HOMOGENEOUS LORENTZ MANIFOLDS WITH SIMPLE ISOMETRY GROUP

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ABSTRACT. Let H be a closed, noncompact subgroup of a simple Lie group G, such that G/H admits an invariant Lorentz metric. We show that if $G = \mathrm{SO}(2,n)$, with $n \geq 3$, then the identity component H° of H is conjugate to $\mathrm{SO}(1,n)^\circ$. Also, if $G = \mathrm{SO}(1,n)$, with $n \geq 3$, then H° is conjugate to $\mathrm{SO}(1,n-1)^\circ$.

1. Introduction

1.1. Definition.
- A *Minkowski form* on a real vector space V is a nondegenerate quadratic form that is isometric to the form $-x_1^2 + x_2^2 + \cdots + x_{n+1}^2$ on $\mathbb{R}^{n+1}$, where $\dim V = n + 1 \geq 2$.
- A *Lorentz metric* on a smooth manifold M is a choice of Minkowski metric on the tangent space T_pM, for each $p \in M$, such that the form varies smoothly as p varies.

A. Zeghib [Ze1] classified the compact homogeneous spaces that admit an invariant Lorentz metric. In this note, we remove the assumption of compactness, but add the restriction that the transitive group G is almost simple. Our starting point is a special case of a theorem of N. Kowalsky.

1.2. Theorem (N. Kowalsky, cf. [Ko3, Thm. 5.1]). *Let G/H be a nontrivial homogeneous space of a connected, almost simple Lie group G with finite center. If there is a G-invariant Lorentz metric on G/H, then either*

1) *there is also a G-invariant Riemannian metric on G/H; or*
2) *G is locally isomorphic to either* $\mathrm{SO}(1,n)$ *or* $\mathrm{SO}(2,n)$, *for some n.*

As explained in the following elementary proposition, it is easy to characterize the homogeneous spaces that arise in Conclusion (1) of Theorem 1.2, although it is probably not reasonable to expect a complete classification.

1.3. Notation. We use $\mathfrak{g}$ to denote the Lie algebra of a Lie group G, and $\mathfrak{h} \subset \mathfrak{g}$ to denote the Lie algebra of a Lie subgroup H of G.

1.4. Proposition (cf. [Ko3, Thm. 1.1]). *Let G/H be a homogeneous space of a Lie group G, such that $\mathfrak{g}$ is simple and* $\dim G/H \geq 2$. *The following are equivalent.*

1) *The homogeneous space G/H admits both a G-invariant Riemannian metric and a G-invariant Lorentz metric.*
2) *The closure of* $\mathrm{Ad}_G H$ *is compact, and leaves invariant a one-dimensional subspace of $\mathfrak{g}$ that is not contained in $\mathfrak{h}$.*

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The two main results of this note examine the cases that arise in Conclusion (2) of Theorem 1.2. It is well known [Ko2, Egs. 2 and 3] that $\mathrm{SO}(1,n)^\circ/\mathrm{SO}(1,n-1)^\circ$ and $\mathrm{SO}(2,n)^\circ/\mathrm{SO}(1,n)^\circ$ have invariant Lorentz metrics. Also, for any discrete subgroup Γ of $\mathrm{SO}(1,2)$, the Killing form provides an invariant Lorentz metric on $\mathrm{SO}(1,2)^\circ/\Gamma$. We show that these are essentially the only examples.

Note that $\mathrm{SO}(1,1)$ and $\mathrm{SO}(2,2)$ fail to be almost simple. Thus, in 1.2(2), we may assume

- G is locally isomorphic to $\mathrm{SO}(1,n)$, and $n \geq 2$; or
- G is locally isomorphic to $\mathrm{SO}(2,n)$, and $n \geq 3$.

2.3′. Proposition. *Let G be a Lie group that is locally isomorphic to $\mathrm{SO}(1,n)$, with $n \geq 2$. If H is a closed subgroup of G, such that*

- *the closure of $\mathrm{Ad}_G H$ is not compact, and*
- *there is a G-invariant Lorentz metric on G/H,*

then either

1) *after any identification of $\mathfrak{g}$ with $\mathfrak{so}(1,n)$, the subalgebra $\mathfrak{h}$ is conjugate to a standard copy of $\mathfrak{so}(1,n-1)$ in $\mathfrak{so}(1,n)$, or*
2) *$n=2$ and H is discrete.*

3.5′. Theorem. *Let G be a Lie group that is locally isomorphic to $\mathrm{SO}(2,n)$, with $n \geq 3$. If H is a closed subgroup of G, such that*

- *the closure of $\mathrm{Ad}_G H$ is not compact, and*
- *there is a G-invariant Lorentz metric on G/H,*

then, after any identification of $\mathfrak{g}$ with $\mathfrak{so}(2,n)$, the subalgebra $\mathfrak{h}$ is conjugate to a standard copy of $\mathfrak{so}(1,n)$ in $\mathfrak{so}(2,n)$.

N. Kowalsky announced a much more general result than Theorem 3.5′ in [Ko2, Thm. 4], but it seems that she did not publish a proof before her premature death. She announced a version of Proposition 2.3′ (with much more general hypotheses and a somewhat weaker conclusion) in [Ko2, Thm. 3], and a proof appears in her Ph.D. thesis [Ko1, Cor. 6.2].

1.5. *Remark.* It is easy to see that there is a G-invariant Lorentz metric on G/H if and only if there is an $(\mathrm{Ad}_G H)$-invariant Minkowski form on $\mathfrak{g}/\mathfrak{h}$. Thus, although Proposition 2.3′ and Theorem 3.5′ are geometric in nature, they can be restated in more algebraic terms. It is in such a form that they are proved in §2 and §3.

Proposition 2.3′ and Theorem 3.5′ are used in work of S. Adams [Ad3] on nontame actions on Lorentz manifolds. See [Zi, Ko3, AS, Ze2, Ad1, Ad2] for some other research concerning actions of Lie groups on Lorentz manifolds.

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2. Homogeneous spaces of $\mathrm{SO}(1,n)$

The following lemma is elementary.

2.1. Lemma. *Let π be the standard representation of $\mathfrak{g} = \mathfrak{so}(1,k)$ on $\mathbb{R}^{k+1}$, and let $\mathfrak{g} = \mathfrak{k} + \mathfrak{a} + \mathfrak{n}$ be an Iwasawa decomposition of $\mathfrak{g}$.*

1) *The representation π has only one positive weight (with respect to $\mathfrak{a}$), and the corresponding weight space is 1-dimensional.*
2) *There are subspaces V and W of $\mathbb{R}^{k+1}$, such that*
   (a) $\dim(\mathbb{R}^{k+1}/V) = 1$;
   (b) $\dim W = 1$;
   (c) $\pi(\mathfrak{n})V \subset W$;
   (d) *for all nonzero $u \in \mathfrak{n}$, we have $\pi(u)^2\mathbb{R}^{k+1} = W$; and*
   (e) *for all nonzero $u \in \mathfrak{n}$ and $v \in \mathbb{R}^{k+1}$, we have $\pi(u)^2 v = 0$ if and only if $v \in V$.*

2.2. Corollary. *Let $\mathfrak{h}$ be a subalgebra of a real Lie algebra $\mathfrak{g}$, let Q be a Minkowski form on $\mathfrak{g}/\mathfrak{h}$, and define $\pi\colon N_G(\mathfrak{h}) \to \mathrm{GL}(\mathfrak{g}/\mathfrak{h})$ by $\pi(g)(v+\mathfrak{h}) = (\mathrm{Ad}_G\, g)v + \mathfrak{h}$.*

1) *Suppose T is a connected Lie subgroup of G that normalizes H, such that $\pi(T) \subset \mathrm{SO}(Q)$ and $\mathrm{Ad}_G\, T$ is diagonalizable over $\mathbb{R}$. Then, for any ordering of the T-weights on $\mathfrak{g}$, the subalgebra $\mathfrak{h}$ contains codimension-one subspaces of both $\mathfrak{g}^+$ and $\mathfrak{g}^-$, where $\mathfrak{g}^+$ is the sum of all the positive weight spaces of T, and $\mathfrak{g}^-$ is the sum of all the negative weight spaces of T.*
2) *If U is a connected Lie subgroup of G that normalizes H, such that $\pi(U) \subset \mathrm{SO}(Q)$ and $\mathrm{Ad}_G\, U$ is unipotent, then there are subspaces $V/\mathfrak{h}$ and $W/\mathfrak{h}$ of $\mathfrak{g}/\mathfrak{h}$, such that*
   (a) $\dim(\mathfrak{g}/V) = 1$;
   (b) $\dim(W/\mathfrak{h}) = 1$;
   (c) $[V, \mathfrak{u}] \subset W$;
   (d) *for each $u \in \mathfrak{u}$, either $W = \mathfrak{h} + (\mathrm{ad}_{\mathfrak{g}}\, u)^2\mathfrak{g}$, or $[\mathfrak{g}, u] \subset \mathfrak{h}$; and*
   (e) *for all $u \in \mathfrak{u}$, we have $(\mathrm{ad}_{\mathfrak{g}}\, u)^2 V \subset \mathfrak{h}$.*

2.3. Proposition. *Let H be a Lie subgroup of $G = \mathrm{SO}(1,n)$, with $n \geq 2$, such that*

- *the closure of H is not compact; and*
- *there is an $(\mathrm{Ad}_G\, H)$-invariant Minkowski form on $\mathfrak{g}/\mathfrak{h}$.*

Then either

1) *H° is conjugate to a standard copy of $\mathrm{SO}(1,n-1)^\circ$ in $\mathrm{SO}(1,n)$, or*
2) *$n = 2$ and H° is trivial.*

Proof. Let $\overline{H}$ be the Zariski closure of H, and note that the Minkowski form is also invariant under $\mathrm{Ad}_G\, \overline{H}$. Replacing H by a finite-index subgroup, we may assume $\overline{H}$ is Zariski connected.

Let $G = KAN$ be an Iwasawa decomposition of G.

Case 1. *Assume $n \geq 3$ and $A \subset \overline{H}$.* From Corollary 2.2(1), we see that $\mathfrak{h}$ contains codimension-one subspaces of both $\mathfrak{n}$ and $\mathfrak{n}^-$. (Note that this implies H° is nontrivial.) This implies that $\overline{H}$ is reductive. (Because $(H \cap N)^\circ\, \mathrm{unip}\, \overline{H}$ is a unipotent subgroup that intersects N nontrivially (and $\mathbb{R}\text{-rank}\, G = 1$), it must be contained in N, so $\mathrm{unip}\, \overline{H} \subset N$. Similarly, $\mathrm{unip}\, \overline{H} \subset N^-$. Therefore $\mathrm{unip}\, \overline{H} \subset N \cap N^- = e$.) Then, since $\overline{H}$ contains a codimension-one subgroup of N, and since $A \subset \overline{H}$, it follows that $\overline{H}$ is conjugate to either $\mathrm{SO}(1,n-1)$ or $\mathrm{SO}(1,n)$. Because H° is a nontrivial, connected, normal subgroup of $\overline{H}$, we conclude that H° is conjugate to either $\mathrm{SO}(1,n-1)^\circ$ or $\mathrm{SO}(1,n)^\circ$. Because $\mathfrak{g}/\mathfrak{h} \neq 0$ (else

$\dim \mathfrak{g}/\mathfrak{h} = 0 < 2$, which contradicts the fact that there is a Minkowski form on $\mathfrak{g}/\mathfrak{h}$), we see that H° is conjugate to $\mathrm{SO}(1, n-1)^\circ$.

Case 2. *Assume $n \geq 3$ and $\overline{H}$ does not contain any nontrivial hyperbolic elements.* The Levi subgroup of $\overline{H}$ must be compact, and the radical of $\overline{H}$ must be unipotent, so choose a compact M and a nontrivial unipotent subgroup U such that $\overline{H} = M \ltimes U$. Replacing H by a conjugate, we may assume, without loss of generality, that $U \subset N$.

Let us show, for every nonzero $u \in \mathfrak{u}$, that $[\mathfrak{g}, u] \not\subset \mathfrak{h}$. From the Morosov Lemma [Ja, Thm. 17(1), p. 100], we know there exists $v \in \mathfrak{g}$, such that $[v, u]$ is hyperbolic (and nonzero). If $[v, u] \in \mathfrak{h}$, this contradicts the fact that $\overline{H}$ does not contain nontrivial hyperbolic elements.

Let $V/\mathfrak{h}$ and $W/\mathfrak{h}$ be subspaces of $\mathfrak{g}/\mathfrak{h}$ as in Corollary 2.2(2). Because $(\mathrm{ad}_{\mathfrak{g}} u)^2 \mathfrak{g} = \mathfrak{n}$ for every nonzero $u \in \mathfrak{n}$, we have $W = \mathfrak{n} + \mathfrak{h}$ (see 2.2(2d)), so $\dim \mathfrak{n}/(\mathfrak{h} \cap \mathfrak{n}) = 1$ (see 2.2(2b)) and

$$[\mathfrak{u}, V] \subset W = \mathfrak{n} + \mathfrak{h} \subset \mathfrak{n} + \overline{\mathfrak{h}} = \mathfrak{n} + \mathfrak{m} \tag{2.4}$$

(see 2.2(2c)).

Assume, for the moment, that $n \geq 4$. Then

$$\begin{aligned} \dim \mathfrak{u} + \dim(V \cap \mathfrak{n}^-) &\geq \dim(\mathfrak{h} \cap \mathfrak{n}) + \dim(V \cap \mathfrak{n}^-) \geq (\dim \mathfrak{n} - 1) + (\dim \mathfrak{n}^- - 1) \\ &= (n-2) + (n-2) \geq n > \dim \mathfrak{n}. \end{aligned}$$

This implies that there exist $u \in \mathfrak{u}$ and $v \in V \cap \mathfrak{n}^-$, such that $\langle u, v \rangle \cong \mathfrak{sl}(2, \mathbb{R})$, with $[u, v]$ hyperbolic (and nonzero). This contradicts the fact that $\mathfrak{m} + \mathfrak{n}$ has no nontrivial hyperbolic elements.

We may now assume that $n = 3$. For any nonzero $u \in \mathfrak{n}$, we have

$$\dim[u, V] \geq \dim[u, \mathfrak{g}] - 1 = \dim \mathfrak{n} + 1 > \dim \mathfrak{n},$$

so $[\mathfrak{u}, V] \not\subset \mathfrak{n}$. Then, from (2.4), we conclude that $\mathfrak{m} \neq 0$, so $\mathfrak{m}$ acts irreducibly on $\mathfrak{n}$. This contradicts the fact that $\mathfrak{h} \cap \mathfrak{n}$ is a codimension-one subspace of $\mathfrak{n}$ that is normalized by $\mathfrak{m}$.

Case 3. *Assume $n = 2$.* We may assume H° is nontrivial (otherwise Conclusion (2) holds). We must have $\dim \mathfrak{g}/\mathfrak{h} \geq 2$, so we conclude that $\dim H^\circ = 1$ and $\dim \mathfrak{g}/\mathfrak{h} = 2$. Because $\mathrm{SO}(1,1)$ consists of hyperbolic elements, this implies that $\mathrm{Ad}_G h$ acts diagonalizably on $\mathfrak{g}/\mathfrak{h}$, for every $h \in H$. Therefore H° is conjugate to A, and, hence, to $\mathrm{SO}(1,1)^\circ$. □

3. Homogeneous spaces of $\mathrm{SO}(2, n)$

3.1. Theorem (Borel-Tits [BT2, Prop. 3.1]). *Let H be an F-subgroup of a reductive algebraic group G over a field F of characteristic zero. Then there is a parabolic F-subgroup P of G, such that* $\operatorname{unip} H \subset \operatorname{unip} P$ *and* $H \subset N_G(\operatorname{unip} H) \subset P$.

3.2. Notation. Let $k = \lfloor n/2 \rfloor$. Identifying $\mathbb{C}^{k+1}$ with $\mathbb{R}^{2k+2}$ yields an embedding of $\mathrm{SU}(1,k)$ in $\mathrm{SO}(2,2k)$. Then the inclusion $\mathbb{R}^{2k+2} \hookrightarrow \mathbb{R}^{2k+3}$ yields an embedding of $\mathrm{SU}(1,k)$ in $\mathrm{SO}(2, 2k+1)$. Thus, we may identify $\mathrm{SU}(1, \lfloor n/2 \rfloor)$ with a subgroup of $\mathrm{SO}(2,n)$.

We use the following well-known result to shorten one case of the proof of Theorem 3.5.

3.3. Lemma ([OW, Lem. 6.8]). *If L is a connected, almost-simple subgroup of $\mathrm{SO}(2,n)$, such that $\mathbb{R}$-rank $L = 1$ and $\dim L > 3$, then L is conjugate under $\mathrm{O}(2,n)$ to a subgroup of either $\mathrm{SO}(1,n)$ or $\mathrm{SU}(1, \lfloor n/2 \rfloor)$.*

3.4. Corollary. *Let L be a connected, reductive subgroup of $G = \mathrm{SO}(2,n)$, such that $\mathbb{R}$-rank $L = 1$. Then $\dim U \leq n-1$, for every connected, unipotent subgroup U of L.*

Furthermore, if $\dim U = n-1$, then either

1) *L is conjugate to $\mathrm{SO}(1,n)^\circ$; or*
2) *n is even, and L is conjugate under $\mathrm{O}(2,n)$ to $\mathrm{SU}(1,n/2)$.*

3.5. Theorem. *Let H be a Lie subgroup of $G = \mathrm{SO}(2,n)$, with $n \geq 3$, such that*

- *the closure of H is not compact, and*
- *there is an $(\mathrm{Ad}_G H)$-invariant Minkowski form on $\mathfrak{g}/\mathfrak{h}$.*

Then H° is conjugate to a standard copy of $\mathrm{SO}(1,n)^\circ$ in $\mathrm{SO}(2,n)$.

Proof. Let $\overline{H}$ be the Zariski closure of H, and note that the Minkowski form is also invariant under $\mathrm{Ad}_G \overline{H}$. Replacing H by a finite-index subgroup, we may assume $\overline{H}$ is Zariski connected.

Let $G = KAN$ be an Iwasawa decomposition of G. For each real root ϕ of $\mathfrak{g}$ (with respect to the Cartan subalgebra $\mathfrak{a}$), let $\mathfrak{g}_\phi$ be the corresponding root space, and let $\mathrm{proj}_\phi \colon \mathfrak{g} \to \mathfrak{g}_\phi$ and $\mathrm{proj}_{\phi \oplus -\phi} \colon \mathfrak{g} \to \mathfrak{g}_\phi + \mathfrak{g}_{-\phi}$ be the natural projections. Fix a choice of simple real roots α and β of $\mathfrak{g}$, such that $\dim \mathfrak{g}_\alpha = 1$ and $\dim \mathfrak{g}_\beta = n-2$ (so the positive real roots are α, β, $\alpha+\beta$, and $\alpha + 2\beta$). Replacing N by a conjugate under the Weyl group, we may assume $\mathfrak{n} = \mathfrak{g}_\alpha + \mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}$. From the classification of parabolic subgroups [BT1, Prop. 5.14, p. 99], we know that the only proper parabolic subalgebras of $\mathfrak{g}$ that contain $\mathfrak{n}_\mathfrak{g}(\mathfrak{n})$ are

$$\mathfrak{n}_\mathfrak{g}(\mathfrak{n}), \quad \mathfrak{p}_\alpha = \mathfrak{n}_\mathfrak{g}(\mathfrak{n}) + \mathfrak{g}_{-\alpha}, \quad \text{and} \quad \mathfrak{p}_\beta = \mathfrak{n}_\mathfrak{g}(\mathfrak{n}) + \mathfrak{g}_{-\beta}. \tag{3.6}$$

Case 1. *Assume $\overline{\mathfrak{h}}$ contains nontrivial hyperbolic elements.* Let $\mathfrak{t} = \overline{\mathfrak{h}} \cap \mathfrak{a}$. Replacing H by a conjugate, we may assume $\mathfrak{t} \neq 0$.

Subcase 1.1. *Assume* $\mathfrak{t} \in \{\ker(\alpha+\beta), \ker\beta\}$.

Subsubcase 1.1.1. *Assume $\overline{H}$ is reductive.* We may assume $\mathfrak{t} = \ker(\alpha+\beta)$ (if necessary, replace H with its conjugate under the Weyl reflection corresponding to the root α). Then, from Corollary 2.2(1), we see that $\mathfrak{h}$ contains a codimension-one subspace of $\mathfrak{g}_{\alpha+2\beta} + \mathfrak{g}_\beta + \mathfrak{g}_{-\alpha}$. (Note that this implies H° is nontrivial.)

Let $\mathfrak{n}' = \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta} + \mathfrak{g}_\beta + \mathfrak{g}_{-\alpha}$, so $\mathfrak{n}'$ is the Lie algebra of a maximal unipotent subgroup of G. (In fact, $\mathfrak{n}'$ is the image of $\mathfrak{n}$ under the Weyl reflection corresponding to the root α.) From the preceding paragraph, we know that

$$\dim(\overline{\mathfrak{h}} \cap \mathfrak{n}') \geq \dim(\mathfrak{g}_{\alpha+2\beta} + \mathfrak{g}_\beta + \mathfrak{g}_{-\alpha}) - 1 = n - 1.$$

Therefore, Corollary 3.4 implies that $\overline{H}$ is conjugate (under $\mathrm{O}(2,n)$) to either $\mathrm{SO}(1,n)$ or $\mathrm{SU}(1,n/2)$. It is easy to see that $\overline{H}$ is not conjugate to $\mathrm{SU}(1,n/2)$. (See [OW, proof of Thm. 1.5] for an explicit description of $\mathfrak{su}(1,n/2) \cap \mathfrak{n}$. If n is even, then $n > 3$, so $\mathfrak{su}(1,n/2)$ does not contain a codimension-one subspace of any $(n-2)$-dimensional root space, but $\overline{\mathfrak{h}}$ does contain a codimension-one subspace of $\mathfrak{g}_\beta$.) Therefore, we conclude that $\overline{H}$ is conjugate to $\mathrm{SO}(1,n)$. Then, because H° is a nontrivial, connected, normal subgroup of $\overline{H}$, we conclude that $H^\circ = (\overline{H})^\circ$ is conjugate to $\mathrm{SO}(1,n)^\circ$.

Subsubcase 1.1.2. *Assume $\overline{H}$ is not reductive.* Let P be a maximal parabolic subgroup of G that contains $\overline{H}$ (see Theorem 3.1). By replacing P and H with conjugate subgroups, we may assume that P contains the minimal parabolic subgroup $N_G(N)$. Therefore, the classification of parabolic subalgebras (3.6) implies that P is either P_α or P_β.

Subsubsubcase 1.1.2.1. *Assume* $\mathfrak{t} = \ker(\alpha+\beta)$. From Corollary 2.2(1), we see that $\mathfrak{h}$ (and hence also $\mathfrak{p}$) contains codimension-one subspaces of $\mathfrak{g}_{\alpha+2\beta} + \mathfrak{g}_\beta + \mathfrak{g}_{-\alpha}$ and $\mathfrak{g}_{-\alpha-2\beta} + \mathfrak{g}_{-\beta} + \mathfrak{g}_\alpha$. Because $\mathfrak{p}_\alpha$ does not contain such a subspace of $\mathfrak{g}_{-\alpha-2\beta} + \mathfrak{g}_{-\beta} + \mathfrak{g}_\alpha$, we conclude that $P = P_\beta$. Furthermore, because the intersection of $\mathfrak{p}_\beta$ with each of these subspaces does have codimension one, we conclude that $\mathfrak{h}$ has precisely the same intersection; therefore $(\mathfrak{g}_{\alpha+2\beta} + \mathfrak{g}_\beta) + (\mathfrak{g}_{-\beta} + \mathfrak{g}_\alpha) \subset \mathfrak{h}$. Hence $\mathfrak{h} \supset [\mathfrak{g}_\alpha, \mathfrak{g}_\beta] = \mathfrak{g}_{\alpha+\beta}$. We now have

$$(\operatorname{ad}_\mathfrak{g} \mathfrak{g}_{\alpha+\beta})^2 \mathfrak{g} = \mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta} \equiv 0 \pmod{\mathfrak{h}},$$

so Corollary 2.2(2d) implies

$$\mathfrak{h} \supset [\mathfrak{g}, \mathfrak{g}_{\alpha+\beta}] \supset [\mathfrak{g}_{-\alpha-\beta}, \mathfrak{g}_{\alpha+\beta}] \supset \ker\beta.$$

This contradicts the fact that $\overline{\mathfrak{h}} \cap \mathfrak{a} = \mathfrak{t} = \ker(\alpha+\beta)$.

Subsubsubcase 1.1.2.2. *Assume* $\mathfrak{t} = \ker\beta$. From Corollary 2.2(1), we see that $\mathfrak{h}$ (and hence also $\mathfrak{p}$) contains a codimension-one subspace of $\mathfrak{g}_{-\alpha} + \mathfrak{g}_{-\alpha-\beta} + \mathfrak{g}_{-\alpha-2\beta}$. Because neither $\mathfrak{p}_\alpha$ nor $\mathfrak{p}_\beta$ contains such a subspace, this is a contradiction.

Subcase 1.2. *Assume* $\mathfrak{t} \in \{\ker\alpha, \ker(\alpha+2\beta)\}$. We may assume $\mathfrak{t} = \ker\alpha$ (if necessary, replace H with its conjugate under the Weyl reflection corresponding to the root β). From Corollary 2.2(1), we see that $\mathfrak{h}$ contains a codimension-one subspace of $\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}$. Because any codimension-one subalgebra of a nilpotent Lie algebra must contain the commutator subalgebra, we conclude that $\mathfrak{h}$ contains $\mathfrak{g}_{\alpha+2\beta}$. Then we have

$$(\operatorname{ad}_\mathfrak{g} \mathfrak{g}_{\alpha+2\beta})^2 \mathfrak{g} = \mathfrak{g}_{\alpha+2\beta} \equiv 0 \pmod{\mathfrak{h}},$$

so Corollary 2.2(2d) implies

$$\mathfrak{h} \supset [\mathfrak{g}, \mathfrak{g}_{\alpha+2\beta}] \supset \mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}.$$

Similarly, we also have $\mathfrak{h} \supset \mathfrak{g}_{-\beta} + \mathfrak{g}_{-\alpha-\beta} + \mathfrak{g}_{-\alpha-2\beta}$. It is now easy to show that $\mathfrak{h} \supset \mathfrak{g}_\phi$ for every real root ϕ, so $\mathfrak{h} = \mathfrak{g}$. This contradicts the fact that $\mathfrak{g}/\mathfrak{h} \neq 0$.

Subcase 1.3. *Assume* $\mathfrak{t}$ *contains a regular element of* $\mathfrak{a}$. Replacing H by a conjugate under the Weyl group, we may assume that $\mathfrak{n}$ is the sum of the positive root spaces, with respect to $\mathfrak{t}$. Then, from Corollary 2.2(1), we see that $\mathfrak{h}$ contains codimension-one subspaces of both $\mathfrak{n}$ and $\mathfrak{n}^-$. Therefore, $\mathfrak{h}$ contains codimension-one subspaces of $\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}$ and $\mathfrak{g}_{-\beta} + \mathfrak{g}_{-\alpha-\beta} + \mathfrak{g}_{-\alpha-2\beta}$, so the argument of Subcase 1.2 applies.

Case 2. *Assume* $\overline{\mathfrak{h}}$ *does not contain nontrivial hyperbolic elements.* The Levi subgroup of $\overline{H}$ must be compact, and the radical of $\overline{H}$ must be unipotent, so choose a compact M and a nontrivial unipotent subgroup U such that $\overline{H} = M \ltimes U$. Choose subspaces $V/\mathfrak{h}$ and $W/\mathfrak{h}$ of $\mathfrak{g}/\mathfrak{h}$ as in Corollary 2.2(2).

Let P be a proper parabolic subgroup of G, such that $U \subset \operatorname{unip} P$ and $H \subset P$ (see Theorem 3.1). Replacing H and P by conjugates, we may assume, without loss of generality, that P contains the minimal parabolic subgroup $N_G(N)$ (so $\operatorname{unip} P \subset N$). From the classification of parabolic subalgebras (3.6), we know that there are only three possibilities for P. We consider each of these possibilities separately.

First, though, let us show that

(3.7) for every nonzero $u \in \mathfrak{u}$, we have $[\mathfrak{g}, u] \not\subset \mathfrak{h}$.

From the Morosov Lemma [Ja, Thm. 17(1), p. 100], we know there exists $v \in \mathfrak{g}$, such that $[v, u]$ is hyperbolic (and nonzero). If $[v, u] \in \mathfrak{h}$, this contradicts the fact that $\overline{\mathfrak{h}}$ does not contain nontrivial hyperbolic elements.

Subcase 2.1. *Assume $P = N_G(N)$ is a minimal parabolic subgroup of G.*

Subsubcase 2.1.1. *Assume* $\operatorname{proj}_\beta \mathfrak{u} \neq 0$. Choose $u \in \mathfrak{u}$, such that $\operatorname{proj}_\beta u \neq 0$, and let $Z = (\operatorname{ad}_\mathfrak{g} u)^2 \mathfrak{g}_{-\alpha-2\beta}$. (So $\dim Z = 1$, $\operatorname{proj}_{-\alpha} Z \neq 0$, and $\operatorname{proj}_{-\alpha-\beta} Z = 0$.) From Corollary 2.2(2d), we know that $Z \subset W$. Then, because $\operatorname{proj}_{-\alpha} \mathfrak{h} \subset \operatorname{proj}_{-\alpha} \mathfrak{p} = 0$, we conclude, from Corollary 2.2(2b), that $W = \mathfrak{h} + Z$.

Because $W = \mathfrak{h} + Z \subset \mathfrak{p} + Z$, we have $\operatorname{proj}_{-\alpha-\beta} W = 0$. Therefore, because $\operatorname{proj}_\beta u \neq 0$, we conclude, from Corollary 2.2(2c), that $\operatorname{proj}_{-\alpha-2\beta} V = 0$, so Corollary 2.2(2a) implies that $V = \ker(\operatorname{proj}_{-\alpha-2\beta})$. In particular, we have $\mathfrak{g}_{-\beta} \subset V$, so Corollary 2.2(2c) implies $[\mathfrak{g}_{-\beta}, u] \subset W$. Therefore, we have

$$\begin{aligned}
[\mathfrak{g}_{-\beta}, \operatorname{proj}_\beta u] &\subset [\mathfrak{g}_{-\beta}, u + (\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta})] = [\mathfrak{g}_{-\beta}, u] + [\mathfrak{g}_{-\beta}, \mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}] \\
&\subset W + (\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta}) = \mathfrak{h} + Z + (\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta}) \subset \mathfrak{m} + \mathfrak{n} + Z.
\end{aligned}$$

Because $\operatorname{proj}_{-\alpha}[\mathfrak{g}_{-\beta}, \operatorname{proj}_\beta u] = 0$, we conclude that $[\mathfrak{g}_{-\beta}, \operatorname{proj}_\beta u] \subset \mathfrak{m} + \mathfrak{n}$. This contradicts the fact that $\mathfrak{m} + \mathfrak{n}$ does not contain nontrivial hyperbolic elements.

Subsubcase 2.1.2. *Assume* $\operatorname{proj}_\beta \mathfrak{u} = 0$. Replacing H by a conjugate under N, we may assume $\mathfrak{m} \subset \mathfrak{g}_0$, so $\operatorname{proj}_\beta \overline{\mathfrak{h}} = 0$.

We have $\mathfrak{u} \subset \mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}$, so $(\operatorname{ad}_\mathfrak{g} u)^2 \mathfrak{g} \subset \mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}$ for every $u \in \mathfrak{u}$. Thus, Corollary 2.2(2d) implies $W \subset (\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}) + \mathfrak{h}$.

We have

$$\operatorname{proj}_{\beta\oplus-\beta} W \subset \operatorname{proj}_{\beta\oplus-\beta}(\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}) + \operatorname{proj}_{\beta\oplus-\beta} \mathfrak{h} = 0,$$

so Corollary 2.2(2c) implies that $\operatorname{proj}_{\beta\oplus-\beta}\big((\operatorname{ad}_\mathfrak{g} \mathfrak{u})V\big) = 0$.

Subsubsubcase 2.1.2.1. *Assume* $\operatorname{proj}_\alpha u \neq 0$, *for some* $u \in \mathfrak{u}$. From the conclusion of the preceding paragraph, we know that $\operatorname{proj}_{-\beta}\big((\operatorname{ad}_\mathfrak{g} u)V\big) = 0$. Because $\operatorname{proj}_\beta u = 0$ and $\operatorname{proj}_\alpha \neq 0$, this implies $\operatorname{proj}_{-\alpha-\beta} V = 0$, so $V = \ker(\operatorname{proj}_{-\alpha-\beta})$ (see 2.2(2a)). In particular, $\mathfrak{g}_{-\alpha} \subset V$, so Corollary 2.2(2c) implies

$$\begin{aligned}
[\mathfrak{g}_\alpha, \mathfrak{g}_{-\alpha}] &\subset [\mathfrak{u} + (\mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}), \mathfrak{g}_{-\alpha}] \subset [\mathfrak{u}, V] + [\mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}, \mathfrak{g}_{-\alpha}] \\
&\subset W + \mathfrak{g}_\beta \subset \mathfrak{h} + \mathfrak{n} \subset \mathfrak{m} + \mathfrak{n}.
\end{aligned}$$

This contradicts the fact that $\mathfrak{m} + \mathfrak{n}$ does not contain nontrivial hyperbolic elements.

Subsubsubcase 2.1.2.2. *Assume* $\operatorname{proj}_{\alpha+\beta} u \neq 0$, *for some* $u \in \mathfrak{u}$. From Subsubsubcase 2.1.2.1, we may assume $\operatorname{proj}_\alpha u = 0$. Because $0 = \operatorname{proj}_{\beta\oplus-\beta}\big((\operatorname{ad}_\mathfrak{g} u)V\big)$ has codimension ≤ 1 in $\operatorname{proj}_{\beta\oplus-\beta}\big((\operatorname{ad}_\mathfrak{g} u)\mathfrak{g}\big)$ (see 2.2(2a)), which contains the 2-dimensional subspace $\operatorname{proj}_{\beta\oplus-\beta}\big([u, \mathfrak{g}_{-\alpha-2\beta} + \mathfrak{g}_{-\alpha}]\big)$, we have a contradiction.

Subsubsubcase 2.1.2.3. *Assume* $\mathfrak{u} = \mathfrak{g}_{\alpha+2\beta}$. (This argument is similar to Subsubsubcase 2.1.2.1.) Because $\operatorname{proj}_\beta\big((\operatorname{ad}_\mathfrak{g} \mathfrak{u})V\big) = 0$, we know that $\operatorname{proj}_{-\alpha-\beta} V = 0$, so $V = \ker(\operatorname{proj}_{-\alpha-\beta})$ (see 2.2(2a)). In particular, $\mathfrak{g}_{-\alpha-2\beta} \subset V$, so Corollary 2.2(2c) implies

$$[\mathfrak{g}_{\alpha+2\beta}, \mathfrak{g}_{-\alpha-2\beta}] \subset [\mathfrak{u}, V] \subset W \subset \mathfrak{h} + \mathfrak{n} \subset \mathfrak{m} + \mathfrak{n}.$$

This contradicts the fact that $\mathfrak{m} + \mathfrak{n}$ does not contain nontrivial hyperbolic elements.

Subcase 2.2. *Assume* $P = P_\alpha$. We may assume there exists $x \in \mathfrak{h}$, such that $\operatorname{proj}_{-\alpha} x \neq 0$ (otherwise, $H \subset N_G(N)$, so Subcase 2.1 applies). Note that, because $U \subset \operatorname{unip} P$, we have $\operatorname{proj}_\alpha \mathfrak{u} = 0$.

Subsubcase 2.2.1. *Assume* $\operatorname{proj}_{\alpha+\beta} \mathfrak{u} \neq 0$. Choose $u \in \mathfrak{u}$, such that $\operatorname{proj}_{\alpha+\beta} u \neq 0$. Then $[x, u] \in [\mathfrak{h}, \mathfrak{u}] \subset \mathfrak{u}$, and $\big[[x, u], u\big]$ is a nonzero element of $\mathfrak{g}_{\alpha+2\beta}$, so we see that $\mathfrak{g}_{\alpha+2\beta} \subset [\mathfrak{u}, \mathfrak{u}]$. Because every unipotent subgroup of $\mathrm{SO}(1,k)$ is abelian, we conclude that $\operatorname{ad}_{\mathfrak{g}} \mathfrak{g}_{\alpha+2\beta}$ acts trivially on $\mathfrak{g}/\mathfrak{h}$, which means $\mathfrak{h} \supset [\mathfrak{g}, \mathfrak{g}_{\alpha+2\beta}]$. This contradicts (3.7).

Subsubcase 2.2.2. *Assume* $\operatorname{proj}_{\alpha+\beta} \mathfrak{u} = 0$. We may assume, furthermore, that $\operatorname{proj}_\alpha \mathfrak{h} \neq 0$ (otherwise, by replacing H with its conjugate under the Weyl reflection corresponding to the root α, we could revert to Subcase 2.1). Then, because $[\mathfrak{h}, \mathfrak{u}] \subset \mathfrak{u}$, we must have $\operatorname{proj}_\beta \mathfrak{u} = 0$. Thus, $\mathfrak{u} = \mathfrak{g}_{\alpha+2\beta}$. From Corollary 2.2(2d), we have

$$W = [\mathfrak{g}, \mathfrak{g}_{\alpha+2\beta}, \mathfrak{g}_{\alpha+2\beta}] + \mathfrak{h} = \mathfrak{g}_{\alpha+2\beta} + \mathfrak{h} \subset \mathfrak{u} + \bar{\mathfrak{h}} = \bar{\mathfrak{h}},$$

so

$$\begin{aligned} W \cap (\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}) &\subset \bar{\mathfrak{h}} \cap (\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}) = (\bar{\mathfrak{h}} \cap \mathfrak{n}) \cap (\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}) \\ &= \mathfrak{u} \cap (\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}) = \mathfrak{g}_{\alpha+2\beta} \cap (\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}) = 0. \end{aligned}$$

On the other hand, from Corollary 2.2(2c), we know that W contains a codimension-one subspace of $[\mathfrak{g}, \mathfrak{g}_{\alpha+2\beta}]$, so W contains a codimension-one subspace of $\mathfrak{g}_\beta + \mathfrak{g}_{\alpha+\beta}$. This is a contradiction.

Subcase 2.3. *Assume* $P = P_\beta$. Note that, because $U \subset \operatorname{unip} P$, we have $\operatorname{proj}_\beta \mathfrak{u} = 0$.

From Corollary 2.2(2d), we have

$$\begin{aligned} W &= \mathfrak{h} + (\operatorname{ad}_{\mathfrak{g}} u)^2 \mathfrak{g} \subset \mathfrak{h} + (\mathfrak{g}_\alpha + \mathfrak{g}_{\alpha+\beta} + \mathfrak{g}_{\alpha+2\beta}) \\ &= \mathfrak{h} + \operatorname{unip} \mathfrak{p}_\beta \subset (\mathfrak{m} + \mathfrak{u}) + \operatorname{unip} \mathfrak{p}_\beta = \mathfrak{m} + \operatorname{unip} \mathfrak{p}_\beta. \end{aligned}$$

Subsubcase 2.3.1. *Assume there is some nonzero* $u \in \mathfrak{u}$*, such that* $\operatorname{proj}_\alpha u = 0$. Replacing H by a conjugate (under $G_{-\beta}$), we may assume $\operatorname{proj}_{\alpha+\beta} u \neq 0$.

Let $V' = V \cap (\mathfrak{g}_{-\alpha} + \mathfrak{g}_{-\alpha-\beta})$. Because V' contains a codimension-one subspace of $\mathfrak{g}_{-\alpha} + \mathfrak{g}_{-\alpha-\beta}$ (see Corollary 2.2(2a)), one of the following two subsubsubcases must apply.

Subsubsubcase 2.3.1.1. *Assume there exists* $v \in V'$*, such that* $\operatorname{proj}_{-\alpha-\beta} v = 0$. From Corollary 2.2(2c), we have $[u, v] \in W$. Then, because $[u, v]$ is a nonzero element of $\mathfrak{g}_\beta$, we conclude that

$$0 \neq W \cap \mathfrak{g}_\beta \subset (\mathfrak{m} + \operatorname{unip} \mathfrak{p}_\beta) \cap \mathfrak{g}_\beta = 0.$$

This contradicts the fact that M, being compact, has no nontrivial unipotent elements.

Subsubsubcase 2.3.1.2. *Assume* $\operatorname{proj}_{-\alpha-\beta} V' = \mathfrak{g}_{-\alpha-\beta}$. For $v \in V'$, we have $\operatorname{proj}_0 [u, v] = [\operatorname{proj}_{\alpha+\beta} u, \operatorname{proj}_{-\alpha-\beta} v]$. Thus, there is some $v \in V'$, such that $\operatorname{proj}_0[u, v]$ is hyperbolic (and nonzero). On the other hand, from Corollary 2.2(2c), we have $[u, v] \in W = \mathfrak{m} + \operatorname{unip} \mathfrak{p}_\beta$. This contradicts the fact that $\mathfrak{m} \subset \bar{\mathfrak{h}}$ does not contain nonzero hyperbolic elements.

Subsubcase 2.3.2. *Assume* $\operatorname{proj}_\alpha u \neq 0$*, for every nonzero* $u \in \mathfrak{u}$. Fix some nonzero $u \in \mathfrak{u}$. Because $\dim \mathfrak{u}_\alpha = 1$, we must have $\dim \mathfrak{u} = 1$ (so $\mathfrak{u} = \mathbb{R} u$). Replacing H by a conjugate (under G_β), we may assume $\operatorname{proj}_{\alpha+\beta} u = 0$. Also, we may assume $\operatorname{proj}_{\alpha+2\beta} u \neq 0$ (otherwise, we could revert to Subsubcase 2.3.1 by replacing H with its conjugate under the Weyl reflection corresponding to the root β).

Let $\mathfrak{t} = [u, \mathfrak{g}_{-\alpha} + \mathfrak{g}_{-\alpha-2\beta}]$. Because $\langle \mathfrak{g}_\alpha, \mathfrak{g}_{-\alpha} \rangle$ and $\langle \mathfrak{g}_{\alpha+2\beta}, \mathfrak{g}_{-\alpha-2\beta} \rangle$ centralize each other, we see that $\mathfrak{t} = [\mathfrak{g}_\alpha, \mathfrak{g}_{-\alpha}] + [\mathfrak{g}_{\alpha+2\beta}, \mathfrak{g}_{-\alpha-2\beta}]$ is a two-dimensional subspace of $\mathfrak{g}$ consisting entirely of hyperbolic elements. Because V contains a codimension-one subspace of $\mathfrak{g}_{-\alpha} + \mathfrak{g}_{-\alpha-2\beta}$ (see Corollary 2.2(2a)), and $[u, V] \subset W$ (see Corollary 2.2(2c)), we see that W contains a codimension-one subspace of $\mathfrak{t}$, so W contains nontrivial hyperbolic elements. This contradicts the fact that $W \subset \mathfrak{m} + \operatorname{unip} \mathfrak{p}_\beta$ does not contain nontrivial hyperbolic elements. □

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