

NON-POSITIVE CURVATURE: GROUP ACTIONS AND COHOMOLOGY

REPORT

The goal of the programme was the study of non-positive curvature, its relationship with rigidity problems (of representations with respect to deformation, of structures, preserved by loose equivalence relations, or with small groups of automorphisms etc), with the opposite situation of very large spaces of representations (like in Teichmüller theories, old and new), the way in which the geometry of non-positively curved spaces reflects on groups acting on them, etc. These topics lie at the nexus of a number of interacting areas of algebra, geometry and analysis, currently at the forefront of mathematical research. This made the research programme both timely and influential. The problems that the programme planned to tackle naturally grouped into four main themes: $CAT(0)$ groups and coarse embeddings; fixed point properties, proper actions and amenability; stability of homomorphisms; bounded cohomology and its applications. Each theme was the subject of a workshop, with 60 to 80 registered participants. In addition, a satellite workshop held in Oxford was destined to young researchers and had a record number of participants: over 170 registered, many more actually following the mini-courses. The main programme too was particularly well attended by young researchers who co-authored several strong results. At our suggestion, two Isaac Newton Prizes for Early Career Researchers were created, to be awarded to the most significant preprint (co-)authored by a young participant.

The central theme bounded cohomology had not yet been part of active research in the UK, despite its great interest, and the fact that the existing areas of expertise made it very accessible to the UK mathematical community. The programme addressed this issue, in a specialized workshop and through series of lectures and talks. The other three central themes were already widely popular in UK, and therefore were from the start at the core of interactions between UK and international experts, from areas ranging from the well established geometric and analytic group theory to its more recent incarnations, in its measured and differential aspects.

The response of the community to the programme was impressive: throughout its duration the programme was attended to the maximal capacity of the working and accommodation space offered (and occasionally beyond), with an incessant stream of activities.

To foster communication between experts from different areas, there were minicourses, offering expertise required to approach the problems set as a goal, and formal and informal seminars.

Five mini-courses (9-12 hours each) were given by full time participants, and the Rotschild Professor; added to these were the Leverhulme and the Rotschild Lectures. The content of at least one of the minicourses will be published as a book, with the first draft already available online. Three preparatory courses most opportunely given by local participants in the Cambridge Mathematics department, made the key topics accessible to non-experts and younger participants.

A weekly research seminar scheduled one to two talks each week. The informal side of the weekly schedule had its starting point at the “Monday@4” teas, an occasion to decide on regular brainstorming meetings, informal reading seminars or working groups tackling particular problems. Several such reading groups were active during the semester and resulted in preprints.

We strived to foster interaction between the three on-going INI programmes too: this went from extending the invitation at “Monday@4” teas to the participants of the parallel programmes, to the organization of a joint meeting of the three programmes featuring a talk by the Field Medalist Vaughan Jones on a topic of common interest.

The programme had various outcomes: new collaborations, complemented by joint research grant applications (several ongoing EPSRC and LMS applications); our Clay Senior Scholar accepted the Sadleirian Chair of Pure Mathematics at the University of Cambridge, etc. By far the most significant though were the advances in several important problems, such as the description of the Caratheodory metric on the Teichmüller space; higher Teichmüller theory; the existence of a renormalization scheme for a certain family of dynamical systems coming from a group action by planar piecewise rotations; the “L-space conjecture”

on 3-manifolds; a unified way of studying growth of groups acting on hyperbolic spaces; a structure theory for compactly generated soluble locally compact groups; a new metric counter-example to the Baum-Connes conjecture and the proof that the same is a metric invariant encoding dynamical features of certain group actions; computations of action dimension for a wide family of groups; applications of topological amenability to large-scale properties of group actions; construction of examples relevant to the problem whether all automatic groups are biautomatic etc.