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Kappa-symmetric non-abelian Born-Infeld actions in three dimensions

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Abstract

A superembedding construction of general non-abelian Born-Infeld actions in three dimensions is described. These actions have rigid target space and local worldvolume supersymmetry (i.e. kappa-symmetry). The standard abelian Born-Infeld gauge multiplet is augmented with an additional worldvolume $SU(N)$ gauge supermultiplet. It is shown how to construct single-trace actions and in particular a kappa-supersymmetric extension of the symmetrised trace action.

1 Introduction

In this paper we use the superembedding method to give a construction of non-abelian brane actions of Born-Infeld type. The superembedding construction guarantees that these actions have manifest local worldvolume supersymmetry (or kappa-symmetry) as well as the rigid supersymmetry of the flat target space. One such action has been constructed before for the case of coincident D-particles [1]. In this case the worldvolume gauge field strength vanishes since the worldvolume is only a one-dimensional worldline.

There has been much discussion in the literature of possible generalisations of abelian Born-Infeld theory to the non-abelian case. This has largely been motivated by the fact that the effective action of coincident D-branes in type II superstring theories is some non-abelian generalisation of the Born-Infeld action. One proposal for this generalisation is to keep the form of the abelian action but replace the $U(1)$ field strengths with $U(N)$ field strengths and then take a symmetrised trace over the orderings of the different field strengths at each order [2]. This correctly reproduces the F^4 terms calculated from string scattering amplitudes but fails at higher orders in the field strengths [3, 4, 5]. In [6] another approach to constructing non-abelian Born-Infeld actions was considered using a generalisation of the partially broken global supersymmetry approach to constructing abelian Born-Infeld actions with linear and non-linear supersymmetry [7, 8, 9].

A different approach to constructing the non-abelian Born-Infeld action has been developed in refs [10, 11, 12] while attempts to find it using ten-dimensional supersymmetry have been given in refs [13, 14, 15] and in ref [16].

Space-filling branes have been considered from the perspective of superembeddings in [17, 18, 19, 20]. The Green-Schwarz action for the space-filling membrane was constructed in [19]. In section 2 we give a brief summary of this method applied to the construction of the abelian Born-Infeld action. In section 3 we show how to generalise this action in the presence of a non-abelian worldvolume $SU(N)$ gauge supermultiplet. By assumption this multiplet is described by a 2-form, F, satisfying the standard Bianchi identity $DF = 0$. We then consider actions which involve the $U(1)$ and $SU(N)$ fields in such a way that they combine into a single $U(N)$ field strength. We concentrate on Lagrangians which have the form a single trace of some function of these field strengths. Our method is similar to that used in [21] to construct kappa-symmetric higher derivative terms in brane actions. The essential idea is to construct a closed worldvolume Lagrangian form using the abelian and non-abelian worldvolume fields.

2 Abelian action

In this section we give a brief review of the superembedding construction of the abelian Born-Infeld action for a supermembrane in flat $N = 2$ three dimensional superspace [19].

2.1 Superembedding formalism

We consider a superembedding, $f : \mathcal{M} \longrightarrow \underline{\mathcal{M}}$. For the purposes of this paper the target space, $\underline{\mathcal{M}}$, will always be flat. Our index conventions are as follows; coordinate indices are taken from the middle of the alphabet with capitals for all, Latin for bosonic and Greek for fermionic, $M = (m, \mu)$, tangent space indices are taken in a similar fashion from the beginning of the alphabet so that $A = (a, \alpha)$. The distinguished tangent space bases are related to coordinate bases by means of the supervielbein, $E_M{}^A$, and its inverse $E_A{}^M$. Coordinates are denoted $z^M = (x^m, \theta^\mu)$. We use exactly the same notation for the target space but with all of the indices underlined. Target space forms are written with an underline, e.g. $\underline{H}$. Their pullbacks are written without an underline, $f^*\underline{H} = H$.

The embedding matrix is the derivative of f referred to the preferred tangent frames, thus

$$E_A{}^{\underline{A}} = E_A{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}. \tag{1}$$

This tells us how to pull back target space forms onto the worldvolume,

$$f^* E^{\underline{A}} = E^A E_A{}^{\underline{A}}. \tag{2}$$

The basic embedding condition is

$$E_\alpha{}^{\underline{a}} = 0. \tag{3}$$

Geometrically, this says that at each point on the brane the odd tangent space of the brane is a subspace of the odd tangent space of the target space. In general, this condition gives constraints on the superfields describing the worldvolume theory. For codimension zero, however, it can be enforced without loss of generality as discussed in [19].

The worldvolume multiplet is described by the transverse target space coordinates considered as superfields on the worldvolume. For the space-filling membrane we embed an $N = 1$ superspace into an $N = 2$ superspace of the same bosonic dimension. Thus, in the absence of further constraints, our worldvolume multiplet is an unconstrained spinor superfield. This superfield, being associated with the breaking of one of the supersymmetries, is referred to as the Goldstone superfield.

The dimensions of the coordinates x and θ are -1 and $-\frac{1}{2}$ respectively while the worldvolume superspace derivatives $\mathcal{D}_a$, $\mathcal{D}_\alpha$ have dimensions $+1$ and $+\frac{1}{2}$ respectively. In the superembedding context it is natural to take the bosonic component of the gauge connection A_a to have dimension -1 so that the non-abelian field strength two-form should be defined by $F = dA + \frac{1}{\alpha'} A^2$ (where α' has dimension minus two). The purely even component, F_{ab}, has dimension zero.

2.2 Space-filling membrane

We now specialise to the membrane in flat three-dimensional $N = 2$ superspace [19]. The bosonic indices for the worldvolume and target space may be identified since we are considering a space-filling brane. The fermionic target space indices are written $\underline{\alpha} = \alpha i$ where $i = 1, 2$ since we embed an $N = 1$ superspace into an $N = 2$ superspace. The index α is a real, two-component Majorana spinor index. The internal index i is an $SO(2)$ index.

Background forms

The Neveu-Schwarz 3-form field strength $\underline{H}$ and the Ramond field strengths $\underline{G}_2$ and $\underline{G}_4$ satisfy the Bianchi identities,

$$d\underline{H} = 0, \qquad d\underline{G}_2 = 0, \qquad d\underline{G}_4 = \underline{G}_2\underline{H}. \tag{4}$$

A solution to these equations in flat space is given by forms whose non-zero components are

$$H_{\alpha i\beta jc} = -i(\gamma_c)_{\alpha\beta}(\tau_1)_{ij}, \qquad G_{\alpha i\beta j} = -i\epsilon_{\alpha\beta}\epsilon_{ij}, \qquad G_{\alpha i\beta jcd} = -i(\gamma_{cd})_{\alpha\beta}(\tau_3)_{ij}. \tag{5}$$

The field strengths $\underline{G}_2$ and $\underline{G}_4$ are related to the potentials $\underline{C}_1$ and $\underline{C}_3$ by

$$\underline{G}_2 = d\underline{C}_1, \qquad \underline{G}_4 = d\underline{C}_3 - \underline{C}_1\underline{H}. \tag{6}$$

The components of the Ramond potentials $\underline{C}_1$ and $\underline{C}_3$ depend on the target space coordinates and can be expressed, in a particular gauge, in terms of $\theta^{\alpha 2}$ only. When the brane is embedded into the $N = 2$ target superspace $\theta^{\alpha 2}$ becomes the Goldstone superfield in the static gauge.

Worldvolume Supergeometry

We parametrise the odd-odd part of the embedding matrix as follows

$$E_\alpha{}^{\beta 1} = \delta_\alpha^\beta \quad \text{and} \quad E_\alpha{}^{\beta 2} = h_\alpha{}^\beta. \tag{7}$$

where $h_{\alpha\beta} = k\epsilon_{\alpha\beta} + h_a(\gamma^a)_{\alpha\beta}$ In addition we can set the even-even part of the embedding matrix to be trivial, $E_a{}^{\underline{b}} = \delta_a{}^b$, since the bosonic dimensions of the brane and the target space are the same. We denote the worldvolume superspace derivatives in the embedding basis by

$$\mathcal{D}_A = E_A{}^M \partial_M \tag{8}$$

We can now calculate the worldvolume torsion by pulling back the standard flat target space torsion. As noted previously for the case of space-filling branes, we do not have to introduce a worldvolume connection [19, 17, 18] so that

$$[\mathcal{D}_A, \mathcal{D}_B] = -T_{AB}{}^C \mathcal{D}_C \,. \tag{9}$$

The dimension zero component of the worldvolume torsion is

$$T_{\alpha\beta}{}^a = -i(\gamma^b)_{\alpha\beta} m_b{}^a, \tag{10}$$

where

$$m_{ab} = (1 + k^2 + h^2)\eta_{ab} - 2h_a h_b - 2\epsilon_{abc} k h^c. \tag{11}$$

The other components of $T_{AB}{}^C$ can be found straightforwardly but we shall not need them in this paper.

To describe the worldvolume multiplet we introduce a worldvolume 2-form $\mathcal{F}$ (the modified field strength). This is satisfies the Bianchi identity

$$d\mathcal{F} = -H, \tag{12}$$

where H is the pullback of $\underline{H}$ onto the worldvolume. To get the required worldvolume $N = 1$ Maxwell multiplet we impose the standard $\mathcal{F}$-constraint $\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\alpha b} = 0$. The constraint $\mathcal{F}_{\alpha\beta} = 0$ tells us that we have an $N = 1$ Maxwell multiplet on the brane as well as the Goldstone fermion of the embedding, while the constraint $\mathcal{F}_{\alpha b} = 0$ eliminates one of these spinor superfields in terms of the other. This leaves us with just the degrees of freedom associated with the (off-shell) Maxwell multiplet. The Bianchi identity then gives a formula for $\mathcal{F}_{ab}$ in terms of the degrees of freedom of the embedding. We find that $k = 0$ and $\mathcal{F}_{ab} = \epsilon_{abc}\mathcal{F}^c$, where

$$\mathcal{F}_a = \frac{2h_a}{1+h^2} \tag{13}$$

The $h_\alpha{}^\beta$ field in the embedding matrix is therefore related to the field strength tensor of the Maxwell multiplet in a non-linear fashion.

Action

To construct an action we start with the closed Wess-Zumino 4-form [22], defined on the worldvolume by $W_4 = G_4 + G_2\mathcal{F}$, where $G_2 = f^*\underline{G}_2$ and $G_4 = f^*\underline{G}_4$. By construction we can write the Wess-Zumino form explicitly as $W_4 = dZ_3$ where $Z_3 = C_3 + C_1\mathcal{F}$ and $C_1 = f^*\underline{C}_1$, $C_3 = f^*\underline{C}_3$. The components of Z_3 depend explicitly on the target space coordinates.

Since W_4 is a form of degree one higher than the bosonic dimension of the worldvolume, the fact that it is closed implies it is exact and we can write $W_4 = dK_3$. The components of K_3 do not explicitly depend on the target space coordinates. The Lagrangian form for the abelian action is then

$$L_3^{U(1)} = K_3 - Z_3, \tag{14}$$

and is closed by construction. We can solve $W_4 = dK_3$ for K_3. We find its only non-zero component is given by

$$K_{abc} = \epsilon_{abc}K, \quad \text{where} \quad K = \frac{1-h^2}{1+h^2}. \tag{15}$$

Given the relation (13) between h_a and $\mathcal{F}_a$ we find that K has the standard Born-Infeld form,

$$K = \sqrt{-\det\left(\eta_{ab} + \mathcal{F}_{ab}\right)} = \sqrt{1-\mathcal{F}^2}, \tag{16}$$

where $\mathcal{F}^2 := \mathcal{F}^a\mathcal{F}_a$. The top component of the Lagrangian form is $L^{U(1)}_{abc} = \epsilon_{abc}L^{U(1)}$ where $L^{U(1)} = (K - Z)$. The first term, K, is the Born-Infeld part and the second, Z, is the Wess-Zumino term. We convert into the coordinate basis using the even-even component of the worldvolume vielbein $E_m{}^a$.

Finally, the Green-Schwarz action for the brane is defined by

$$S^{U(1)} = \int d^3x\, (\det E) L^{U(1)}. \tag{17}$$

The superfields in the integrand are evaluated at $\theta = 0$. The closure of $L_3^{U(1)}$ ensures that this action is invariant under general diffeomorphisms of the worldvolume. The odd diffeomorphisms are identified with kappa-symmetry as described in [22].

3 Non-abelian actions

In this section we consider ways of generalising the approach of the previous section in the presence of non-abelian worldvolume fields. We start by considering the general structure of such Lagrangians and then show how to construct non-abelian Born-Infeld Lagrangians.

3.1 General structure of non-abelian actions

We now discuss possible generalisations to the abelian Born-Infeld action. We look for Lagrangians which are functions of a non-abelian field strength, $\hat{F}$, taking values in the Lie algebra of $U(N)$. We take a similar point of view to [1] whereby we look for actions which are invariant under a single kappa-symmetry and regard any other kappa-symmetries to have been gauge-fixed. This is in contrast to [23] where the parameter of the kappa-symmetry is taken to trasform under the adjoint representation of $U(N)$ adjoint. This adjoint kappa-symmetry was shown to be inconsistent after a certain order in the field strengths [24]. Accordingly, we keep the picture of a single brane embedded in a flat target space but introduce an extra worldvolume $SU(N)$ field strength, F and construct Lagrangians which are functions of $\hat{F} = \mathcal{F}\mathbb{1} + F$.

Our main interest in these actions arises from the fact that the effective action of coincident D-branes in type II string theories is some generalisation of the abelian Born-Infeld action which is invariant under two target space supersymmetries and one local worldvolume supersymmetry (kappa-symmetry). As such we shall restrict our attention to actions which take the form of a single trace of some function of the worldvolume fields.

We recall that in three dimensions we can write the pure $\mathcal{F}$ terms of the (super) Born-Infeld Lagrangian as (16),

$$\sqrt{-\det\left(\eta_{ab} + \mathcal{F}_{ab}\right)} = \sqrt{1 - \mathcal{F}_a\mathcal{F}^a} = \sum_{n=0}^{\infty} c_n \mathcal{F}_{a_1}\mathcal{F}^{a_1} \ldots \mathcal{F}_{a_n}\mathcal{F}^{a_n}. \tag{18}$$

This Lagrangian can be defined as the effective Lagrangian given by open strings ending on a D-brane excluding derivative corrections. One ambiguity in passing to the non-abelian case is that one can exchange terms with antisymmetric pairs of covariant derivatives with terms involving commutators of the field strength, using the relation $\alpha' \nabla_{[a}\nabla_{b]}\hat{F}_{cd} \sim [\hat{F}_{ab}, \hat{F}_{cd}]$. (The explicit α' is a consequence of our definition of F in section 2.1.) We use the term derivative corrections to refer to terms with derivatives which cannot be cast into commutator form. The pure $\hat{F}$ terms in any non-abelian generalisation can then be written as

$$\mathrm{Str}\sqrt{\mathbb{1} - \hat{F}_a\hat{F}^a} + \text{ commutator terms} + \text{ derivative corrections}, \tag{19}$$

where the $U(N)$ field strengths are denoted by $\hat{F}_{ab} = \epsilon_{abc}\hat{F}^c$ and Str denotes the symmetrised trace. The symmetrised trace part can be written as

$$\sum_{n=0}^{\infty} c_n \hat{F}^{a_1 R_1} \hat{F}^{R_2}_{a_1} .... \hat{F}^{a_n R_{2n-1}} \hat{F}^{R_{2n}}_{a_n} D_{R_1..R_{2n}}, \tag{20}$$

where the coefficients c_n are the same as those in the abelian case and $D_{R_1....R_{2n}} = \mathrm{Str}(T_{R_1}....T_{R_1})$.

Any terms with commutators, including any pure $\hat{F}$ terms of odd powers, vanish in the abelian limit, while the symmetrised trace part reduces to the usual abelian Lagrangian.

3.2 Superembedding construction of non-abelian actions

To construct actions of the type discussed above, we use a similar method to that used to construct kappa symmetric higher derivative terms in brane actions [21]. We introduce non-abelian fields onto the worldvolume and construct another closed Lagrangian 3-form L_3 out of these fields by specifying its lower-dimensional components and solving $dL_3 = 0$ [25, 26].

The $U(1)$ multiplet is given by a 2-form field strength $\mathcal{F}$ satisfying the modified Bianchi identity $d\mathcal{F} = -H$. We shall assume that the non-abelian fields are given by a worldvolume $SU(N)$ 2-form field strength $F = dA + \frac{1}{\alpha'}A^2$, $F = F^R t_R$, where the t_R are the generators of the Lie algebra of $SU(N)$. The 2-form F satisfies the standard Bianchi identity,

$$DF = 0. \tag{21}$$

We construct the Lagrangian form from the components of this 2-form along with the abelian field $\mathcal{F}^a$ introduced in the previous section. We shall combine these in such a way as to define a Lagrangian whose purely bosonic part is a function of $\hat{F}_{ab} = \mathcal{F}_{ab}\mathbb{1} + F_{ab}$. For this reason we drop any terms with fermions in the action but it is possible to calculate these terms straightforwardly from the equations given below.

In components the Bianchi identity for F (21) reads

$$\nabla_{[A}F_{BC]} + T_{[AB}{}^{D}F_{|D|C]} = 0. \tag{22}$$

where $\nabla_A = \mathcal{D}_A + A_A$ is the $SU(N)$ gauge covariant derivative, with A_A being the components of the gauge potential one form, A.

We can take $F_{\alpha\beta} = 0$ without loss of generality by shifting the bosonic part of the potential. The solution to the Bianchi identity is then

$$F_{\alpha\beta} = 0, \tag{23}$$

$$F_{\alpha b} = m^{-1}{}_b{}^c(\gamma_c\psi)_\alpha, \tag{24}$$

$$F_{ab} = im^{-1}{}_a{}^c m^{-1}{}_b{}^d \epsilon_{cde}(\gamma^e)^{\alpha\beta}\nabla_\alpha\psi_\beta + \psi \text{ terms} \tag{25}$$

where we have used the fact that the dimension zero component of the worldvolume torsion is given by

$$T_{\alpha\beta}{}^c = -im_d{}^c(\gamma^d)_{\alpha\beta} \qquad \text{with} \qquad m_{ab} = (1+h^2)\eta_{ab} - 2h_a h_b. \tag{26}$$

We can use the field $\psi_\alpha = \psi_\alpha^R t_R$ to construct the closed Lagrangian three form L_3 which we require for our action.

The top component of L_3, i.e. L_{abc}, can be written as $L_{abc} = \epsilon_{abc} L$. This defines a kappa-invariant action in the same way as (17),

$$S = \int d^3x \, (\det E) L. \tag{27}$$

A shift of the form $L_3 \longrightarrow L_3 + dX_2$ leaves the action S unchanged and this allows us to set $L_{\alpha\beta\gamma} = 0$. We can also use this freedom to set the antisymmetric part of $(\gamma_b)^{\alpha\beta} L_{\alpha\beta c}$ to zero.

In components $dL_3 = 0$ reads

$$\mathcal{D}_{[A} L_{BCD]} + \tfrac{3}{2} T_{[AB}{}^E L_{|E|CD]} = 0. \tag{28}$$

Using this and the above constraints on L_3 we find that

$$L_{\alpha\beta c} = i m^{-1}{}_c{}^d (\gamma_d)_{\alpha\beta} L_o. \tag{29}$$

The idea is then to choose L_o to be a suitable function of the abelian and non-abelian world-volume fields and use the closure of L_3 (28) to compute the remaining components of L_3 and hence the action (27).

For now we ignore derivative corrections in the action. We therefore include no explicit α's in L_o. Since L_o has dimension -1 and the only negative dimension field on the worldvolume is ψ_α^R, with dimension $-\frac{1}{2}$, we can take L_o to be given by the formula

$$L_o = i \psi_\alpha^R \psi_\beta^S (J_{RS}^o \epsilon^{\alpha\beta} + J_{RS}^a (\gamma_a)^{\alpha\beta}) + \psi\psi\psi\partial\psi .... \tag{30}$$

Here J_{RS}^o,J_{RS}^a are functions of the dimension zero worldvolume fields, $\mathcal{F}_{ab}, F_{ab}$.

The other components of L_3 are then given by (28). We obtain

$$L_{\alpha\beta\gamma} = 0, \tag{31}$$
$$L_{\alpha\beta c} = i m^{-1}{}_c{}^d (\gamma_d)_{\alpha\beta} L_o, \tag{32}$$
$$L_{\alpha bc} = m^{-1}{}_b{}^d m^{-1}{}_c{}^e \epsilon_{dea} (\gamma^a)_\alpha{}^\beta \mathcal{D}_\beta L_o + \psi\psi \text{ terms}, \tag{33}$$
$$L_{abc} = i \epsilon_{abc} (\det m^{-1}) \mathcal{D}_\alpha \mathcal{D}^\alpha L_o + \psi \text{ terms}. \tag{34}$$

We see from the final equation that the Lagrangian L is then determined up to ψ terms by acting with derivatives on L_o. We ignore the ψ terms for now as we are interested in the pure $\hat{F}$ contribution to the Lagrangian. The terms which have only $\mathcal{F}_{ab} = \epsilon_{abc}\mathcal{F}^c$ and $F_{ab} = \epsilon_{abc} F^c$ are those where the two derivatives in (34) each act on one of the ψs in L_o. To calculate the Lagrangian the relevant equations are

$$L = i (\det m^{-1}) \mathcal{D}_\alpha \mathcal{D}^\alpha L_o + \psi \text{ terms}, \tag{35}$$
$$F^a = i (\det m^{-1}) m_b{}^a (\gamma^b)^{\alpha\beta} \nabla_\alpha \psi_\beta + \psi \text{ terms}. \tag{36}$$

We can absorb the factors of $\det m^{-1}$ into L_o and ψ at the expense of generating more ψ terms by defining the quantities

$$\tilde{\psi}_\alpha = (\det m^{-1})\psi_\alpha \tag{37}$$

and

$$\tilde{L}_o = (\det m^{-1})L_o = i\tilde{\psi}^R_\alpha \tilde{\psi}^S_\beta (\tilde{J}^o_{RS}\epsilon^{\alpha\beta} + \tilde{J}^a_{RS}(\gamma_a)^{\alpha\beta}) + \tilde{\psi}\tilde{\psi}\tilde{\psi}\partial\tilde{\psi}... \tag{38}$$

The equations now read

$$L = i\mathcal{D}_\alpha \mathcal{D}^\alpha \tilde{L}_o + \tilde{\psi}... \tag{39}$$

$$F^a m^{-1}{}_a{}^b = i(\gamma^b)^{\alpha\beta}\nabla_\alpha \tilde{\psi}_\beta + \tilde{\psi}... \tag{40}$$

Since $\tilde{L}_o$ is a gauge scalar we can replace $\mathcal{D}$ with ∇ in the above equation (39). Employing (40) and noting that the antisymmetric part of $\nabla_\alpha \tilde{\psi}_\beta$ gives rise to fermion terms we find

$$L = F^{aR}F^{bS}m^{-1}{}_a{}^c m^{-1}{}_b{}^d(\eta_{cd}\tilde{J}^o_{RS} - \epsilon_{cde}\tilde{J}^e_{RS}) + \tilde{\psi}... \tag{41}$$

The second term involves commutators of F^a since the anti-symmetric contraction of the Lorentz indices on the Fs implies antisymmetry of the gauge indices. We call this term L_A. The first term is symmetric in Lorentz and gauge indices and we call this L_S. We shall combine L_S with pure $\mathcal{F}$ part of the abelian Lagrangian, K, to give the symmetrised trace term of the non-abelian Born-Infeld Lagrangian. The second term, L_A, can be used to construct arbitrary commutator terms.

The full kappa-invariant action for the non-abelian brane is

$$S^{U(N)} = NS^{U(1)} + S, \tag{42}$$

each term being separately kappa symmetric.

Expanding these terms we have

$$S^{U(N)} = \int d^3x\ (\det E)\big(N(K-Z) + L_S + L_A + \psi \text{ terms}\big). \tag{43}$$

We now explain how the two terms, L_S and L_A, can be used to construct respectively the symmetrised trace part and arbitrary commutator terms in the non-abelian Born-Infeld Lagrangian.

Symmetrised trace Lagrangian

The first term in (41) is

$$L_S = F^{aR}F^{bS}m^{-1}{}_a{}^c m^{-1}{}_b{}^d \eta_{cd}\tilde{J}^o_{RS} \tag{44}$$

$$= F^{aR}F^{bS}\frac{1}{(1+h^2)^2}\Big(\eta_{ab} + \frac{4h_a h_b}{(1-h^2)^2}\Big)\tilde{J}^o_{RS} \tag{45}$$

$$= F^{aR}F^{bS}\frac{1}{(1+h^2)^2}\Big(\eta_{ab} + \frac{\mathcal{F}_a\mathcal{F}_b}{1-\mathcal{F}^2}\Big)\tilde{J}^o_{RS} \tag{46}$$

$$= F^{aR}F^{bS}\Big(\eta_{ab}(1-\mathcal{F}^2) + \mathcal{F}_a\mathcal{F}_b\Big)A_{RS}. \tag{47}$$

In the last line we have absorbed a factor of $(1-h^2)^{-2}$ by defining $A_{RS} = \tilde{J}^o_{RS}(1-h^2)^{-2}$.

To look for a symmetrised trace solution we require

$$NK + L_S = \mathrm{Str}\sqrt{-\det\left(\eta_{ab}\mathbb{1} + \hat{F}_{ab}\right)}, \tag{48}$$

i.e.

$$N\sqrt{1-\mathcal{F}^2} + \mathrm{Str}\left(F^aF^b\Big(\eta_{ab}(1-\mathcal{F}^2) + \mathcal{F}_a\mathcal{F}_b\Big)\mathbb{A}\right) = \mathrm{Str}\sqrt{\mathbb{1} - (\mathcal{F}_a\mathbb{1} + F_a)(\mathcal{F}^a\mathbb{1} + F^a)}. \tag{49}$$

The first term on the left hand side is the purely abelian contribution, the second being the new non-abelian parts of the Lagrangian. $\mathbb{A}$ is an $N \times N$ matrix which will be a power series in the $SU(N)$ generators t_R. We have

$$\mathbb{A} = A_0\mathbb{1} + A^S t_S + A^{ST} t_S t_T + \ldots \tag{50}$$

where each coefficient function is symmetric. The tensor A_{RS} is then given by

$$A_{RS} = \delta_{RS}A_0 + d_{RST}A^T + d_{RSTU}A^{TU} + \ldots \tag{51}$$

Equation (49) can now be solved for $\mathbb{A}$. We introduce the variables

$$X = \mathcal{F}_a\mathcal{F}^a\mathbb{1}, \qquad Y = \mathcal{F}_aF^a, \qquad Z = F_aF^a. \tag{52}$$

The equation now reads

$$\mathrm{Str}\left(\sqrt{\mathbb{1}-X} + \Big(Z(\mathbb{1}-X) + Y^2\Big)\mathbb{A}\right) = \mathrm{Str}\sqrt{\mathbb{1} - (X + 2Y + Z)}. \tag{53}$$

If we expand out the square root on the right hand side we notice that the trace will kill terms which are linear in Y and have no Z. These terms can be written explicitly as

$$-Y(\mathbb{1}-X)^{-\frac{1}{2}}. \tag{54}$$

If we explicitly remove these terms from the equation we find that $\mathbb{A}$ is given by

$$\sqrt{\mathbb{1}-X} + \Big(Z(\mathbb{1}-X) + Y^2\Big)\mathbb{A} = \sqrt{\mathbb{1} - (X+2Y+Z)} + Y(\mathbb{1}-X)^{-\frac{1}{2}}. \tag{55}$$

We do not have to worry about the ordering of the non-abelian quantities Y and Z because under the symmetric trace operation everything effectively commutes. This equation defines $\mathbb{A}$ as a Taylor expansion in X, Y, Z about $X = 0, Y = 0, Z = 0$,

$$\mathbb{A} = \sum_{l,m,n=0}^{\infty} a_{l,m,n}X^lY^mZ^n. \tag{56}$$

Note that this would not be the case if the relative value of the coefficients in equation (49) were different. The existence of a non-singular solution to this equation shows that this construction can be used to obtain a supersymmetric, kappa-invariant action whose pure $\hat{F}$ terms give the symmetrised trace non-abelian Born-Infeld Lagrangian. The first few terms in the Taylor expansion of $\mathbb{A}$ are

$$\begin{aligned}\mathbb{A} = &-\tfrac{1}{2}\mathbb{1} - \tfrac{3}{4}X - \tfrac{1}{2}Y - \tfrac{1}{8}Z - \tfrac{15}{16}X^2 - \tfrac{5}{4}XY \\ &- \tfrac{5}{8}Y^2 - \tfrac{5}{16}XZ - \tfrac{3}{8}YZ - \tfrac{1}{16}Z^2 + ...\end{aligned} \tag{57}$$

This solution corresponds to the following expression for A_{RS} in equation (47):

$$\begin{aligned}A_{RS} = -\frac{1}{2}\Big\{&\delta_{RS}\left(1 + \frac{3}{2}\mathcal{F}^2 + \frac{15}{8}\mathcal{F}^4 + \ldots\right) \\ &+ d_{RST}\left(\mathcal{F}_a F^{aT} + \frac{5}{2}\mathcal{F}^2\mathcal{F}_a F^{aT} + \ldots\right) \\ &+ d_{RSTU}\left(\frac{1}{4}F_a^T F^{aU} + \frac{5}{4}\mathcal{F}_a F^{aT}\mathcal{F}_b F^{bU} + \frac{5}{8}\mathcal{F}^2 F_a^T F^{aU} + \ldots\right) \\ &+ d_{RSTUV}\left(\frac{3}{4}\mathcal{F}_a F^{aT} F_b^U F^{bV} + \ldots\right) \\ &+ d_{RSTUVW}\left(\frac{1}{8}F_a^T F^{aU} F_b^V F^{bW} + \ldots\right) + \ldots\Big\}\end{aligned} \tag{58}$$

where $d_{RST} = \mathrm{Str}(t_R t_S t_T)$ etc. are the $SU(N)$ d-symbols.

Commutator terms

To obtain commutator terms we use the second term in equation (41),

$$L_A = -F^{aR}F^{bS}m^{-1}{}_a{}^c m^{-1}{}_b{}^d \epsilon_{cde}\tilde{J}^e_{RS}. \tag{59}$$

If we define A^a_{RS} by $\tilde{J}^a_{RS} = (\det m)m^{-1}{}_b{}^a A^b_{RS}$ then we find

$$L_A = -F^{aR}F^{bS}\epsilon_{abc}A^c_{RS}. \tag{60}$$

We can now choose A^c_{RS} to be any function of the $U(N)$ field strength $\hat{F}^a = \mathcal{F}^a + F^a\mathbb{1}$ so as to incorporate any desired commutator terms in the Lagrangian. For example at order $\hat{F}^3$ in the Lagrangian we can have

$$A^c_{RS} = -F^{cT}f_{RST}, \tag{61}$$

which gives a term in the Lagrangian of the form,

$$\mathrm{Tr}F^aF^bF^c\epsilon_{abc} = \mathrm{Tr}\hat{F}^a\hat{F}^b\hat{F}^c\epsilon_{abc}. \tag{62}$$

At order $\hat{F}^4$ in the Lagrangian we can have

$$A^c_{RS} = -\epsilon^{cde}F^U_d F^V_e f_{RST}f^T{}_{UV}, \tag{63}$$

which gives a term in the Lagrangian of the following type,

$$\mathrm{Tr}[F^a, F^b][F_a, F_b] = \mathrm{Tr}[\hat{F}^a, \hat{F}^b][\hat{F}_a, \hat{F}_b]. \tag{64}$$

Full action

In summary the full action we have constructed is as follows:

$$S^{U(N)} = \int d^3x (\det E)(NL^{U(1)} + L_S + L_A + \psi \text{ terms}). \tag{65}$$

We have shown that L_S can be chosen such that the purely bosonic parts of the first and second terms combine to give the symmetrised trace Lagrangian and that L_A can give any choice of commutator terms for $\hat{F}$ including those with odd powers of $\hat{F}$. We therefore have

$$S^{U(N)} = \int d^3x (\det E)\left(\mathrm{Str}\sqrt{-\det(\eta_{ab}\mathbb{1} + \hat{F}_{ab})} - NZ + \text{ commutator terms } + \psi \text{ terms}\right), \tag{66}$$

where the superfield integrand is evaluated at $\theta = 0$ as usual. The kappa-symmetry of this action is guaranteed by the fact that it is constructed from the sum of two closed forms (the abelian one and the new one).

4 Conclusions

In this paper we have shown how to construct a manifestly kappa-symmetric non-abelian action for the space-filling brane in three dimensions. The invariance under a single kappa-symmetry is equivalent to the local worldvolume supersymmetry of the system in the superembedding picture.

The basic idea is to extend the abelian action by adding a new invariant involving an $SU(N)$ worldvolume gauge supermultiplet in such a way that the resulting action is a single trace over a function of the $U(N)$ field strength. The $SU(N)$ field strength multiplet is described by a spinorial superfield ψ of dimension $-1/2$ which allows one to construct a closed Lagrangian 3-form from a scalar superfield, L_o, of dimension -1. Note that, in three dimensions, the Wess-Zumino term is the same as in the abelian case. This means that the $SU(N)$ part of the non-abelian Born-Infeld action comes entirely from L_o and is not determined by the WZ term. In higher dimensions the latter will have non-abelian contributions and this will necessitate a slightly modified approach to the problem.

We note that kappa symmetry (and target space supersymmetry) does not determine the non-abelian action uniquely, at least in the model under consideration. The symmetrised trace contribution seems to be fixed but it is possible to add many different commutator terms as we have seen. For each of these actions our method guarantees supersymmetry and the fermion contributions could be worked out straightforwardly. However, we have not worked these out in detail. It might be that the structure of these terms could imply further restrictions on the form of the action.

If we include derivative terms in the functions A_{RS} of equation (47) and A^a_{RS} of equation (60) we can produce derivative corrections in the Lagrangian. Such terms would be invariant

independently of the action constructed above in equation (66)and would in some sense be analogous to those found in [21] using a similar procedure.

It should be easy to generalise the discussion in this paper to a 2-brane embedded in a curved $N = 2, D = 3$ supergravity background. This would modify the background curvatures but would not substantially alter the procedure for obtaining the non-abelian Born-Infeld action, althought there would clearly be couplings to the supergravity fields. We could also look for terms involving higher derivatives of the background curvature [27]. However, the dimension of spacetime is too low for there to be background curvature corrections to the Wess-Zumino term of the type found in [28, 29].

It should also be possible to dimensionally reduce the action (66) in different ways. For example, reducing the worldvolume to a worldline would give an action describing a D-particle moving in three spacetime dimensions while double dimensional reduction followed by a reduction of the worldvolume to a worldline would allow a comparison with the results of [1].

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