HYD2 Short Final Report

Dispersive hydrodynamics is the study of nonlinear conservation laws subject to dispersive corrections encompassing both dynamic and stochastic aspects of multiscale wave propagation. A characteristic feature of dispersive hydrodynamics is the dispersive shock wave (DSW), a rank-ordered, expanding, rapidly oscillating wave form that results from the dispersive regularisation of a hydrodynamic singularity. Mathematical challenges in the field abound, including the development of a general analytical framework for DSWs that is akin to conservation law theory for classical shocks, modulation theory and inverse scattering theory for multidimensional dispersive hydrodynamics, the solution of initial value problems for integrable equations with random data, boundary value problems for integrable and non-integrable equations, and the quantitative modelling of physical problems.

The HYD2 programme that took place from July–December 2022 attracted applied mathematicians and scientists from a variety of disciplines to engage in the mathematics and applications of dispersive hydrodynamics. The programme aimed to increase the visibility of this burgeoning field and inspire new collaboration with four major themes: modulation theory and DSWs, analysis of dispersive hydrodynamic problems, integrable turbulence and soliton gases, and physical applications. The participation of 115 junior (25%) and senior (75%) researchers and the research surveyed across 44 weekly seminars demonstrated the field's broad reach and scope, including synergies with concurrent INI programmes like Applicable Resurgent Asymptotics. Approximately 70% of participants[1] initiated new collaborations, began research in new directions, and learned of new applications of dispersive hydrodynamics because of their participation in the programme. Research achievements included the solution of generalised Riemann problems, new classes of breathers and rogue wave solutions, a variety of applications including dispersive hydrodynamics from first-order hyperbolic systems, and many striking experiments. There were recurring discussions of soliton/breather gases and their numerical synthesis, multidimensional modulation theory, experimental observations, connections to the field of generalised hydrodynamics from theoretical physics and N-soliton solutions with $N \to \infty$ identified with soliton condensates. The generated body of work motivated a successful application for a 2024, one-month INI satellite programme in Newcastle.

On their own initiative, long-term junior participants established a weekly, early-career-only seminar that facilitated robust discussion. The Women in Dispersive Equations event showcased a range of research in the area by female mathematicians and culminated in a panel discussion on issues of diversity and equity. In addition to the 115 programme participants, HYD2 hosted an additional 75 workshop-only participants across five week-long workshops, including one satellite workshop at Loughborough University, two Open for Business events, a visitor programme at OxPDE, University of Oxford and a joint seminar with INI-RIMS Kyoto University. The four workshops held at the Institute served to kick-off each of the four programme themes and averaged more than 70 registered, in-person participants each.

In summary, the hyd2 programme provided a huge boost to research efforts, new collaborations, and mentoring of junior researchers in the field of dispersive hydrodynamics.

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[1] Based on 48 programme survey respondents out of 115 in-person participants (42%).