

Effective Supergravity for Supergravity Domain Walls

M. Cvetič^{1,3} * and N.D. Lambert^{2†}

¹ *Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104, USA*

² *Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854-8019, USA*

³ *Isaac Newton Institute for Mathematical Sciences, University of Cambridge, Cambridge, UK*

Abstract

We discuss the low energy effective action for the Bosonic and Fermionic zero-modes of a smooth BPS Randall-Sundrum domain wall, including the induced supergravity on the wall. The result is a pure supergravity in one lower dimension. In particular, and in contrast to non-gravitational domain walls or domain walls in a compact space, the zero-modes representing transverse fluctuations of domain wall have vanishing action.

arXiv:hep-th/0205247 v1 23 May 2002

*email: cvetic@cvetic.hep.upenn.edu

†email: nlambert@physics.rutgers.edu

I. INTRODUCTION

Supergravity domain walls [1] have been recently the subject of much attention from a variety of different points of view (for example see [1–11]; for an earlier review [12]). A central area of study has been that of the so-called Randall-Sundrum domain walls [13], which trap gravity to their worldvolumes. It was recognized early on [2] that the tuning used to stabilize such domain walls is simply a supersymmetry condition. However it has proven very difficult to obtain smooth four-dimensional Randall-Sundrum domain walls from the known supergravities. This led to several no-go theorems [4,14,5,15]. More recently a five-dimensional supergravity which admits a smooth Randall-Sundrum domain wall was obtained in [16].

In [5] it was observed that in a Randall-Sundrum background the proposed Goldstone Fermion from broken supersymmetry diverges on the wall. This result was interpreted as an explanation for the absence of smooth Randall-Sundrum domain walls in a large class of supergravities. However there are smooth domain walls of that type in four-dimensional supergravity [1] for which the the problem described in [5] still occurs, as it also does in the recent five-dimensional example [16]. In addition, to the extent of our knowledge, the coupling of the zero-modes to the gravity on the wall has not yet been addressed.

In this paper we would like to extend the discussion of the Bosonic and Fermionic zero-modes and obtain the effective action associated with these modes. In particular, we will reexamine the dynamics of the zero-modes of a supergravity domain wall (of the Randall-Sundrum type). The wall will be viewed as a solitonic object and thus our analysis will be analogous to the classic treatment of the low energy motion of monopoles in a gauge theory [17], which was first applied to gravity in [18–20]. We will see that in this context there is no problem with Fermion zero-modes, indeed these modes do not blow up on the wall. However, the dynamics of Randall-Sundrum type domain walls are qualitatively different to that of non-gravitational theories, or domain walls in compact spaces [21–24]. In particular, in contrast to the Higg’s mechanism observed in [21,22] for domain walls on a circle, we will see that the zero-modes that represent the transverse fluctuations are, in a sense, removed from the physical spectrum. This is caused by an exact cancellation between the positive tension of the domain wall and the negative energy density of bulk anti-de Sitter space. In effect the domain wall behaves as if it were tensionless.

The rest of this paper is organized in the following way. In Section II the supersymmetric action (up to bi-linear Fermionic terms) in D dimensions is given: it contains a gravity supermultiplet and a matter supermultiplet, whose real scalar field creates a wall. There the form of the Fermionic and Bosonic zero-modes is given and their effective action, whose prefactors turn out to be zero, is discussed. In Section III we discuss the properties of the effective action and supersymmetry transformations for the theory reduced on the wall. In Section IV we conclude with some remarks, interpreting the results and suggesting further investigations.

II. DOMAIN WALLS AND ZERO-MODES

For simplicity we assume that we are in D -dimensions ($D > 3$) with only gravity, one scalar ϕ and their superpartners ψ_m^i and λ^i active. Here i an internal spinor index which

we include for generality. We will restrict our attention here to supergravities where the supersymmetries takes the form

$$\begin{aligned}
\delta e_m^{\underline{n}} &= -\bar{\epsilon}_i \Gamma^{\underline{n}} \psi_m^i + c.c. , \\
\delta \phi &= \bar{\epsilon}_i \lambda^i + c.c. , \\
\delta \psi_m^i &= \nabla_m \epsilon^i + \kappa^{D-2} W \Gamma_m \epsilon^i , \\
\delta \lambda^i &= \left(-\frac{1}{2} \Gamma^m \partial_m \phi + (D-2) \frac{\partial W}{\partial \phi} \right) \epsilon^i ,
\end{aligned} \tag{2.1}$$

where an underlined index refers to the tangent frame, $m, n = 0, 1, 2, \dots, D-1$ and κ is the D -Dimensional Planck length. This is certainly not the most general form that one can imagine. In particular we have restricted the supersymmetry transformations to be diagonal in the i indices. However supergravities of this type have received considerable attention in recent years and the extension of our analysis to other supergravities is clear. In particular for the case of $D = 5$ a more detailed discussion can be found in [26]. In even dimensions one expects that there are terms involving Γ_{D+1} . However we expect that the analysis presented here is fairly insensitive to the precise form of the supersymmetry.

An action which is invariant under these supersymmetries, at least to lowest order in the Fermions, has the form [9]

$$\begin{aligned}
S &= \frac{1}{\kappa^{D-2}} \int d^D x e \left\{ R + \bar{\psi}_{mi} \Gamma^{mnp} \nabla_n \psi_p^i - \kappa^{D-2} \partial_m \phi \partial^m \phi + \kappa^{D-2} \bar{\lambda}_i \Gamma^m \nabla_m \lambda^i - \kappa^{D-2} V(\phi) \right. \\
&\quad + 2(D-2) \kappa^{D-2} \frac{\partial^2 W}{\partial \phi^2} \bar{\lambda}_i \lambda^i - (D-2) \kappa^{2D-4} W \bar{\lambda}_i \lambda^i - (D-2) W \kappa^D \bar{\psi}_{mi} \Gamma^{mn} \psi_n^i \\
&\quad \left. + \frac{1}{2} \kappa^D \partial_n \phi (\bar{\psi}_{mi} \Gamma^n \Gamma^m \lambda^i + \bar{\lambda}_i \Gamma^m \Gamma^n \psi_m^i) + (D-2) \kappa^D \frac{\partial W}{\partial \phi} (\bar{\psi}_{mi} \Gamma^m \lambda^i - \bar{\lambda}_i \Gamma^m \psi_m^i) + \dots \right\} ,
\end{aligned} \tag{2.2}$$

where the ellipsis denotes higher order terms in the Fermions. The scalar potential is

$$V = 4(D-2)^2 \left[\left(\frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left(\frac{D-1}{D-2} \right) W^2 \right] . \tag{2.3}$$

This action reproduces the equations of motion used in [5].

We note that the supersymmetric vacuum configurations are simply AdS spacetimes with ϕ fixed at a critical point of the superpotential, W , so that $V = -4(D-1)(D-2)\kappa^{D-2}W^2 \leq 0$. We will use coordinates in which the metric is

$$ds^2 = dr^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu , \tag{2.4}$$

where $A = -2\kappa^{D-2}W(\phi_0)r$, $r = x^{D-1}$ and $\mu, \nu = 0, \dots, D-2$. The Killing spinors are of the form

$$\epsilon_1^i = e^{\frac{1}{2}A} \tilde{\eta}_+^i , \quad \epsilon_2^i = \left(e^{-\frac{1}{2}A} - 2\kappa^{D-2}W(\phi_0)e^{\frac{1}{2}A} x^\mu \Gamma_\mu \right) \tilde{\eta}_-^i . \tag{2.5}$$

Here $\tilde{\eta}_{\pm}^i$ are constant spinors which satisfy $\Gamma^r \tilde{\eta}_{\pm}^i = \pm \tilde{\eta}_{\pm}^i$.

Our interest here is in supersymmetric domain wall solutions which have the same form for the metric 2.4 but the scalar field ϕ is not constant and A is not a linear function

$$\phi' = 2(D-2) \frac{\partial W}{\partial \phi}, \quad A' = -2\kappa^{D-2} W, \quad (2.6)$$

where a prime denotes differentiation with respect to r . This is merely a gravitational version of a BPS kink solution that interpolates between two supersymmetric vacua. Broken Poincare invariance implies that it has a Bosonic zero-mode \tilde{r} corresponding to the location of the kink, i.e. the general scalar field profile has the form $\phi(r - \tilde{r})$ for any \tilde{r} . A particular class of domain walls are the so-called Randall-Sundrum type [13], where W changes sign between the two vacua. These walls have the interesting feature that $e^{2A} \sim e^{-4\kappa^{D-2}|Wr|}$ as $r \rightarrow \infty$ leading to localized gravity on the wall. For the rest of our discussion we will restrict our attention to these types of domain walls.

Next we wish to construct the low energy dynamics associated to the zero-mode \tilde{r} . From the field theory perspective this is achieved by allowing \tilde{r} to depend on the walls' coordinates x^μ . One then simply evaluates the D -dimensional Lagrangian around such a background to lowest order in derivatives. In a theory with local diffeomorphism invariance one must be a little more careful. Following [25] we first note that the transformation

$$\delta g_{rr} = 0, \quad \delta g_{\mu r} = 0, \quad \delta g_{\mu\nu} = -2A' \tilde{r} e^{2A} \eta_{\mu\nu}, \quad (2.7)$$

that corresponds to an infinitesimal but constant shift $r \rightarrow r - \tilde{r}$ is simply a diffeomorphism. However if we now let \tilde{r} depend on x^μ then 2.7 is not a diffeomorphism and therefore we expect it to represent a physical mode.

To continue we consider a variation of the form 2.7 with an arbitrary fluctuation $\tilde{r}(x^\mu)$. To obtain the effective Lagrangian we substitute the domain wall solution back into 2.2, and then integrate over r . Note that this procedure does not imply that the full D -dimensional equations of motion are satisfied. Hence solutions to the effective action equations of motion do not lift to full solutions of the D -dimensional equations of motion. Instead they represent an effective description, analogous to the way that motion on monopole moduli space is an effective description of the behaviour of monopoles. However we must also check that the g_{rr} and $g_{\mu r}$ equations of motion are satisfied identically since the effective action we construct does not have any fields that represents them (i.e. they act as constraints on the low energy effective action).

With this prescription we obtain the $(D-1)$ -dimensional Lagrangian (we postpone including variations of the metric on the wall until the next section)

$$\tilde{\mathcal{L}}_B = \tilde{\mathcal{L}}_W - 4(D-2)^2 \int dr e^{(D-3)A} \left(\left(\frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left(\frac{D-3}{D-2} \right) W^2 \right) \eta^{\mu\nu} \partial_\mu \tilde{r} \partial_\nu \tilde{r}, \quad (2.8)$$

where $\tilde{\mathcal{L}}_W$ is the integral over r of the Lagrangian evaluated on the wall solution

$$\tilde{\mathcal{L}}_W = -8(D-2)^2 \int dr e^{(D-1)A} \left(\left(\frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left(\frac{D-1}{D-2} \right) W^2 \right). \quad (2.9)$$

However we note that, for any d ,

$$2(D-2) \left(\left(\frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left(\frac{d}{D-2} \right) W^2 \right) e^{dA} = \frac{d}{dr} (W e^{dA}) . \quad (2.10)$$

Hence for a Randall-Sundrum domain wall, due to the exponential fall-off of the metric at large r , $\tilde{\mathcal{L}}_B = \tilde{\mathcal{L}}_W = 0$. Thus the effective action for fluctuations of the wall vanishes, even though these fluctuations are not diffeomorphisms. Note that this integral vanishes only if $\kappa \neq 0$.

Now we wish to consider the Fermionic properties of a domain wall. One can readily check that ϵ_1^i is still a Killing spinor of the domain wall background, whereas ϵ_2^i is not. The fact that half of the supersymmetries of the AdS vacuum are preserved by the wall implies that the Bosonic zero-mode has a superpartner, so that the preserved supersymmetry is linearly realized. In particular the broken supersymmetry creates this Fermionic zero-mode. However in supergravity, or any theory with local symmetry, there are an infinite number of broken supersymmetries. Any such spinor is a linear combination

$$\epsilon^i = F_+ \tilde{\eta}_+^i + F_- \tilde{\eta}_-^i , \quad (2.11)$$

with arbitrary coefficients $F_+(x^m)$ and $F_-(x^m)$. To continue then let us outline two natural choices for the physical Goldstino mode.

At first thought we should choose the resulting Goldstino to respect the same symmetries as the wall, i.e. $\partial_\mu = 0$, $\psi_\mu^i = 0$. This is obtained by acting with supersymmetry that is preserved by AdS space but broken by the wall: $F_+ = 0$, $F_- = \kappa^{1-D} W^{-1} (e^{-\frac{1}{2}A} - 2\kappa^{D-2} W e^{\frac{1}{2}A} x^\mu \Gamma_\mu)$ and yields

$$\lambda^i = \frac{2(D-2)}{\kappa^{D-1}} \frac{1}{W} \frac{\partial W}{\partial \phi} e^{-\frac{1}{2}A} \tilde{\eta}_-^i , \quad \psi_r^i = -\frac{2(D-2)}{\kappa^{D-1}} \left(\frac{1}{W} \frac{\partial W}{\partial \phi} \right)^2 e^{-\frac{1}{2}A} \tilde{\eta}_-^i , \quad \psi_\mu^i = 0 . \quad (2.12)$$

This is precisely the Goldstino found in [5] and diverges if the superpotential W changes sign. This will mean that when we construct the effective action for the Fermionic zero-mode, found by letting $\tilde{\eta}_-^i$ become a field which depends on x^μ , we find

$$\tilde{\mathcal{L}}_F = 4(D-2)^2 \kappa^{2-2D} \int dr e^{(D-3)A} \left(\frac{1}{W} \frac{\partial W}{\partial \phi} \right)^2 \tilde{\eta}_{-i} \Gamma^\mu \partial_\mu \tilde{\eta}_-^i + \dots , \quad (2.13)$$

where the ellipsis denotes cross terms involving $\tilde{\eta}_{-i} \partial_\nu \tilde{\eta}_-^i$. The existence of these terms indicates that a field redefinition is needed to put the action into a standard form. Thus in a Randall-Sundrum type of domain wall, where W passes through zero, the kinetic term for such a Fermionic zero-modes diverges. This seems contradictory since the kinetic term for the Bosonic zero-mode \tilde{r} is well behaved, indeed it vanishes. In [5] this was used as an indication that W cannot change sign in a well-defined supergravity. However it is clear that the divergence is caused by the choice $\epsilon^i = W^{-1} \epsilon_2^i$.

A better choice is to find a Goldstino mode with $\partial_\mu = 0$ but $\psi_\mu^i \neq 0$. This can be done by simply acting on the domain wall with the supersymmetry generated by $e^{-\frac{1}{2}A} \tilde{\eta}_-^i$ and yields

$$\lambda^i = 2(D-2) \frac{\partial W}{\partial \phi} e^{-\frac{1}{2}A} \tilde{\eta}_-^i, \quad \psi_r^i = 0, \quad \psi_\mu^i = 2\kappa^{D-2} W \Gamma_\mu e^{\frac{1}{2}A} \tilde{\eta}_-^i. \quad (2.14)$$

This solution is much nicer. Indeed if we evaluate the effective action for it we find

$$\tilde{\mathcal{L}}_F = 4(D-2)^2 \int dr e^{(D-3)A} \left(\left(\frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left(\frac{D-3}{D-2} \right) W^2 \right) \bar{\tilde{\eta}}_{-i} \Gamma^\mu \partial_\mu \tilde{\eta}_-^i. \quad (2.15)$$

Thus we encounter precisely the same, vanishing, integral that we obtained for the Bosonic zero-mode. In addition the cross terms involving $\bar{\tilde{\eta}}_{-i} \partial_\nu \tilde{\eta}_-^i$ are total derivatives and can be discarded. Therefore 2.14 seems to be the correct choice of Goldstino that is linearly related to \tilde{r} .

It is instructive to contrast this discussion with the case of a domain wall in non-gravitating theory. Specifically, we take the flat space limit $\kappa = 0$ and set $e_m^{\underline{n}} = \delta_m^{\underline{n}}$, $\psi_m^i = 0$. Thus the original D -dimensional action 2.2 simplifies to

$$S_{\kappa=0} = - \int d^D x \left\{ \partial_m \phi \partial^m \phi + 4(D-2)^2 \left(\frac{\partial W}{\partial \phi} \right)^2 - \bar{\lambda}_i \Gamma^m \partial_m \lambda^i - 2(D-2) \frac{\partial^2 W}{\partial \phi^2} \bar{\lambda}_i \lambda^i \right\} \quad (2.16)$$

The supersymmetry transformations are easily determined from 2.1 by setting $\kappa = 0$ and they reduce to the rigid supersymmetry $\delta \tilde{\eta}_-^i = \frac{1}{2} \tilde{\Gamma}^\mu \partial_\mu \tilde{r}$, $\delta \tilde{r} = -\bar{\tilde{\epsilon}}_{+i} \tilde{\eta}_-^i + c.c.$. However, integrals of terms of the form 2.10 no longer vanish and we instead find the effective action of a free scalar \tilde{r} and its superpartner $\tilde{\eta}_-^i$

$$\tilde{\mathcal{L}}_{\kappa=0} = -8(D-2)^2 \int dr \left(\frac{\partial W}{\partial \phi} \right)^2 \left(1 + \frac{1}{2} \partial_\mu \tilde{r} \partial^\mu \tilde{r} - \frac{1}{2} \bar{\tilde{\eta}}_{-i} \tilde{\Gamma}^\mu \partial_\mu \tilde{\eta}_-^i \right). \quad (2.17)$$

The integral over r is again a total derivative and can be evaluated to be $4(D-2)|W(r=\infty) - W(r=-\infty)|$, which is simply the tension of the domain wall. Thus the low energy dynamics of the zero-modes in the supergravity domain wall can not be continuously reduced to the flat space limit by making κ arbitrarily small.

III. SUPERGRAVITY ON THE WALL

In this section we are interested in understanding how the zero-modes found above, which describe the fluctuations of the wall, couple to the gravitational fields. We will be primarily interested in whether or not the full effective action of the wall can be identified with a $(D-1)$ -dimensional supergravity. Previous studies have discussed the reduction of the supergravity to the wall [6,8,7,8] however these have not included the zero-modes, i.e. they treat the wall as rigid. We also seek to further justify the choice 2.14 as the correct Fermionic zero-mode.

More precisely we wish to reduce the D -dimensional supersymmetry involving $e_m^{\underline{n}}, \phi, \lambda^i, \psi_m^i$ to $(D-1)$ -dimensional supersymmetry involving $\tilde{r}, \tilde{\eta}_-^i$ and the bulk supergravity fields, suitably dimensionally reduced. In particular we consider the following standard ansatz for the Bosonic fields

$$\begin{aligned}
\phi &= \phi(r - \tilde{r}) , \\
A &= A(r - \tilde{r}) , \\
e_m^{\underline{n}} &= \begin{pmatrix} 1 & e^{\alpha A} \tilde{A}_\mu \\ 0 & e^A \tilde{e}_\mu^{\underline{\nu}} \end{pmatrix} ,
\end{aligned} \tag{3.1}$$

where ϕ and A continue to satisfy the domain wall Bogomoln'yi equations 2.6 and α is a parameter that we will fix shortly. In what follows we use tildes to denote $(D-1)$ -dimensional fields and $\tilde{\Gamma}_\mu = \tilde{e}_\mu^{\underline{\nu}} \Gamma_\nu$. Our next task is to determine the correct form for the Fermions so as to obtain a supersymmetry acting only on the $(D-1)$ -dimensional fields living on the wall.

First we note that the action 2.2, when expressed in terms of the new fields, is still invariant under the supersymmetry 2.1. Although the transformations given in 2.1 are symmetries for any ϵ^i , we are only interested in those that preserve the domain wall. Thus we are led to introduce the $(D-1)$ -dimensional supersymmetry generator $\tilde{\epsilon}_+^i$ as

$$\epsilon^i = e^{\frac{1}{2}A} \tilde{\epsilon}_+^i , \tag{3.2}$$

where $\Gamma_r \tilde{\epsilon}_+^i = \tilde{\epsilon}_+^i$.

With the form for λ^i given in 2.14 the $\delta\phi$ and $\delta\lambda^i$ variations simply reduce to

$$\begin{aligned}
\delta\tilde{r} &= -\tilde{\epsilon}_{+i} \tilde{\eta}_-^i + c.c. , \\
\delta\tilde{\eta}_-^i &= \frac{1}{2} \tilde{\Gamma}^\mu \left(\partial_\mu \tilde{r} + e^{\alpha A} \tilde{A}_\mu \right) \tilde{\epsilon}_+^i .
\end{aligned} \tag{3.3}$$

Recall that we restrict to terms that are at most quadratic in the Fermi fields. The supersymmetry reduces to an expression that involves only the $(D-1)$ -dimensional fields only if $\alpha = 0$. That this occurs at all is due to the precise form for the λ^i Goldstino in 2.14 and does not occur if we used any other form. The appearance of \tilde{A}_μ is reminiscent of a Higg's mechanism where A_μ "eats" $\partial_\mu \tilde{r}$. Thus we see that \tilde{r} and $\tilde{\eta}_-^i$ defined by 2.14 are indeed superpartners on the wall.

Next we must ensure that the gauge choice $e_r^{\underline{r}} = 1$ and $e_\mu^{\underline{r}} = 0$ is preserved by the supersymmetries generated by $\tilde{\epsilon}_+^i$. This implies that $\psi_r^i = 0$. However to preserve $\psi_r^i = 0$ we must have

$$0 = \delta\psi_r^i = -\frac{1}{8} \tilde{F}^{\mu\nu} \tilde{\Gamma}_{\mu\nu} e^{-\frac{3}{2}A} \tilde{\epsilon}_+^i , \tag{3.4}$$

where here, and in what follows, we have set $\alpha = 0$. Thus to preserve supersymmetry on the wall we must set $\tilde{F}_{\mu\nu} = 0$. Note that this does not necessarily imply $\tilde{A}_\mu = 0$. Another way to see this restriction arises in the physically interesting cases of $D \leq 5$. It is not hard to see that the reduction of the Einstein-Hilbert action in D dimensions using the ansatz 3.1 leads to the kinetic term

$$-\frac{1}{4\kappa^{D-2}} \int dr e^{(D-5)A} \tilde{F}^2 , \tag{3.5}$$

for the graviphoton. Clearly in $D \leq 5$ the integral over r is infinite. From the point of view of the theory on the wall this can be interpreted as saying that the $(D-1)$ -dimensional electromagnetic coupling constant vanishes and hence Maxwell's equation is simply $\tilde{F}_{\mu\nu} = 0$.

Next we consider the gravitini ψ_μ^i . In particular, using 2.14 as a guide, we let $\psi_\mu^i = 2\kappa^{D-2}W e^{\frac{1}{2}A}\tilde{\Gamma}_\mu\tilde{\eta}_-^i + e^{\frac{1}{2}A}\tilde{\psi}_{\mu+}^i + e^{\beta A}\tilde{\psi}_{\mu-}^i + e^{\gamma A}\tilde{\Gamma}_\mu\tilde{\chi}_+^i$, where β and γ are to be determined. Note that $\tilde{\chi}_+^i$ and $\tilde{\eta}_-^i$ can be distinguished from each other by their chiralities and the coefficient of $\tilde{\psi}_{\mu+}^i$ will be justified by the calculations which follow. Considering the variation of ψ_μ^i we find

$$\delta\tilde{\psi}_{\mu+}^i + e^{(\beta-\frac{1}{2})A}\delta\tilde{\psi}_{\mu-}^i = \tilde{\nabla}_\mu\tilde{\epsilon}_+^i, \quad \delta\tilde{\chi}_+^i = 0, \quad (3.6)$$

where we have used the restriction $\tilde{F}_{\mu\nu} = 0$. Next we substitute this ansatz into the variation of $e_\mu{}^r = \tilde{A}_\mu$ to find

$$\delta\tilde{A}_\mu = e^{(\frac{1}{2}+\beta)A}\tilde{\epsilon}_{+i}\tilde{\psi}_{\mu-}^i + e^{(\frac{1}{2}+\gamma)A}\tilde{\epsilon}_{+i}\tilde{\Gamma}_\mu\tilde{\chi}_+^i + c.c. . \quad (3.7)$$

To ensure that the right hand side is independent of r we must set $\beta = \gamma = -1/2$. Thus $\tilde{\chi}_+^i$ plays the role of a graviphotini since $\delta\tilde{A}_\mu = \tilde{\epsilon}_{+i}\tilde{\Gamma}_\mu\tilde{\chi}_+^i$ while $\tilde{\psi}_{\mu-}^i$ is sterile in the sense that 3.6 implies $\delta\tilde{\psi}_{\mu-}^i = 0$. However, as with \tilde{A}_μ above, it is easy to see that the $\tilde{\psi}_{\mu-}^i$ and $\tilde{\chi}_+^i$ kinetic terms in the effective action are infinite if $D \leq 5$ and hence they must be set to zero. This also follows by supersymmetry for any D since we must have $\tilde{F}_{\mu\nu} = 0$ for all variations $\tilde{\epsilon}_+^i$. Thus we set $\tilde{\psi}_{\mu-}^i = \tilde{\chi}_+^i = 0$.

Lastly we can obtain the variation of the vielbein on the wall

$$\delta\tilde{e}_\mu{}^\nu = -\tilde{\epsilon}_{+i}\tilde{\Gamma}^\nu\tilde{\psi}_{\mu+}^i - 2\kappa^{D-2}W\tilde{\epsilon}_{+i}\tilde{\Gamma}^{\nu\lambda}\tilde{\eta}_-^i\tilde{e}_{\mu\lambda} + c.c. . \quad (3.8)$$

Here we find that the variation involves the r -dependent term $2\kappa^{D-2}W\tilde{\epsilon}_{+i}\tilde{\Gamma}^{\nu\lambda}\tilde{\eta}_-^i\tilde{e}_{\mu\lambda} + c.c..$ To continue let us first summarize our calculations so far. We have found the following ansatz for the Fermions

$$\begin{aligned} \lambda^i &= 2(D-2)\frac{\partial W}{\partial\phi}e^{-\frac{1}{2}A}\tilde{\eta}_-^i, \\ \psi_\mu^i &= 2\kappa^{D-2}W e^{\frac{1}{2}A}\tilde{\Gamma}_\mu\tilde{\eta}_-^i + e^{\frac{1}{2}A}\tilde{\psi}_{\mu+}^i, \\ \psi_r^i &= 0, \end{aligned} \quad (3.9)$$

which leads to the symmetry

$$\begin{aligned} \delta\tilde{e}_\mu{}^\nu &= -\tilde{\epsilon}_{+i}\tilde{\Gamma}^\nu\tilde{\psi}_{\mu+}^i - 2\kappa^{D-2}W\tilde{\epsilon}_{+i}\tilde{\Gamma}^{\nu\lambda}\tilde{\eta}_-^i\tilde{e}_{\mu\lambda} + c.c. , \\ \delta\tilde{r} &= -\tilde{\epsilon}_{+i}\tilde{\eta}_-^i + c.c. , \\ \delta\tilde{A}_\mu &= 0, \\ \delta\tilde{\eta}_-^i &= \frac{1}{2}\tilde{\Gamma}^\mu\left(\partial_\mu\tilde{r} + \tilde{A}_\mu\right)\tilde{\epsilon}_+^i, \\ \delta\tilde{\psi}_{\mu+}^i &= \tilde{\nabla}_\mu\tilde{\epsilon}_+^i, \end{aligned} \quad (3.10)$$

where we have imposed $\tilde{F}_{\mu\nu} = 0$.

The additional term in the vielbein variation is puzzling. However it is in fact rather harmless. To see this we may obtain the effective action for the $(D-1)$ -dimensional fields by substituting in the ansatz 3.1,3.1 into 2.2. For a Randall-Sundrum domain wall this yields

$$\tilde{\mathcal{L}} = \frac{1}{\kappa^{D-2}} \int dr e^{(D-3)A} \tilde{e} \left\{ \tilde{R} + \tilde{\psi}_{\mu+i} \tilde{\Gamma}^{\mu\nu\rho} \tilde{\nabla}_\nu \tilde{\psi}_{\rho+}^i \right\} . \quad (3.11)$$

Note that in deriving 3.11 we have discarded several terms whose integral over r vanishes due to the asymptotic fall off of the metric. We have also checked that the g_{rr} and $g_{\mu r}$ equations of motion are satisfied identically.

The action 3.11 is the minimal supergravity Lagrangian in $(D-1)$ dimensions. The integral over r in 3.11 is finite and is simply absorbed into Planck's constant in $(D-1)$ -dimensions. In addition \tilde{r} , \tilde{A}_μ and $\tilde{\eta}_-^i$ have all disappeared from the action, i.e. they do not represent any physical modes of the low energy effective dynamics. Therefore without loss of generality we may set $\tilde{A}_\nu = -\partial_\nu \tilde{r}$ and $\tilde{\eta}_-^i = 0$. In this case the final supersymmetry on the wall is just that of a minimal supergravity

$$\begin{aligned} \delta \tilde{e}_\mu^\nu &= -\tilde{e}_{+i} \tilde{\Gamma}^\nu \tilde{\psi}_{\mu+}^i + c.c. , \\ \delta \tilde{\psi}_{\mu+}^i &= \tilde{\nabla}_\mu \tilde{\epsilon}_+^i . \end{aligned} \quad (3.12)$$

We note that for an arbitrary choice of $\tilde{r}(x)$ and $\delta g_{\mu r} = \tilde{A}_\mu$ the corresponding variation 2.7 of the wall is not a diffeomorphism. However in the special case that $\tilde{A}_\mu = -\partial_\mu \tilde{r}$, where the supersymmetry of transformation rules are simple, then the variation is a diffeomorphism.

In a sense the Bosonic zero-mode is eaten by the graviphoton, in a manner similar to [21,22] and similarly the gravitini has, in a sense, eaten the Fermionic zero-mode. However the graviphoton and (part of the) gravitini are not really massive but rather have been frozen out all together. Furthermore, and in contrast to the Higg's mechanism, the kinetic terms for \tilde{r} and $\tilde{\eta}_-^i$ have disappeared. Thus, rather than becoming components of some massive fields, the zero-modes are completely removed from the physical spectrum. We can understand the vanishing of the kinetic terms for \tilde{r} and $\tilde{\eta}_-^i$ as an exact cancellation between the positive tension of the domain wall, given by the first term on the left hand side of 2.10, and the negative energy density of anti-de Sitter space, given by the second term on the left hand side of 2.10.

We would also like to contrast our results to those obtained in [23,24] where the effective action for (infinitely thin) Randall Sundrum domain walls, where the transverse dimension was a compact Z_2 orbifold. In this case prefactor for the a kinetic energy term of the modulus is non-zero. Note however, that in this case the transverse direction is finite (due to the orbifold compactification) whereas here the vanishing of the kinetic terms arises because of the infinite extra dimension. In addition in this case, the choice of the boundary conditions seems to remove the the Bosonic zero mode \tilde{r} . Consequently the authors of [23,24] allowed for a more general fluctuation of the metric component g_{rr} and this may be an interesting direction to further explore within our context. We also note that the action that we have considered should be viewed as a truncation of supergravity to the sector most relevant to the domain wall. In general we expect that there will be other scalars and p -form fields which will contribute to the low energy dynamics. However we do not expect that these fields will affect the dynamics of \tilde{r} that we discussed here.

IV. CONCLUSIONS

In this paper we have evaluated the effective action for the Bosonic and Fermionic zero-modes of a Randall-Sundrum domain wall. Our result was simply 3.11, i.e. pure supergravity. In particular the zero-mode, and its superpartner, that are normally identified with transverse fluctuations of the wall were found to have a vanishing action.

Given this somewhat surprising result it seems appropriate to mention some aspects of the approximation that we have used to obtain the effective action. In particular we considered slow motion on the moduli space by allowing the domain wall to fluctuate and then evaluating the action to second order in derivatives. As a consequence some modes, such as the graviphoton, are frozen, i.e. their kinetic terms diverge. This is a familiar effect in soliton dynamics and implies that the low energy dynamics is restricted to a subspace of the full moduli space. In particular the freezing of half of the gravitini leads to the reduced supersymmetry in the effective theory. Other modes, such as the fluctuations of gravity along the wall, are described by the appropriate action in the lower dimension and lead to non-trivial dynamics at low energy. However the transverse fluctuations have not been frozen out, nor do they have a kinetic term. Instead they disappear from the action regardless of the form they take. In effect, even though these modes are not diffeomorphisms in the full theory, they behave like diffeomorphisms from the perspective of the effective theory.

Note that the effective action does not contain the full dynamics of the theory, just those that are relevant to small fluctuations about the soliton. It would be interesting to further understand the validity of this approximation and determine if there are applications to brane-world scenarios where the low energy dynamics of our world are insensitive to the presence of the extra-dimension through any of the fluctuations of the domain wall.

Acknowledgments

We would like to thank N. Arkani-Hamed, H. Liu, R. Myers, J-L. Lehners, D. Tong and particularly K. Stelle for discussions. N.D.L. Would like to thank the University of Pennsylvania for its hospitality where this work was initiated and was also partially supported by a PPARC Advanced Fellowship at the Department of Mathematics, King's College London and the Isaac Newton Institute of Cambridge University. M.C. is supported by the DOE grant EY-76-02-3071 and by the UPenn Class of 1965 Endowed Term Chair and would like to thank High Energy Theory group of Rutgers University, and the Isaac Newton Institute of Cambridge University for hospitality and support during the course of the work.

REFERENCES

- [1] M. Cvetič, S. Griffies and S. J. Rey, *Static domain walls in $N=1$ supergravity*, Nucl. Phys. B **381**, 301 (1992), hep-th/9201007.
- [2] K. Behrndt and M. Cvetič, *Supersymmetric domain wall world from $D = 5$ simple gauged supergravity*, Phys. Lett. B **475**, 253 (2000), hep-th/9909058.
- [3] K. Skenderis and P. K. Townsend, *Gravitational stability and renormalization-group flow*, Phys. Lett. B **468**, 46 (1999), hep-th/9909070.
- [4] R. Kallosh and A. D. Linde, *Supersymmetry and the brane world*, JHEP **0002**, 005 (2000), hep-th/0001071.
- [5] G.W. Gibbons and N.D. Lambert, *Solitons and Domain Walls in Odd Dimensions*, Phys. Lett. **B488** (2000) 90, hep-th/0003197.
- [6] H. Lü and C. N. Pope, *Branes on the brane*, Nucl. Phys. B **598**, 492 (2001), hep-th/0008050.
- [7] M. Cvetič, H. Lü and C. N. Pope, *Brane-world Kaluza-Klein reductions and branes on the brane*, hep-th/0009183.
- [8] M. J. Duff, J. T. Liu and W. A. Sabra, *Localization of supergravity on the brane*, Nucl. Phys. B **605**, 234 (2001), hep-th/0009212.
- [9] F. Brito, M. Cvetič and S-C. Yoon, *From Thick to Thin Domain Walls in Supergravity*, Phys. Rev. D **64**, 064021 (2001), hep-ph/0105010.
- [10] C.M. Hull, *Domain Wall and de Sitter Solutions of Gauged Supergravity*, JHEP **0111**, 061 (2001), hep-th/0110048.
- [11] P.K. Townsend, *Quintessence from M-theory*, JHEP **0111**, 042 (2001), hep-th/0110072.
- [12] M. Cvetič and H. H. Soleng, *Supergravity domain walls*, Phys. Rept. **282**, 159 (1997), hep-th/9604090.
- [13] L. Randall and R. Sundrum, *An Alternative to Compactification*, Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.
- [14] M. Wijnholt and S. Zhukov, *On the uniqueness of black hole attractors*, hep-th/9912002.
- [15] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. A **16**, 822 (2001), hep-th/0007018.
- [16] K. Behrndt and G. Dall'Agata, *Vacua of $N = 2$ gauged supergravity derived from non-homogeneous quaternionic spaces*, hep-th/0112136.
- [17] N. S. Manton, *A Remark On The Scattering Of Bps Monopoles*, Phys. Lett. B **110**, 54 (1982).
- [18] G. W. Gibbons and P. J. Ruback, *The Motion Of Extreme Reissner-Nordstrom Black Holes In The Low Velocity Limit*, Phys. Rev. Lett. **57**, 1492 (1986).
- [19] R. C. Ferrell and D. M. Eardley, *Slow Motion Scattering And Coalescence Of Maximally Charged Black Holes*, Phys. Rev. Lett. **59**, 1617 (1987).
- [20] K. Shiraishi, *Moduli space metric for maximally charged dilaton black holes*, Nucl. Phys. B **402**, 399 (1993).
- [21] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The Hierarchy Problem and New Dimensions at a Millimeter*, Phys. Lett. **B429** (1999) 263, hep-th/9803315.
- [22] G. Dvali, I.I. Kogan and M. Shifman, *Topological Effects in Our Brane World From Extra Dimensions*, Phys. Rev. D **62** (2000) 106001, hep-th/0006213.
- [23] M. A. Luty and R. Sundrum, *Hierarchy stabilization in warped supersymmetry*, Phys. Rev. D **64**, 065012 (2001), hep-th/0012158.

- [24] J. Bagger, D. Nemeschansky and R. J. Zhang, *Supersymmetric radion in the Randall-Sundrum scenario*, JHEP **0108**, 057 (2001), hep-th/0012163.
- [25] T. Adawi, M. Cederwall, U. Gran, B. E. Nilsson and B. Razaznejad, *Goldstone tensor modes*, JHEP **9902**, 001 (1999), hep-th/9811145.
- [26] E. Cremmer, *Supergravities In 5 Dimensions*, in C80-06-22.1.1 LPTENS 80/17 *Invited paper at the Nuffield Gravity Workshop, Cambridge, Eng., Jun 22 - Jul 12, 1980.*