Operator algebras: subfactors and their applications

Vaughan Jones initiated the study of subfactors in the early 1980's, in the theory of von Neumann algebras of operators on Hilbert spaces. Subfactor theory rapidly led to connections with link and 3-manifold invariants, quantum groups and exactly solvable models in statistical mechanics reinforcing connections with physics. Subsequently deep applications and connections have been uncovered with algebraic, topological and conformal quantum field theory CFT.

This INI programme was driven by recent breakthroughs on two fronts – the classification of subfactors up to a bit beyond index five, involving the newer planar algebra tools, and the growing evidence that subfactors, hence those previously thought to be exotic, such as the Haagerup subfactor, yields natural conformal field theories.

The programme began with an opening two week primer workshop to introduce graduate students and postdocs to the programme, and then structured around three periods of two months each. These focused on: I structure of operator algebras: subfactors and fusion categories; II subfactors, higher geometry, higher twists and almost Calabi-Yau algebras; III subfactors, K-theory and CFT - each with a corresponding workshop.

On the structure theory of operator algebras, there has traditionally been symbiosis in developments in von Neumann algebras (noncommutative measure theory) and C*-algebras (noncommutative topology), which was fully exploited in this programme. The classification of Connes of amenable factors had a recent climax in the parallel Elliott classification programme of amenable which was highlighted by Elliott, White and Winter in the Structure Workshop. The classification of amenable subfactor by Popa can be viewed as a classification of amenable tensor categories on a hyperfinite II_1-factor. Arano worked on a C*-analogue of the classification of Rokhlin actions of fusion categories on Kirchberg algebras via Kasparov KK-theory. By serendipity, it was discovered at INI that the same chain complex, which Popa, Shlakahko and Vaes had used to compute a homology theory for the Temperley-Lieb planar algebra, was also arising independently in work of HTL participant Hogancamp, in low dimensional topology and that the right point of view is given by Thurston's train tracks.

The middle portion of the programme included higher twists as higher Dixmier-Douady C*-bundles, which may be of relevance to CFT as the work of OAS participant Teleman, with Freed and Hopkins had earlier shown that the Verlinde ring is the K-theory of equivariant Dixmier-Douady compact operator bundles. Izumi and Matui have been working on a conjecture of Izumi formulated seven years ago on the classification of a class of group actions on the Kirchberg algebras in terms of certain principal bundles. The survey talk by Henriques about the Dadarlat-Pennig generalisation of Dixmier-Douady theory to more exotic C*-bundles, led Izumi and Matui to adapt a homotopy fixed point notion from homotopy theory to their programme of classifying group actions on C*-algebras.

In an attempt to construct a CFT directly from a subfactor Jones introduced a construction of a "semicontinuous limit" by reversing the idea of block spin renormalization. This involves a general procedure for constructing actions of groups of fractions of certain categories. Although this method has failed to produce CFT -and for good reasons, it has taken on a life of its own and may conceivably have applications to real life quantum spin chains. The first spinoff of the construction was a way of producing all knots and links in 3-dimensional space from elements of the Thompson groups. This was presented during the final workshop in a special talk arranged by the other two programs (on topology and geometric group theory). Invariants of knots and links arise as coefficients of these unitary representations. Jones also constructs transfer matrices (generators of dynamics) on the semiconitnuous limit. This involves the study of a classical dynamical system and its behaviour under iteration. In the simplest case of the Temperley Lieb algebra this dynamical system is just a rational function on the complex numbers which thus has Julia and Fatou sets and a Mandelbrot set in the parameter space defined by the spin-blocking operator. One can identify quantum phase transitions when the varying physical parameter is the spectral parameter of the transfer matrix. The average of the log of the vacuum expectation value of the transfer matrix is smooth on the connected components of the Fatou set but is discontinuous on the Julia set. This is reminiscent of the hierarchical models of Derida, Eckmann et al in the 1980's as was pointed out by Saleur at the last workshop.

Further interaction between disparate fields involved the physicist Saleur formulating a precise conjecture between the classification of Lehrer on non-semisimple Temperley-Lieb algebras and logarithmic, non-unitary CFT. Evans and Gannon pushed forward their programme to understand CFT via twisted equivariant K-theory. This has resulted in not only understanding module categories and modular invariants for the twisted doubles of finite groups through correspondences as KK elements, but also in realising for the first time the twisted doubles of finite groups and doubles of Tambara-Yamagami systems as a CFT. The latter has also resulted in a better understanding of the double of the Haagerup subfactor indeed as the Z_2-grafting of the orbifolds of two Tamabara-Yamagami systems on two finite groups giving further evidence to their conjecture that the Haagerup subfactor arises from a natural CFT.

Other exciting prospects for the future was the work of Liu and Jaffe on connections between subfactor theory, planar algebras on one side and quantum information and quantum computing on the other.