Special Holonomy Spaces and M-theory *

M. Cvetič[1,5], G.W. Gibbons[2], H. Lü[3] and C.N. Pope[4,5]

[1]*Department of Physics and Astronomy*
University of Pennsylvania, Philadelphia, PA 19104, USA
[2] *DAMTP, Centre for Mathematical Sciences, Cambridge University*
Wilberforce Road, Cambridge CB3 0WA, UK
[3] *Michigan Center for Theoretical Physics, University of Michigan*
Ann Arbor, MI 48109, USA
[4] *Center for Theoretical Physics, Texas A&M University, College Station, TX 77843, USA*
[5] *Isaac Newton Institute for Mathematical Sciences,*
20 Clarkson Road, University of Cambridge, Cambridge CB3 0EH, UK

Abstract

We review the construction of regular p-brane solutions of M-theory and string theory with less than maximal supersymmetry whose transverse spaces have metrics with special holonomy, and where additional fluxes allow for brane resolutions via transgression terms. We summarize properties of resolved M2-branes and fractional D2-branes, whose transverse spaces are Ricci flat eight-dimensional and seven-dimensional spaces of special holonomy. Recent developments in the construction of new G_2 holonomy spaces are also reviewed.

I. INTRODUCTION

Regular supergravity solutions with less than maximal supersymmetry may provide viable gravity duals to strongly coupled field theories with less than maximal supersymmetry. In particular, the regularity of such solutions at small distances sheds light on confinement and chiral symmetry breaking in the infrared regime of the dual strongly coupled field theory [1].

We shall briefly review the construction of such regular supergravity solutions with emphasis on resolved M2-branes of 11-dimensional supergravity and fractional D2-branes of Type IIA supergravity, which provide viable gravity duals of strongly coupled three-dimensional theories with $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetry.

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This construction has been referred to as a "resolution via transgression" [2]. It involves the replacement of the standard flat transverse space by a smooth space of special holonomy, i.e. a Ricci-flat space with fewer covariantly constant spinors. Furthermore, additional field strength contributions are involved, which are provided by harmonic forms in the space of special holonomy. Transgression–Chern-Simons terms modify the equation of motion and/or Bianchi identity for the original p-brane field strength. In Section II the construction will be reviewed in general and then applied to resolved M2-branes and D2-branes.

The explicit construction of such solutions has led to mathematical developments, for example obtaining harmonic forms for a large class of special holonomy metrics. As a prototype example we shall review the construction of the metric and the middle-dimensional forms for the Stenzel manifolds in $D = 2n$ (with $n \geq 2$ integer) [3,4]. We shall also briefly mention examples of known G_2 holonomy spaces and their associated harmonic forms. We also discuss the old as well as the new two-parameter metric with Spin(7) holonomy [5,6] and the associated harmonic forms. We shall then briefly summarize the properties of resolved M2-branes [2,4] and fractional D2-branes [2,7] as well as fractional M2-brane whose transverse space is that of the new Spin(7) holonomy metrics [5]. All these developments will be reviewed in in Section III.

In the subsequent Section IV we review the most recent progress on explicit constructions of G_2 holonomy spaces. In particular, we highlight the construction of general G_2 holonomy spaces whose principal orbits are S^3 bundles over S^3 and the intriguing connection of those spaces to a unified description of deformed and resolved conifolds in six-dimensions.

In Section V we also outline directions of current and future research, in particular the study of singular G_2 holonomy spaces and their implications for four-dimensional non-Abelian chiral theories that can arise from a compactification of M-theory on such classes of special holonomy spaces.

The work presented in these lectures was initiated in [2] and further pursued in a series of papers that provide both new technical mathematical results and physics implications for resolved p-brane configurations [3–7]. Recent progress in the construction of new special holonomy spaces was initiated in [5,6], resulting in the first example of asymptotically locally conical metric with Spin(7) holonomy. A subsequent series of papers, which appeared after the lectures had been given, developed these techniques and provided explicit analyses of classes of cohomogeneity-one G_2 holonomy metrics, with the primary focus on those whose principal orbits are S^3 bundles over S^3 [8–23].

II. RESOLUTION VIA TRANSGRESSION

A. Motivation

The $\text{AdS}_{D+1}/\text{CFT}_D$ correspondence [24–26] provides a quantitative insight into strongly coupled superconformal gauge theories in D dimensions, by studying the dual supergravity solutions. The prototype supergravity dual is the D3-brane of Type IIB theory, with the classical solution

$$ds_{10}^2 = H^{-1/2} \, dx \cdot dx + H^{1/2} \left(dr^2 + r^2 \, d\Omega_5^2 \right),$$
$$F_{(5)} = d^4x \wedge dH^{-1} + \hat{*}(d^4x \wedge dH^{-1}),$$

$$H = 1 + \frac{R^4}{r^4} \; .$$
(1)

In the decoupling limit $H = 1 + \frac{R^4}{r^4} \rightarrow \frac{R^4}{r^4}$ this reduces to $AdS_5 \times S^5$, which provides a gravitational dual of the strongly coupled $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory.

Of course, the ultimate goal of this program is to elucidate strongly coupled YM theory, such as QCD, that has no supersymmetry. But for the time being important steps have been taken to obtain viable (regular) gravitational duals of strongly coupled field theories with less than maximal supersymmetry. In particular, within this framework we shall shed light on gravity duals of field theories in $D = \{2, 3, 4\}$ with $\mathcal{N} = \{1, 2\}$ supersymmetry.

As a side comment, within $D = 5$ $\mathcal{N} = 2$ gauged supergravity progress has been made (see [27–29] and references therein) in constructing domain wall solutions, both with vector-multiplets *and* hyper-multiplets, which lead to smooth solutions that provide viable gravity duals of $D = 4$ $\mathcal{N} = 1$ conformal field theories. Note however, that often the higher dimensional interpretation of this approach, and thus a direct connection to string and M-theory, is not clear. The aim in these lectures is to discuss the string and M-theory embeddings of configurations with less than maximal supersymmetry, and the field theory interpretation of such gravity duals.

A procedure for obtaining a supergravity solution with lesser supersymmetry is to replace the flat transverse 6-dimensional space $ds_6^2 = dr^2 + r^2 \, d\Omega_5^2$ of the D3-brane in (1) with a smooth non-compact Ricci-flat space with fewer Killing spinors. In this case the metric function H still satisfies $\Box H = 0$, but now $\Box$ is the Laplacian in the new Ricci-flat transverse space. This procedure ensures one has a solution with reduced supersymmetry; however the solution for H can be singular at the inner boundary of the transverse space, signifying the appearance of the (distributed) D3-brane source there.

A resolution of the singularity (and the removal of the additional source) can take place if one turns on additional fluxes ("fractional" branes). Within the D3-brane context, the Chern-Simons term of type IIB supergravity modifies the equations of motion:

$$dF_{(5)} = d{*}F_{(5)} = F_{(3)}^{\mathrm{NS}} \wedge F_{(3)}^{\mathrm{RR}} = \tfrac{1}{2i} F_{(3)} \wedge \bar{F}_{(3)} \, ,$$
$$F_{(3)} \equiv F_{(3)}^{\mathrm{RR}} + \mathrm{i}\, F_{(3)}^{\mathrm{NS}} = m L_{(3)} \, ,$$
(2)

where $L_{(3)}$ is a complex harmonic self-dual 3-form on the 6-dimensional Ricci-flat space. Depending on the properties of L_3, this mechanism may allow for a smooth and thus viable supergravity solution. This is precisely the mechanism employed by Klebanov and Strassler, which in the case of the deformed conifold yields a supergravity dual of $D = 4$ $\mathcal{N} = 1$ SYM theory. (For related and follow up work see, for example, [30–38]. For earlier work see, for example, [39–42].)

In a general context, the resolution via transgression [2] is a consequence of the Chern-Simons-type (transgression) terms that are ubiquitous in supergravity theories. Such terms modify the Bianchi identities and/or equations of motion when additional field strengths are turned on. p-brane configurations with $(n + 1)$-transverse dimensions, i.e. with "magnetic" field strength $F_{(n)}$, can have additional field strengths $F_{(p,q)}$ which, via transgression terms, modify the equations for $F_{(n)}$:

$$dF_{(n)} = F_{(p)} \wedge F_{(q)} \, ; \quad (p + q = n + 1) \, .$$
(3)

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If the $(n + 1)$-dimensional transverse Ricci-flat space admits a harmonic p-form $L_{(p)}$ then the equations of motion are satisfied if one sets $F_{(p)} = m L_{(p)}$, and by duality $F_{(q)} \sim \mu * L_{(p)}$. Depending on the L^2 normalizability properties of $L_{(p)}$, one may be able to obtain resolved (non-singular) solutions.

B. Resolved M2-brane

The transgression term in the 4-form field equation in 11-dimensional supergravity is given by

$$d * F_{(4)} = \tfrac{1}{2} F_{(4)} \wedge F_{(4)} \,, \tag{4}$$

and the modified M2-brane Ansatz takes the form

$$
\begin{aligned}
d\hat{s}_{11}^2 &= H^{-2/3}\, dx^\mu\, dx^\nu\, \eta_{\mu\nu} + H^{1/3}\, ds_8^2 \,, \\
F_{(4)} &= d^3x \wedge dH^{-1} + m\, L_{(4)} \,,
\end{aligned}
\tag{5}
$$

where $L_{(4)}$ is a harmonic self-dual 4-form in the 8-dimensional Ricci-flat transverse space. The equation for H is then given by

$$\Box H = -\tfrac{1}{48} m^2\, L_{(4)}^2 \,. \tag{6}$$

For related work see, for example, [34,44–48].

C. Resolved D2-brane

The transgression modification in the 4-form field equation in type IIA supergravity is

$$d\big(e^{\frac{1}{2}\phi}\, \hat{*} F_4\big) = F_{(4)} \wedge F_{(3)} \,, \tag{7}$$

and the modified D2-brane Ansatz takes the form:

$$
\begin{aligned}
ds_{10}^2 &= H^{-5/8}\, dx^\mu\, dx^\nu\, \eta_{\mu\nu} + H^{3/8}\, ds_7^2 \,, \\
F_{(4)} &= d^3x \wedge dH^{-1} + m\, L_{(4)} \,, \quad F_{(3)} = m\, L_{(3)} \,, \quad \phi = \tfrac{1}{4} \log H \,,
\end{aligned}
\tag{8}
$$

where $G_{(3)}$ is a harmonic 3-form in the Ricci-flat 7-metric ds_7^2, and $L_{(4)} = * L_{(3)}$, with $*$ the Hodge dual with respect to the metric ds_7^2. The function H satisfies

$$\Box H = -\tfrac{1}{6} m^2 L_{(3)}^2 \,, \tag{9}$$

where $\Box$ denotes the scalar Laplacian with respect to the transverse 7-metric ds_7^2. Thus the deformed D2-brane solution is completely determined by the choice of Ricci-flat 7-manifold, and the harmonic 3-form supported by it.

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D. Other Examples

In general the transgression terms modify field equations or Bianchi identities as given in (3), thus allowing resolved branes with $(n+1)$ transverse dimensions for the following additional examples in M-theory and string theory:

- (i) D0-brane: $d*F_{(2)} = *F_{(4)} \wedge F_{(3)}$,

- (ii) D1-brane: $d*F_{(3)}^{RR} = F_{(5)} \wedge F_{(3)}^{NS}$,

- (iii) D4-brane: $dF_{(4)} = F_{(3)} \wedge F_{(2)}$,

- (iv) IIA string: $d*F_{(3)} = F_{(4)} \wedge F_{(4)}$,

- (v) IIB string: $d*F_{(3)}^{NS} = F_{(5)} \wedge F_{(3)}^{RR}$,

- (vi) heterotic 5-brane: $dF_{(3)} = F_{(2)}^i \wedge F_{(2)}^i$.

In what follows, we shall focus on resolved M2-branes and briefly mention fractional D2-branes. For details of other examples and their properties, see e.g., [2,3,49,50].

III. SPECIAL HOLONOMY SPACES, HARMONIC FORMS AND RESOLVED BRANES

The construction of resolved supergravity solutions necessarily involves the explicit form of the metric on the Ricci-flat special holonomy spaces. These spaces fall into the following classes:

- Kähler spaces in $D = 2n$ dimensions (n-integer) with $SU(n)$ holonomy, and two covariantly constant spinors. There are many examples, with the Stenzel metric on T^*S^n providing a prototype. They are typically asymptotically conical (AC).

- Hyper-Kähler spaces in $D = 4n$ with $Sp(n)$ holonomy, and $n + 1$ covariantly constant spinors. Subject to certain technical assumptions, Calabi's metric on the co-tangent bundle of $\mathbf{CP}^n$ is the only complete irreducible cohomogeneity one example [51].

- In $D = 7$ there are exceptional G_2 holonomy spaces with one covariantly constant spinor. Until recently only three AC examples were known [52,53], but new metrics have been recently constructed in [10,9,12–23] and will be discussed in a separate Section IV.

- In $D = 8$ there are exceptional $Spin(7)$ holonomy spaces with one covariantly constant spinor; until recently only one AC example was known [52,53]. New metrics were recently constructed in [5,6,12,14,15].

Here the focus is on a construction of cohomogeneity one spaces that are typically asymptotic to cones over Einstein spaces. Recent mathematical developments evolved in two directions: (i) construction of harmonic forms on known Ricci-flat spaces (see in particular [3,7]), (ii) construction of new exceptional holonomy spaces [5,6,9,10,12–23] . In the following subsections we illustrate these developments by

- Summarizing results on the construction of harmonic forms on the Stenzel metric [3],

- Briefly mentioning the results for the old G_2 holonomy metrics [2,7],

- Presenting results for the new Spin(7) two-parameter metrics [5,6], and

- Summarizing the implications of these prototype special holonomy spaces for resolved M2-branes and D2-branes.

In Section IV we shall then summarize the recent new constructions of G_2 holonomy spaces and the implications for M-theory dynamics on such spaces.

A. Harmonic forms for the Stenzel metric

The Stenzel [54] construction provides a class of complete non-compact Ricci-flat Kähler manifolds, one for each even dimension, on the co-tangent bundle of the $(n+1)$-sphere, $T^\star S^{n+1}$. These are asymptotically conical, with principal orbits that are described by the coset space $SO(n+2)/SO(n)$, and they have real dimension $d = 2n+2$.

1. Stenzel metric

In the following we summarize the relevant results for the construction of the Stenzel metric. (For more details see [3].) This construction [3,54] of the Stenzel metric starts with L_{AB}, which are left-invariant 1-forms on the group manifold $SO(n+2)$. By splitting the index as $A = (1, 2, i)$, we have that L_{ij} are the left-invariant 1-forms for the $SO(n)$ subgroup, and so the 1-forms in the coset $SO(n+2)/SO(n)$ will be

$$\sigma_i \equiv L_{1i}\,, \qquad \tilde{\sigma}_i \equiv L_{2i}\,, \qquad \nu \equiv L_{12}\,. \tag{10}$$

The metric Ansatz takes the form:

$$ds^2 = dt^2 + a^2 \sigma_i^2 + b^2 \tilde{\sigma}_i^2 + c^2 \nu^2\,, \tag{11}$$

where a, b and c are functions of the radial coordinate t. One defines Vielbeine

$$e^0 = dt\,, \qquad e^i = a\,\sigma_i\,, \qquad e^{\tilde{i}} = b\,\tilde{\sigma}_i\,, \qquad e^{\tilde{0}} = c\,\nu\,, \tag{12}$$

for which one can introduce a holomorphic tangent-space basis of complex 1-forms ϵ^α:

$$\epsilon^0 \equiv -e^0 + \mathrm{i}\,e^{\tilde{0}}\,, \qquad \epsilon^i = e^i + \mathrm{i}\,e^{\tilde{i}}\,. \tag{13}$$

Defining $a = e^\alpha$, $b = e^\beta$, $c = e^\gamma$, and introducing the new coordinate η by $a^n b^n c\,d\eta = dt$, one finds [3] that the Ricci-flat equations can be obtained from a Lagrangian $L = T - V$ which can be written as a "supersymmetric Lagrangian": $L = \frac{1}{2} g_{ij}\,(d\alpha^i/d\eta)\,(d\alpha^j/d\eta) - \frac{1}{2} g^{ij}\,\frac{\partial W}{\partial \alpha^i}\,\frac{\partial W}{\partial \alpha^j}$. The solution of the first-order equations yields the explicit solution:

$$a^2 = R^{\frac{1}{n+1}}\,\coth r\,;\, b^2 = R^{\frac{1}{n+1}}\,\tanh r\,;\, h^2 = c^2 = \frac{1}{n+1} R^{-\frac{n}{n+1}}\,\sinh^n(2r)\,, \tag{14}$$

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where $R(r) \equiv \int_0^r (\sinh 2u)^n \, du$, and the radial coordinate r is introduced as $dt = h \, dr$.

For each n the result is expressible in relatively simple terms. For example,

$$R = \sinh^2 r \, ; R = \tfrac{1}{8}(\sinh 4r - 4r) \, ; R = \tfrac{2}{3}(2 + \cosh 2r) \sinh^4 r \, , \tag{15}$$

for $n = 1, 2, 3$, respectively. The case $n = 1$ is the Eguchi-Hanson metric [43], and $n = 2$ it is the deformed conifold [55].

As r approaches zero, the metric takes the form

$$ds^2 \sim dr^2 + r^2 \tilde{\sigma}_i^2 + \sigma_i^2 + \nu^2 \, , \tag{16}$$

which has the structure locally of the product $\mathbf{R}^{n+1} \times S^{n+1}$, with S^{n+1} being a "bolt." As r tends to infinity, the metric becomes

$$ds^2 \sim d\rho^2 + \rho^2 \{ \tfrac{n^2}{(n+1)^2} \nu^2 + \tfrac{n}{2(n+1)^2} (\sigma_i^2 + \tilde{\sigma}_i^2) \} \, , \tag{17}$$

representing a cone over the Einstein space $SO(n + 2)/SO(n)$.

2. Harmonic middle-dimension (p, q) forms

An Ansatz compatible with the symmetries of the Stenzel metric is of the form:

$$\begin{aligned}
L_{(p,q)} &= f_1 \, \epsilon_{i_1 \cdots i_{q-1} j_1 \cdots j_p} \, \bar{\epsilon}^0 \wedge \bar{\epsilon}^{i_1} \wedge \cdots \wedge \bar{\epsilon}^{i_{q-1}} \wedge \epsilon^{j_1} \wedge \cdots \wedge \epsilon^{j_p} \\
&+ f_2 \, \epsilon_{i_1 \cdots i_{p-1} j_1 \cdots j_q} \, \epsilon^0 \wedge \epsilon^{i_1} \wedge \cdots \wedge \epsilon^{i_{p-1}} \wedge \bar{\epsilon}^{j_1} \wedge \cdots \wedge \bar{\epsilon}^{j_q} \, ,
\end{aligned} \tag{18}$$

with f_1, f_2 being functions of r, only. The harmonicity condition becomes $dL_{(p,q)} = 0$, since $*L_{(p,q)} = i^{p-q} L_{(p,q)}$. The functions f_1, f_2 are solutions of coupled first-order homogeneous differential equations, yielding a solution that is finite as $r \to 0$:

$$f_1 = q \, {}_2F_1 \left[\tfrac{1}{2}p, \tfrac{1}{2}(q+1), \tfrac{1}{2}(p+q) + 1; -(\sinh 2r)^2 \right] \, , \tag{19}$$

$$f_2 = -p \, {}_2F_1 \left[\tfrac{1}{2}q, \tfrac{1}{2}(p+1), \tfrac{1}{2}(p+q) + 1; -(\sinh 2r)^2 \right] \, . \tag{20}$$

For any specific integers (p, q), these are elementary functions of r.

For the two special cases of greatest interest, they have the following properties:

- (p, p)-forms in $4p$-dimensions: $f_1 = -f_2 = \frac{p}{(\cosh r)^{2p}}$ with $|L_{(p,p)}|^2 = \frac{const.}{(\cosh r)^{4p}}$ falls-off fast enough as $r \to \infty$. This turns out to be the only L^2 normalizable form.

- $(p + 1, p)$-forms in $(4p + 2)$-dimensions. As $r \to \infty$: $|L_{(p+1,p)}|^2 \sim \frac{1}{[\sinh(2r)]^{2p}}$ which is marginally L^2-non-normalizable.

From the viewpoint of physics, the case in $2(n + 1) = 4$ dimensions with an L^2-normalizable $L_{(1,1)}$-form is precisely the example of the resolved self-dual string discussed in [2].

In $2(n + 1) = 6$ dimensions, the $L_{(2,1)}$-form was constructed in [1], and provides a resolution of the D3-brane. Since $L_{(2,1)}$ is only *marginally non-normalizable* as $r \to \infty$, the

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decoupling limit of the space-time does not give an AdS$_5$, but instead there is a logarithmic modification. In particular, this modification accounts for a renormalization group running of the difference of the inverse-squares of the two gauge group couplings in the dual $SU(N) \times SU(N + M)$ SYM [41].

On the other hand in $2(n + 1) = 8$ dimensions the L^2 normalizable $L_{(2,2)}$-form supports additional fluxes that resolve the original $M2$-brane, whose details are given in [3].

It turns out that one can construct regular supersymmetric resolved M2-branes for many other examples of 8-dimensional special holonomy transverse spaces, such as the original Spin(7) holonomy transverse space [2], a number of new Kähler spaces [2,7], and hyper-Kähler spaces [4].

B. Old G_2 holonomy metrics and their harmonic forms

1. Resolved cones over $S^2 \times S^4$ and $S^2 \times \mathbf{CP}^2$

The first complete Ricci-flat 7-dimensional metrics of G_2 holonomy were obtained in [52,53]. There were two types. The first type comprises two examples of R^3 bundles over four-dimensional quaternionic-Kähler Einstein base manifolds M. These spaces are of co-homogeneity one, with principal orbits that are S^2 bundles over M (sometimes referred to as the twistor space over M). For the two examples that arise, M is S^4 or $\mathbf{CP}^2$. The two G_2 manifolds have principal orbits that are $\mathbf{CP}^3$ (S^2 bundle over S^4), or the flag manifold $SU(3)/(U(1) \times U(1))$ (S^2 bundle over $\mathbf{CP}^2$), respectively. These two manifolds are the bundles of self-dual 2-forms over S^4 or $\mathbf{CP}^2$ respectively. They approach $\mathbf{R}^3 \times S^4$ or $\mathbf{R}^3 \times \mathbf{CP}^2$ locally near the origin.

The derivations for the two cases, with the principal orbits being S^2 bundles either over S^4 or over $\mathbf{CP}^2$, proceed essentially identically. In the notation of [53], the G_2 metrics are given by

$$d\hat{s}_7^2 = h^2 \, dr^2 + a^2 \, (D\mu^i)^2 + b^2 \, ds_4^2 \,, \tag{21}$$

where μ^i are coordinates on $\mathbf{R}^3$ subject to $\mu^i \, \mu^i = 1$, and ds_4^2 is the metric on S^4 or $\mathbf{CP}^2$ scaled to have $R_{\alpha\beta} = 3g_{\alpha\beta}$. The 1-forms A^i are $su(2)$ Yang-Mills instanton potentials, and

$$D\mu^i \equiv d\mu^i + \epsilon_{ijk} A^j \, \mu^k \,. \tag{22}$$

The field strengths $J^i \equiv dA^i + \frac{1}{2}\epsilon_{ijk} A^j \wedge A^k$ satisfy the algebra of the unit quaternions, $J^i_{\alpha\gamma} J^j_{\gamma\beta} = -\delta_{ij} \delta_{\alpha\beta} + \epsilon_{ijk} J^k_{\alpha\beta}$. The harmonic 3-form (other than the covariant constant one), was constructed in [4]: it is smooth and L^2-normalizable.

2. Resolved cone over $S^3 \times S^3$

The second type of complete 7-dimensional manifold of G_2 holonomy obtained in [52,53] is again of cohomogeneity one, with principal orbits that are topologically $S^3 \times S^3$. The manifold is the spin bundle of S^3; near the origin it approaches locally $\mathbf{R}^4 \times S^3$.

The Ricci-flat metric on the spin bundle of S^3 is given by [52,53].

$$ds_7^2 = \alpha^2\, dr^2 + \beta^2\, (\sigma_i - \tfrac{1}{2}\Sigma_i)^2 + \gamma^2\, \Sigma_i^2\,,\tag{23}$$

where the functions α, β and γ are given by

$$\alpha^2 = \left(1 - \frac{1}{r^3}\right)^{-1}, \qquad \beta^2 = \tfrac{1}{9}r^2\left(1 - \frac{1}{r^3}\right), \qquad \gamma^2 = \tfrac{1}{12}r^2\,.\tag{24}$$

Here Σ_i and σ_i are the two sets of left-invariant 1-forms on two independent $SU(2)$ group manifolds. The principal orbits $r =$ constant are therefore S^3 bundles over S^3. Since the bundle is trivial, they are topologically $S^3 \times S^3$, although not with the standard product metric. The radial coordinate runs from $r = a$ to $r = \infty$.

This metric admits a regular harmonic 3-form, explicitly constructed in [2]: it is square-integrable at short distance, but gives a linearly divergent integral at large distance. The short-distance square-integrability is enough to give a regular deformed D2-brane solution, even though $L_{(3)}$ is not L^2-normalizable.

C. New Spin(7) holonomy metrics and their harmonic forms

1. The old metric and harmonic 4-forms

Until recently only one explicit example of a complete non-compact metric on a Spin(7) holonomy space was known [52,53]. The principal orbits are S^7, viewed as an S^3 bundle over S^4. The solution (25) is asymptotic to a cone over the "squashed" Einstein 7-sphere, and it approaches $\mathbf{R}^4 \times S^4$ locally at short distance (i.e. $r \approx \ell$). The metric is of the form:

$$ds_8^2 = \left(1 - \frac{\ell^{10/3}}{r^{10/3}}\right)^{-1} dr^2 + \tfrac{9}{100}r^2\left(1 - \frac{\ell^{10/3}}{r^{10/3}}\right)h_i^2 + \tfrac{9}{20}r^2\, d\Omega_4^2\,,\tag{25}$$

where $h_i \equiv \sigma_i - A_{(1)}^i$, and the σ_i are left-invariant 1-forms on $SU(2)$, $d\Omega_4^2$ is the metric on the unit 4-sphere, and $A_{(1)}^i$ is the $SU(2)$ Yang-Mills instanton on S^4. The σ_i can be written in terms of Euler angles as $\sigma_1 = \cos\psi\, d\theta + \sin\psi\, \sin\theta\, d\varphi$, $\sigma_2 = -\sin\psi\, d\theta + \cos\psi\, \sin\theta\, d\varphi$, $\sigma_3 = d\psi + \cos\theta\, d\varphi$. A regular L^2 normalizable harmonic 4-form in this metric was obtained in [2].

2. New Spin(7) holonomy metric

The generalization that we shall consider involves allowing the S^3 fibers of the previous construction themselves to be "squashed." Namely, the S^3 bundle is itself written as a $U(1)$ bundle over S^2 leading to the following "twice squashed" Ansatz:

$$d\hat{s}_8^2 = dt^2 + a^2\, (D\mu^i)^2 + b^2\, \sigma^2 + c^2\, d\Omega_4^2\,,\tag{26}$$

where a, b and c are functions of the radial variable t. (The previous Spin(7) example has $a = b$.) Here

$$\mu_1 = \sin\theta\, \sin\psi\,, \qquad \mu_2 = \sin\theta\, \cos\psi\,, \qquad \mu_3 = \cos\theta\,,\tag{27}$$

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are the S^2 coordinates, subject to the constraint $\mu_i \mu_i = 1$, and

$$D\mu^i \equiv d\mu^i + \epsilon_{ijk} A^j_{(1)} \mu^k, \quad \sigma \equiv d\varphi + \mathcal{A}_{(1)}, \quad \mathcal{A}_{(1)} \equiv \cos\theta \, d\psi - \mu^i A^i_{(1)}, \tag{28}$$

where the field strength $\mathcal{F}_{(2)}$ of the $U(1)$ potential $\mathcal{A}_{(1)}$ turns out to be given by: $\mathcal{F}_{(2)} = \frac{1}{2}\epsilon_{ijk} \mu^k D\mu^i \wedge D\mu^j - \mu^i F^i_{(2)}$.

The Ricci-flatness conditions can be satisfied by solving the first-order equations coming from a supersymmetric Lagrangian, yielding the following special solution (for details see [5,6]):

$$\begin{aligned}
ds_8^2 &= \frac{(r-\ell)^2 \, dr^2}{(r-3\ell)(r+\ell)} + \frac{\ell^2 \, (r-3\ell)(r+\ell)}{(r-\ell)^2} \sigma^2 \\
&+ \tfrac{1}{4}(r^2 - 3\ell)(r+\ell) \, (D\mu^i)^2 + \tfrac{1}{2}(r^2 - \ell^2) \, d\Omega_4^2,
\end{aligned} \tag{29}$$

The quantity $\frac{1}{4}[\sigma^2 + (D\mu^i)^2]$ is the metric on the unit 3-sphere, and so in this case we find that the metric smoothly approaches $\mathbf{R}^4 \times S^4$ locally, at small distance ($r \to 3\ell$), in the same way that it does in the previously-known example. Therefore it has the same topology as the old Spin(7) holonomy space. On the other hand, it locally approaches $\mathcal{M}_7 \times S^1$ at large distance. Here $\mathcal{M}_7$ denotes the 7-manifold of G_2 holonomy on the $\mathbf{R}^3$ bundle over S^4 [52,53]. Asymptotically the new metric behaves like a circle bundle over an asymptotically conical manifold in which the length of the $U(1)$ fibers tends to a constant; in other words, it is ALC.

If one takes r to be negative, or instead analytically continues the solution so that $\ell \to -\ell$ (keeping r positive), one gets a different complete manifold. Thus instead of (29), the quantity $\frac{1}{4}(\sigma^2 + (D\mu^i)^2 + d\Omega_4^2)$ is precisely the metric on the unit 7-sphere, and so as r approaches ℓ the metric ds_8^2 smoothly approaches $\mathbf{R}^8$. At large r the function b, which is the radius in the $U(1)$ direction σ, approaches a constant, and so the metric tends to an S^1 bundle over a 7-metric of the form of a cone over $\mathbf{CP}^3$; it has the same asymptotic form as (29). The manifold in this case is topologically $\mathbf{R}^8$.

In [5,6] the general solution to the first-order system of equations is obtained, leading to additional families of regular metrics of Spin(7) holonomy, which are complete on manifolds $\mathbf{B}_8^\pm$ that are similar to $\mathbf{B}_8$. These additional metrics have a non-trivial integration constant which parameterizes inequivalent solutions. (For details see [6] and Appendix A of [5]).

L^2 normalizable harmonic 4-forms for the new Spin(7) 8-manifolds were obtained in [5].

D. Applications: resolved M2-branes and D2-branes

The explicit construction of harmonic 4-forms on 8-dimensional Ricci flat spaces led to analytic expressions for resolved M2-brane solutions, while the 3-forms (and dual 4-forms) of G_2 holonomy spaces led to analytic expressions for fractional D2-branes. Their properties, such as supersymmetry conditions, flux integrals and aspects of the dual field theories, were discussed in [2–4,7,48].

Resolved M2-branes on a suitable eight-dimensional space can be supported by L^2-normalizable harmonic forms and thus they are regular at short distance and have decoupling limits at large distance that yield AdS_4. They have no conserved additional (fractional)

charges. The dual 3-dimensional field theory is superconformal (with $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetry), and is in turn perturbed by marginal operators associated with pseudo-scalar fields [48]. On the other hand the fractional D2-branes have conserved fractional charges corresponding to D4-branes wrapping the 2-cycles dual to S^4 or $\mathbf{CP}^2$ in $\mathcal{M}_7$, or to NS-NS 5-branes wrapping the 3-cycle dual to S^3 in $\mathcal{M}_7$.

An interesting application of these new Spin(7) holonomy spaces is the construction of fractional M2-branes. After reduction on S^1 these give D2-branes with additional fractional magnetic charge associated with D4-branes wrapping 2-cycles *and* D6-branes wrapping 4-cycles. The fact that the resolved M2-brane on the new Spin(7) holonomy space has non-zero fractional charge is a consequence of the asymptotically locally conical structure of the new Spin(7) holonomy space.

IV. NEW G_2 HOLONOMY METRICS

Subsequent to these lectures there have been major developments the construction of new holonomy spaces and the study of their implications for M-theory dynamics. In part motivated by the construction of the new two-parameter Spin(7) holonomy metrics with ALC structure [5,6] (described in the previous Section III), new constructions of G_2 (as well as Spin(7)) holonomy spaces [9,10,12–23] have been given. The implications from M-theory on such spaces for the dynamics of the resulting $\mathcal{N} = 1$, $D = 4$ field theory [56–58] are attracting considerable attention. Specifically, it has been proposed that M-theory compactified on a certain singular seven-dimensional space with G_2 holonomy might be related to an $\mathcal{N} = 1$, $D = 4$ gauge theory [26,56–58,60] that has no conformal symmetry. The quantum aspects of M-theory dynamics on spaces of G_2 holonomy can provide insights into non-perturbative aspects of four-dimensional $\mathcal{N} = 1$ field theories, such as the preservation of global symmetries and phase transitions. For example, Ref. [56] provides an elegant exposition and study of these phenomena using the three original manifolds of G_2 holonomy that were obtained in [52,53].

One related development in this direction is the discovery of M3-brane configurations [8,9]. These have a flat 4-dimensional world-volume and a transverse space that is a deformation of the G_2 manifold, and with the 4-form field strength is turned on. They turn out to have zero charge and ADM mass, leading to naked singularities at small distances.

A. Classification of G_2 holonomy spaces with $S^3 \times S^3$ orbits

In another recent development, G_2 metrics have been obtained that make contact with the six-dimensional resolved and deformed conifolds. This work is described in detail in [20] (See also [18,19]). We consider a generalization of the original Ansatz [52,53] (23) for metrics of cohomogeneity one with $S^3 \times S^3$ principal orbits. The more general Ansatz is given by

$$
\begin{aligned}
ds_7^2 = dt^2 &+ c^2 \left(\Sigma_3 - \sigma_3\right)^2 + f^2 \left(\Sigma_3 + g_3 \, \sigma_3\right)^2 \\
&+ a^2 \left[\left(\Sigma_1 + g \, \sigma_1\right)^2 + \left(\Sigma_2 + g \, \sigma_2\right)^2\right] + b^2 \left[\left(\Sigma_1 - g \, \sigma_1\right)^2 + \left(\Sigma_2 - g \, \sigma_2\right)^2\right],
\end{aligned}
\tag{30}
$$

where Σ_i and σ_i are again two sets of left-invariant 1-forms of $SU(2)$, and the six coefficients a, b, c, f, g and g_3 depend only on t. In the orthonormal basis

$$e^0 = dt, \quad e^1 = a\,(\Sigma_1 + g\,\sigma_1), \quad e^2 = a\,(\Sigma_2 + g\,\sigma_2), \quad e^3 = c\,(\Sigma_3 - \sigma_3),$$
$$e^4 = b\,(\Sigma_1 - g\,\sigma_1), \quad e^5 = b\,(\Sigma_2 - g\,\sigma_2), \quad e^6 = f\,(\Sigma_3 + g_3\,\sigma_3), \tag{31}$$

there is a natural candidate for an invariant associative 3-form, namely

$$\Phi = e^0 \wedge (e^1 \wedge e^4 + e^2 \wedge e^5 + e^3 \wedge e^6) - (e^1 \wedge e^2 - e^4 \wedge e^5) \wedge e^3$$
$$+ (e^1 \wedge e^5 - e^2 \wedge e^4) \wedge e^6. \tag{32}$$

Requiring the closure and co-closure of this 3-form gives a set of first-order equations for G_2 holonomy [20],

$$\dot{a} = \frac{c^2\,(a^2 - b^2) + [4a^2\,(a^2 - b^2) - c^2\,(5a^2 - b^2) - 4a\,b\,c\,f]\,g^2}{16a^2\,b\,c\,g^2},$$

$$\dot{b} = -\frac{c^2\,(a^2 - b^2) + [4b^2\,(a^2 - b^2) + c^2\,(5b^2 - a^2) - 4a\,b\,c\,f]\,g^2}{16a\,b^2\,c\,g^2},$$

$$\dot{c} = \frac{c^2 + (c^2 - 2a^2 - 2b^2)\,g^2}{4a\,b\,g^2}, \tag{33}$$

$$\dot{f} = -\frac{(a^2 - b^2)\,[4a\,b\,f^2\,g^2 - c\,(4a\,b\,c + a^2\,f - b^2\,f)\,(1 - g^2)]}{16a^3\,b^3\,g^2},$$

$$\dot{g} = -\frac{c\,(1 - g^2)}{4a\,b\,g},$$

together with an algebraic equation for g_3:

$$g_3 = g^2 - \frac{c\,(a^2 - b^2)(1 - g^2)}{2a\,b\,f}. \tag{34}$$

There are two combinationsof the equations (33) that can be integrated explicitly, giving two invariants built out of the metric functions. These two constants are nothing but the coefficients in front of the volume forms for the respective three-spheres in the associated three-form: $\Phi = m\,\sigma_1 \wedge \sigma_2 \wedge \sigma_3 + n\,\Sigma_1 \wedge \Sigma_2 \wedge \Sigma_3 + \cdots$, which may be seen to be constant by imposing closure of Φ (see [19]). Ultimately, the system (33) can be reduced to a single non-linear second-order differential equation.

The general solution of the first-order equations (33) is not known. Of course the asymptotically conical G_2 metric (23) is a solution. An explicit, singular, solution was found in [18,19]. Another exact solution, found earlier in [10], is

$$ds_7^2 = \frac{(r^2 - \ell^2)}{(r^2 - 9\ell^2)}\,dr^2\,\tfrac{1}{12}(r - \ell)(r + 3\ell)[(\Sigma_1 - \sigma_1)^2 + (\Sigma_2 - \sigma_2)^2]$$
$$+ \tfrac{1}{12}(r + \ell)(r - 3\ell)[(\Sigma_1 + \sigma_1)^2 + (\Sigma_2 + \sigma_2)^2]$$
$$+ \tfrac{1}{9}r^2\,(\Sigma_3 - \sigma_3)^2 + \tfrac{4}{9}\ell^2\,\frac{r^2 - 9\ell^2}{r^2 - \ell^2}\,(\Sigma_3 + \sigma_3)^2. \tag{35}$$

The radial coordinate runs from an S^3 bolt at $r = 3\ell$ to an asymptotic region as r approaches infinity. The metric is asymptotically locally conical, with the radius of the circle with coordinate $(\psi + \tilde{\psi})$ stabilising at infinity. The metric is closely analogous to an ALC Spin(7) metric on the $\mathbf{R}^4$ bundle over S^4 that was found previously [5,6].

Although explicit solutions to the first-order system (33) are not in general known, it is nevertheless possible to study the system by a combination of approximation and numerical methods. Specifically, one can perform a Taylor expansion around the bolt at a minimum radius where the $S^3 \times S^3$ orbits degenerate, and use this to set initial data just outside the bolt for a numerical integration towards large radius. The criterion for a complete non-singular metric is that the metric functions should be well-behaved at large distance, either growing linearly with distance as in an AC metric, or else with one or more metric coefficients stabilising to fixed values asymptotically, as in an ALC metric such as (35). This method is discussed in detail in [13,18,20], and it is established there that there exist three families of non-singular ALC metrics, each with a non-trivial parameter λ that gives the size of a stabilising circle at infinity relative to the size of the bolt at short distance. The metrics, denoted by $\mathbf{B}_7$, $\mathbf{D}_7$ and $\widetilde{\mathbf{C}}_7$, have bolts that are a round S^3, a squashed S^3 and $T^{p,q} = S^3 \times S^3/U(1)_{(p,q)}$ respectively, where $p/q = \sqrt{m/n}$ and m, n are the two explicit integration constants of the first-order system (33) that we discussed previously. The radius of the stabilising circle ranges from zero at $\lambda = 0$ to infinity at $\lambda = \infty$. As one takes the limit $\lambda \to 0$, the ALC G_2 metric approaches the direct product of a six-dimensional Ricci-flat Kähler metric and a vanishing circle. This limit is known mathematically as the Gromov-Hausdorff limit.

The cases of most immediate interest are $\mathbf{B}_7$ and $\mathbf{D}_7$. Their Gromov-Hausdorff limits are a vanishing circle times the deformed conifold, and a vanishing circle times the resolved conifold, respectively [18,20]. On the other hand, as λ goes to infinity, they both approach the original AC metric of [52,53]. If, therefore, we begin with a solution (Minkowski)$_4 \times Y_7$ in M-theory, with Y_7 being a $\mathbf{B}_7$ or $\mathbf{D}_7$ metric, then we can dimensionally reduce it on the circle that stabilises at infinity, thereby obtaining a solution of the type IIA string. The radius of the M-theory circle, R, is related to the string coupling constant g_{str} by $g_{\text{str}} = R^{3/2}$. This means that taking the Gromov-Hausdorff limit in $\mathbf{B}_7$ or $\mathbf{D}_7$ corresponds to the weak-coupling limit in the type IIA string, and the ten-dimensional solution becomes the product of (Minkowski)$_4$ with the deformed or resolved conifold. In the strong-coupling domain, where λ goes to infinity, these two ten-dimensional solutions become unified via the $\mathbf{B}_7$ and $\mathbf{D}_7$ solutions in M-theory.

A yet more general system of cohomogeneity one G_2 metrics with $S^3 \times S^3$ principal orbits was obtained recently in [23]. The construction was based on an approach developed recently by Hitchin [61], in which one starts from an Ansatz for an associative 3-form, and derives first-order equations via a system of Hamiltonian flow equations. These first-order equations can be shown to imply that a certain metric derived from the 3-form has G_2 holonomy. By applying this procedure to the case of $S^3 \times S^3$ principal orbits, it was shown in [23] that the metric

$$ds_7^2 = dt^2 \tag{36}$$
$$+\frac{1}{y_1}[(n\,x_1+x_2\,x_3)\Sigma_1^2+(m\,n+x_1^2-x_2^2-x_3^2)\Sigma_1\,\sigma_1+(m\,x_1+x_2\,x_3)\sigma_1^2]$$
$$+\frac{1}{y_2}[(n\,x_2+x_3\,x_1)\Sigma_2^2+(m\,n+x_2^2-x_3^2-x_1^2)\,\Sigma_2\sigma_2+(m\,x_2+x_3\,x_1)\sigma_2^2]$$
$$+\frac{1}{y_3}[(n\,x_3+x_1\,x_2)\Sigma_3^2+(m\,n+x_3^2-x_1^2-x_2^2)\,\Sigma_3\sigma_3+(m\,x_3+x_1\,x_2)\sigma_3^2]$$

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has G_2 holonomy if the functions x_i and y_i, which depend only on t, satisfy the first-order Hamiltonian system of equations

$$\dot{x}_1 = \sqrt{\frac{y_2\, y_3}{y_1}}\,, \qquad \dot{y}_1 = \frac{m\, n\, x_1 + (m+n)\, x_2\, x_3 + x_1\, (x_2^2 + x_3^2 - x_1^2)}{\sqrt{y_1\, y_2\, y_3}}\,, \tag{37}$$

and cyclically for the 2 and 3 directions. In addition, the conserved Hamiltonian must vanish, which implies that

$$4y_1\, y_2\, y_3 + m^2\, n^2 - 2m\, n\, (x_1^2 + x_2^2 + x_3^2) - 4(m+n)\, x_1\, x_2\, x_3$$
$$+x_1^4 + x_2^4 + x_3^4 - 2x_1^2\, x_2^2 - 2x_2^2\, x_3^2 - 2x_3^2\, x_1^2 = 0\,. \tag{38}$$

The above first-order system encompasses all the previous cases as specialisations. In particular, the first-order system for the metrics (30) is obtained by making the specialisation $x_1 = x_2$, $y_1 = y_2$. If, instead, one sets $m = n = 1$, the system reduces to one studied in [9,10].

In addition to these $SU(2) \times SU(2)$ invariant metrics with principal orbits $S^3 \times S^3$, one may take various Inönü-Wigner contractions to give metrics with principal orbits $T^3 \times S^3$, or other orbit types constructed from the possible contractions of $SU(2)$ [23]. In the particular case of $T^3 \times S^3$ orbits, the resulting first-order system is that of [22].

We find that the general set of equations (37) does not seem to yield new classes of regular solutions, other than those already classified [20] for the first-order system (33).

V. CONCLUSIONS AND OPEN AVENUES

In these lectures we have presented a summary of some recent developments in the construction of regular p-brane configurations with less than maximal supersymmetry. In particular, the method involves the introduction of complete non-compact special holonomy metrics and additional fluxes, supported by harmonic-forms in special holonomy spaces, which modify the original p-brane solutions via Chern-Simons (*transgression*) terms.

The work led to a number of important *mathematical developments* which we have also summarized. Firstly, the construction of harmonic forms for special holonomy spaces in diverse dimensions was reviewed, and the explicit construction of harmonic forms for Stenzel metrics was summarized. Secondly, a construction of new two-parameter Spin(7) holonomy spaces was discussed. These have the property that they interpolate asymptotically between a local $S^1 \times \mathcal{M}_7$, where the length of the circle is finite and $\mathcal{M}_7$ is the G_2 holonomy space with the topology of the S^2 bundle over S^4, while at small distance they approach the "old" Spin(7) holonomy space with the topology of the chiral spin bundle over S^4.

These mathematical developments also led to a number of important *physics implications*, relevant for the properties of the resolved p-brane solutions. In particular, the focus was on the properties of resolved M2-branes with 8-dimensional special holonomy transverse spaces, for example Stenzel, hyper-Kähler and Spin(7) holonomy spaces, and the results for the fractional D2-branes with three 7-dimensional G_2 holonomy transverse spaces.

After the lectures were given, there was major progress in constructing new G_2 holonomy spaces and studying the M-theory dynamics on such spaces. We have summarized this

progress in Section IV, and in particular highlighted the classification of general G_2 holonomy spaces with $S^3 \times S^3$ principal orbits.

Until recently, the emphasis has been on finding new G_2 manifolds that are complete and non-singular. However, M-theory compactified on such spaces necessarily gives only Abelian and non-chiral $\mathcal{N} = 1$ theories in four dimensions. To obtain non-Abelian chiral theories from M-theory, one needs to consider compactifications on singular G_2 manifolds. One explicit realisation of such an M-theory compactification has an interpretation as an S^1 lift of Type IIA theory, compactified on an orientifold, with intersecting D6-branes and O6 orientifold planes [62]. Non-Abelian gauge fields arise at the locations of coincident branes, and chiral matter arises at the intersections of D6-branes. Interestingly, these constructions provide [63] the first three-family supersymmetric standard-like models with intersecting D6-branes. The S^1 lift of these configurations results in singular G_2 holonomy metrics in M-theory. Co-dimension four ADE-type singularities are associated with the location of the coincident D6-branes, and co-dimension seven singularities are associated with the location of the intersection of two D6-branes in Type IIA theory [62,56,64–66].

Further analyses of co-dimension seven singularities of the G_2 holonomy spaces, leading to chiral matter, were given in [64–66] and subsequent work [67–73]. It is expected that there exists a wide classe of singular 7-manifolds with G_2 holonomy that yield non-Abelian $\mathcal{N} = 1$ supersymmetric four-dimensional theories with chiral matter. The explicit construction of such metrics would provide a starting point for further studies of chiral M-theory dynamics.

A recent study of an explicit class of singular G_2 holonomy spaces was given in [74]. These are cohomogeneity two metrics foliated by twistor spaces, that is S^2 bundles over self-dual Einstein four-dimensional manifolds M_4. The 4-manifold is chosen to be a self-dual Einstein space with orbifold singularities. An investigation of this construction was carried out in [68]. In [74] the most general self-dual Einstein metrics of triaxial Bianchi IX type, which have an $SU(2)$ isometry acting transitively on 3-dimensional orbits that are (locally) S^3, were considered.

Specialisation to biaxial solutions with positive cosmological constant yields manifolds that are compact, but in general with singularities. The radial coordinate ranges over an interval that terminates at endpoints where the $SU(2)$ principal orbits degenerate; to a point (a NUT) at one end, and to a two-dimensional surface (a bolt) that is (locally) S^2 at the other. Only for very special values of the NUT parameter is the metric regular at both ends. In general, however, one encounters singularities at both endpoints of the radial coordinate. In the generic case, a specific choice of the period for the azimuthal angle allows the singularity at the S^2 bolt to be removed, but then the NUT has a co-dimension four orbifold singularity. Alternatively, choosing the periodicity appropriate for regularity at the NUT, there will be a co-dimension two singularity on the S^2 bolt. The associated seven-dimensional G_2 holonomy spaces therefore have singularities of the same co-dimensions. The co-dimension four NUT singularities may admit an M-theory interpretation associated with the appearance of non-Abelian gauge symmetries, and the circle reduction of M-theory on these G_2 holonomy spaces may have a Type IIA interpretation in terms of coincident D6-branes [68]. On the other hand, the co-dimension two singularities at the bolts do not seem to have a straightforward interpretation in M-theory.

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