Report on the program INV

1 Introduction

Many important real world problems give rise to an Inverse problem (IP). These include medical imaging, non-destructive testing, oil and gas exploration, landmine detection and process control. For example, in the exploration for oil and gas, one needs to assess the structure of the interior of the earth from observations made at the surface. Typically, an explosion is created and the resulting shockwaves together with their reflections are used to build a model of the structure of the earth. In magnetoencephalography one needs to determine the electric current in the neurones from the measurement of the magnetic field outside the head. In the field of medical imaging IP forms an important tool in diagnostic investigations. For example, PET and SPECT are two modern imaging techniques whose success is dependent on solving IPs. At their simplest level IP are concerned with obtaining information about the interior of a body from data which is available at its surface. Mathematically, this is a parameter identification problem: given a set of data representing the behaviour of solutions, identify the unknown parameters of the model. The mathematical machinery needed for solving various IPs is mainly founded in mathematical analysis and uses tools from functional analysis, function theory, conformal maps, spectral theory, theory of PDEs, integral equations, and micro-local and global analysis. In recent years tools from differential geometry, stochastic analysis, etc., are becoming important. Moreover, in order to realise the solution to many applied problems in a useful way, the tools of numerical analysis and scientific computing are needed.

1.1 Organisation

The organisers were Malcolm Brown (Cardiff), Thanasis Fokas (Cambridge), Yaroslav Kurylev (University College London), Bill Lionheart (Manchester) and William Symes (Rice). In addition they were aided be a group of Scientific Advisors: Simon Arridge (University College London), Christer Bennewitz (Lund), Margaret Cheney (Rensselaer Polytechnic Institute), Chris Farmer (Oxford), Jari Kaipio

(Auckland), Andreas Kirsch (Karlsruhe), Lassi Päivärinta (Helsinki) and Gunther Uhlmann (Washington and Irvine)

The program attempted to (and in the view of the organizers succeed in) assemble a diverse group of researchers from various areas of mathematics, physics and earth scientists, as well as graduate students and junior scientists, in order to create cross-pollination of ideas and methods and thus further the development of a community and facilitate progress in, this important and fascinating new field, full of rich and challenging problems.

1.2 Participants

The program involved around 130 long term participants, with additional visitors to the five workshops, from Austria, Canada, Czech Republic, Finland, France, Germany, Holland, Ireland, Israel, Italy, Japan, Korea, Poland, Russia, Spain, Sweden, Switzerland, UK and USA . This included world leaders, such as members of leading world scientific Academies (e.g., E. B. Davies, FRS, Gunther Uhlmann NA US,. L.D.Faddeev –Academician-Secretary of Division of Mathematics, Russian academy of Sciences, Tony Chan– President of the Hong Kong University of Science and Technology) and many other leading experts, as well as many junior researchers (dozens of graduate students and postdocs have participated in the program). The participants represented many areas of mathematics (analysis, ODEs, PDEs, mathematical physics, operator theory, probability, dynamical systems, differential geometry, numerical analysis and scientific computing), as well as other scientists. Support was provided by various sources: EPSRC (programme and workshops):£239,041 Leverhulme Trust: £22,848 Scottish Trust: £10,086 LMS: £8,169 Microsoft Research: £6,617 Cambridge Philosophical Society: £500 , together with some unspecified funds to support Canadian participants.

2 Workshops

Besides the regular activities involving long and short time visitors, the program included five workshops. In addition two of them hosted embedded *Open for business days.*

2.1 25-29 July 2011 Introductory Workshop Organisers: Malcolm Brown (Cardiff), Matti Lassas (chair) (Helsinki) and Roland Potthast (Reading)

The aim of the introductory workshop of the Inverse Problems research programme was to provide an overview on several topics that will studied by the leading experts

of inverse problems. The workshop was intended for postgraduate students, postdocs, and researchers working on inverse problems and the related areas.

The workshop contained five four hour courses on inverse problems:

C1. Gunther Uhlmann (University Washington and University of California at Irvine) -Overview on hybrid imaging problems and microlocal methods C2. Adrian Nachman (University of Toronto) -An course on electrical impedance tomography and PDE methods C3. Andreas Kirsch (Karlsruhe Institute of Technology) -A course on inverse scattering problems C4. Samuli Siltanen (University of Helsinki) -A course on numerical and statistical methods for inverse problems C5. Liliana Borcea (Rice University) -A course on imaging in random medium.

In addition to this, the workshop contained some highlighted lectures, each giving a one hour overview on focused topics in inverse problems. These lectures were given by: Rainer Kress (University of Gttingen), Yaroslav Kurylev (University College London), and Martin Burger (University of M?nster). More than 80 participants attended the workshop and courses. The lectures were very successful and interesting and provided both introductory material for the newcomers in the area and the more advanced information for specialists of the area. The staff of the Newton institute were very helpful and were essential for the success of the workshop.

2.2 1-5 August 2011 Inverse Problems in Analysis and Geometry
Organisers: Malcolm Brown (Cardiff), Hiroshi Isozaki (Tsukuba), Yaroslav Kurylev (UCL), Lassi Pivrinta (Chair of the Organising Committee) (Helsinki) and Gunther Uhlmann (Washington)

The conference "Inverse Problems in Analysis and Geometry" took place 1-5 August at INI, Cambridge. It followed immediately after the Introductory Workshop on Inverse Problems (25-29 July) and was logically and logistically related to it. The goal of the Introductory Workshop was to provide a detailed picture of the current actively used methods in Inverse Problems. The goal of the conference "Inverse Problems in Analysis and Geometry" was two-fold:

1. To provide a forum for world-leading researchers working at the forefront of theoretical inverse problems to report on their recent activities and results. 2. To broaden the spectrum of techniques crucial to the development of modern inverse problems in addition to those exposed at the Introductory Workshop.

In this connection, a special effort was taken not to concentrate on a particular subarea within theory of inverse problems but rather attract speakers who (a) de-

velop different methods within inverse problems, (b) provide some new unexpected applications of inversion techniques, (c) work in the areas of analysis and geometry related to the future advancement of inverse problems. Thus, the talk by A. Ruiz was devoted to the relations of inverse problems with harmonic analysis with a new important result regarding stability of IP obtained in this direction. The talk by B. Davies was concentrated on the generalization of the classical Ambarzumyan uniqueness result using methods of spectral theory. In the talk by L.Robbiano new techniques within Carleman-type estimates was developed and applied to solve inverse problems. The talk by H. Isozaki was about scattering for asymptotically hyperbolic manifolds and corresponding inverse problems. The talk of U. Leonhardt was about the relations between quantum and electromagnetic invisibility, metamaterials and inverse problems. The above 6 examples (of more than 20 talks presented at the conference) shows the breadth of the methods relevant to the area and made it possible for the participants to envisage possible new correlations between their problems and methods and those reported at the conference. For example, the talk by Leonhardt inspired collaboration between A. Greenleaf, Y. Kurylev, M.Lassas and G.Uhlmann, on one side, and U.Leonhardt, on the other, which resulted in a paper on new development of mathematical theory of cloaking and its physical implications (submitted). The talks by A. Kirsch and A. Ruiz made them understand the complimentarity of their methods and gave rise to their cooperation on developing a technique based on harmonic analysis within the factorization method. The talk by M. Salo about the recovery of the conformal factor in anisotropic inverse problem elucidated the relations between the method of limiting Carleman weights and the Boundary control methods which is currently being investigated by D. Dos Santos Ferreira, Y. Kurylev, M. Lassas and M. Salo. Again, these 3 examples are only a small selection of inter-fertilization due to the exposure to a wide variety of ideas and techniques at the conference.

2.3 22-26 August 2011 Analytic and Geometric Methods in Medical Imaging
Organisers: Malcolm Brown (Cardiff),Thanasis Fokas (Cambridge),Eldad Haber (UBC), Yaroslav Kurylev (UCL), Bill Lionheart (Manchester) and Adrian Nachman (Chair)(Toronto) partially supported by MITACS (Mathematics of Information Technology and Complex Systems) and the University of Toronto.

The Workshop on "Analytic and Geometric Methods in Medical Imaging" took place at the Isaac Newton Institute from 22 to 26 August 2011, as part of the In-

verse Problems programme. The workshop was also organized in conjunction with the 2011-2012 Mitacs International Focus Period on the Mathematics of Medical Imaging. (The Canadian participants were supported by Mitacs and the University of Toronto).

The workshop aimed to establish connections between two distinct areas of research in the Mathematics of Medical Imaging : Inverse Problems (which arise in the development of next generation image acquisition modalities) and Image Analysis (of images obtained with current instrumentation in hospitals across the world). Both areas have seen deep recent analytical and geometric advances, which were the focus of the workshop.

The range of topics surprised even seasoned researchers, who were happy to learn about new developments and discoveries in areas neighboring their own.

For instance, in one novel research direction, techniques from the theory of quasiconformal maps were shown to yield breakthroughs in the registration of biological surfaces, in the keynote lecture by Tony Chan (President of the Hong Kong University of Science and Technology and one of the most cited researchers in image analysis) while in another direction, quasiconformal maps were used in an essential way in the solution of the inverse problem of Calderon in two dimensions for very rough coefficients, presented in the talk by of Kari Astala (from University of Helsinki, one of the top world experts in quasiconformal maps). These developments led to lively discussions between the specialists in the two areas.

The study of geodesics was another fruitful point of contact for the two communities: in his talk, Gunther Uhlmann (University of Washington and University of California, Irvine) explained how deep understanding of geodesic flow led to the solution (in two dimensions) of the classic open problem of recovering a Riemannian metric from distances between the points on the boundary, as well as the solution of open problems in tomography, while on the image analysis side, Laurent Cohen (Universit Paris-Dauphine) showed how computing geodesics can yield novel efficient methods in biological image segmentation.

Further advanced geometric methods in imaging arise in the study of Riemannian metrics defined on Shape Spaces, introduced so as to be able to measure similarities between shapes and perform statistical analysis on data sets for automatic medical diagnostics. The workshop included three lectures by some of the foremost researchers working in this area: Laurent Younes (John Hopkins University, author of the central textbook in the subject), Alain Trouve (cole Normale Suprieure) and Mario Micheli (UCLA).

Graph-theoretic methods in imaging were presented in the talks by Nikos Paragios (Ecole Centrale de Paris) and Jerome Darbon (cole Normale Suprieure).

On the analytic side, the lectures by Guillaume Bal (Columbia University), Otmar Scherzer (University of Vienna), Simon Arridge (University College Lon-

don) and Habib Amari (Ecole Polytechnique) described work on Hybrid Methods, a very active new area in Inverse Problems in which one seeks to significantly improve both the quantitative accuracy and the resolution of traditional approaches by coupling of two different kinds of physical measurements. The talk by Amir Moradifam (University of Toronto) showed the connection between some of these problems and well-known total variation minimization techniques in image analysis. This led to fruitful discussions also involving Adina Ciomaga, Jerome Darbon and Adrian Nachman.

An important analytic theme of the workshop was the research in compressed sensing, reconstruction under sparsity constraints and its relationship to imaging and tomography. Recent results in this area were presented in the talks by Carola Schoenlieb (Cambridge), Honkai Zhao (University of California at Irvine), Matti Lassas (University of Helsinki) and Anders Hansen (Cambridge).

In addition to the 30 presentations on theoretical analytic and geometric work in Medical Imaging, the workshop also featured an "Open for Business" afternoon, which showcased the relevance of some of these developments in clinical and business applications. The guests from various companies invited to the event were excited by the amazing new tissue images being obtained by novel hybrid methods such as photoacoustics, presented by Lihong Wang (University of Washington (winner of the Joseph W. Goodman Book Writing Award for the textbook on Biomedical Optics, of the NIHs FIRST and NSFs CAREER awards, and of the C. E. K. Mees Medal for seminal contributions to photoacoustic tomography). As well, quite a few of the academics present were surprised by the advanced geometrical techniques (involving, eg. Ricci flow and conformal maps) being developed in some medical imaging companies with clinical focus. The animated panel discussion led to further valuable exchanges of ideas, in keeping with the overall atmosphere which the workshop sought to create.

Many of the participants praised this as one of the most interesting and successful meetings they had attended.

2.4   17-21 October 2011 Inverse Spectral Problems in One Dimension (A Satellite Meeting at ICMS, Edinburgh) Christer Bennwitz (Lund), Malcolm Brown (Cardiff) Chair, F. Gesztesy (Missouri), Rudi Weikard (Birmingham AL)

This meeting consisted of a number of keynote talks given by J Behrndt (Technische Universitat Graz), F Gesztesy (Missouri), P Kurasov (Stockholm), M Malamud (Donetsk), S Naboko (St Petersburg), B Rundell (Texas A and M University), G Teschi (Vienna), R Weikard (Birmingham, Alabama) focused on diverse aspects of

inverse spectral problems for one dimensional problems. These included the role of the Dirichlet to Neumann operator, or $m-$function Inverse spectral problems that occur as a consequence of integrable systems: in particular to Comasa Horn problem on the real line and inverse problems on graphs. A feature of the organisation of the meeting, which seemed to be much appreciated by the participants, was to schedule after every 2-3 talks, an extended session in the coffee room where the speakers could engage in an in-depth and structured discussion with other participants.

2.5 12-16 December 2011 Inverse Problems in Science and Engineering
Organisers: Malcolm Brown (Cardiff), Thanasis Fokas (Cambridge) and William Symes (Rice)

The programme included presentations on a wide variety of inverse problems occuring in scientific and engineering applications, including geophysical prospecting and reservoir engineering, radar imaging, x-ray computed tomography, hybrid imaging modalities in biomedical and other applications, statistical estimation in inverse problems, numerical simulation methods, and a number of other topics. Three hour-long tutorial lectures overviewing seismic (Wim Mulder, Shell Global Solutions and Delft University of Technology), radar (Margaret Cheney, Rensselaer Polytechnic Institute), and biomedical (Simon Arridge, University College London) inversion and imaging, began the workshop program. Following these, thirty-seven half-hour talks presented recent research progress on all of the above topics and more.

The program included an afternoon-long Open for Business event on Wednesday 14 December, featuring four excellent overview lectures on "Inverse Problems in Oil and Gas Exploration" by leading research managers in the oil and gas business. Johan Robertsson (Schlumberger Cambridge Research), David Ellis (BP), Henri Calandra (Total E and P), and Fons ten Kroode (Shell International Exploration and Production) gave the participants a broad sense of the size, scope, and economic meaning of oil industry data acquisition and processing, the central role that inverse problems play in the industry, and some of the future prospects and challenges facing exploration technology. The afternoon's activities ended with a lively panel discussion.

One of the themes that emerged from discussions during the sessions is the relative importance power of deterministic vs. statistical methods in inversion and imaging. Another pervasive theme was the importance of high-performance computation and its interaction with analytical developments. Both of these themes will surely be featured in future workshops on applied inverse problems.

3   Publications

The program has lead to many articles published, submitted, or in preparation, full list of which is not available to the organizers at the moment.

4   Further activities

Following from this meeting at Cambridge which showed the importance of long-term collaboration programs in the highly interdisciplinary and rapidly growing area of Inverse problems there have been several succsessful applications for further extended workshops. For example there have been a successful application for a semester in IP at Mittag-Leffler, Stockholm (organising committee: J.Boman, M. de Hoop, P. Kurasov, Y. Kurylev, L. Paivarinta, G. Uhlmann). The semester will be in Jan.-May, 2013. Moreover, directly inspired by our programme, D. Dos Santos Ferreira, C. Gillarmou, J.LeRousseau and M.Lassas (all the participants of our program) are applying for a semester in Paris (tentatively in 2014) at Institut Henry Poincare.

1) A conference at the Paris School of Mines in July "100 years of electrical imaging" in which we will bring together those working in geophysics, medicine and process electrical imaging

2) A study group on inverse problems challenges in land mine detection in collaboration with the charity "Find a better way", which we are planning early summer.

3) A bid for "Building Global Engagements" from Manchester focussed on translating mathematical developments to a broad range of applied problems in tomographic imaging.

4) An application for "Center in Inverse Problems" at UCL was completed during INI to involve a broad range of interdisciplinary research in the UK. This application involved organising of a "town meeting" at UCL on perspectives of inverse problems (29 September, 2011). It attracted more that 60 people from 11 Departments across UCL with a number of participants at the INI playing the leading role. In particular, G. Uhlmann gave a plenary talk on relation between theory of IP and its various applications.

To our knowledge, this program has given rise to many new international collaborative groups bidding for various national and international funding. Summarising, we believe that the program has played a very fruitful role in developing new ideas and directions in IP, promoting the area of IP among a wider range of researchers in mathematics and reshaping IP around the most important and prospective directions of study.