

Singularity Resolution and Gravitational Chern-Simons terms

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Abstract

We show that gravitational Chern-Simons corrections, associated with the sigma-model anomaly on the M5-brane world-volume, can resolve the singularity of the M2-brane solution with Ricci-flat, special holonomy transverse space. We explicitly find smooth solutions in the cases when the transverse space is a manifold of Spin(7) holonomy and SU(4) holonomy. We comment on the consequences of these results for the holographically related three-dimensional theories living on the world volume of a stack of such resolved M2-branes.

1 Introduction

The original AdS/CFT correspondence is an equivalence between Type IIB strings moving in the near horizon geometry created by a stack of N D3 branes and $\mathcal{N} = 4$ SU(N) gauge theory which is the infra-red limit of the theory living on the world volume of the branes [1]. In the large N limit, with $g_{\text{YM}}^2 N$ fixed but large, tree level supergravity is a good approximation to string theory. The α' and g_s expansions in string theory map to $1/\sqrt{g_{\text{YM}}^2 N}$ and $1/N$ expansions in the gauge theory. Stringy effects in AdS spaces are hard to compute since it is difficult to quantize strings in RR backgrounds. However, we know some α' corrections to the low energy space-time effective action in the form of higher curvature corrections of the form $\alpha'^3 R^4$. Such an α' expansion should contain information about the strong coupling expansion of the gauge theory [2].

We can study similar questions in the context of M-theory on $\text{AdS}_4 \times X^7$ with AdS scale $R \approx l_p N^{1/6}$. In this setup, l_p/R corrections to supergravity go like $1/N^{1/6}$.

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In this paper, we consider higher curvature corrections to 11 dimensional supergravity action which are the gravitational Chern-Simons terms arising from the σ -model anomaly on the M5 brane world volume, thereby modifying the Bianchi identity for the five brane. Such corrections, schematically go like $l_p^6 R^4$ and hence correspond to $1/N$ effects in the corresponding holographic field theories. Specifically, we consider the correction to classical M2-brane solutions arising from such gravitational Chern-Simons terms, when such M2-branes have transverse spaces which are Ricci-flat deformations of cones over seven dimensional Einstein manifolds. Branes at such conical singularities have been studied extensively for the past few years and we briefly recap some of the features salient for our study.

In the search of a string dual for four dimensional gauge theories with lesser supersymmetry, variations of the original correspondence involve placing D3 branes at the apex of a Ricci flat six dimensional cone whose base is a five dimensional Einstein manifold X^5 [3]. This leads to the conjecture that type IIB string theory on $AdS_5 \times X^5$ is dual to the low energy limit of the theory on the D3 branes at the singularity. For the conifold singularity, X^5 is the Einstein space $T^{1,1}$. Adding fractional D3 branes, (D5 branes wrapped over two-cycles of X_5) introduces a non-zero three-form RR flux through the three-cycle of $T^{1,1}$ and results in a non-AdS bulk solution. Correspondingly, on the gauge theory side, fractional branes lead to a non-conformal gauge theory with running couplings. Supergravity solutions for such configurations of D3 branes were considered in [4, 5]. The solution in [5] that includes fractional D3-brane, was singular. In [6], the singularity was resolved by replacing the singular conifold by a (smooth) deformed conifold [7]. This regular supergravity solution realized the chiral symmetry breaking and confinement of the dual $\mathcal{N} = 1$ supersymmetric four-dimensional gauge theory geometrically.

It is possible to generalize the above setup to other p-branes placed at tips of other cones, *i.e.* the transverse space to the p-branes is a cone $ds^2 = dr^2 + r^2 ds_X^2$, where ds_X^2 is the metric on an Einstein manifold X , which is called the base of the cone [8]. Turning on additional F_{6-p} field strengths corresponds to deforming the holographically related field theory. In [9, 10] (for a review see [11]), it was shown that after addition of such fluxes regular solutions can be obtained which have the feature that the singular conical transverse space is resolved in the IR region of the field theory. Also, these solutions generically do not have horizons, implying that a mass gap has been generated in the dual field theory.

In finding smooth, non-singular solutions, a crucial role is played by the Chern-Simons type terms with additional field strengths which modify the Bianchi identities and/or equations of motions for the original field strength. Note that the additional field strengths F_{6-p} are supported by harmonic forms on the special holonomy space, and in particular the L^2 normalizability of these harmonic forms ensures that the solutions are regular both in the interior and at large radial coordinate r . For most of the cases and in particular for resolved M2-branes and D2 branes [9, 10, 12, 13], with

the transverse space asymptotically conical (AC) Spin(7) and with G_2 holonomy, the asymptotic form of the field strength was such that it did not produce any new flux at infinity. As such, these configurations do not describe gravity dual of fractional branes [14, 13] but perturbations of the boundary field theory living on a large stack of regular branes by relevant operators. The power law fall-off of the field strength determines the dimension of the operator to be added to the gauge theory action.

In this paper we focus on another aspect of resolved brane solutions which is due to the gravitational Chern-Simons type corrections. These terms, first established within Type II string theory [15], have their M-theory analog due to σ -model anomaly on M5-brane. Within eleven dimensional supergravity we shall find explicit solutions for resolved M2-brane due to these higher derivative corrections.¹

Some of the aspects in connection with M2-branes and gravitational Chern-Simons terms were studied earlier in [16, 19]. We extend this analysis by finding explicit solutions for such M2-branes whose eight-dimensional transverse space is a Ricci-flat deformation of cones over seven dimensional Einstein manifolds. These are non-compact, smooth spaces with special Spin(7) and SU(4) holonomy, i.e. Ricci flat spaces with at least one covariantly constant spinor, and whose metrics are explicitly known.

The starting point is the Ansatz for the original M2-brane solution:

$$\begin{aligned} ds_{11}^2 &= H^{-2/3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/3} ds_8^2, \\ F_{0123r} &= \partial_r H^{-1}, \end{aligned}$$

where ds_8^2 is a Ricci-flat transverse metric of Spin(7) or SU(4) holonomy. Without taking into account higher curvature corrections, the function H is harmonic, satisfying the equation $\square H = 0$ where \square is the Laplacian on the Ricci-flat transverse space. H turns out to be singular at the inner boundary of the transverse space. We shall however see that the inclusion of the gravitational Chern-Simons-type corrections, which are of the type $\propto \text{Tr}(R^4) - \frac{1}{4} \text{Tr}(R^2)^2$, the singularity of the solution can be resolved. No inclusion of the four-form G_4 supported by the special holonomy transverse space is needed. In addition, the gravitational Chern-Simons term induces a bulk M2 brane charge and the solution asymptotically approaches $\text{AdS}_4 \times X^7$ where X^7 is the base of the transverse cone. However, since the only scale in the problem is l_p , the solution has a characteristic curvature scale of order l_p and thus supergravity approximation cannot be trusted. As we will see later, to get a good supergravity description, we indeed need to turn on four-form field strength G_4 in such a way that the curvature of the solution is everywhere much smaller than l_p .

The paper is organized as follows. In section 2, we discuss generalities about the gravitational Chern-Simons eight-form σ model anomaly. In section 3, taking

¹There are of course other higher derivative corrections which we are not considering in this paper. However, since the gravitational Chern Simons term is associated with an anomaly, and is topological in nature, it plays a special role.

into account the correction to the M2-brane equations of motion from this term, we explicitly construct smooth M2-brane solutions for the case with the transverse space is one of two different metrics of Spin(7) holonomy, one originally constructed in [20, 21, 22] and the other recently found in [23, 24, 25] which is asymptotically locally conical (ALC). The corresponding dual (2+1)-dimensional field theories have $\mathcal{N} = 1$ supersymmetry. In section 4, we repeat the analysis when the transverse space is T^*S^4 , with Stenzel metric which has SU(4) holonomy. The holographically dual (2+1)-dimensional field theory on the world volume of the M2-brane has $\mathcal{N} = 2$ supersymmetry. In concluding section 5, we comment on the interpretation of our result in the dual field theory. In Appendix A we present the details of the calculation for the Ricci tensor and curvature for a class of Spin(7) holonomy metric and in Appendix B we collected the details for the calculation of the harmonic functions.

2 Gravitational Chern-Simons corrections and M2-branes

The bosonic sector of $d = 11$ supergravity [26] is given by

$$S_{11} = \frac{1}{2} \int d^{11}x \sqrt{g} R - \int \left(\frac{1}{2} F \wedge *F + \frac{1}{6} A \wedge F \wedge F \right), \quad (1)$$

where g_{MN} is the space time metric and A is a three form with field strength $F = dA$. The field strength obeys the Bianchi identity $dF = 0$ and its equation of motion is

$$d * F = -\frac{1}{2} F \wedge F.$$

The gravitational Chern-Simons corrections associated with the σ -model anomaly on the M5-brane [16, 15], modify the equation:

$$d * F = -\frac{1}{2} F \wedge F + (2\pi)^4 \beta \mathcal{X}_8, \quad (2)$$

where β is related to the five-brane tension as $T_6 = \beta / (2\pi)^3$ and the eight-form anomaly \mathcal{X}_8 can be expressed in terms of the curvature two-form Θ :

$$\mathcal{X}_8 = \frac{1}{(2\pi)^4} \left\{ -\frac{1}{768} (\text{Tr } \Theta^2)^2 + \frac{1}{192} (\text{Tr } \Theta^4) \right\}. \quad (3)$$

The equation of motion (3) can be derived from the action (1) with the addition of the following term:

$$\Delta S_{11} = \int A \wedge \left\{ -\frac{1}{768} (\text{Tr } \Theta^2)^2 + \frac{1}{192} (\text{Tr } \Theta^4) \right\}. \quad (4)$$

We look for solutions with (2+1)-dimensional Lorentz invariance:

$$\begin{aligned} ds_{11}^2 &= H^{-2/3} dx^\mu dx^\nu \eta_{\mu\nu} + H^{1/3} ds_8^2, \\ F &= d^3x \wedge dH^{-1} + mG_4. \end{aligned} \quad (5)$$

Here, we have also added a harmonic four-form G_4 on the transverse eight-manifold. With this Ansatz, eq(2) yields

$$\square H = -\frac{1}{48} m^2 |G_{(4)}|^2 + (2\pi)^4 \beta X_8, \quad (6)$$

where $\mathcal{X}_8 = X_8 \text{dvol}_8$, dvol_8 is the volume form and \square is the Laplacian on the eight-fold. Such solutions were studied in [9, 10]. For vacuum solutions $m = 0$ and if X_8 is trivial, we can choose H to be a harmonic function on the eight-fold. In fact, a constant H will then be a solution, whence we get a product manifold with metric $ds^2 = dx^\mu dx^\nu \eta_{\mu\nu} + ds_8^2$. However, if the holonomy of the eight-manifold is non-trivial, the anomaly term may not vanish, and $H = \text{constant}$ is not a solution anymore. This can be interpreted as a distribution of background charge over the eight-fold, induced by the anomaly term.

As we will see, a smooth solution for H can be found even when the four-form G_4 is *not* turned on. However, such solutions fail to have a good AdS/CFT interpretation since the solution has a curvature scale of order l_p and supergravity is not a good description. To get a good gravity description, we can turn on background fluxes through G_4 which is an (anti) self-dual four-form, supported on the special holonomy space, such that the length scale associated with the metric is everywhere $\gg l_p$.

In what follows, we will study specific examples of transverse eight manifolds to be (i) two different manifolds of Spin(7) holonomy (one is the original one [20, 21] with AC structure and another set, recently constructed in [23, 24], has ALC structure) where the (2+1)-dimensional field theory has $\mathcal{N} = 1$ SUSY, (ii) T^*S^4 , with Stenzel metric, which has SU(4) holonomy, and the (2+1)-dimensional field theory on the world volume of the M2-brane has $\mathcal{N} = 2$ SUSY.

3 Resolved M2-brane and Spin(7) holonomy

In this section, we will find explicit M2-brane solutions when the transverse space is a manifold of Spin(7) holonomy. As such, there is one covariantly constant spinor on the manifold and the corresponding holographic (2+1)-dimensional field theory has $\mathcal{N} = 1$ supersymmetry. We consider the general Ansatz for a Spin(7) manifold introduced in [23], with special cases yielding metrics constructed in [20, 21]:

$$ds_8^2 = h^2 dr^2 + a^2 (D\mu^i)^2 + b^2 \sigma^2 + c^2 d\Omega_4^2, \quad i = 1, 2, 3 \quad (7)$$

where h, a, b and c are functions of a radial coordinate r , μ_i parameterize an S^2 and satisfy $\mu^i \mu^i = 1$,

$$\mu_1 = \sin \theta \sin \psi, \quad \mu_2 = \sin \theta \cos \psi, \quad \mu_3 = \cos \theta. \quad (8)$$

and

$$D\mu_i = d\mu_i + \epsilon_{ijk} A^j u^k, \quad \sigma = d\varphi + \mathcal{A}, \quad \mathcal{A} \equiv \cos \theta d\psi - \mu^i A^i. \quad (9)$$

The 1-form A^i is the $SU(2)$ Yang-Mills instanton on S^4 . In terms of coordinates (θ, ψ) on S^2 , we have

$$\begin{aligned} \sum_i (D\mu^i)^2 &= (d\theta - A^1 \cos \psi + A^2 \sin \psi)^2 \\ &\quad + \sin^2 \theta (d\psi + A^1 \cot \theta \sin \psi + A^2 \cot \theta \cos \psi - A^3)^2. \end{aligned} \quad (10)$$

The Vielbeine are given by

$$\begin{aligned} \hat{e}^0 &= h dr, & \hat{e}^\alpha &= c e^\alpha, \\ \hat{e}^1 &= a(d\theta - A^1 \cos \psi + A^2 \sin \psi), \\ \hat{e}^2 &= a \sin \theta (d\psi + A^1 \cot \theta \sin \psi + A^2 \cot \theta \cos \psi - A^3), \\ \hat{e}^3 &= b \sigma, \end{aligned} \quad (11)$$

where e^α , with $\alpha=4, 5, 6, 7$, is an orthonormal basis of the tangent-space 1-forms on the unit S^4 .

The spin connection ω_{ab} satisfying $de^a + \omega^a_b \wedge e^b = 0$ and the curvature two form $\Theta_{ab} = d\omega_{ab} + \omega_a^c \wedge \omega_{cb}$ for this are given in Appendix A. In what follows, we will study two cases of such manifolds with $\text{Spin}(7)$ holonomy.

3.1 Old $\text{Spin}(7)$ holonomy space

For the special case when $a = b$, the Ansatz given in (7) reduces to [20, 21, 22]:

$$ds_8^2 = h^2 dr^2 + a^2 (\sigma_i - A^i)^2 + c^2 d\Omega_4^2, \quad (12)$$

where we have used the relation

$$\sum_i (\sigma_i - A^i)^2 = \sum_i (D\mu^i)^2 + \sigma^2. \quad (13)$$

Conditions for Ricci flatness and $\text{Spin}(7)$ holonomy for this Ansatz have the following solution [20, 21, 22]:

$$h^2(r) = \left(1 - \frac{l^{10/3}}{r^{10/3}}\right)^{-1}, \quad a^2(r) = b^2(r) = \frac{9}{100} r^2 \left(1 - \frac{l^{10/3}}{r^{10/3}}\right), \quad c^2(r) = \frac{9}{20} r^2. \quad (14)$$

This metric is a resolution of a cone with a squashed seven sphere base. (Indeed, when $l = 0$, (14) becomes $ds_8^2 = dr^2 + r^2 ds_{\text{squashed } S^7}^2$.) The space is asymptotically conical (AC) with the principal orbits S^7 , viewed as an S^3 bundle over S^4 .

Singular cone ($l = 0$): By placing M2-branes at the tip of this cone and taking an appropriate scaling limit, we can arrive at a correspondence between M-theory on $\text{AdS}_4 \times S^7_{\text{squashed}}$ and the $\mathcal{N} = 1$ field theory living on the world volume of such M2-branes. The scaling limit corresponds to looking for the solution to the equation of motion $\square H = c\delta(r)$ which approaches zero asymptotically. Notice that there is a δ function source term on the right hand side of the EOM. The solution thus obtained is $H = \frac{32\pi^2 N l_p^6}{r^6}$ for N M2-branes placed at the singularity. The space-time metric is $\text{AdS}_4 \times S^7_{\text{squashed}}$, with the AdS scale given by $R = (32\pi^2 N)^{1/6} l_p$.

Resolved cone with Spin(7) holonomy ($l \neq 0$): We will now find M2-branes solutions with transverse space a smooth Spin(7) holonomy manifold given by (14), arising from resolution of the conical singularity by replacing the singular "tip" of the cone by a bolt. As we will discuss later, such smooth solutions are gravity dual of $\mathcal{N} = 1$ field theory living on the world volume of the M2-branes perturbed with relevant operators (associated with the pseudoscalar fields of the dual field theory [14]).

We will first look for vacuum solutions, *i.e.* solutions with no four-form flux turned on ($m = 0$). There is, however, a four-form bulk charge induced by the anomaly term X_8 which is now non-zero:

$$(2\pi)^4 X_8 = \frac{20 l^3}{3^7} (1530 r^{20/3} l^{31/3} + 3120 r^{10} l^7 - 1228 r^{50/3} l^{1/3} - 697 l^{17} - 1185 r^{40/3} l^{11/3} - 1540 l^{41/3} r^{10/3}) / r^{64/3} (-r^{10/3} + l^{10/3})^2, \quad (15)$$

The equation of motion (6) can be solved explicitly and details are given in Appendix B. The solution is in general singular at $r = l$. However, we can remove this singularity and find a smooth solution if we choose a specific integration constant as discussed in Appendix B. The full regular solution is

$$H(r) = \frac{\beta}{34904520} (-5758444 l^4 r^{32/3} + 23942926 r^{14} l^{2/3} - 11848824 l^{22/3} r^{22/3} - 834309 l^{14} r^{2/3} - 5501349 l^{32/3} r^4) / l^{2/3} r^{50/3} (r^{10/3} - l^{10/3}) + c_2. \quad (16)$$

Asymptotically, as $r \rightarrow \infty$ we have

$$H(r) \sim \frac{90011}{131220} \frac{\beta}{r^6} + \frac{53171}{102060} \frac{\beta l^{10/3}}{r^{28/3}} + \dots + c_2. \quad (17)$$

As is usual in the AdS/CFT correspondence, we will choose $c_2 = 0$. The space is asymptotically $\text{AdS}_4 \times S^7_{\text{squashed}}$. The length scale of AdS is $O(l_p)$ so we cannot trust supergravity.

We can, however, get an AdS radius $\gg l_p$, by turning on an anti-self-dual harmonic four-form [9]:

$$|G_{(4)}|^2 = \frac{35840000 l^4/3}{729 r^{28/3}}. \quad (18)$$

Now the equation of motion (6) for H has a source term arising from G_4 as well as X_8 . Solving for H [9], we find a smooth solution which near $r \approx l$ and $r \rightarrow \infty$ behaves:

1. As $r \rightarrow l$ we find

$$H(r) \sim c - \left[\frac{8500}{729} \frac{\beta}{l^7} + \frac{112 \times 10^3 m^2}{729 l^7} \right] (r - l) + \left[\frac{383750}{6561} \frac{\beta}{l^8} + \frac{952 \times 10^3 m^2}{3^7 l^8} \right] (r - l)^2.$$

The function H approaches a constant at $r = l$.

2. As $r \rightarrow \infty$:

$$H(r) = \left[\frac{90011}{131220} \beta + \frac{2 \times 10^5}{3^7} m^2 \right] \frac{1}{r^6} - \frac{28 \times 10^4 l^{4/3} m^2}{2673 r^{22/3}} + \frac{53171}{102060} \frac{\beta l^{10/3}}{r^{28/3}} \dots \quad (19)$$

This solution is supposed to describe the gravity dual of the theory living on the world volume of N M2-branes placed at the conical singularity perturbed by relevant operators, whose conformal dimension is determined by the subleading term [14] in the harmonic function. Note that the term proportional to β does not contribute at this subleading order. On the other hand the leading term in the harmonic function indeed gives an $\text{AdS}_4 \times S^7_{\text{squashed}}$. To get the right AdS scale, we need

$$\frac{90011}{131220} \beta + \frac{2 \times 10^5}{3^7} m^2 = 32\pi^2 N l_p^6 \quad (20)$$

The metric has no horizon, implying the existence of a mass gap in the dual field theory.

3.2 New Spin(7) holonomy space: \mathbb{B}_8

In [23, 24], new metrics of Spin(7) holonomy, whose structure is asymptotically locally conical (ALC), were found by starting with the Ansatz (7), and allowing for the S^3 fibers of the old Spin(7) construction themselves to be “squashed”. Namely, the S^3 bundle is itself written as a $U(1)$ bundle over S^2 . The general two-parameter metrics were given analytically (up to quadratures) and analyzed in [23, 24]. We will use one explicit example from these new metrics, namely the manifold labeled \mathbb{B}_8 in [23]. For this manifold, solution to the Spin(7) conditions is:

$$\begin{aligned} h^2(r) &= \frac{(r-l)^2}{(r-3l)(r+l)}, & a^2(r) &= \frac{1}{4}(r-3l)(r+l), \\ b^2(r) &= \frac{l^2(r-3l)(r+l)}{(r-l)^2}, & c^2(r) &= \frac{1}{2}(r^2-l^2). \end{aligned} \quad (21)$$

We calculate the spin connection ω_{ab} and the curvature two form Θ_{ab} in Appendix A. The anomaly eight-form, \mathcal{X}_8 is non-zero. In fact

$$\begin{aligned} (2\pi)^4 X_8 &= -\frac{3l^2}{8}(-55r^7 + 491r^6l - 1795r^5l^2 + 1871r^4l^3 + 2579r^3l^4 \\ &\quad - 5431r^2l^5 - 1369rl^6 + 4733l^7) / (r+l)^2 (-r+l)^{15}. \end{aligned} \quad (22)$$

In addition, we turn on a self-dual four-form [23]²:

$$|G_{(4)}|^2 = \frac{96(75r^6 - 350r^5 + 829r^4 - 932r^3 + 885r^2 - 414r + 99)}{(r-1)^6(r+1)^8} \quad (23)$$

The explicit solution for the harmonic function H can be found in Appendix B. Its limits are:

1. As $r \rightarrow 3l$ we have

$$H(r) \sim c - \left[\frac{75}{8192}\beta + \frac{9}{2048}m^2 \right] (r-3) + \left[\frac{141}{8192}\beta + \frac{33}{8192}m^2 \right] (r-3)^2 + \dots (24)$$

2. As $r \rightarrow \infty$,

$$\begin{aligned} H(r) = & \left(\frac{10323}{98560}\beta + \frac{63}{20}m^2 \right) \frac{1}{r^5} + \left(\frac{3441}{19712}\beta - \frac{79}{4}m^2 \right) \frac{1}{r^6} \\ & + \left(\frac{134199}{137984}\beta + \frac{317}{4}m^2 \right) \frac{1}{r^7} + \left(\frac{34675}{19712}\beta - \frac{953}{4}m^2 \right) \frac{1}{r^8} + \dots \quad (25) \end{aligned}$$

Some field theory aspects of the original M2-brane solution with this Spin(7) holonomy transverse space were studied in [17]. In [23] the fractional M2-brane ($m \neq 0$, $\beta = 0$) and a relation to the fractional D2-brane, which is obtained via a reduction along the S^1 isometry of the Spin(7) holonomy space, was discussed. Namely, due to the ALC structure of the space there is now a conserved magnetic M2-brane charge $\propto \int_{S^4} G_4$. With $\beta \neq 0$ our results for the harmonic function demonstrate that the gravitational Chern-Simons corrections contribute to the leading as well as the subleading terms, along with the terms $\propto m^2$, but with the alternating relative signs.

4 M2-branes with transverse space T^*S^4

Ricci-flat Kähler metrics on T^*S^{n+1} were constructed for general n by Stenzel [18]. Those are asymptotically conical spaces with the principal orbits described by a coset space $SO(n+2)/SO(n)$.

The case of $n = 2$ corresponds to the deformed conifold with metric given originally by Candelas and de la Ossa [7]. Such spaces are asymptotically conical. We will specifically be interested in the case $n = 3$ when the Einstein Sasakian seven manifold is $V_{5,2} = SO(5)/SO(3)$. The (2+1)-dimensional field theory living on the world volume of M2-branes with transverse space T^*S^4 with SU(4) holonomy Stenzel metric has $\mathcal{N} = 2$ supersymmetry in three dimensions. In the following, we find explicit M2-brane solutions with transverse space T^*S^4 , taking into account the gravitational

²In the remainder of this section, we have set $l = 1$

Chern-Simons σ -model anomaly corrections. We follow closely the notation and the explicit form of the metric for the eight-manifold as given in [10].

We define left invariant 1-forms L_{AB} on the group manifold $SO(n+2)$. By splitting the index as $A = (1, 2, i)$, we have that L_{ij} are the left-invariant 1-forms are the $SO(n)$ subgroup, and so the 1-forms in the coset $SO(n+2)/SO(n)$ will be

$$\sigma_i \equiv L_{1i}, \quad \tilde{\sigma}_i \equiv L_{2i}, \quad \nu \equiv L_{12}. \quad (26)$$

The metric takes the form (for $n = 4$):

$$ds_8^2 = h^2 dr^2 + a^2 \sigma_i^2 + b^2 \tilde{\sigma}_i^2 + c^2 \nu^2, \quad i = 1, 2, 3. \quad (27)$$

We define the Vielbeine:

$$e^0 = h dr, \quad e^i = a \sigma_i, \quad e^{\tilde{i}} = b \tilde{\sigma}_i, \quad e^{\bar{0}} = c \nu, \quad (28)$$

The functions a, b, c and h are given by

$$\begin{aligned} a^2 &= \frac{1}{3}(2 + \cosh 2r)^{1/4} \cosh r, & b^2 &= \frac{1}{3}(2 + \cosh 2r)^{1/4} \sinh r \tanh r, \\ h^2 = c^2 &= (2 + \cosh 2r)^{-3/4} \cosh^3 r. \end{aligned} \quad (29)$$

As r approaches zero, the metric takes the form

$$ds^2 \sim dr^2 + r^2 \tilde{\sigma}_i^2 + \sigma_i^2 + \nu^2$$

which has the structure locally of the product $R^4 \times S^4$, with R^4 corresponding to the "cotangent directions". As r tends to infinity, the metric becomes

$$ds^2 \sim d\rho^2 + \rho^2 \left(\frac{9}{16} \nu^2 + \frac{3}{32} (\sigma_i^2 + \tilde{\sigma}_i^2) \right),$$

representing a cone over the seven-dimensional Einstein space $V_{5,2} = SO(5)/SO(3)$.

The spin connection $\omega_{\alpha\beta}$ and the curvature two-form $\Theta_{\alpha\beta}$ were given in [10]. Then, using the expression for $\Theta_{\alpha\beta}$ in (3), the σ -model anomaly correction to the equations of motion \mathcal{X}_8 can be calculated (Appendix B):

$$\begin{aligned} (2\pi)^4 X_8 &= -\frac{5}{16} (2385 \cosh^{20} r + 10467 \cosh^{18} r + 21966 \cosh^{16} r + 28296 \cosh^{14} r \\ &+ 24687 \cosh^{12} r + 15300 \cosh^{10} r + 6880 \cosh^8 r + 2216 \cosh^6 r \\ &+ 486 \cosh^4 r + 65 \cosh^2 r + 4) / [\cosh^{20} r (1 + 2 \cosh^2 r)^5]. \end{aligned} \quad (30)$$

In addition, we can turn on a harmonic four-form G_4 , which was explicitly derived in [10] and its magnitude is given by

$$|G_{(4)}|^2 = \frac{360}{\cosh^8 r}. \quad (31)$$

The solution to the equation of motion (6) for H can be found exactly and is given explicitly in Appendix B. One of the two integration constants has been chosen to yield a non-singular solution. H has the following properties:

1. $r \rightarrow 0$

$$H(r) \sim c - \left(\frac{5m^2}{16} + \frac{145\beta}{24} \right) 3^{1/4} r^2 + \left(\frac{35m^2}{96} + \frac{5365\beta}{432} \right) 3^{1/4} r^4 + \dots, \quad (32)$$

2. $r \rightarrow \infty$

$$H(r) \sim \left(\frac{640m^2}{3^7} + \frac{205\beta}{243} \right) \frac{1}{\rho^6} - \frac{20480 \cdot 2^{1/3} m^2}{28431 \cdot 3^{2/3} \rho^{26/3}} - \left(\frac{103180 \cdot 6^{2/3}}{1003833} \beta + \frac{396800}{1003833} \frac{4^{1/3}}{3} m^2 \right) \frac{1}{\rho^{34/3}} + \dots \quad (33)$$

where ρ is the proper distance defined as $h dr = d\rho$. Again note, that this solution describes a the gravity dual of the theory living on the world volume of N M2-branes placed at the conical singularity perturbed by relevant operators, whose conformal dimension is determined by the subleading term. Note that the term proportional to β does not contribute at this subleading order.

5 Conclusions

We have studied M2-brane solutions with special holonomy transverse space, taking into account the gravitational Chern-Simons corrections arising from the σ -model anomaly on the M5 brane world volume. For the cases when the transverse space has the (i) original AC Spin(7) holonomy space and (ii) Stenzel metric with SU(4) holonomy on T^*S^4 , we have a clear interpretation as a deformation of the field theory on M2-branes placed at a conical singularity. Field theory living on the world volume of M2-branes placed at the tip of these cones is known for the Stenzel case [27, 28]. The M2-brane solution with the resolved cone as the transverse space is perfectly smooth, and corresponds to adding a relevant operator to the dual field theory [14]. The solution has no horizon implying the existence of a mass gap in the field theory.

The gravitational Chern-Simons term effectively generates a bulk M2-brane charge. So for the asymptotically flat cases, the solution still approaches $\text{AdS}_4 \times X^7$. The AdS scale is set by the strength of the background four form turned on (m^2) and by the bulk charge generated through the eight-form anomaly. The leading correction to $\text{AdS}_4 \times X^7$ asymptotically still arises from the background four-form. The gravitational Chern-Simons term contributes at higher order. Hence the interpretation of the gravity solution in terms of relevant operators remains as in [14]. The gravitational Chern-Simons term effects should correspond to $1/N$ effects in the renormalization group flow driven by addition of the relevant operator in the dual field theory.

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A Spin connection, curvature and Ricci tensor for Spin(7) holonomy metric

In this appendix we calculate the spin connection, curvature two-form and the Ricci tensor components for the metric in (7) with the veilbein given by (11).

$\hat{\omega}_{ab}$

The spin connection $\hat{\omega}_{ab}$ satisfying $d\hat{e}^a + \hat{\omega}^a{}_b \wedge \hat{e}^b = 0$ is given by:

$$\begin{aligned}
\hat{\omega}_{01} &= -\frac{a'}{a h} \hat{e}^1, & \hat{\omega}_{02} &= -\frac{a'}{a h} \hat{e}^2, & \hat{\omega}_{03} &= -\frac{b'}{b h} \hat{e}^3, & \hat{\omega}_{0\alpha} &= -\frac{c'}{c h} \hat{e}^\alpha, \\
\hat{\omega}_{12} &= \frac{b}{2a^2} \hat{e}^3 + \frac{\mu^1 A^1 + \mu^2 A^2}{\sin^2 \theta} - \frac{\cot \theta}{a} \hat{e}^2, \\
\hat{\omega}_{13} &= \frac{b}{2a^2} \hat{e}^2, & \hat{\omega}_{23} &= -\frac{b}{2a^2} \hat{e}^1, \\
\hat{\omega}_{1\alpha} &= \frac{a}{2c^2} (\sin \psi F_{\alpha\beta}^2 - \cos \psi F_{\alpha\beta}^1) \hat{e}^\beta, \\
\hat{\omega}_{2\alpha} &= \frac{a}{2c^2} \left(-\sin \theta F_{\alpha\beta}^3 + \cos \theta \sin \psi F_{\alpha\beta}^1 + \cos \theta \cos \psi F_{\alpha\beta}^2 \right) \hat{e}^\beta, \\
\hat{\omega}_{3\alpha} &= -\frac{b}{2c^2} \mu^i F_{\alpha\beta}^i \hat{e}^\beta, \\
\hat{\omega}_{\alpha\beta} &= \frac{a}{2c^2} \left(-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1 \right) \hat{e}^1 + \frac{b}{2c^2} \mu^i F_{\alpha\beta}^i \hat{e}^3 \\
&\quad + \frac{a}{2c^2} \left(\sin \theta F_{\alpha\beta}^3 - \cos \theta \sin \psi F_{\alpha\beta}^1 - \cos \theta \cos \psi F_{\alpha\beta}^2 \right) \hat{e}^2 + \omega_{\alpha\beta}. \quad (\text{A.1})
\end{aligned}$$

where ω_{ab} is the spin connection on the S^4 .

$\hat{\Theta}_{ab}$

The curvature two-form $\hat{\Theta}_{ab} = d\hat{\omega}_{ab} + \hat{\omega}_{ac} \wedge \hat{\omega}_{cb}$ can be computed by using the spin connection calculated above:

$$\begin{aligned}
\hat{\Theta}_{01} &= -\left(\frac{a''}{ah^2} - \frac{a'h'}{ah^3}\right)\hat{e}^0 \wedge \hat{e}^1 + \frac{b}{2a^2h}\left(\frac{a'}{a} - \frac{b'}{b}\right)\hat{e}^2 \wedge \hat{e}^3 - \left(\frac{a'}{h} - \frac{ac'}{ch}\right)(-\cos\psi F^1 + \sin\psi F^2), \\
\hat{\Theta}_{02} &= -\left(\frac{a''}{ah^2} - \frac{a'h'}{ah^3}\right)\hat{e}^0 \wedge \hat{e}^2 + \frac{b}{2a^2h}\left(\frac{a'}{a} - \frac{b'}{b}\right)\hat{e}^3 \wedge \hat{e}^1 \\
&\quad + \left(\frac{a'}{h} - \frac{ac'}{ch}\right)(-\sin\psi \cos\theta F^1 - \cos\psi \cos\theta F^2 + \sin\theta F^3), \\
\hat{\Theta}_{03} &= -\left(\frac{b''}{bh^2} - \frac{b'h'}{bh^3}\right)\hat{e}^0 \wedge \hat{e}^3 + \frac{b}{a^2h}\left(\frac{a'}{a} - \frac{b'}{b}\right)\hat{e}^2 \wedge \hat{e}^1 + \left(\frac{b'}{h} - \frac{bc'}{ch}\right)\mu^i F^i, \\
\hat{\Theta}_{0\alpha} &= -\left(\frac{c''}{ch^2} - \frac{c'h'}{ch^3}\right)\hat{e}^0 \wedge \hat{e}^\alpha \\
&\quad - \frac{1}{2c^2}\left(\frac{ac'}{ch} - \frac{a'}{h}\right)(-\sin\psi \cos\theta F_{\alpha\beta}^1 - \cos\psi \cos\theta F_{\alpha\beta}^2 + \sin\theta F_{\alpha\beta}^3)\hat{e}^2 \wedge \hat{e}^\beta \\
&\quad + \left(\frac{ac'}{2c^3h} - \frac{a'}{2c^2h}\right)(-\cos\psi F_{\alpha\beta}^1 + \sin\psi F_{\alpha\beta}^2)\hat{e}^1 \wedge \hat{e}^\beta + \frac{1}{2c^2}\left(\frac{b'}{h} - \frac{bc'}{ch}\right)\mu^i F_{\alpha\beta}^i \hat{e}^3 \wedge \hat{e}^\beta, \\
\hat{\Theta}_{12} &= \left(\frac{b'}{a^2h} - \frac{a'b}{a^3h}\right)\hat{e}^0 \wedge \hat{e}^3 - \left(\frac{3b^2}{4a^4} - \frac{1}{a^2} + \frac{a'^2}{a^2h^2}\right)\hat{e}^1 \wedge \hat{e}^2 + \left(1 - \frac{a^2}{2c^2} - \frac{b^2}{2a^2}\right)\mu^i F^i, \\
\hat{\Theta}_{13} &= \left[\frac{b}{4ac^2}(\cos\theta \sin\psi F_{\alpha\beta}^1 + \cos\theta \cos\psi F_{\alpha\beta}^2 - \sin\theta F_{\alpha\beta}^3)\right. \\
&\quad \left. - \frac{ab}{4c^4}(\mu^i F_{\eta\alpha}^i(\sin\psi F_{\eta\beta}^2 - \cos\psi F_{\eta\beta}^1))\right]\hat{e}^\alpha \wedge \hat{e}^\beta \\
&\quad + \left(\frac{b'}{2a^2h} - \frac{a'b}{2a^3h}\right)\hat{e}^0 \wedge \hat{e}^2 + \left(\frac{b^2}{4a^4} - \frac{a'b'}{abh^2}\right)\hat{e}^1 \wedge \hat{e}^3, \\
\hat{\Theta}_{23} &= \left[\frac{b}{4ac^2}(-\sin\psi F_{\alpha\beta}^2 + \cos\psi F_{\alpha\beta}^1)\right. \\
&\quad \left. - \frac{ab}{4c^4}(\mu^i F_{\eta\beta}^i(\sin\theta F_{\eta\alpha}^3 - \cos\theta \sin\psi F_{\eta\alpha}^1 - \cos\theta \cos\psi F_{\eta\alpha}^2))\right]\hat{e}^\alpha \wedge \hat{e}^\beta \\
&\quad - \left(\frac{b'}{2a^2h} - \frac{a'b}{2a^3h}\right)\hat{e}^0 \wedge \hat{e}^1 + \left(\frac{b^2}{4a^4} - \frac{a'b'}{abh^2}\right)\hat{e}^2 \wedge \hat{e}^3, \\
\hat{\Theta}_{1\alpha} &= \left(\frac{a'}{2c^2h} - \frac{ac'}{2c^3h}\right)(-\cos\psi F_{\alpha\beta}^1 + \sin\psi F_{\alpha\beta}^2)\hat{e}^0 \wedge \hat{e}^\beta + \left(\frac{a^2}{4c^4} - \frac{a'c'}{ach^2}\right)\hat{e}^1 \wedge \hat{e}^\alpha \\
&\quad + \frac{a}{2c^3}(-\cos\psi D_\gamma F_{\alpha\beta}^1 + \sin\psi D_\gamma F_{\alpha\beta}^2)\hat{e}^\gamma \wedge \hat{e}^\beta \\
&\quad - \left(\frac{a^2}{4c^4} - \frac{1}{2c^2} + \frac{b^2}{4a^2c^2}\right)\mu^i F_{\alpha\beta}^i \hat{e}^2 \wedge \hat{e}^\beta + \left[\frac{ab}{4c^4}\mu^i F_{\alpha\eta}^i(-\sin\psi F_{\eta\beta}^2 + \cos\psi F_{\eta\beta}^1)\right. \\
&\quad \left. - \frac{b}{4ac^2}(\sin\psi \cos\theta F_{\alpha\beta}^1 + \cos\psi \cos\theta F_{\alpha\beta}^2 - \sin\theta F_{\alpha\beta}^3)\right]\hat{e}^\beta \wedge \hat{e}^3, \\
\hat{\Theta}_{2\alpha} &= -\left(\frac{a'}{2c^2h} - \frac{ac'}{2c^3h}\right)(-\sin\psi \cos\theta F_{\alpha\beta}^1 - \cos\psi \cos\theta F_{\alpha\beta}^2 + \sin\theta F_{\alpha\beta}^3)\hat{e}^0 \wedge \hat{e}^\beta \\
&\quad + \left(\frac{a^2}{4c^4} - \frac{a'c'}{ach^2}\right)\hat{e}^2 \wedge \hat{e}^\alpha - \frac{a}{2c^3}(-\sin\psi \cos\theta D_\gamma F_{\alpha\beta}^1 - \cos\psi \cos\theta D_\gamma F_{\alpha\beta}^2
\end{aligned}$$

$$\begin{aligned}
& + \sin \theta D_\gamma F_{\alpha\beta}^3 \hat{e}^\gamma \wedge \hat{e}^\beta + \left(\frac{a^2}{4c^4} - \frac{1}{2c^2} + \frac{b^2}{4a^2c^2} \right) \mu^i F_{\alpha\beta}^i \hat{e}^1 \wedge \hat{e}^\beta \\
& + \left[\frac{ab}{4c^4} \mu^i F_{\alpha\eta}^i (-\sin \psi \cos \theta F_{\eta\beta}^1 - \cos \psi \cos \theta F_{\eta\beta}^2 + \sin \theta F_{\eta\beta}^3) \right. \\
& \left. - \frac{b}{4ac^2} (-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1) \right] \hat{e}^\beta \wedge \hat{e}^3, \\
\widehat{\Theta}_{3\alpha} = & - \left(\frac{b'}{2c^2 h} - \frac{bc'}{2c^3 h} \right) \mu^i F_{\alpha\beta}^i \hat{e}^0 \wedge \hat{e}^\beta + \left[- \frac{b}{4ac^2} (\sin \psi \cos \theta F_{\alpha\beta}^1 \right. \\
& + \cos \psi \cos \theta F_{\alpha\beta}^2 - \sin \theta F_{\alpha\beta}^3) + \frac{ab}{4c^4} \mu^i F_{\gamma\beta}^i (-\sin \psi F_{\gamma\alpha}^2 + \cos \psi F_{\gamma\alpha}^1) \left. \right] \hat{e}^1 \wedge \hat{e}^\beta \\
& + \left[- \frac{b}{4ac^2} (-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1) - \frac{ab}{4c^4} \mu^i F_{\gamma\beta}^i (\sin \psi \cos \theta F_{\gamma\alpha}^1 \right. \\
& + \cos \psi \cos \theta F_{\gamma\alpha}^2 - \sin \theta F_{\gamma\alpha}^3) \left. \right] \hat{e}^2 \wedge \hat{e}^\beta + \frac{b^2}{4c^4} (\mu^i F_{\gamma\beta}^i) (\mu^j F_{\gamma\alpha}^j) \hat{e}^3 \wedge \hat{e}^\beta \\
& - \frac{b'c'}{bc h^2} \hat{e}^3 \wedge \hat{e}^\alpha - \frac{b}{2c^3} \mu^i D_\gamma F_{\alpha\beta}^i \hat{e}^\gamma \wedge \hat{e}^\beta, \\
\widehat{\Theta}_{\alpha\beta} = & \Theta_{\alpha\beta} - \frac{c'^2}{c^2 h^2} \hat{e}^\alpha \wedge \hat{e}^\beta + \left\{ - \frac{a^2}{4c^4} (\sin \psi F_{\alpha\eta}^2 - \cos \psi F_{\alpha\eta}^1) (\sin \psi F_{\beta\gamma}^2 - \cos \psi F_{\beta\gamma}^1) \right. \\
& - \frac{a^2}{4c^4} (\sin \psi \cos \theta F_{\alpha\eta}^1 + \cos \psi \cos \theta F_{\alpha\eta}^2 - \sin \theta F_{\alpha\eta}^3) \times \\
& (\sin \psi \cos \theta F_{\beta\gamma}^1 + \cos \psi \cos \theta F_{\beta\gamma}^2 - \sin \theta F_{\beta\gamma}^3) + \frac{b^2}{4c^4} \left[(\mu^i F_{\alpha\eta}^i) (\mu^j F_{\beta\gamma}^j) \right. \\
& \left. - 2(\mu^i F_{\alpha\beta}^i) (\mu^j F_{\eta\gamma}^j) \right] + \frac{a^2}{4c^4} (-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1) (\sin \psi F_{\eta\gamma}^2 - \cos \psi F_{\eta\gamma}^1) \\
& + \frac{a^2}{4c^4} (-\sin \psi \cos \theta F_{\alpha\beta}^1 - \cos \psi \cos \theta F_{\alpha\beta}^2 + \sin \theta F_{\alpha\beta}^3) \times \\
& (\sin \psi \cos \theta F_{\eta\gamma}^1 + \cos \psi \cos \theta F_{\eta\gamma}^2 - \sin \theta F_{\eta\gamma}^3) \left. \right\} \hat{e}^\eta \wedge \hat{e}^\gamma \\
& + \left(\frac{a'}{c^2 h} - \frac{ac'}{c^3 h} \right) (-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1) \hat{e}^0 \wedge \hat{e}^1 \\
& - \left(\frac{a'}{c^2 h} - \frac{ac'}{c^3 h} \right) (\sin \psi \cos \theta F_{\alpha\beta}^1 + \cos \psi \cos \theta F_{\alpha\beta}^2 - \sin \theta F_{\alpha\beta}^3) \hat{e}^0 \wedge \hat{e}^2 \\
& + \frac{a}{2c^3} (-\sin \psi D_\gamma F_{\alpha\beta}^2 + \cos \psi D_\gamma F_{\alpha\beta}^1) \hat{e}^\gamma \wedge \hat{e}^1 \\
& - \frac{a}{2c^3} (\sin \psi \cos \theta D_\gamma F_{\alpha\beta}^1 + \cos \psi \cos \theta D_\gamma F_{\alpha\beta}^2 - \sin \theta D_\gamma F_{\alpha\beta}^3) \hat{e}^\gamma \wedge \hat{e}^2 \\
& + \left(\frac{b'}{c^2 h} - \frac{bc'}{c^3 h} \right) \mu^i F_{\alpha\beta}^i \hat{e}^0 \wedge \hat{e}^3 + \left[\frac{b}{2ac^2} (\sin \psi \cos \theta F_{\alpha\beta}^1 + \cos \psi \cos \theta F_{\alpha\beta}^2 \right. \\
& - \sin \theta F_{\alpha\beta}^3) + \frac{ab}{4c^4} \mu^i F_{\gamma\beta}^i (-\sin \psi F_{\alpha\gamma}^2 + \cos \psi F_{\alpha\gamma}^1) \\
& - \frac{ab}{4c^4} \mu^i F_{\alpha\gamma}^i (-\sin \psi F_{\gamma\beta}^2 + \cos \psi F_{\gamma\beta}^1) \left. \right] \hat{e}^1 \wedge \hat{e}^3 + \left[\frac{b}{2ac^2} (-\sin \psi F_{\alpha\beta}^2 + \cos \psi F_{\alpha\beta}^1) \right. \\
& \left. - \frac{ab}{4c^4} \mu^i F_{\gamma\beta}^i (\sin \psi \cos \theta F_{\alpha\gamma}^1 + \cos \psi \cos \theta F_{\alpha\gamma}^2 - \sin \theta F_{\alpha\gamma}^3) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{ab}{4c^4} \mu^i F_{\alpha\gamma}^i (\sin \psi \cos \theta F_{\gamma\beta}^1 + \cos \psi \cos \theta F_{\gamma\beta}^2 - \sin \theta F_{\gamma\beta}^3) \Big] \hat{e}^2 \wedge \hat{e}^3 \\
& + \left(\frac{1}{c^2} - \frac{a^2}{2c^4} - \frac{b^2}{2a^2 c^2} \right) \mu^i F_{\alpha\beta}^i \hat{e}^1 \wedge \hat{e}^2 + \frac{b}{2c^3} \mu^i D_\gamma F_{\alpha\beta}^i \hat{e}^\gamma \wedge \hat{e}^3.
\end{aligned} \tag{A.2}$$

$F^i \equiv \frac{1}{2} F_{\alpha\beta}^i e^\alpha \wedge e^\beta$ and $D_\gamma F_{\alpha\beta}^i = \nabla_\gamma F_{\alpha\beta}^i + \epsilon_{ijk} A_\gamma^j F_{\alpha\beta}^k$ is the gauge-covariant derivative of $F_{(2)}^i$, ∇_γ is the Riemannian covariant derivative on S^4 . $\Theta_{\alpha\beta} = e^\alpha \wedge e^\beta$ is the curvature two-form on S^4 .

The non-zero components of the Ricci tensor in the orthonormal basis $\widehat{R}_{ab} = \widehat{R}_{acb}{}^c$ are

$$\begin{aligned}
\widehat{R}_{00} &= -2 \left(\frac{a''}{a h^2} - \frac{a' h'}{a h^3} \right) - \left(\frac{b''}{b h^2} - \frac{b' h'}{b h^3} \right) - 4 \left(\frac{c''}{c h^2} - \frac{c' h'}{c h^3} \right), \\
\widehat{R}_{11} &= - \left(\frac{a''}{a h^2} - \frac{a' h'}{a h^3} \right) - \left(\frac{3 b^2}{4 a^4} - \frac{1}{a^2} + \frac{a'^2}{a^2 h^2} \right) + \left(\frac{b^2}{4 a^4} - \frac{a' b'}{a b h^2} \right) + 4 \left(\frac{a^2}{4 c^4} - \frac{a' c'}{a c h^2} \right), \\
\widehat{R}_{13} &= \frac{ab}{4c^4} \mu^i F_{\eta\alpha}^i (-\sin \psi F_{\eta\alpha}^2 + \cos \psi F_{\eta\alpha}^1), \\
\widehat{R}_{22} &= \widehat{R}_{11}, \\
\widehat{R}_{23} &= \frac{ab}{4c^4} \mu^i F_{\alpha\eta}^i (-\sin \psi \cos \theta F_{\eta\alpha}^1 - \cos \psi \cos \theta F_{\eta\alpha}^2 + \sin \theta F_{\eta\alpha}^3), \\
\widehat{R}_{33} &= - \left(\frac{b''}{b h^2} - \frac{b' h'}{b h^3} \right) + 2 \left(\frac{b^2}{4 a^4} - \frac{a' b'}{a b h^2} \right) - \frac{4 b' c'}{b c h^2} + \frac{b^2}{4 c^4} (\mu^i F_{\eta\alpha}^i) (\mu^j F_{\eta\alpha}^j), \\
\widehat{R}_{1\alpha} &= \frac{a}{2c^3} (-\sin \psi D_\beta F_{\alpha\beta}^2 + \cos \psi D_\beta F_{\alpha\beta}^1), \\
\widehat{R}_{2\alpha} &= \frac{a}{2c^3} (\sin \psi \cos \theta D_\beta F_{\alpha\beta}^1 + \cos \psi \cos \theta D_\beta F_{\alpha\beta}^2 - \sin \theta D_\beta F_{\alpha\beta}^3), \\
\widehat{R}_{3\alpha} &= -\frac{b}{2c^3} \mu^i D_\beta F_{\alpha\beta}^i, \\
\widehat{R}_{\alpha\beta} &= \left(-\frac{c''}{c h^2} + \frac{c' h'}{c h^3} + \frac{a^2}{2c^4} - \frac{2a' c'}{a c h^2} - \frac{b' c'}{b c h^2} - \frac{3c'^2}{c^2 h^2} \right) \delta_{\alpha\beta} + \frac{R_{\alpha\beta}}{c^2} \\
& - \frac{b^2}{2c^4} (\mu^i F_{\alpha\delta}^i) (\mu^j F_{\beta\delta}^j) + \frac{3 a^2}{4 c^4} (-\sin \psi F_{\alpha\delta}^2 + \cos \psi F_{\alpha\delta}^1) (\sin \psi F_{\beta\delta}^2 - \cos \psi F_{\beta\delta}^1) \\
& + \frac{3 a^2}{4 c^4} (-\sin \psi \cos \theta F_{\alpha\delta}^1 - \cos \psi \cos \theta F_{\alpha\delta}^2 + \sin \theta F_{\alpha\delta}^3) \times \\
& (\sin \psi \cos \theta F_{\beta\delta}^1 + \cos \psi \cos \theta F_{\beta\delta}^2 - \sin \theta F_{\beta\delta}^3).
\end{aligned} \tag{A.3}$$

Two of the Ricci flat metrics with cohomogeneity one [10, 21], are

$$h^2(r) = \left(1 - \frac{l^{10/3}}{r^{10/3}} \right)^{-1}, \quad a^2(r) = b^2(r) = \frac{9}{100} r^2 \left(1 - \frac{l^{10/3}}{r^{10/3}} \right), \quad c^2(r) = \frac{9}{20} r^2, \tag{A.4}$$

obtained in [21] and

$$\begin{aligned}
h^2(r) &= \frac{(r-l)^2}{(r-3l)(r+l)}, & a^2(r) &= \frac{1}{4}(r-3l)(r+l), \\
b^2(r) &= \frac{l^2(r-3l)(r+l)}{(r-l)^2}, & c^2(r) &= \frac{1}{2}(r^2-l^2),
\end{aligned} \tag{A.5}$$

obtained recently for the \mathbb{B}_8 manifold [23]. Here r is a radial coordinate defined as $r > l$ and $r > 3l$, where $l > 0$, for the first and second solutions respectively. Also, in our conventions,

$$F^1 = -(e^5 \wedge e^6 + e^4 \wedge e^7), \quad F^2 = -(e^6 \wedge e^4 + e^5 \wedge e^7), \quad F^3 = -(e^4 \wedge e^5 + e^6 \wedge e^7),$$

where $e^\alpha = (e^4, e^5, e^6, e^7)$ is the basis of the tangent-space 1-forms on the unit S^4 .

B Detailed Calculations

In this appendix we provide details for the calculation of the harmonic function H which is the solution to eq. (6).

Old Spin(7) holonomy metric

For the metric [20, 21] given in (12), X_8 is given in (15). Then, with $m = 0$ in (6), we have

$$(\sqrt{g}h^{-2}H_1')' = (2\pi)^4\beta X_8\sqrt{g} = (2\pi)^4\beta X_8 \frac{3^7}{4 \times 10^5} \left(1 - \frac{l^{10/3}}{r^{10/3}}\right) r^7. \quad (\text{B.1})$$

which can be solved:

$$H(r) = \frac{4 \times 10^5}{3^7} \int I_1(r) \left[\left(1 - \frac{l^{10/3}}{r^{10/3}}\right)^2 r^7 \right]^{-1} dr, \quad (\text{B.2})$$

where $I_1(r)$ is given by

$$\begin{aligned} I_1(r) &= \frac{(2\pi)^4 3^7 \beta}{4 \times 10^5} \int \left[\left(1 - \frac{l^{10/3}}{r^{10/3}}\right) r^7 \right] X_8 dr. \\ &= -\frac{\beta}{4 \times 10^6} (-8364 l^{50/3} - 33555 l^{40/3} r^{10/3} + 72390 l^{20/3} r^{10} + 73680 r^{40/3} l^{10/3} \\ &\quad - 14140 l^{10} r^{20/3} + 73680 r^{50/3}) / r^{50/3} + c_1. \end{aligned} \quad (\text{B.3})$$

As $r \rightarrow l$, we have

$$\begin{aligned} H(r) &\sim -\frac{4 \times 10^3}{243 l^5} \left[\frac{163691}{(4 \times 10^6)} \beta + c_1 \right] \frac{1}{(r-l)} - \frac{32 \times 10^3}{729 l^6} \left[\frac{163691}{(4 \times 10^6)} \beta + c_1 \right] \ln(r-l) \\ &+ \frac{437 \times 10^3}{6561 l^7} \left[c_1 - \frac{234467033}{1748 \times 10^6} \beta \right] (r-l) - \frac{731500}{19683 l^8} \left[c_1 - \frac{8970520067}{5852 \times 10^6} \beta \right] (r-l)^2 \\ &+ \dots + c. \end{aligned} \quad (\text{B.4})$$

The solution is regular at $r = l$ if we choose the integration constant $c_1 = -(163691/4 \times 10^6)\beta$, and it then tends to a finite constant. In fact the full regular solution is given

by

$$H_1(r) = \frac{\beta}{34904520} (-5758444 l^4 r^{32/3} + 23942926 r^{14} l^{2/3} - 11848824 l^{22/3} r^{22/3} - 834309 l^{14} r^{2/3} - 5501349 l^{32/3} r^4) / l^{2/3} r^{50/3} (r^{10/3} - l^{10/3}) + c_2. \quad (\text{B.5})$$

The asymptotic behavior, as $r \rightarrow \infty$, is

$$H(r) \sim \frac{90011}{131220} \frac{\beta}{r^6} + \frac{53171}{102060} \frac{\beta l^{10/3}}{r^{28/3}} + \dots + c_2. \quad (\text{B.6})$$

\mathbb{B}_8 -metric

The ALC Spin(7) holonomy metric on \mathbb{B}_8 [23, 24] is given in (A.5) and X_8 is found in (22). Eq. (6), with $m = 0$ then takes the form:

$$(\sqrt{g} h^{-2} H')' = (2\pi)^4 \beta X_8 \sqrt{g} = (2\pi)^4 \beta X_8 (1/16) (r - 3l) (r + l) (r^2 - l^2)^2 l. \quad (\text{B.7})$$

This equation can be solved to yield

$$H(r) = 16 \int I_1(r) [l(r - 3l)^2 (r + l)^4]^{-1} dr, \quad (\text{B.8})$$

where $I_1(r)$ is the first integration of H_1 and is given by

$$\begin{aligned} I_1(r) &= \frac{(2\pi)^4 \beta}{16} \int [(r - 3l) (r + l) (r^2 - l^2)^2 l] X_8 dr \\ &= -\frac{\beta l^3}{98560} (1837308 l^4 r^5 + 3530010 l^5 r^4 - 2019600 l^6 r^3 - 4883604 l^7 r^2 \\ &\quad + 2013318 l^8 r - 442365 l r^8 + 42350 r^9 + 1914528 l^2 r^7 - 3768996 l^3 r^6 \\ &\quad + 2565531 l^9) / (r - l)^{12} + c_1. \end{aligned} \quad (\text{B.9})$$

At short distance, *i.e.*, as $r \rightarrow 3l$, we have

$$\begin{aligned} H(r) &\sim c_2 - \frac{1}{16 l^5} \left[\frac{10323 \beta}{315392} + c_1 \right] \frac{1}{(r - 3l)} - \frac{1}{16 l^6} \left[\frac{10323 \beta}{315392} + c_1 \right] \ln(r - 3l) \\ &+ \frac{5}{128 l^7} \left[-\frac{63597 \beta}{315392} + c_1 \right] (r - 3l) + \frac{5}{512 l^8} \left[\frac{2727777}{1576960} \beta - c_1 \right] (r - 3l)^2 + \dots \end{aligned} \quad (\text{B.10})$$

We can see that this solution is regular at $r = 3l$ for $c_1 = -(10323/315392)\beta$ and tends to a finite constant. The regular solution is given by

$$\begin{aligned} H(r) &= -\frac{\beta}{35481600} (3453165 r^{13} + 205750648 l^{13} - 27625320 r^{12} l + 87480180 r^{11} l^2 \\ &\quad - 119709720 r^{10} l^3 - 12234087 r^9 l^4 + 285056136 r^8 l^5 - 414630720 r^7 l^6 \\ &\quad + 252973584 r^6 l^7 - 121143233 r^5 l^8 + 214217512 r^4 l^9 - 107560436 r^3 l^{10} \\ &\quad - 184835000 r^2 l^{11} - 9583109 r l^{12}) / l^5 (r + l)^3 (r - l)^{11} \\ &\quad + \frac{76737 \beta}{1576960 l^6} \ln \left(\frac{r + l}{r - l} \right) + c_2. \end{aligned} \quad (\text{B.11})$$

The asymptotic behavior, i.e., at $r \rightarrow \infty$, is

$$H(r) \sim \frac{10323}{98560} \frac{\beta}{l r^5} + \frac{3441}{19712} \frac{\beta}{r^6} + \frac{134199}{137984} \frac{l\beta}{r^7} + \frac{34675}{19712} \frac{l^2\beta}{r^8} + \dots + c_2. \quad (\text{B.12})$$

Stenzel metric

The Stenzel metric on T^*S^4 is given in the main text by eq.(27) and X_8 is given by eq. (30). We now look for solutions to (6), with $m = 0$ Assuming that H only depends on the coordinate r , we find that eq. (6) takes the form:

$$(\sqrt{g}h^{-2}H')' = (2\pi)^4\beta X_8\sqrt{g} = (2\pi)^4\beta X_8 \sinh^3 2r/216, \quad (\text{B.13})$$

This equation can be solved for H :

$$H(r) = 216 \int I_1(r) \left[\frac{(2 + \cosh 2r)^{-3/4} \cosh^3 r}{\sinh^3 2r} \right] dr, \quad (\text{B.14})$$

where $I_1(r)$ is the first integration of H_1 and is given by

$$\begin{aligned} I_1(r) &= (2\pi)^4\beta \int \frac{\sinh^3 2r}{216} X_8 dr. \\ &= -\frac{5\beta}{27648} (-715906 \cosh^2 2r - 1660504 \cosh 2r + 72432 \cosh^9 2r \\ &\quad + 489417 \cosh^8 2r + 7660840 \cosh^5 2r + 4778536 \cosh^6 2r \\ &\quad + 1920696 \cosh^7 2r + 7506142 \cosh^4 2r + 3485960 \cosh^3 2r \\ &\quad + 4770 \cosh^{10} 2r - 587631) / [(\cosh 2r + 2)^4 (\cosh 2r + 1)^8] \\ &\quad + c_1 \end{aligned} \quad (\text{B.15})$$

The solution Eq.(B.14) for H has a small r expansion:

$$\begin{aligned} H(r) \sim &\left\{ -\frac{9}{2} \left(\frac{205\beta}{1024} + c_1 \right) \frac{3^{1/4}}{r^2} - 9 \left(\frac{205\beta}{1024} + c_1 \right) 3^{1/4} \ln r \right. \\ &+ \left[\frac{93}{40} \left(\frac{205\beta}{1024} + c_1 \right) - \frac{145\beta}{24} \right] 3^{1/4} r^2 + \left[-\frac{641}{1680} \left(\frac{205\beta}{1024} + c_1 \right) + \frac{5365\beta}{432} \right] 3^{1/4} r^4 \\ &\left. + \dots \right\} + c. \end{aligned} \quad (\text{B.16})$$

Note that the solution at $r = 0$ is regular if $c_1 = -205\beta/1024$ and tends to a finite constant. We also see that this exactly agrees with the condition $I_1(0) = 0$. Indeed, for this specific choice of c_1 the integration in Eq.(B.14), can be performed exactly by redefining the coordinate r as $2 + \cosh 2r = y^4$. In terms of the new coordinate y we find

$$\begin{aligned} H(y) = &c_2 - \frac{7995\sqrt{2}\beta}{1232} F(\arcsin(\frac{1}{y}) | -1) + \frac{\sqrt{2}\beta}{59136} (383760 y^{44} - 2839824 y^{40} \\ &+ 9224385 y^{36} - 16750755 y^{32} + 18533340 y^{28} - 12563700 y^{24} + 4910886 y^{20} \\ &- 850770 y^{16} - 29300 y^{12} + 29340 y^8 - 14175 y^4 + 6237) / (y^4 - 1)^{15/2} y^{15}, \end{aligned} \quad (\text{B.17})$$

where c_2 is an integration constant, and $F(\phi|m)$ is the incomplete elliptic integral of the first kind, i.e.,

$$F(\phi|m) \equiv \int_0^\phi (1 - m \sin^2 \theta)^{-1/2} d\theta. \quad (\text{B.18})$$

For large r the solution can be expressed as

$$H(r) \sim \frac{205}{243} \frac{\beta}{\rho^6} - \frac{103180}{1003833} \frac{6^{2/3}}{\rho^{34/3}} \beta + \dots + c_2, \quad (\text{B.19})$$

where ρ is the proper distance defined as $h dr = d\rho$. We can see that there is no divergence at large distance and the M2-brane has a well-defined ADM mass.

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