Final report on the Newton Institute Programme

Mathematical theory and applications of multiple wave scattering

04 January – 30 June 2023

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1. Brief Background and Historical Information

Multiple wave scattering is a vibrant and expanding research area of interest to mathematicians, physicists, engineers and others concerned with the properties of waves in complex materials. It is both profoundly mathematical and deeply practical, with applications from imaging methods in medicine to materials science. The mathematical theory has a long history and was studied by the luminaries of 19th-century science, such as Maxwell and Rayleigh. The current mathematical challenges are extensive, ranging from the modelling challenges of complex metamaterials to the numerical difficulties associated with massive scattering simulations.

Within recent years, the mathematical theory of multiple scattering has undergone considerable advancements. New applications, especially in metamaterials, have greatly expanded the types of scattering problems considered. Seminal theories, such as the Pendry superlens or invisibility cloak, have found many practical applications and realisations. Computational methods can now handle hundreds of thousands of scatterers. Powerful asymptotic homogenisation methods have been developed for both structured and random scattering problems. Advances in inverse scattering for image reconstruction have led to new applications in transient elastography, supersonic shear imaging and multi-wave imaging.

The mathematical theory of multiple scattering is widely applicable, which has led to detachment of communities. One critical purpose of the MWS programme was to bring together researchers from diverse communities, which can be divided according to discipline (electromagnetism, hydrodynamics, seismology/elasticity, acoustics, etc.), and subdiscipline (metamaterial design, homogenisation techniques, inverse methods, computational methods, etc.). The aim was to disseminate knowledge of the current state-of-the-art and to exchange information about the most pressing research questions.

2. Programme Timeliness, Scope and Outline

Such an exchange was timely and required, as the same or similar methods were used for solving problems by different research communities. For example, different approximation methods are used in physics, mathematics and engineering and even within subdisciplines, such as Mie scattering, the Born approximation, homogenisation theory or probabilistic approximations. The precise assumptions used may have different meanings and implications in different contexts, and it is often unclear how these assumptions are related and which method works best in which circumstances. The plan for the programme was to provide an interdisciplinary joining of forces, empowering researchers to address critical developments most directly. Over 100 programme participants were involved, including mathematicians, engineers and physicists, from the UK, Europe, North America, Asia, Africa, Australasia … and Cambridge.

The key to multiple scattering is to exploit symmetry or structure within the underlying problem. In this simplest case, this leads to the celebrated Bragg scattering and this idea has been extended in numerous directions. However, the scattering objects themselves have become highly sophisticated, especially in the context of metamaterials, and the methods require an extension to cope with this complexity. Concurrently, applications such as in medical imaging, chemical engineering and modern complex composites require the development of methods that can accommodate different degrees of disorder. Even in the case of metamaterials, there will exist some imperfection in the fabricated structures. Therefore, an essential and pressing question is how stable the effects are with respect to perturbations. On the other hand, natural materials are often approximated as having a specific structure (e.g., periodic) to make the problems amenable to powerful (analytical or numerical) solution methods, even though they do not genuinely possess this structure (e.g., porous media or vast fields of ice floes in the marginal ice zone). Sometimes these approximations lead to good results, and sometimes they give entirely wrong answers. Therefore, it is indispensable to understand the limits of the estimates. Both issues arise in all application fields, such as photonics, phononics, acoustics, elastic crystals and water waves.

Therefore, the main scientific aim of the programme was to elucidate the fundamental mathematical aspects of wave scattering in complex media. We considered a variety of contexts, which are governed by diverse equations, aiming for an understanding of the commonalities. The overarching aims of the programme were twofold:

- To bring together internationally leading experts in multiple wave scattering to consolidate recent developments and to catalyse new links between them.
- To enable early-career researchers to meet and interact with established researchers, especially through a winter school and the four workshops.

The investigated topics were divided into four main themes: Canonical scattering problems; Theory of wave scattering in complex and random media; Computational methods for multiple scattering; and Multiple scattering in engineering and applied sciences. Besides being elucidated at over twenty weekly seminar talks, each of these topics was addressed at a designated workshop.

Winter School (MWS05)

09–13 January 2023

Organisers: I. David Abrahams (Cambridge), Luke G. Bennetts (Adelaide), Michael H. Meylan (Newcastle)

The programme was kickstarted (in its second week) with a Winter School designed for early-career researchers, but also appreciated by the more experienced programme participants. Three leading experts presented the fundamentals of their mathematical approach to multiple scattering problems over the course of the week.

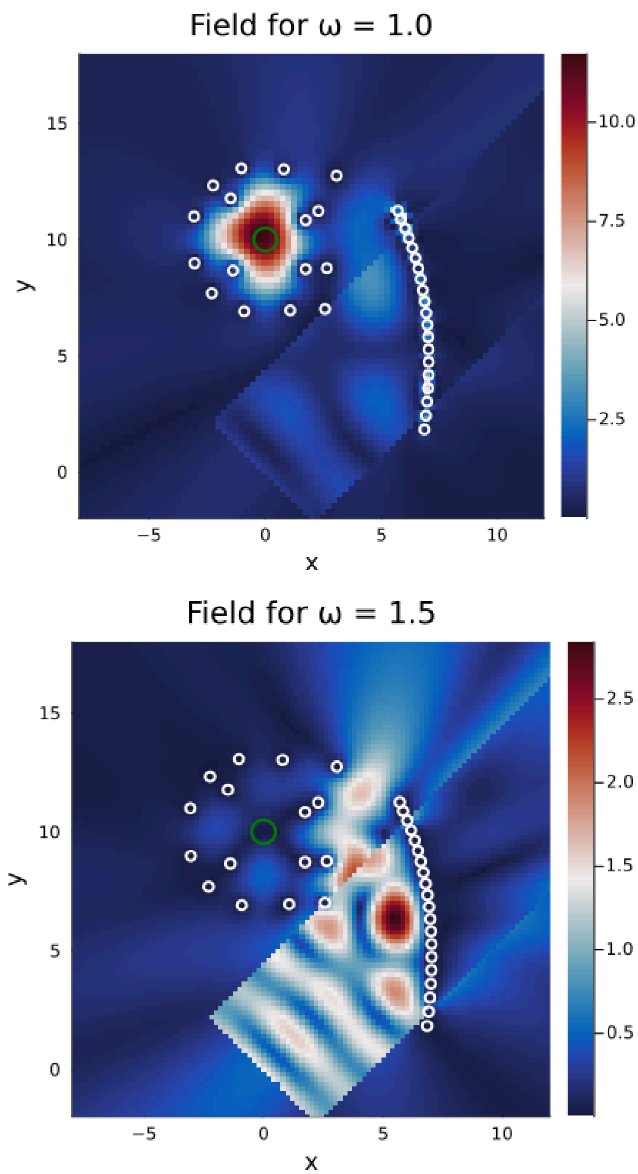
Prof Paul Martin (Colorado School of Mines, USA) presented the fundamentals of scattering in systems of arbitrary numbers of scatterers, from underlying PDEs to boundary value problems and radiation conditions, integral representations and T-matrix methods.

Dr Sonia Fliss (ENSTA Paris Tech, France) introduced methods for the analysis of wave propagation in unbounded media, in particular the scattering in Helmholtz problems, due to a local perturbation in a periodic medium. Appropriate radiation conditions were formulated and Dirichlet-to-Neumann operators were demonstrated.

Prof David Abrahams (University of Cambridge, UK) presented homogenisation methods for multiple wave scattering, for applications in electromagnetics and acoustics, solving illustrative problems in 1D for effective reflection coefficients and effective wavenumbers to highlight important features of such problems in more complex materials.

ECRs were given the opportunity to attempt a hands-on design problem in multiple wave scattering using two pieces of numerical simulation code, kindly provided and supported by Dr Stuart Hawkins (Tmatrom3; Macquarie University, Australia) and Dr Artur Gower (Wave Software; University of Sheffield, UK). The Newton Gateway awarded prizes to the ECRs for their project outcomes and their oral presentations.

Finally, a collection of industrial and academic talks set the context for the multiple scattering theory by highlighting a range of applications for these theories in diverse sectors, from radar to food.



Two solutions from the winning team of ECRs in the hands-on design challenge. The images show an optimised arrangement of scatterers (white circles) that maximise the wave field at the target region (green circle) on frequency 1.0 (top panel) and minimise it at frequency 1.5 (bottom).

Canonical Scattering Problems (MWS01)

06–10 February 2023

Organisers: Raphael Assier (Manchester), Viktoriia Babicheva (New Mexico), Michael H. Meylan (Newcastle)

The main aim of this workshop was to discuss recent developments in analytical methods for wave scattering theory, whilst ensuring that numerical and experimental aspects were discussed as well. The word `canonical' was to be understood in the broad sense of wave scattering by simple obstacles. Everyone played the game and fitted the brief very nicely.

Various themes were treated, notably scattering within periodic media, discrete lattices or networks, high-frequency asymptotics, applications of complex analysis and multidimensional complex analysis to diffraction theory through integral transforms, and how to describe deterministic waves through solving stochastic differential equations. One afternoon was dedicated to experimental work in light scattering. The three talks preceding the formal dinner, given by eminent figures of the field (Abrahams, Martin and Smyshlyaev), were, by design, dedicated to give a historical perspective to scattering theory.

We had 30 speakers from 7 countries (UK, France, India, New Zealand, Russia, USA and Italy) and about 70 participants in total. We also had posters that were showcased throughout the week. It brought together recognised international leaders and early-career researchers. We are grateful that the more senior researchers really took the time to interact with those more early in their career in such an inspiring yet friendly way. Rare enough to be mentioned these days: all talks took place in person and lively discussions happened. However, we could not quite reach full agreement on a proper definition of the words "canonical" or "diffraction"!

One of the objectives of the workshop was to inspire participants to apply the advanced theoretical techniques presented to significant practical problems. Time will tell if this does indeed happen, but through the conversations overheard during (and since!) the workshop between theoreticians, experimentalists and engineers, we are confident that this will be the case.

Overall, we felt it was a great success and that the workshop consolidated an already diverse and vibrant community.

Theory of wave scattering in complex and random media (MWS02)

20–24 March 2023

Organisers: Artur Gower (Sheffield), Valerie Pinfield (Loughborough), Katherine Tant (Strathclyde), Kevin Vynck (Lyon)

Waves propagating in complex and random media are key to many areas of physics and engineering. Although the theoretical tools to understand wave phenomena can be applied to all linear waves, such as elastic and electromagnetic, the associated research communities are somewhat disconnected. The central aim of this workshop was to bridge the gap between researchers across disciplines and applications and encourage crossover of research methods and tools. For instance, various techniques such as diagrammatic approaches, strong fluctuation theory and stochastic differential equations have been employed in addressing a common problem, namely, determining the moments and distribution of waves propagating through random media. Through the workshop, we gained insights into the merits and limitations of these methods. The workshop also delved into different themes, such as scattering from random particulates, wave localization, the impacts of correlation in random media, variational approaches and leveraging methods from periodic lattices to gain insights into phenomena in random materials.

The diversity in participants' expertise and background was clear throughout the week and this was positively commented on regularly. As the workshop progressed, more cross-disciplinary references were made within the talks indicating a good degree of cross-fertilization between participants across the different research areas. Whilst communication across different disciplines can be challenging, and takes time to develop, this workshop was a valuable step towards developing a shared understanding and language. Wednesday afternoon was dedicated to informal discussions and researchers made good use of the time. A 'poster pitch' session (rapid 2-slide, 2-minute talks) was held prior to the posters being displayed and discussed on Tuesday afternoon and this was particularly successful in a) generating interest in the individual posters and b) allowing those not speaking to introduce themselves to the group. The poster pitch was extended to anyone not otherwise doing an oral presentation at the meeting, so that all who attended had the opportunity to present their work to all the workshop participants. Following the poster session, an informal social event was held at the Institute for early-career researchers, to support networking building and sharing of ideas and experiences. Although Covid19 continued to loom in the background, with some participants unable to join in person due to the presence of a few positive cases, it was clear that the participants enjoyed being able to connect in person once again. Only one talk (Prof K Solna, University of California, Irvine), was delivered online and although this went smoothly, due to time constraints, there was no opportunity for live interaction with the speaker. This validated the organisers' decision not to offer a remote presentation option as standard.

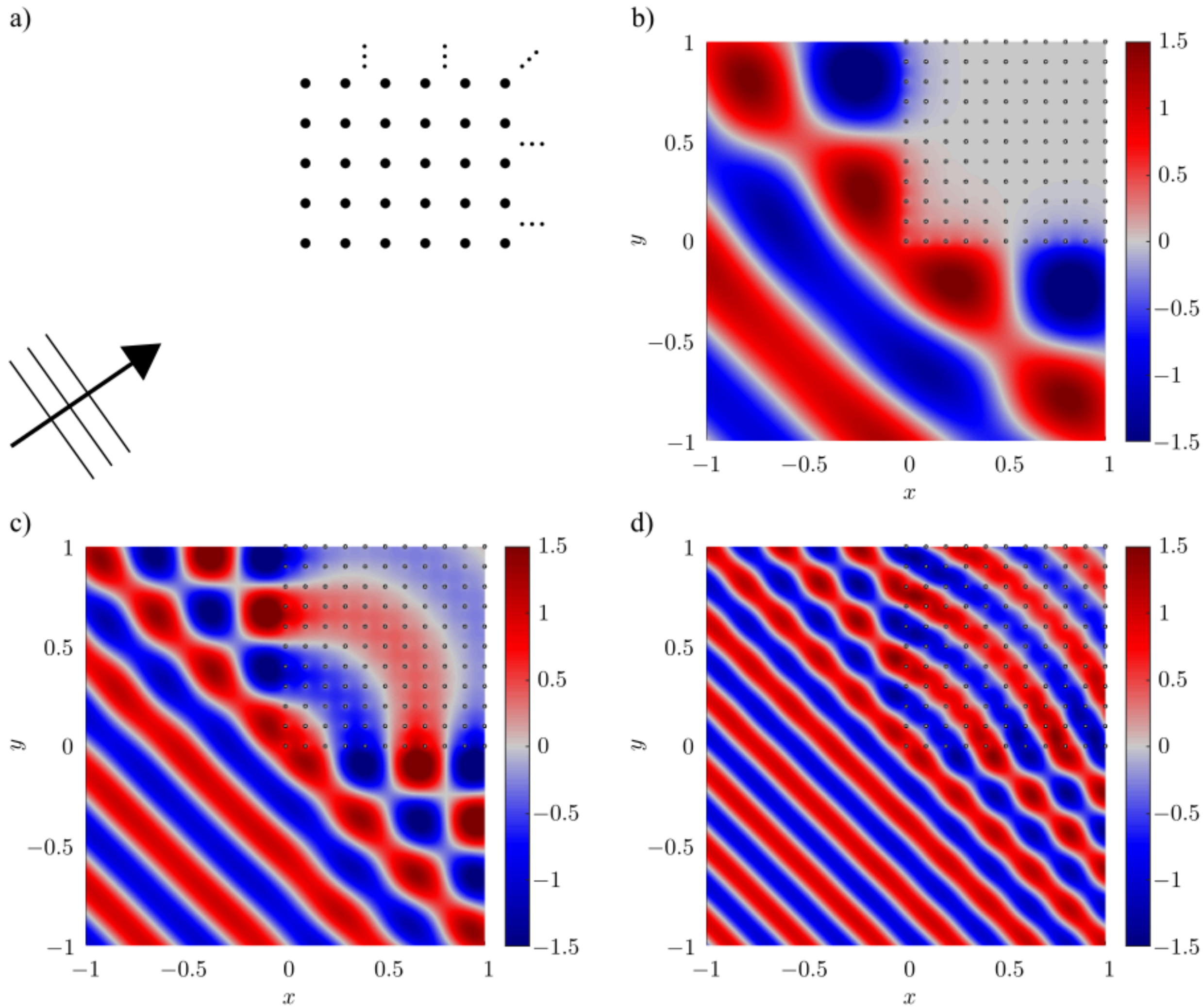
Highlights of the week included the Kirk lecture, delivered by Agnes Maurel on "Control of Water Waves by Metamaterial-Based Devices" and the formal workshop dinner which was held at Christ's College and much appreciated. Several of the workshop participants were also programme participants, so the workshop acted as a catalyst for discussions and collaborations which continued throughout the remainder of the programme. The workshop built on the foundations of Workshop 1 on Canonical Scattering Problems since these problems form the basis for the multiple wave scattering effects explored in this workshop. Future workshops explored the numerical methods in greater depth (WS3), and science/engineering applications (WS4).

Multiple Wave Scattering @ British Applied Mathematics Colloquium

03–06 April 2023

Organisers: Matthew Nethercote (Manchester), Kevish Nepal (Sheffield)

Two of our early-career researchers ran a mini-symposium at the British Applied Mathematics Colloquium in Bristol, with talks from eleven of the programme participants.



Multiple scattering problem where an acoustic pressure wave is impacting on a doubly periodic quarter lattice of simple circular cylinders with sound-soft boundary conditions, as presented at the BAMC MWS mini-symposium. Panel (a) is the problem schematic and panels (b–d) are solutions for the total pressure field where the incident wave field is at three different frequencies, including ones within a stop band and a pass band (Nethercote et al., J. Acoust. Soc. Am., doi.org/10.1121/10.0020844).

Computational Methods for Multiple Scattering (MWS03)

17–21 April 2023

Organisers: Alex Barnett (Flatiron Institute), Sonia Fliss (ENSTA Paris), David Hewett (UCL), Malte A. Peter (Augsburg)

This 5-day workshop comprised 26 talks covering the latest research on numerical methods to simulate the interaction of waves with geometries that induce multiple scattering. An overarching goal of such methods is to approximate a given number of correct digits in the solution wavefield in the fastest computation time. The geometries discussed in the workshop included:

- A large number of simple scatterers (Hawkins, Giovangigli)
- One or a small number of complicated scatterers with realistic geometries from industry or medicine (Betcke, Rachh, Chaillat, Ganesh, Urzua-Torres)
- Spatially periodic or unbounded geometries such as corrugated surfaces and waveguides (Chandler-Wilde, Zhang, Delorme, Felix)
- Spatially “quasiperiodic” media—a deterministic model for disorder achieved by slicing a higher-dimensional function (Joly, Amenoagbadji)
- Singular geometries such as fractal screens that scatter at all length scales (Moiola, Gibbs) or screens meeting at non-Lipschitz junctions (Urzua-Torres)
- Water waves interacting with solid or elastic structures with depth (Bonnet-BenDhia, Meylan)
- Resonant cavities with strong trapping (Spence)
- Potentials for the Schrödinger equation (Peterseim)
- Anisotropic elastic media with unusual Green’s functions (Lu)

The application areas included underwater acoustics, optics, electromagnetic antennae, metamaterials and screens, plasma physics, ultrasound, water waves and Bose–Einstein condensates.

Unusual for a single workshop in applied mathematics, there were representatives pushing the state-of-the-art on several distinct numerical approaches, varying from low-order to high-order to asymptotic. These included:

- Finite element and finite difference methods (Imbert-Gerard, Verfuerth, Urzua-Torres, Fortino), including their fast direct solution (Gillman)
- Boundary integral or boundary element methods, aka BEM (Rachh, Chandler-Wilde, Bonnet-BenDhia, Betcke, Hawkins), and their high-frequency asymptotics (Langdon, Spence)
- Volume integral, aka Lippman–Schwinger, equations (Perez-Arancibia)
- Modal expansion methods in space (Felix) or time (Meylan)
- Homogenisation of small-scale structures by asymptotic matching (Peterseim, Giovangigli)

The workshop provided a rare chance for these sometimes inward-facing numerical communities to interact, and the broad audience ensured that each method was to some extent overviewed. As expected for the high-order methods, accurate and efficient quadrature schemes for integrals and integral equations was a common theme. Modern methods for unbounded geometries—needed for example when a half-space is locally perturbed—were presented, including “perfectly matched layers” (PML), the closely related “complex scaling” (bending physical coordinates into the complex plane) and operator Riccati equations for the response of a semi-infinite periodic waveguide. There was (as expected) an interplay between frequency-domain and time-domain approaches, with the latter solved in many presentations via convolution quadrature (combining a set of complex-frequency-domain solves). For example, Chaillat presented a case where such a time-domain acoustic solver had to be coupled with a “black box” industrial time-domain code for the elastic response of a submarine, leading to a global-in-time coupling iteration. In the other direction, Fortino showed how an existing time-domain solver can be used as a “linear filter” to solve frequency-domain scattering problems iteratively.

Some talks were largely numerical analysis, such as high-frequency finite-element asymptotic errors (Spence) and coercivity of new rough-surface BEM formulations (Chandler-Wilde). The limiting-absorption principle came up in many settings where waves were confined. Other talks were very applied, focussing on industrial problems, high-order convergence and correct digits achieved, and fast algorithms (computational effort close to linear scaling in the problem size). There was a mix of early-career and senior speakers, and participants from all six continents apart from Antarctica.

We received several positive comments about the format (40 minute talks, 5 minutes for questions and change-over, at most two talks in a block, and lengthy coffee and lunch breaks) which allowed speakers to get into some of the technical depth, while still giving time for plenty of interaction between participants.

MWS04 Multiple Scattering in Engineering and Applied Sciences

22–26 May 2023

Organisers: Malin Goteman (Uppsala), Vicente Romero Garcia (Valencia), Daniel Torrent (Jaume I), Olga Umnova (Salford)

This workshop brought together researchers applying multiple scattering methods to solve various problems in engineering and applied sciences. The application areas included, among others, mechanics and thermal physics, interaction of electromagnetic waves with matter and acoustics. The workshop comprised 35 talks, 45 minutes each, with only three of them being presented remotely. Topics covered by the workshop could be roughly divided into the following themes.

1. Wave interactions with metamaterials, metasurfaces and periodic structures

Various types of metamaterial (acoustic, EM, water and elastic) were considered. Several talks were devoted to topological effects in mechanical chains, elastic structures and chiral waveguides. Nonlinear models were presented for elastic prestressed materials and mechanical chains where a phase including an edge mode was shown to appear as the amplitude increased. A multitude of approaches to problem solutions was demonstrated including simplified effective medium approximation, homogenisation of periodic media, Foldy model, jump conditions for interfaces and numerical models, to mention just a few. The applications mentioned in the talks included design of ocean-wave energy parks, noise mitigation from underwater structures, moving objects in scattering media, therapeutical ultrasound and structures designed to delay transition to turbulent flow.

2. Waves in disordered and hyperuniform media

Several talks dealt with multiple scattering effects in random media. Hyperuniform composites exhibiting complete bandgaps in random structures attracted significant attention. Propagation of elastic waves in random distribution of scatterers was considered in several talks, using mostly numerical approaches. Another interesting application of numerical models included investigating chaotic behaviour in nonlinear disordered lattices. Propagation of waves in bubbly media were considered using classical multiple scattering models and showing that the Foldy model is accurate enough for many purposes. Interesting applications of the models presented included manipulation of the bubbles in water, design of composite materials for EM wave manipulations and phonon amplification via Anderson-localised modes.

3. Inverse problems

Approximately one quarter of the talks dealt with multiple scattering phenomena in imaging problems. The reconstruction of the wave speed profiles in a generic piecewise medium from the reflected pulse using echo-to-impedance algorithm was considered in one of the talks. The incident wavefront shaping to improve the resolution of the image was another approach presented. Data-driven models were presented in several talks, e.g., the use of a generative deep learning approach for shape recognition. Reduced-order modelling was another approach discussed. For instance, Data-to-Born transform was presented in one of the talks which means construction of the highly nonlinear transform taking full waveform data to single scattering data. This approach can be used for pre-processing and integrated into existing imaging workflows. Reduced-order modelling was also the topic of the public lecture given by Rothschild fellow Liliana Borcea on Wednesday afternoon.

Perhaps unusual for a mathematics programme, many presentations included descriptions of the laboratory and even clinical experiments. Posters were exhibited throughout the week and discussion continued during the coffee/tea breaks. There was a mix of established scientists and ECRs (including PhD students) among the speakers and also representatives from North and South America, Australia, Europe and Asia.

Wave scattering in complex matter: Advances in material characterisation and design of materials (OFBW60)

31st May 2023

Organisers: Valerie Pinfield (Loughborough), Olga Umnova (Salford)

This one-day Open for Business event was organised by the Newton Gateway and aimed to build links between mathematical formulations of wave scattering, in direct and inverse forms, and practical implementations, focussing on acoustic, elastic and electromagnetic waves. The event explored the latest innovations in imaging at different length scales (X-ray imaging and radar), material characterisation (by ultrasonics and microscopy) and design of novel materials for defence applications and communications (optical fibres). A broad range of applications was addressed, from non-destructive testing of pipes, analysis of food structure, ultrasonic characterisation of liquids and sound-absorbing materials for defence applications.

Kirk Distinguished Visiting Fellow

The MWS programme Kirk Fellow was **Professor Agnes Maurel**, CNRS Senior Researcher (Directrice de Recherche) at the Institute Langevin in Paris, France. She participated in the MWS programme for several months and gave a Kirk public lecture entitled Control of Water Waves by Metamaterial-Based Devices on 23rd March 2023, as part of Workshop 2 (MWS02). She is author/co-author of 120 articles in peer reviewed journals and 8 book chapters dedicated to problems of wave propagation in complex media. She graduated in physics from Ecole Normale Supérieure of Lyon and was subsequently awarded her PhD in Fluid Mechanics in 1994 from Université Pierre et Marie Curie (now Sorbonne Université) studying jet instability.

Professor Maurel has a strong track record of research in both the mathematical theory and applications of wave propagation problems. Her work demonstrates a sound theoretical approach to address problems with mathematical techniques, combined with the pursuit of their practical application in real systems, including the development of experimental investigations and methodologies. Professor Maurel's research has focussed on three overarching themes, each of which relates to our programme: (a) *elastodynamics*: the properties of wave propagation in elastic media with singularities, stretched vortices and dislocations and the development of inverse methods for the characterisation of such materials; (b) *water waves*: control of linear and non-linear swell and the development of an experimental characterisation method for water surface profiles and various trapped mode, time reversal and Helmholtz-analogue systems; and (c) *metamaterials and metasurfaces*: multi-scale homogenisation methods with matched asymptotics to derive effective properties, with diverse applications in acoustics, electromagnetism, elastodynamics and water waves.

Professor Maurel's talk focussed on methods for controlling water waves using many resonant docks (cavities) in a linked structure forming a metasurface or metainterface designed to protect a region inside the structure by creating a quiet zone for water waves despite the conditions outside the interface. Professor Maurel presented the mathematical formulation to design such a system, starting with the single resonant element (dock) and progressing to the metamaterial of many resonators by applying a homogenisation in one direction combined with a Bloch–Floquet analysis in the other.

Rothschild Distinguished Visiting Fellow

The MWS programme Rothschild Distinguished Fellow was Professor Liliana Borcea from the University of Michigan. She participated in both MWS and the Rich and Nonlinear Tomography programme. She gave her Rothschild public lecture entitled When Data Driven Reduced Order Modelling Meets Full Waveform Inversion on May 24, 2023, as part of Workshop 4 (MWS04).

She is research leader in the fields of wave scattering in random media and effective properties of composite materials. She has published over ninety, highly cited journal articles, of which she is the lead author of over 80 articles, and has received multiple research awards, such as being chosen as the 2017 AWM–SIAM Sonia Kovalevsky Lecturer. More generally, she is a leader in the applied mathematics community, particularly through her prominent roles in SIAM, for which she is on the Board of Trustees and the Editor-in-Chief of the SIAM Journal on Multiscale Modeling and Simulation.

Professor Borcea's research interests are motivated by her view of applied mathematics as a science that reaches across disciplines to physics, statistics and numerical simulations. Her recent work has two themes: (i) wave propagation and sensor imaging from wave scattering in complex media; and (ii) reduced order model approaches to solving inverse problems. These are areas in applied mathematics with a broad spectrum of applications and unique challenges. Her goal is to advance the physical knowledge of these problems and to develop rigorous mathematical theory and efficient algorithms for their solution.

Professor Borcea's talk focussed on the problem of extracting information about the wave speed map of a region from sensor data detecting backscattered signals received subsequent to a transmitted signal. The problem is of relevance in geophysical exploration, radar imaging, non-destructive evaluation of materials, etc. The use of velocity estimation methods using PDE-constrained optimisation has accelerated in line with increased computational power, but the optimisation problem is non-convex and subject to many local minima even before experimental noise is considered. Professor Liliana presented a method for the development of a reduced order model for the wave operator, obtained directly from the observed data. The technique leads to an improved objective function that enables gradient-based optimisers to achieve velocity estimation successfully.

3. Scientific Outcomes and Highlights

The MWS programme met all of its goals set by the organisers and INI. It had outstanding success in forming new collaborations (far too many to list in this report). Some have already resulted in research articles and many other collaborations are moving towards outputs. In particular, numerous new cross-disciplinary collaborations were formed, which was strongly needed to overcome the detachment of communities outlined above. Even where no formal collaborations have resulted, the programme fostered cross-fertilisation of fields as intended. All workshops and other activities strongly contributed to this success. More details on the outcomes and highlights are given below.

At the end of the programme, we polled the participants to gain their input on the most useful outcomes of the MWS programme. The top responses were: knowledge transfer across different branches of wave science; creation of a unified language across the wave science branches; rediscovering of problems (e.g., engine noise); discovery of new problems arising from applications of models and methods in other areas; education of early-career researchers (and some more senior researchers) about applications for mathematical theories.

Technical Outcomes and Highlights

The technical outcomes were centred around the themes of the four Workshops, where each workshop was designed to address different goals of the programme. Below, we highlight one main outcome from each workshop.

Workshop 1 identified a strong need for a common set of canonical problems (and their analytic solutions) in the different application areas involving fully multiple wave scattering, as benchmarks for advanced numerical methods in particular. This would also be an important step towards transferring the methods for canonical problems to the solutions of significant practical problems.

Workshop 2 highlighted the strong and often unexpected effects of the degree of disorder in a multiple scattering medium on its properties, ranging from perfectly ordered to fully random, and the need to understand these behaviours in a unified setting to empower design strategies in applications. Moreover, the workshop provided the basis for consolidating the diverse methodologies used by different communities who treat different degrees of disorder.

Workshop 3 showed a strong positive impact of communications between numerical and analytical communities and increased understanding of the progress being made on both sides. On the one hand, numerics benefits from incorporation of analytical (and semi-analytical) methods. On the other hand, analytical approaches are being greatly advanced by numerical testbeds available through open-source software. The Workshop (and, more broadly, the MWS programme) demonstrated how progress in multiple wave scattering is accelerated by collaborative inputs from those with expertise in the numerical and analytical aspects of the problems.

Workshop 4 served as the apex of the MWS programme, by bringing together researchers from the different application areas of multiple wave scattering, and from different disciplines (engineering, physics and mathematics) to share the latest advances in their fields. In addition to a clear desire from all to learn from one another, there was recognition that the most rapidly advancing areas were developing mathematical theory and physical experiments simultaneously.

Broader Outcomes and Highlights

Alongside the cross-fertilisation of wave science fields, a particular success of the MWS programme was the integration of early-career researchers. The Winter School kick-started the intent of the programme in this direction by forming a close community between the ECRs and facilitating interactions with more senior members of the programme. By opening applications for the Winter School to any ECRs interested in its content and advertising broadly, we attracted many ECRs we were previously not connected with. Moreover, by scheduling the winter school at the very beginning of the programme (early in the new year), we were able to encourage many of the ECRs to conduct longer-term participation in the MWS programme. In total, we had a large proportion of ECR programme participants, some of whom will likely be the future leaders of the field. Some of the ECR participants have already found jobs through the contacts they made at the programme.

Beyond the regular workshops within the programme (detailed above), some of the stand-out features of the MWS programme that have brought the community together and generated outcomes were:

- We were able to combine INI's residential long-term character with the open-plan space we had available for the programme to create a close collaborative atmosphere. In particular, all programme participants joined a communal "coffee break" every morning, complemented by activity on the blackboards, and with at least one scheduled (but informal) presentation per week during the breaks (usually from an early-career participant).
- The hands-on design challenge during Winter School was a great success, which all ECRs actively engaged in (along with some of the more senior programme participants), encouraging them to put their knowledge of mathematical theory into practice, work in teams and present their findings. The code used during the challenge also served as the basis for collaborations during the programme.
- The MWS programme had a strong presence at other major meetings, including the formal MWS mini-symposium at the BAMC in Bristol (see above), but also many programme participants presenting at meetings, particularly at Waves and Imaging in London in March, Phononics in Manchester in June and Mathematics and Mechanics of Solids and Structures in Aberystwyth also in June.
- There were strong interactions with the other two programmes taking place in parallel (Rich and Nonlinear Tomography and Data-driven Engineering). Many participants from these other programmes attended our weekly seminars and workshops (as well as our

participants attending their seminars and workshops), and the programme jointly ran a one-day event based around short presentations from ECRs involved in all programmes.

- There will be a Special Feature of *Proceedings of the Royal Society A* dedicated to research outcomes from the MWS programme, with an emphasis on collaborations developed during the programme. At the time of writing, ten articles have been submitted or accepted, and five more submissions are expected soon.

In addition to asking the MWS participants about the programme successes (see above), we also asked them about the outstanding challenges in the field and promising future directions. The responses can be summarised as follows.

Mathematical challenges:

- Finding a unified approach for treating degrees of disorder that encompasses perfectly ordered systems to fully random.
- Moving away from overreliance on standard geometries (usually axisymmetric), such as resonant scatterers used in metamaterials.
- Better connecting current theories for very large multiple scattering systems (including advanced asymptotic methods and multi-dimensional complex analysis) with state-of-the-art computational methods, with a particular focus on the discrepancies caused by comparing infinite media (common in theoretical approaches) with finite media (as required in most computations).

Future applications foci of multiple-wave scattering :

- Quasi-periodic and time-varying metamaterials.
- Active matter, particularly in bio-engineering applications.
- Optimisation and inverse problems, leveraging on commonalities established with members of the Rich and Nonlinear Tomography programme.

Publications

Details, commentary and photos for all of the non-workshop talks during the MWS programme are contained in a Google Document. (Details and photos for the workshops can be found here.)

Research resulting from collaborations formed during the MWS programme will appear in a Special Feature of the *Proceedings of the Royal Society A*. We expect the Special Feature to be completed in the second half of 2024.

Media

Almost all of the formal presentations given during the programme (i.e. at the winter school, the workshops and the weekly seminars) were live streamed and made publicly available permanently. The programme was also featured in a *Plus Magazine* collection, containing articles and a podcast.

Future activities

There is a strong demand from the participants for a follow-on workshop at INI. Following the above Scientific Outcomes and Highlights, a likely focus for the workshop will be metamaterials, including fractal and quasi-periodic variants and imperfections, leading to an allied focus on degrees of disorder. Some participants proposed smaller, related programmes of focus topics at international venues, e.g., MATRIX in Australia. We hope that this will lead to a future INI programme, possibly preceded by a satellite programme, with Manchester a likely host.

Acknowledgements

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