

Metric and Analytic Aspects of Moduli Spaces

Four week program at the Isaac Newton Institute, July 20 - August 15, 2015

The goal of this program was to draw together experts working on moduli spaces arising in geometric analysis and mathematical physics, with an emphasis on new analytic and geometric techniques to study these problems and new perspectives from physics. The program included an intensive workshop in its second week; during the first week of the program, when quite a few people were in attendance, there were two seminars a day, while during the third and fourth weeks there were considerably fewer talks.

There were three main directions of research represented in this program: various types of solitons and their moduli spaces, the classical Teichmueller and Riemann moduli spaces, and the study of gravitational instantons and other Riemannian manifolds with special geometry. Top experts in each of these areas took part, and important new progress was announced in each of these areas.

In the first week, Manton gave a lengthy survey describing his work over the past decade or more on solutions of the Bogomolny equation as well as of the soliton equation on surfaces. Kottke reported on his work with Singer which provides a substantial generalization of the Gibbons-Manton construction of monopoles, and Swoboda and Witt described their joint work with Mazzeo and Weiss about a new construction of solutions of Hitchin equations on Riemann surfaces. Hein gave a survey describing his work on the classification of gravitational instantons in four dimensions, and Guilarmou presented a number of results about the renormalized volume functional on the space of geometrically finite hyperbolic three-manifolds.

During the workshop, dramatic new results about the Weil-Peterson metric on the Riemann moduli space were described by Melrose and Zhu, and by Takhtajan.

In terms of the physical aspects, both supersymmetric and supersymmetry-breaking objects were discussed. Moore discussed his new work on the dimensions of the singular monopole moduli spaces with general gauge group, and Cherkis talked about Kahler potentials on moduli spaces of instantons on ALF spaces, while Maldonado's lecture was on periodic monopole moduli spaces, their geodesic submanifolds, and periodic monopole scattering probing the interior of these moduli spaces.

Moduli spaces of quantum gauge theories, in particular, Coulomb branches of three-dimensional theories with eight supercharges were central to the talks of Amihay Hanany, Hiraku Nakajima, Greg Moore, and Sergey Cherkis. The proper mathematical definition of these spaces is currently emerging, and was described by Nakajima, as well as their relation to the classical moduli spaces of singular monopoles (Moore), bow varieties (Cherkis), and quiver varieties (Hanany). These are all important advances in both geometry and theoretical physics.

There were a number of researchers representing closely related fields, and presenting

their work. One of the most dramatic new results announced here was by LeBrun, who with Hein has succeeded in extending the positive mass theorem to the setting of Kaehler manifolds, as well as to ALE spaces. This work had commenced some months earlier, but the bulk of the research and writing on this project took place during the program.

Hunsicker described her work on representing weighted cohomology by harmonic forms, which has relevance to the study of harmonic forms on monopole moduli spaces. Biquard described the new class of folded hyperKaehler metrics and how they arise as the limit of the Hitchin equations as n tends to infinity. A different interpretation of this same n tends to infinity problem was the subject of Hitchin's lecture.

The program was very successful in providing a clear view of the current state of these various problems, and even more importantly, of exposing researchers in one area to the important questions and advances in other areas. The exit survey provided evidence of numerous new interactions and collaborations which began during this program and a follow-up workshop will take place at the Banff International Research Station in August 2017.