

Final Report on the Newton Institute Programme “Polynomial Optimisation”

July 15th to August 19th 2013

1 Committees

The organising committee consisted of:

- Adam N. Letchford (Department of Management Science, Lancaster)
- Jörg Fliege (School of Mathematical Sciences, Southampton)
- Jean-Bernard Lasserre (LAAS-CNRS, Toulouse)
- Markus Schweighofer (School of Mathematics and Statistics, Konstanz)

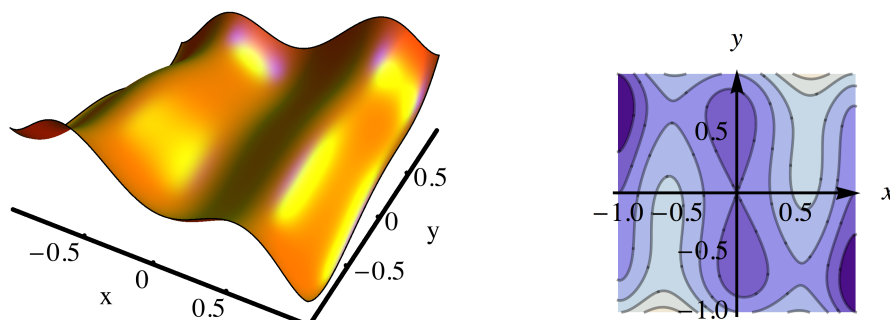
The international advisors were:

- Kurt Anstreicher (Department of Computer Science, Iowa)
- Monique Laurent (CWI, Amsterdam).

2 Scope and Purpose

The programme was concerned with optimisation problems in which the objective and constraint functions are all polynomials. This topic has attracted a great deal of interest recently, for three reasons. First, it is an extremely general model, including not only classical well-solved optimisation problems such as linear and convex quadratic programming, but also a range of NP-hard problems, both continuous and discrete. Second, there are a host of important practical applications, for example in operational research, statistics, computer science, engineering and the physical sciences. Third, it is inherently inter-disciplinary, attracting researchers from all of the above, and from branches of pure mathematics such as algebraic geometry, commutative algebra and moment theory. Indeed, particular care was taken to ensure that the programme participants came from a variety of academic backgrounds.

Figure 1: Graph of the polynomial $15x^2 - 30x^4 + 15x^6 + 2xy - 3y^2 + 4y^4$.



3 Summary of the Four Weeks

3.1 Week 1: Summer School and Workshop

The first-week was dedicated to a Summer School and a Workshop. Both were very successful, with about 80 in attendance at each. The Summer School was aimed mainly at PhD students and post-docs, and consisted of a series of invited lectures from prominent international speakers, as follows:

- *Some Recent Uses of Convex Programming in Data Science*
Emmanuel Candès (Stanford University)
- *Nonlinear Semidefinite Optimization*
Michal Kocvara (University of Birmingham)
- *Algebraic and Geometric Ideas in the Theory of Discrete Optimisation*
Jesüs De Loera (University of California, Davis)
- *Semidefinite Programming and its Feasible Sets*
Tim Netzer (Leipzig University)
- *Convex Algebraic Geometry*
Bernd Sturmfels (University of California, Berkeley)

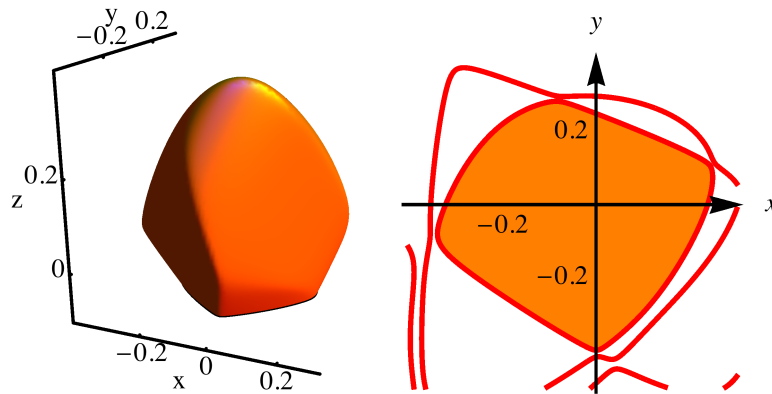
These invited lectures reflected a good blend of Mathematics (pure, applied and computational in general, and Optimization and Operational Research). The Workshop was intended for a broader audience, and consisted of 20 talks and a poster session. Again, there was an encouraging mix of topics, from theory to applications to software. Feedback was very positive.

3.2 Week 2 to 4: Specific Topics

The remaining three weeks were devoted to three specific topics: (i) Algebraic Approaches, (ii) Convex Relaxations and Approximations, and (iii) Algorithms and Software. In each of these weeks, about 25 leading academics were in attendance. The format of each day was as follows: each morning, an invited speaker gave a 1-hour talk on a topic of his choice. The following hour was for discussions between participants, who also decided whether someone else would present an informal talk in the afternoon. This format was much appreciated, as it left a lot of time for discussion and interactions.

In the second week, it became abundantly clear that Polynomial Optimization continues to motivate several highly interesting problems in mathematics (concerning, e.g., representations of convex semi-algebraic sets as spectahedra, representations of polynomials as sums of squares, bounds on the algebraic degree of optimal solutions).

Figure 2: An example of a spectrahedron.



In the third week, Semidefinite Programming (and closely-related techniques, such as copositive programming and eigenvalue optimisation) was highlighted several times as a key tool in obtaining non-trivial relaxations and approximations of NP-hard polynomial optimisation problems. This was also a theme of the fourth week, but additional techniques, such as algorithmic versions of Hilbert's Nullstellensatz, and methods for computing Gröbner bases, also played a role.

4 The “Open for Business” Event

In week 4, an “Open for Business” event was held, in order to facilitate interactions between academia and industry. Among the attendees were representatives from IBM, MOSEK, Radio Design, the Numerical Algorithms Group (NAG), the European Space Agency (ESA), and the Industrial Mathematics KTN, along with an independent consultant. The representatives from MOSEK and NAG explained the current features of their optimisation software. The delegate from IBM discussed some optimisation problems arising in the production and transmission of electricity. The delegate from ESA explained a multitude of very complex optimisation problems arising in space engineering, in particular spacecraft trajectory optimisation. Finally, a particular highlight of the event was the talk by the speaker from Radio Design, and the following discussion. It turned out that his problem, that of designing and tuning radio frequency filters, was essentially a problem of optimising the degree and coefficients of a rational function. The academics present were able to offer immediate helpful advice, which has led to a joint project between Radio Design and Professor Kocvara (University of Birmingham).

5 Outputs

This is a list of outputs to date. More are likely to follow.

5.1 Videos

Videos of most of the talks have been made available on the web site of the Institute, at the following address:

<http://www.newton.ac.uk/programmes/POP/seminars/>

5.2 Papers

All papers started or worked on during the programme will be submitted to the Institute’s preprint server, to the repository Optimisation Online, and then to appropriate international journals. Papers that have been brought to the attention of the committee to date are:

- I. Bomze (2013): Copositivity for second-order optimality conditions in general smooth optimization problems. Working paper, ISOR, University of Vienna.
- J. De Loera, R. Hemmecke, J. Lee: On augmentation methods for linear and integer programming. Working paper, Department of Mathematics, UC Davis.

- M.H.C. Fampa & J. Lee: On optimization with indefinite quadratics. Working paper, Department of Industrial and Operations Engineering, University of Michigan.
- L. Galli & A.N. Letchford: A discretisation approach to non-convex quadratically constrained quadratic programs. Working paper, Department of Computer Science, University of Pisa.
- L. Galli & A.N. Letchford: Lift-and-project cutting planes for disjointly-constrained bilinear programs. Working paper, Department of Computer Science, University of Pisa.
- J.B. Lasserre: Tractable approximations of sets defines with quantifiers. LAAS-CNRS, to be submitted.
- J. Peña, J.C. Vera & L.F. Zuluaga (2013) Completely positive reformulations for polynomial optimization. Submitted.

5.3 Special Issue

A special issue of the premier optimisation journal, Mathematical Programming, has been organised around the theme of the programme. The deadline for submission is the end of November 2013. The guest editors are:

- Jean-Bernard Lasserre (LAAS-CNRS, Toulouse)
- Adam Letchford (Lancaster, UK)
- Pablo Parrilo (MIT)
- David Steurer (Cornell)

5.4 Workshop

A workshop on “Optimisation in Space Engineering” has been organised for 5th-6th November at the University of Birmingham. This is a direct outcome of the “Open for Business” event, and will involve both academics and delegates from ESA and the space industry in the UK.