Short report on the Newton Institute programme
– *New connections in number theory and physics* –

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Brief summary

The purpose of this interdisciplinary programme was to initiate novel scientific interactions between various fields of mathematics and theoretical physics, with particular emphasis on connections between number theory (modular forms, automorphic forms, mock modular forms) and string theory.

Report

During the 20th century, interactions between geometers and theoretical physicists were revolutionary in both areas, leading to such unexpected developments as mirror symmetry and the impact of topological field theory on invariants of

4-manifolds. Another momentum is now developing around new connections between number theory and physics, mainly but not exclusively through the theory of automorphic forms. We have reached a time when contacts between number theorists and physicists have the potential for another revolution.

This field allows cross-disciplinary collaborations with great benefit to both areas. Recent work has revealed, on the one hand, a profound significance of automorphic forms and automorphic representations for string scattering amplitudes and, on the other hand, an unexpected appearance of mock theta functions in answering key questions pertaining to quantum properties of black holes. There has also been a surge of interest in analyzing the integrands of string theory amplitudes. In the case of genus one string amplitudes these integrands are doubly periodic functions on the torus and have been dubbed "modular graph functions." Mathematically, this gives rises to a new class of modular objects which exhibit interesting special values. A full understanding of these relations is an open problem and was one of the topics discussed at the programme.

The event was kick-started with a workshop designed to introduce the key topics and facilitate interactions and collaborations. The workshop featured a series of cross-disciplinary introductory lectures, which provided seeds for discussions that could be further cultivated during the remainder of the programme. The main themes included automorphic forms, representation theory, the Langlands program, string scattering amplitudes and non-perturbative string theory.

During each non-workshop week, there were one or two formal daytime seminars, leaving time for informal discussions and collaborations. In addition, we organized a very successful informal evening seminar series at the Møller Institute which was well-attended. Another highlight of the programme was a recurring discussion session on "QFT for mathematicians." Here the aim was to facilitate interactions between physicists and mathematicians, using the framework of quantum field theory as a unifying canvas.

A number of new collaborations were initiated and others were continued during the programme. We view this programme as part of a series of events over the course of several years. It was preceded by an online scientific workshop in 2021 hosted by the Newton Institute, and we plan to have recurring follow-up events in the coming years. To build collaborations across scientific boundaries requires sustained opportunities for interactions. This workshop enabled us to build a strong foundation for new interactions between mathematics and physics.