A'priori estimates for higher order multipliers on a circle

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ABSTRACT: We present an elementary proof of an a'priori estimate of Bourgain for a general class of multipliers on a circle using an extension of methods developed in our previous work. The main tool is a suitable version of a counting argument of Zygmund for unbounded regions.

1 Introduction and the result

In a series of papers beginning in early 90-ties Bourgain derived various periodic analogues of Strichartz inequalities. Apart from their intrinsic interest such inequalities have become a powerful tool for establishing well-posedness results for various nonlinear partial differential equations (see for example [B1], [B2], [B3], [HM1], [ST]). In [B1] and [B2] Bourgain provided explicit proofs of these inequalities in the quadratic ($\nu = 2$) and cubic ($\nu = 3$) cases and used them to study the periodic Cauchy problem for the nonlinear Schrödinger and KdV equations respectively. In [B3] he stated them for a much larger class of multipliers with arbitrary integer $\nu \geq 2$ (see (1.1) below). In our previous paper [HM2] we gave an elementary proof of these inequalities for the case when ν is even. Our approach was motivated by the work of Fang and Grillakis [FG] on

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*Both authors were partially supported by the NSF under grant number DMS-9970857 and by the Faculty Research Program of the University of Notre Dame. The second author was also supported by the Isaac Newton Institute for Mathematical Sciences of the University of Cambridge.

1991 Mathematics Subject Classification. Primary 42B15, Secondary 35G25.

Key words and phrases. Fourier transform, multiplier inequalities, interpolation

the Boussinesq equation. In this paper we extend those methods to complete the picture by establishing the inequalities for odd ν.

Our main result is contained in the following

Theorem 1.1 *Let $\nu \geq 2$ be a positive integer. Then there is a constant $c_\nu > 0$ such that for any test function f, we have*

$$\|(1+|\tau-\xi^\nu|)^{-\frac{\nu+1}{\nu}\frac{1}{r}}\hat{f}\|_{L^q(\mathbb{Z}\times\mathbb{R})} \leq c_\nu \|f\|_{L^p(\mathbb{T}\times\mathbb{R})}, \tag{1.1}$$

where $2 \leq q \leq \infty$ and $\frac{1}{p}+\frac{1}{q} = 1+\frac{1}{r}$ and $4 \leq r \leq \infty$.

It is of interest to determine what is the best possible estimate of this type for any given $\nu \geq 2$ above. In [B1] Bourgain constructed an example showing that if $\nu = 2$ then the corresponding inequality fails for $p = 6/5$ and $q = 2$. He suggested however that it may continue to hold if we allow $p = (6-\epsilon)/(5-\epsilon)$, for any sufficiently small positive ϵ. In a similar vein one may speculate that when $\nu = 2$ inequality (1.1) will hold for $p = 6/5$ and $q = (2-\epsilon)/(1-\epsilon)$.

The proof of Theorem 1.1 in the next section follows the approach used in our previous paper [HM2]. Using standard harmonic analysis we first reduce the proof to a certain bilinear estimate (see Lemma 2.2) and then develop a new counting argument ala Zygmund [Z] (see also [FG]) that enables us to suitably estimate the number of integer points located on certain intersections of straight lines with unbounded regions (see Lemma 2.3 and Lemma 2.4 below). These constructions are the main technical device used in the paper.

2 Proof of Theorem 1.1

The principal step in our proof of the Theorem will be to establish the following inequality.

Proposition 2.1 *For an arbitrary integer $\nu \geq 2$ and for any test function f, we have*

$$\|(1+|\tau-\xi^\nu|)^{-\frac{\nu+1}{4\nu}}\hat{f}\|_{L^2(\mathbb{Z}\times\mathbb{R})} \leq c_\nu \|f\|_{L^{4/3}(\mathbb{T}\times\mathbb{R})}. \tag{2.1}$$

As mentioned above (2.1) was proved in [B1] for $\nu = 2$ and in [B3] for $\nu = 3$. Theorem 1.1 will now follow by interpolating between the above estimate and the trivial Fourier transform estimate using Stein's interpolation theorem (see for example [SW]).

Proof of Proposition 2.1 The proof of the proposition in the case of even ν can be found in [HM2] and it will not be reproduced here. We shall therefore concentrate on the case when ν is odd. Observe that dualizing (2.1) gives

$$\|f\|_{L^4} \leq C_\nu \|(1 + |\tau - \xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}\|_{L^2}.$$

We proceed to derive this inequality beginning with some standard preliminaries (see [HM2] for more details, if necessary). First, without loss of generality we may assume that

$$\operatorname{supp} \hat{f} \subseteq \{(\xi, \tau) : \tau - \xi^\nu \geq 0\}.$$

Next, we introduce a dyadic decomposition of the Fourier frequency (ξ, τ)-space using a cut-off function in $C^\infty[1/2, 2]$ with the property that $\varphi(x) + \varphi(2x) = 1$ for all $x \in [1/2, 1]$. Defining

$$\varphi_j(x) = \varphi\left(\frac{x}{2^j}\right), \quad \varphi_0(x) = 1 - \sum_{j=1}^{\infty} \varphi_j(x),$$

and setting

$$\hat{f}_j(\xi, \tau) = \varphi_j(\tau - \xi^\nu) \hat{f}(\xi, \tau),$$

we can conveniently decompose

$$f = \sum_{j=0}^{\infty} f_j$$

in such a way that

$$\operatorname{supp} \hat{f}_j \subseteq \left\{(\xi, \tau) : 2^{j-1} \leq \tau - \xi^\nu \leq 2^{j+1}\right\}, \quad j = 1, 2 \ldots \tag{2.2}$$

and

$$\operatorname{supp} \hat{f}_o \subseteq \{(\xi, \tau) : 0 \leq \tau - \xi^\nu \leq 2\}.$$

Using the above decomposition we have

$$\|f\|_{L^4}^2 = \|f^2\|_{L^2} \le \sum_{j,k=0}^{\infty} \|f_j f_k\|_{L^2}.$$

A straightfoward manipulation using Cauchy-Schwarz inequality and the fact that

$$\sum_{k=0}^{\infty} 2^{-\frac{\nu-1}{4\nu}|j-k|} \le 2(1-2^{(1-\nu)/4\nu})^{-1}$$

makes it possible to reduce the proof of Proposition 2.1 to the proof of the following lemma.

Lemma 2.2 *There is a positive constant c such that*

$$\|f_j f_k\|_{L^2(\mathbb{T}\times\mathbb{R})} \le \frac{c}{2^{\frac{\nu-1}{4\nu}|j-k|}} \|(1+|\tau-\xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_j\|_{L^2(\mathbb{Z}\times\mathbb{R})} \|(1+|\tau-\xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_k\|_{L^2(\mathbb{Z}\times\mathbb{R})}.$$

Proof of Lemma 2.2. It suffices to consider $k \le j$, since the case $k \ge j$ is analogous. Applying the inverse Fourier transform and setting $\tau = \tau_1 + \tau_2$, $q = \tau_2 - \xi_2^\nu$ and $\xi = \xi_1 + \xi_2$ we can represent the product $f_j f_k$ in the following form

$$f_j f_k(x,t) = \int_{\mathbb{R}} \sum_{\xi\in\mathbb{Z}} e^{i(t\tau + x\xi)} \hat{G}_{jk}(\xi,\tau)\, d\tau,$$

where

$$\hat{G}_{jk}(\xi,\tau) = \int_{\mathbb{R}} \sum_{\xi_2\in\mathbb{Z}} \hat{f}_j(\xi-\xi_2, \tau - q - \xi_2^\nu) \hat{f}_k(\xi_2, q + \xi_2^\nu)\, dq.$$

Notice that the restriction imposed on the supports of $\hat{f}_l$ in (2.2) leads to the following relations for q and ξ_2

$$q \in \Delta_k = [2^{k-1}, 2^{k+1}] \qquad \text{and} \qquad \xi_2 \in \Lambda_j(\tau,\xi,q),$$

where, for $a = \tau - q - 2^{j+1}$ we put

$$\Lambda_j(\tau,\xi,q) = \left\{ \xi_2 \in \mathbb{Z} : a \le \xi_1^\nu + \xi_2^\nu \le a + \frac{3}{2} 2^j,\ \ \xi_1 + \xi_2 = \xi \right\}.$$

Finding suitable estimates of the cardinality of the set $\Lambda_j(\tau,\xi,q)$ will be crucial in what follows. First, however, using Plancherel equality and Jensen inequality we obtain

$$\begin{aligned}\|f_jf_k\|_{L^2}^2 &= \|\hat{G}_{jk}\|_{L^2}^2 \le \int_{\mathbb{R}} \sum_{\xi\in\mathbb{Z}} \left(\int_{\Delta_k} \sum_{\xi_2\in\Lambda_j} |\hat{f}_j\hat{f}_k|\,dq \right)^2 d\tau \\ &\le \int_{\mathbb{R}} \sum_{\xi\in\mathbb{Z}} \mathrm{meas}(\Delta_k) \int_{\Delta_k} \left(\sum_{\xi_2\in\Lambda_j} |\hat{f}_j\hat{f}_k| \right)^2 dq\,d\tau = I.\end{aligned}$$

It is at this point that we need to change our strategy as compared with the case of even ν. As mentioned above, to estimate I we will require a suitable bound on the cardinality of the set $\Lambda_j(\tau,\xi,q)$. In order to overcome the difficulty caused by the fact that the region defined by the inequalities $a \le \xi_1^\nu + \xi_2^\nu \le a + \frac{3}{2}2^j$ is unbounded we shall consider two cases.

Case (1). $2\left(\frac{a}{2}\right)^{1/\nu} \le 2^{j/\nu} \iff \tau - q \le 2^{j-\nu+1} + 2^{j+1}$.

and

Case (2). $2\left(\frac{a}{2}\right)^{1/\nu} > 2^{j/\nu} \iff \tau - q > 2^{j-\nu+1} + 2^{j+1}$.

According to these, we decompose I into two pieces

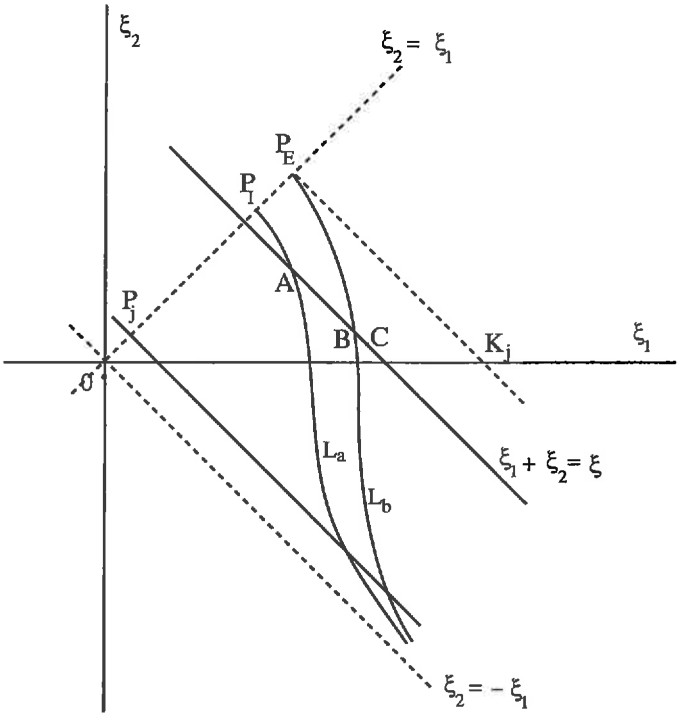
$$\begin{aligned} I = |\Delta_k| \int\int_{\tau-q\le 2^{j-\nu+1}+2^{j+1}} \sum_{\xi\in\mathbb{Z}} \left(\sum_{\xi_2\in\Lambda_j} |\hat{f}_j\hat{f}_k| \right)^2 dq\,d\tau + \\ +|\Delta_k| \int\int_{\tau-q> 2^{j-\nu+1}+2^{j+1}} \sum_{\xi\in\mathbb{Z}} \left(\sum_{\xi_2\in\Lambda_j} |\hat{f}_j\hat{f}_k| \right)^2 dq\,d\tau \\ = I_1 + I_2. \end{aligned}$$

Case (1). This case corresponds to estimating I_1. To proceed we will need the following "counting lemma".

Lemma 2.3 *The set A_j of all ξ for which the integrand in I_1 is not zero satisfies the estimate*

$$\sup_{\tau,q} \mathrm{card}\,(A_j(\tau,q)) \le 6\cdot 2^{j/\nu}.$$

Proof of Lemma 2.3. Observe that in this case the line $\xi_1 + \xi_2 = \left(\frac{a}{2}\right)^{1/\nu}$ lies "below" the line $\xi_1 + \xi_2 = 2^{j/\nu}$, as shown on the following picture.



From the picture we also find that

$$d(0, P_E) = d(0, P_I) + d(P_I, P_E) \leq d(0, P_j) + d(P_I, P_E),$$

where the various points have coordinates

$$P_I = (\left(\frac{a}{2}\right)^{1/\nu}, \left(\frac{a}{2}\right)^{1/\nu}),$$

$$P_j = (\frac{1}{2}2^{j/\nu}, \frac{1}{2}2^{j/\nu}), \text{ and } P_E = (\left(\frac{a}{2} + \frac{3}{4}2^j\right)^{1/\nu}, \left(\frac{a}{2} + \frac{3}{4}2^j\right)^{1/\nu}).$$

Since, clearly

$$d(P_I, P_E) \leq \sqrt{2}\left(\frac{3}{4}\right)^{1/\nu} 2^{j/\nu},$$

we must have

$$d(0, P_E) \leq 2^{j/\nu} + \sqrt{2}\left(\frac{3}{4}\right)^{1/\nu} 2^{j/\nu} < 3 \cdot 2^{j/\nu}.$$

However, since the set A_j is contained in the interval $[0, K_j]$ (refer again to the picture above) and since, necessarily, $K_j \leq \sqrt{2}d(0, P_E)$, we conclude that A_j must satisfy the desired estimate. $\square$

We now return to estimating the contribution I_1. Using Minkowski's inequality we find that

$$I_1 \lesssim |\Delta_k| \int \int_{\tau - q \leq 2^{j-\nu+1} + 2^{j+1}} \sum_{\xi \in A_j} \left(\sum_{\xi_2 \in \Lambda_j} |\hat{f}_j \hat{f}_k| \right)^2 dq\, d\tau$$

$$\lesssim |\Delta_k| \int_{\mathbb{R}} \int_{\mathbb{R}} \sum_{\xi \in A_j} \left(\sum_{\xi_2 \in \Lambda_j} |\hat{f}_j \hat{f}_k| \right)^2 d\tau\, dq$$

$$\lesssim |\Delta_k| \sum_{\xi \in A_j} \int_{\mathbb{R}} \left(\sum_{\xi_2 \in \Lambda_j} \left(\int_{\mathbb{R}} |\hat{f}_j(\xi - \xi_2, \tau - q - \xi_2^\nu)|^2 |\hat{f}_k(\xi_2, q + \xi_2^\nu)|^2\, d\tau \right)^{1/2} \right)^2 dq$$

$$= c|\Delta_k| \sum_{\xi \in A_j} \int_{\mathbb{R}} \left(\sum_{\xi_2 \in \Lambda_j} |\hat{f}_k(\xi_2, q + \xi_2^\nu)| \left(\int_{\mathbb{R}} |\hat{f}_j(\xi - \xi_2, \tau - q - \xi_2^\nu)|^2\, d\tau \right)^{1/2} \right)^2 dq.$$

Applying Cauchy-Schwarz and changing appropriate variables it follows that the last integral is bounded by

$$|\Delta_k| \sum_{\xi \in A_j} \int_{\mathbb{R}} \left(\sum_{\xi_2 \in \Lambda_j} |\hat{f}_k(\xi_2, q + \xi_2^\nu)|^2 \right) \left(\sum_{\xi_2 \in \Lambda_j} \int_{\mathbb{R}} |\hat{f}_j(\xi - \xi_2, \tau - q - \xi_2^\nu)|^2\, d\tau \right) dq$$

$$\begin{aligned} &= c|\Delta_k||A_j| \left(\int_{\mathbb{R}} \sum_{\xi_2 \in \mathbb{Z}} |\hat{f}_k(\xi_2, \eta_2)|^2\, d\eta_2 \right) \left(\sum_{\xi_2 \in \mathbb{Z}} \int_{\mathbb{R}} |\hat{f}_j(\xi - \xi_2, \eta_1)|^2\, d\eta_1 \right) \\ &= c 2^k 2^{j/\nu} \|\hat{f}_k\|_{L^2}^2 \|\hat{f}_j\|_{L^2}^2. \end{aligned}$$

Therefore, since $\tau - \xi^\nu \simeq 2^j$, we immediately find that

$$\begin{aligned} I_1 &\leq \frac{c}{2^{\frac{\nu-1}{4\nu}(j-k)}} 2^{\frac{\nu+1}{4\nu} j} 2^{\frac{\nu+1}{4\nu} k} \|\hat{f}_j\|_{L^2} \|\hat{f}_k\|_{L^2} \\ &\simeq \frac{c}{2^{\frac{\nu-1}{4\nu}(j-k)}} \|(1 + |\tau - \xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_j\|_{L^2} \|(1 + |\tau - \xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_k\|_{L^2}. \end{aligned}$$

Case (2). This time we need to estimate the contribution of I_2 which requires a different argument. Note, however, that now the region in question is bounded and so a version of a "counting lemma" employed in [HM2] for even ν should do the job. More precisely, we have

Lemma 2.4 *Let* $2\,(a/2)^{1/\nu} > 2^{j/\nu}$. *Then there is a constant* $c > 0$ *such that*

$$\sup_{\tau,\xi,q} \operatorname{card}\left(\Lambda_j(\tau,\xi,q)\right) \leq c 2^{j/\nu}.$$

Proof of Lemma 2.4. It will once more be convenient to refer to the picture above. Begin by observing that the line $\xi_1 + \xi_2 = 2\left(\frac{a}{2}\right)^{1/\nu}$ is tangent to the inner level curve L_a: $\xi_1^\nu + \xi_2^\nu = a$ at the point

$$P_I = \left(\left(\frac{a}{2}\right)^{1/\nu}, \left(\frac{a}{2}\right)^{1/\nu}\right),$$

while the line $\xi_1 + \xi_2 = 2^{j/\nu}$ intersects the diagonal $\xi_1 = \xi_2$ at the point

$$P_j = \left(\frac{1}{2}2^{j/\nu}, \frac{1}{2}2^{j/\nu}\right).$$

Assume $s \geq \left(\frac{a}{2}\right)^{j/\nu}$. Let A be the point on the inside level curve L_a, for which $\xi_1 = s$. Then the other coordinate of A is $\xi_2 = (a - s^\nu)^{1/\nu}$ and the equation of the line passing through A and having the slope -1 is $\xi_2 = -\xi_1 + s + (a - s^\nu)^{1/\nu}$. Consider now the following function

$$h(\xi_1) = \xi_1^\nu + \left(-\xi_1 + s + (a - s^\nu)^{1/\nu}\right)^\nu - a - \frac{3}{2}2^j.$$

Note that $h(\xi_1) = 0$ if and only if the point $B = \left(\xi_1, -\xi_1 + s + (a - s^\nu)^{1/\nu}\right)$ lies on the curve L_b: $\xi_1^\nu + \xi_2^\nu = a + \frac{3}{2}2^j$. On the one hand we easily find that

$$h(s) = s^\nu + \left(-s + s + (a - s^\nu)^{1/\nu}\right)^\nu - a - \frac{3}{2}2^j = -\frac{3}{2}2^j < 0.$$

On the other hand, we claim that

$$h(s + \left(\frac{3}{2}2^j\right)^{1/\nu}) \geq 0.$$

In order to obtain this inequality we need to call on our assumptions. First, we substitute

$$h(s+\left(\frac{3}{2}2^j\right)^{1/\nu})=$$

$$=\left(s+\left(\frac{3}{2}2^j\right)^{1/\nu}\right)^{\nu}+\left(-\left(\frac{3}{2}2^j\right)^{1/\nu}+(a-s^\nu)^{1/\nu}\right)^{\nu}-a-\frac{3}{2}2^j.$$

Next, noting that $(a-s^\nu)^{1/\nu}\le s$ and recalling that ν is an odd integer, we can estimate

$$\begin{aligned} h(s+\left(\frac{3}{2}2^j\right)^{1/\nu}) &\ge s^\nu+\frac{3}{2}2^j+a-s^\nu-\frac{3}{2}2^j \\ &+ \nu s\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}+\nu(a-s^\nu)^{1/\nu}\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}-a-\frac{3}{2}2^j \\ &= \nu s\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}+\nu(a-s^\nu)^{1/\nu}\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}-\frac{3}{2}2^j \\ &\ge \nu s\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}-\frac{3}{2}2^j. \end{aligned}$$

Now, since $s\ge\left(\frac{a}{2}\right)^{1/\nu}$, the last inequality gives

$$h(s+\left(\frac{3}{2}2^j\right)^{1/\nu})\ge\nu\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}\left(\frac{a}{2}\right)^{1/\nu}-\frac{3}{2}2^j.$$

However, bringing in the assumption that $2\left(\frac{a}{2}\right)^{1/\nu}\ge 2^{j/\nu}$ and using the fact that $\nu\ge 3$, we find that

$$h(s+\left(\frac{3}{2}2^j\right)^{1/\nu})\ge\nu\left(\frac{3}{2}2^j\right)^{\frac{\nu-1}{\nu}}\frac{1}{2}2^{j/\nu}-\frac{3}{2}2^j=\frac{3}{2}\left(\frac{\nu}{3}\left(\frac{3}{2}\right)^{\frac{\nu-1}{\nu}}-1\right)2^j\ge 0.$$

Denote by C the point on the line $\xi_1+\xi_2=\xi$ with coordinates

$$C=\left(s+\left(\frac{3}{2}2^j\right)^{1/\nu},-\left(\frac{3}{2}2^j\right)^{1/\nu}+(a-s^\nu)^{1/\nu}\right).$$

A quick check shows that

$$d(A,B)\le d(A,C)=\sqrt{2}\left(\frac{3}{2}2^j\right)^{1/\nu}$$

and so the Lemma is proved. ☐

Using Lemma 2.4 we are now ready to estimate the second contribution to I. Namely we have

$$\begin{aligned}
I_2 &= |\Delta_k| \int\int_{\tau-q>2^{j-\nu+1}+2^{j+1}} \sum_{\xi\in\mathbb{Z}} \left(\sum_{\xi_2\in\Lambda_j} |\hat{f}_j \hat{f}_k| \right)^2 dq\, d\tau \\
&\leq c|\Delta_k|\, 2^{\frac{j}{\nu}} \int\int_{\tau-q>2^{j-\nu+1}+2^{j+1}} \sum_{\xi\in\mathbb{Z}} \sum_{\xi_2\in\Lambda_j} |\hat{f}_j \hat{f}_k|^2\, dq\, d\tau \\
&\leq c\, 2^k\, 2^{\frac{j}{\nu}} \int_{\mathbb{R}} \sum_{\xi\in\mathbb{Z}} \int_{\mathbb{R}} \sum_{\xi_2\in\mathbb{Z}} |\hat{f}_j(\xi-\xi_2, \tau-q-\xi_2^\nu)|^2 |\hat{f}_k(\xi_2, q+\xi_2^\nu)|^2\, dq\, d\tau \\
&= c\, 2^k\, 2^{\frac{j}{\nu}} \sum_{\xi\in\mathbb{Z}} \sum_{\xi_2\in\mathbb{Z}} \int_{\mathbb{R}} |\hat{f}_j(\xi-\xi_2, \eta_1)|^2\, d\eta_1 \cdot \int_{\mathbb{R}} |\hat{f}_k(\xi_2, \eta_2)|^2\, d\eta_2 \\
&= c\, 2^k\, 2^{\frac{j}{\nu}} \|\hat{f}_j\|_{L^2}^2 \|\hat{f}_k\|_{L^2}^2.
\end{aligned}$$

Therefore, since $\tau - \xi^\nu \simeq 2^j$, we get as before

$$I_2 \leq \frac{c}{2^{\frac{\nu-1}{4\nu}(j-k)}} \|(1+|\tau-\xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_j\|_{L^2} \|(1+|\tau-\xi^\nu|)^{\frac{\nu+1}{4\nu}} \hat{f}_k\|_{L^2}.$$

The above estimate combined with the corresponding estimate for I_1 yields the desired inequality in Lemma 2.2 and concludes the proof. ☐

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