Interaction and Growth in Complex Stochastic Systems

21 July to 19 December 2003

**Report from the Organisers: E Bolthausen (Zurich), K Khanin (Newton Institute), G Lawler (Duke),
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Scientific Background

Random growth processes play an increasingly important role in various parts of pure and applied mathematics, with applications in many areas of natural science (physics, chemistry, biology, environmental studies, computer science), engineering (including material sciences and management), humanities (economics, finance, social sciences) and medicine. Traditionally, growth processes were studied in many disciplines in a rather isolated fashion which can be explained by a broad variety of models and phenomena. However, major recent advances have fundamentally changed our perceptions of well established problems, and these have tended to unify the area. The aim of this programme was to put together, in a more systematic way, the various strands which have been discovered and studied over the last 10 years, with the main focus on the probabilistic study of random growth processes emerging in theoretical and mathematical physics.

Random processes are often understood by means of an asymptotic limit in which the number of basic random components becomes large. Classical examples of behaviour which can be manifest in such a limit are the law of large numbers and the central limit theorem. These two examples apply in contexts where the interaction between typical random components becomes negligible in the limit. However, there are many extremely important physical models where such an asymptotic decorrelation does not hold. Such models correspond to a critical behaviour where interaction is strong and statistical scaling properties are highly nontrivial. Understanding of the asymptotic properties for such systems is one of the most important problems in modern probability theory.

This programme was focused on problems from a number of contexts where strong random interactions give rise to non-classical limiting behaviour. These include random walks in a random environment and interactive random walks, interfaces in growth models and equilibrium statistical mechanics, coagulation and fragmentation processes, conformally invariant scaling limits and random matrices. The range of possible limiting behaviours is rich and introduces to probability new connections with other areas of mathematics and physics. Of particular importance is a connection with conformal field theory. Renormalization group ideology, which was developed by physicists over the last 30 years, predicts the appearance of universal scaling and conformally invariant random fields characterizing statistical behaviour in critical situations.

However, rigorous results in this direction were out of reach until very recently when new connections with complex analysis and the Loewner differential equation were discovered. Powerful new techniques are emerging which make possible the computation of exact asymptotics along with a rigorous understanding of convergence. These new ideas and techniques were one of the main topics of the programme.

Structure of the Programme

The programme attracted a very strong contingent of nearly eighty long-term and sixty short-term visitors. The highest periods of attendance were built around the four workshops, one of which (the NATO ASI) had a substantial instructional component. In between, a regular seminar series was run with two talks weekly. The inspiring atmosphere of the Isaac Newton Institute stimulated intensive interaction between the participants and quite a few new collaborations started during the programme.

Workshops

Conformal Invariance and Random Spatial Processes
NATO Advanced Study Institute and Satellite Workshop at the International Centre for Mathematical Sciences, Edinburgh,
8-19 July 2003
Organisers: K Khanin, G Lawler and JR Norris

This NATO Advanced Study Institute (ASI) and EPSRC-funded workshop took place at the University of Edinburgh and was organised by the International Centre for Mathematical Sciences (ICMS).
Recent exciting advances at the intersection of probability and complex variable theory have made it possible to understand mathematically ideas that arose in theoretical physics. It had been conjectured that a number of two-dimensional systems in statistical physics such as percolation, the Ising model and self-avoiding walks have scaling limits that are invariant under conformal transformations (i.e., they look similar when subjected to a change that locally looks like a scaling factor and a rotation). Physicists had used this conjecture and a mathematically non-rigorous technique, conformal field theory, to make predictions about these systems. Numerical simulations had given strong support to these conjectures, but until a few years ago they were unproven.
The meeting started with an intensive instructional NATO ASI with five mini-courses given by the leading experts in the field. This structure gave the chance for many participants to get enough understanding of the subject to follow the research part of the meeting. Two short courses were given by G Lawler and W Werner on Schramm-Loewner evolution (SLE), a process introduced by Oded Schramm, that is the missing ingredient needed to understand these limits rigorously. Other short courses were given in closely related areas: J Cardy, a theoretical physicist, gave a course for mathematicians on Coulomb gas methods; K Johannson discussed another exciting area in probability, namely random matrix theory and its applications; and R Kenyon lectured on combinatorial and conformal geometric methods to understand lattice dimers and related models.
The workshop which followed the ASI featured experts in probability, theoretical physics and

analysis who gave the state of the art of this area. A number of the lecturers discussed recent results in a programme to construct conformal field theories using Schramm-Loewner evolution. This is an exciting development for both the physics and mathematics communities. Physicists are, for the first time, understanding the process that arises as boundaries in statistical physics systems. Mathematicians can use SLE to understand the non-rigorous arguments of conformal field theory that heretofore appeared very mysterious.
Other talks focused on properties of SLE and a number of discrete models - there are still many open questions concerning the convergence of discrete models to SLE. There were also talks on related areas, e.g., new results on spin glasses and dynamical systems.
The last session of the workshop was entirely dedicated to open problems in this very new and active research area. A web-based e-publication is in preparation which will contain all the materials of minicourses and most of the research talks. This will be available for free access by anyone interested in the subject.

Random Walks in Random Environment
Workshop, 18-22 August 2003
Organisers: E Bolthausen and A-S Sznitman

The workshop brought together leading scientists working on random walks in random environments (RWREs) and on topics closely related to this, such as random walks in random potentials, directed polymers in random environments, random walks on percolation clusters, reinforced random walks, and passive tracer models.
RWREs have proved to be a source of challenging problems in past years, and much progress has taken place recently. New developments were presented at the workshop. For instance, regeneration times have proved to be a powerful tool. These allow the introduction of a kind of independence structure, but the application is delicate.
Of considerable interest are random walks in random potentials. For time independent potentials, the asymptotic behaviour can be non-diffusive, and particles move to 'traps' where they stay for a long time. This is now quite well-understood. The theory in the important time-dependent case is much less developed, but there has recently been considerable progress here too. This case is closely related to directed polymers in random environments, a model with a long history which is of particular importance because of its relations with turbulence, with random growth models, with first passage percolation and with spin glass theory, to name only a few. Rigorous results have been quite rare, especially for the high disorder situation. At the workshop, there were several talks reporting on recent results in this area. Some striking results have recently been obtained using martingale methods. These were presented in detail by several authors. There were also a number of talks on different but closely related models, including random catalysts, heat kernel estimates on percolation clusters, and reinforced random walks.
The Newton Institute provided an excellent environment for an extremely lively, communicative and informative workshop which gave a fairly comprehensive overview of the most important recent developments in this rapidly growing field.

Random Matrix Theory
Satellite Workshop at Gregynog, University of Wales, 14-19 September 2003
Organisers: I Davies, L Pastur, YM Suhov and A Truman

This workshop, supported by the European Science Foundation through its project Random Dynamics in Spatially Extended Systems, brought together about thirty leading researchers in this exciting field of modern mathematics. They discussed methods and results from classical

and functional analysis, combinatorics, integrable systems, number theory, operator algebras and probability, with applications in quantum mechanics, quantum field theory, statistical mechanics and condensed matter theory. The list of problems under consideration included the Ulam problem of the longest increasing subsequence in a random permutation, related problems about asymptotics of Toeplitz determinants, directed first-passage percolation, asymptotic distributions in queueing systems, random Young tableaux, partitions of natural numbers, zeros of the Riemann zeta function and Anderson localisation. According to the common opinion of many participants, the main success of the workshop was that it brought together a wide circle of interested academics including P Bleher (Purdue), N O'Connell (Warwick), K Johansson (Stockholm), K McLaughlin (North Carolina), A Soshnikov (California), H Spohn (Munich) and others, which stimulated exchange of ideas and contributed to an inspiring meeting.
It was the unanimous opinion that Gregynog Hall (associated with the University of Wales and located in beautiful local countryside) provided excellent working and recreational facilities, and that the local organisers and the staff demonstrated the highest degree of hospitality. The workshop will certainly be remembered by all present with affection and appreciation.

Stochastic Methods in Coagulation and Fragmentation
Euroworkshop, 8-12 December 2003
Organisers: JR Norris, M Kraft and W Wagner

This workshop was the first substantial international meeting to bring together probabilists and analysts working on coagulation and fragmentation. Activity in this area has intensified in the last ten years for a number of independent reasons. Probabilists and analysts have developed a greater facility in understanding the evolution of large and complex systems, and have recognised the scientific and mathematical interest of coagulation and fragmentation. At the same time modern computing resources offer the possibility of simulating and predicting the behaviour of these systems. The workshop revealed significant advances in the structural understanding of probabilistic models and in the behaviour of these models in long time limits, as well as in scaling limits and gelation for analytic models.
The workshop attracted seventy participants. There was also input from industry in the form of a special session. All of the participants were exposed to this subject from at least one new perspective and many took up the opportunity to explore new directions with the experts present at the workshop.
The programme comprised four lectures of one hour on most days. On the probabilistic and combinatorial side the speakers were J Bertoin, J Schweinsberg, G Miermont, D Aldous, P Chassaing, M Deaconu, N Fournier, A Lushnikov, O Zeitouni, J Berestycki and J-F Le Gall. On the analytic side there were talks by P Laurencot, M Escobedo, B Niethammer, F Leyvraz and V Kolokoltsov. W Wagner addressed some numerical issues. M Kraft led a session in which A Bayly and P Mort, both of Proctor & Gamble, and C von Toerne of Bayer gave a valuable industrial perspective.

Outcome and Achievements

The programme attracted a large number of participants including many of the leading experts in probability theory and related fields.
Feedback from the participants was positive, praising the research environment of the

Institute and its facilitation of interaction and the development of new collaborations. In addition to the four workshops described above, a seminar was held twice weekly and a study group on SLE was held weekly.

A significant outcome of the programme was the dissemination of recent work on SLE to a wide audience of probabilists, beginning with the courses by Lawler and Werner and continuing in seminars and the study group. Many participants mentioned the impact of this exposure to their ongoing work. During the programme, progress was made by Lawler on some new discrete models which converge to SLE, and by Werner on the Brownian loop soup and SLE curves when the central charge is positive. Extensions to Riemann surfaces were the focus of work by Makarov and Friedrich.

Progress was made in a number of important problems for lattice-based statistical mechanics, notably in metastability, droplet formation and random walks in a random environment. Bovier and den Hollander proved metastability results for Kawaski dynamics at low temperature and low density in dimensions two and three, describing the behaviour of the critical droplet. Hryniv, Ioffe and Kotecky obtained sharp asymptotics for the critical region for droplet formation and also central limit-type results for certain functionals of random phase separation lines, such as area or local magnetisation. Shlosman completed a work with van Enter on phase transitions in gauge theories with continuous symmetry and started a project with Ioffe modelling the process of droplet condensation of hot ideal gas on a cold substrate. Slade and van der Hofstad obtained new results for percolation critical values in high dimensions. J van den Berg completed a work on self-destruction percolation. Comets collaborated with Yoshida and Zeitouni on some problems in random media and in random walks in a random environment.

A class of random media termed quasi-stationary was studied by Boldrighini, Khanin, Minlos, Molchanov and Pellegrinotti. In this model the random potential is formed by the product of two factors: one depending on space and the other on time. It turns out that if the probability distribution for the space factor has exponential tails then the corresponding random walk will be localised in the trapping region, while in the bounded case numerical studies suggest that random walk exhibits KPZ scalings.

Hydrodynamic limits of many-particle systems and of interfaces were studied by many participants, including Conlon, Funaki, Ioffe and Norris.

Koenig and Moerters completed a work on the intersections of several Brownian motions.

A major theme was random matrix theory and its relation to stochastic growth models. Prähofer and Spohn made progress in understanding the fluctuations of stationary, as well as droplet-type, one-dimensional growth models and related multi-matrix ensembles. Bleher worked on a Gaussian random matrix model with external source. O'Connell and Johansson were able to interpret a probabilistic model for the Riemann zeros in terms of the eigenvalues of Brownian motion on a certain symmetric cone.

Hambly and Martin studied a certain two-dimensional last passage percolation problem, which may also be considered as a growth model, obtaining good estimates on the passage time.

Goldsheid proved a generalisation of the Thouless formula to non-Hermitian Jacobi matrices.

Kesten and Sidoravicius worked on a model for the spread of infection by random walkers on a multi-dimensional lattice, with infection on contact. They determined the asymptotic shape of the epidemic and also showed that if individuals can recuperate, and do so fast enough, then the infection dies out.

M Penrose made studies of directed geometrical random graphs, making links with the PoissonDirichlet distribution and fragmentation processes. He also collaborated with Yukich on random deposition models.

The mass-flow process is used in the simulation of certain coagulation processes. Work by

Wagner established that this process explodes for any coagulation kernel homogeneous of degree greater than one. This is relevant to the detection of gelation for such kernels.
Above we have described some of the results obtained by the participants during the programme. The programme has certainly stimulated an intensive exchange of ideas between participants; new collaborations have started and we are confident that they will lead to new important results in the near future, which will stand as the main achievement of the programme.