among them our Rothschild Distinguished Visiting Fellow W. Hugh Woodin from Harvard University. In addition to the established core of the set theoretic research community, we also had researchers from bordering research areas among our visiting fellows (e.g., theoretical computer science, philosophy of mathematics, proof theory, or constructive mathematics) as well as 13 doctoral students and many junior postdoctoral researchers who greatly benefitted from working in direct contact with the people who formed our field. We believe that for these young researchers, HIF will prove to be a catalyst for their mathematical development and their careers.

The programme linked to the ESTS by hosting the 5th European Set Theory Conference (5ESTC) at the Newton Institute, during which Istvan Juhász, the president of the ESTS, awarded Ronald Jensen and John Steel with the 2015 Hausdorff Medal for their work in set theory; it also linked to the general logic community in the UK by hosting the annual British Logic Colloquium (BLC 2015) in association with the programme at the Newton Institute.

Using the excellent infrastructure of the Newton Institute, it brought together researchers for scientific collaborations: in a mathematical field where papers often take years to finish, it is remarkable to see that there are already now (May 2016) thirty-six HIF entries in the INI preprint series, among them several preprints of authors who did not have any joint publications before they collaborated at the Newton Institute during HIF: the topics of these papers range from very foundational (e.g., on the philosophical notion of restrictiveness for axiom systems of set theory) via

traditional technical contributions to set theory (e.g., on descriptive set theory, combinatorial set theory, or cardinal arithmetic) to applications of set theory (e.g., in measure theory or the theory of infinitary computation).

Our satellite workshop \emph{Independence Results in Mathematics and Challenges in Iterated Forcing} in Norwich, held at the Sportspark of the University of East Anglia, was a very successful research event stressing the technical contributions and allowed the participants to enjoy the somewhat unique experience of discussing mathematics while watching the swimming pool or the climbing wall.

After four months of research in set theory, the programme put a spotlight on the applications of set theory in other parts of mathematics and other disciplines during the final workshop \emph{The Role of the Higher Infinite in Mathematics and Other Disciplines} in the week before Christmas. With talks including links to topological dynamics, automata theory, infinite matroids and program analysis, the programme came to an end, just before the Christmas party of the Newton Institute, with a presentation proving, with techniques from descriptive set theory, that the question whether two knots are the same is very difficult: a fitting final talk for a programme that positioned set theory centrally among many areas of the mathematical sciences.

The programme HIF was a key step in the consolidation of a European set theory research network with strong links to the leading researchers in North America and has served as a springboard for further actions, such as the upcoming research program \emph{Large Cardinals and Strong Logics} at the Centre de Recerca Matem\`atica near Barcelona from 5 September to 16 December 2016.