A NOTE ON GLOBAL p^{th} POWERS OF RIGID ANALYTIC FUNCTIONS

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Abstract. Assume that K is a perfect field of characteristic $p > 0$ that is complete with respect to an ultrametric valuation, and let X be a rigid analytic variety over K. Suppose that X is smooth and connected with respect to its Grothendieck topology. Let f be a (global) function on X the differential of which vanishes locally at some point of X; then f is the p^{th} power of a (global) function.

1. Introduction

This note is an answer to a question that was posed to me by David Goss, whom I thank for several helpful comments. Assume that K is a perfect field of characteristic $p > 0$ that is complete with respect to an ultrametric valuation, and let X be a rigid analytic variety over K. Suppose that X is smooth and connected with respect to its Grothendieck topology. Is a (global) function on X the differential of which vanishes in a neighborhood of a point the p^{th} power of a (global) function? Theorem 2.6, below, is the positive answer to this question. It is a consequence of the basic sheaf theory and of the fact that affinoid algebras are excellent rings. This note provides a convenient reference in the literature for this useful means of detecting global p^{th} powers.

In Section 3, we assume that K is algebraically closed, and use Lemma 3.1 to give a direct treatment of some special cases of Theorem 2.6. In Examples 3.2 and 3.3, we treat the cases of the closed unit polydisc and an annulus, respectively. In Example 3.4, we use the affinoid Mittag-Leffler Theorem to give a direct treatment of the case of a Zariski-connected, open K-affinoid subvariety of the affine line, as well as of the case of a connected affinoid subset of the projective line over K.

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2. Global p^{th} powers

Let K be a field that is complete with respect to an ultrametric absolute value $|\cdot|\colon K \to \mathbb{R}_+$. A *Tate ring* $T_n(K)$ *over* K is a ring of power series

$$T_n = K\langle \xi_1, \ldots, \xi_n \rangle = \left\{ \sum a_\nu \xi^\nu : a_\nu \in K \text{ and } \lim_{|\nu|\to\infty} a_\nu = 0 \right\}.$$

The ring T_n is a Noetherian UFD ([1], Theorem 5.2.6.1,) and it satisfies a Nullstellensatz: for every $\mathfrak{m} \in \operatorname{Max} T_n$, the quotient field $T_n/\mathfrak{m}$ is a finite extension of K ([1], Theorem 7.1.1.1.) Any quotient of a Tate ring is called a K*-affinoid algebra* (see [1] or [4].)

We assume throughout that K is a perfect field of positive characteristic p that is complete in an ultrametric absolute value $|\cdot|\colon K \to \mathbb{R}_+$. The main point is contained in Lemma 2.1, below, which under mild conditions shows that, for elements of K-affinoid algebras, the property of being a p^{th} power is quite local.

The proof of Lemma 2.1 relies on properties of G-rings. A Noetherian ring A is said to be a *G-ring* if for every prime ideal $\mathfrak{p}$ of A, the natural homomorphism $A_\mathfrak{p} \to \widehat{A}_\mathfrak{p}$ is regular (where $\widehat{A}_\mathfrak{p}$ is the maximal-adic completion of the local ring $A_\mathfrak{p}$.) By [10], Theorem 8.8, a map from a Noetherian local ring to its completion is faithfully flat, hence by [10], Theorem 32.2(i), the ring $A_\mathfrak{p}$ is reduced if, and only if, $\widehat{A}_\mathfrak{p}$ is reduced. This fact will be used in the proof of the following lemma.

A K-affinoid algebra A is an excellent ring by [2], Theorem 3.3.3, hence, in particular, it is a G-ring.

Lemma 2.1. *Suppose that A is a regular, excellent ring that is an integral domain. Let $f \in A$ and suppose that there is a maximal ideal $\mathfrak{m}$ of A and an element $g \in \widehat{A}_\mathfrak{m}$ such that $g^p = f$. Then $g \in A$.*

Proof. Note that since A is an integral domain, we may regard A as a subring of the localization $A_\mathfrak{m}$, and since $A_\mathfrak{m} \to \widehat{A}_\mathfrak{m}$ is faithfully flat, we may regard $A_\mathfrak{m}$ as a subring of $\widehat{A}_\mathfrak{m}$. Since A is a regular ring, it is normal ([10], Theorem 19.4;) hence, it suffices to show that $g \in A_\mathfrak{m}$.

Suppose $g \in \widehat{A}_\mathfrak{m} \setminus A_\mathfrak{m}$, and put $R := A_\mathfrak{m}[g]$. Since A is a regular ring, $A_\mathfrak{m}$ is a UFD ([10], Theorem 20.3,) so the polynomial $X^p - f = X^p - g^p$ is irreducible in $A_\mathfrak{m}[X]$, and

$$R = A_\mathfrak{m}[X]/(X^p - g^p).$$

Since $\widehat{A}_\mathfrak{m}$ is reduced ([10], Theorem 32.2(i)) and $R \subset \widehat{A}_\mathfrak{m}$, also R is reduced.

Put $\mathfrak{M} := \mathfrak{m} \cdot R$, and let $\widehat{R}$ denote the $\mathfrak{M}$-adic completion of R, so

$$\widehat{R} = \widehat{A}_{\mathfrak{m}}[X]/(X^p - g^p) \cdot \widehat{A}_{\mathfrak{m}}[X].$$

Hence $X - g$ is a non-zero nilpotent element of $\widehat{R}$. Now, R is a finite extension of the local ring $A_{\mathfrak{m}}$, so by [10], Theorem 8.15, $\widehat{R}$ is the direct sum of finitely many maximal-adic completions of R. Thus, some maximal-adic completion of R is not reduced. Since R is reduced, it follows from [10], Theorem 32.2(i) that R cannot be a G-ring.

For any G-ring A, the polynomial ring $A[X]$ is also a G-ring ([9], Theorem 77.) Moreover, it follows from the definition that any quotient or localization of a G-ring is again a G-ring. Therefore, R is also a G-ring, a contradiction. □

Remark 2.2. (i) A ring R similar to the one in the proof above is found in [11], Section A.1, Examples 6 and 3.2, and in [7], Example 4.2.4.

(ii) By [6], Theorem 3.3, any affinoid algebra is an excellent ring; thus Lemma 2.1 (as well as Proposition 2.4, below) holds for any complete, valued field of characteristic $p > 0$.

(iii) Consider the class of K-algebras A that are quotients of a ring $S_{m,n}(E, K)$ of separated power series (see [7], Definition 2.1.1 and [7], Remark 2.1.8.) Under any of the hypotheses of [7], Proposition 4.2.5 (e.g., when $[K : K^p] < \infty$ and E is a complete DVR contained in the valuation ring of K such that E is a finite extension of E^p,) the quasi-affinoid algebra A is an excellent ring. Thus the corresponding analogue of Lemma 2.1 holds. This leads to a stronger form of [8], Theorem 3.2.

The basic properties of the sheaf of functions on a rigid analytic variety over K yield Proposition 2.4, below, which generalizes Lemma 2.1. For the definition of rigid analytic variety, we refer the reader to [1], Definition 9.3.1.4. Among these are the affinoid varieties $X = \operatorname{Sp} A$, A a K-affinoid algebra, which may be glued together as in [1], Section 9.3.2 to form rigid analytic varieties. The simplest admissible open sets of the affinoid variety $X = \operatorname{Sp} A$ are the rational subdomains (see [1], Definition 7.2.3.5 and [1], Corollary 7.3.5.3.) These are sets of the form

$$X\left(\frac{f}{g}\right) := \{x \in X : |f_i(x)| \le |g(x)|,\ 1 \le i \le r\},$$

where $g, f_1, \dots, f_r \in A$ have no common zero. Consider the affinoid algebra

$$B := A\left\langle \frac{f_1}{g}, \cdots, \frac{f_r}{g} \right\rangle = A\langle \eta_1, \dots, \eta_r\rangle/(f_1 - \eta_1 g, \dots, f_r - \eta_r g);$$

by [1], Section 7.2.3, $X = \operatorname{Sp} B$. The simplest admissible open covers of X are the finite covers by rational subdomains; these are called affinoid covers. By Lemma 2.3, below, the covering of an affinoid variety by its Zariski-connected components is an affinoid cover. This permits the proof of Proposition 2.4, below, to be reduced to Lemma 2.1.

Lemma 2.3. *Let $X = \operatorname{Sp} A$ be an affinoid variety, and let Z be a Zariski-connected component of X. Then Z is a rational subdomain of X. It follows that the covering of X by its Zariski-connected components is an affinoid cover.*

Proof. Since Z is Zariski-closed, there are $f_1, \dots, f_r \in A$ such that

$$Z = \{x \in X : f_1(x) = \cdots = f_r(x) = 0\}.$$

Since Z is Zariski-open, $Z' := X \setminus Z$ is a closed affinoid set, and by [1], Lemma 7.3.4.7,

$$\alpha(x) := \max_{1 \le i \le r} |f_i(x)|$$

assumes its minimum on Z'. Hence $0 < \min_{x \in Z'} \alpha(x)$. Let $\varepsilon \in \sqrt{|K \setminus \{0\}|}$ with

$$0 < \varepsilon < \min_{x \in Z'} \alpha(x);$$

then

$$Z = X\left(\frac{f_1}{\varepsilon}, \dots, \frac{f_r}{\varepsilon}\right)$$

is a rational subdomain of X, as desired. □

Proposition 2.4. *Let X be a smooth rigid analytic variety over K that is connected with respect to its Grothendieck topology. Let $f \in \mathcal{O}_X(X)$ and suppose that there is a point $x \in X$ and an element $g \in \widehat{\mathcal{O}}_{X,x}$ such that $g^p = f$. Then there is an element $g \in \mathcal{O}_X(X)$ such that $g^p = f$.*

Proof. Let $\{X_i\}_{i \in I}$ be an admissible affinoid covering of X. Passing to a refinement if necessary, by Lemma 2.3, we may assume that each affinoid variety X_i is Zariski-connected. Thus since X is smooth, each K-affinoid algebra $\mathcal{O}_X(X_i)$ is a regular ring and an integral domain (by [10], Theorem 14.3.) Put

$$\begin{aligned} I_1 &:= \{i \in I : \text{for some } h \in \mathcal{O}_X(X_i), h^p = f|_{X_i}\}, \text{ and} \\ I_2 &:= I \setminus I_1. \end{aligned}$$

Suppose that $i \in I_1$, $j \in I_2$ and that there is a point $y \in X_i \cap X_j$. Since $i \in I_1$, there is an $h \in \mathcal{O}_X(X_i)$ such that $h^p = f|_{X_i}$; whence there is an $h \in \widehat{\mathcal{O}}_{X_j,y}$ such that $h^p = f$. Since $y \in X_j$, by Lemma 2.1, there is an $h \in \mathcal{O}_X(X_j)$ such that $h^p = f$, contradicting $j \in I_2$. Since X

is connected, it follows that $I_2 = \varnothing$. Since $\mathcal{O}_X$ is a sheaf, there is an element $g \in \mathcal{O}_X(X)$ with $g^p = f$. □

As in [4], Theorem 3.6.1, to each K-affinoid algebra A, there is canonically associated a universal finite differential module $\Omega^f_{A/K}$. (Note that since we have assumed that K is perfect, $\Omega^f_{A/K}$ coincides with the module $\Omega_{A/K}$ of Kähler differentials of A over K, defined as in [10], Section 25. Indeed to see this, by the universal properties defining these two modules, it suffices to show that $\Omega_{A/K}$ is a finite A-module. Since the derivation d vanishes on A^p, by [10], Theorem 25.1, it follows that $\Omega_{A/K} = \Omega_{A/A^p}$. Since K is perfect, A is finite over A^p, thus, $\Omega_{A/A^p} = \Omega_{A/K}$ is a finite A-module, as desired.)

From $\Omega^f_{A/K}$, one obtains the sheaf $\Omega_{Y/K}$ on the affinoid variety $Y = \operatorname{Sp} A$ as in [4], Definition 4.2.1. Let U be a rational subdomain of Y; then $\Omega^f_{\mathcal{O}(U)/K}$ is canonically isomorphic to $\Omega_{Y/K}(U)$ (see [4], Example 4.4.1.) Since each rigid analytic variety X has an admissible affinoid covering, this yields the gluing data for the sheaf $\Omega_{X/K}$ of differentials on X over K.

By [2], Theorem 2.1.4, to each ring $S := \mathcal{O}_{X,x}$ there is canonically associated a universal finite differential module $\Omega^f_{S/K}$. By [2], Theorem 2.4.4, the stalk $\Omega_{X/K,x}$ is canonically isomorphic to $\Omega^f_{S/K}$.

Now suppose that the rigid analytic variety X is smooth, let $x \in X$ and consider the stalk $\mathcal{O}_{X,x}$. By the Nullstellensatz ([1], Proposition 7.1.1.1,) the residue field of the local ring $\mathcal{O}_{X,x}$ is a finite field extension K_1 of K, and by [5], Theorem II.5.6, the stalk $\mathcal{O}_{X,x}$ is isomorphic to a ring $K_1\{\xi_1, \dots, \xi_n\}$ of convergent power series over K_1.

Since K is perfect, K_1 is a separable algebraic extension of K, so $\Omega^f_{K_1\{\xi\}/K} = \Omega^f_{K_1\{\xi\}/K_1}$. By [2], Paragraph 2.2.5, we have:

$$d\colon K_1\{\xi\} \to \Omega^f_{K_1\{\xi\}/K_1} = \bigoplus_i K_1\{\xi\} d\xi_i \colon f \mapsto \sum_i \frac{\partial f}{\partial \xi_i} d\xi_i. \tag{2.5}$$

Let $f \in \mathcal{O}_X(X)$, let $x \in X$, and let $d_x\colon \mathcal{O}_{X,x} \to \Omega_{X/K,x}$ be the corresponding derivation. Suppose that $d_x(f) = 0$. Since K is perfect, also K_1 is perfect, hence from Equation (2.5), it follows that there is an element $g \in \mathcal{O}_{X,x}$ such that $g^p = f$.

From the above and from Proposition 2.4, we now deduce the following.

Theorem 2.6. *Let X be a smooth rigid analytic variety over K that is connected with respect to its Grothendieck topology. Let $f \in \mathcal{O}_X(X)$, let $x \in X$, and let $d_x\colon \mathcal{O}_{X,x} \to \Omega_{X/K,x}$ be the corresponding derivation.*

Then $d_x(f) = 0$ if, and only if, there is an element $g \in \mathcal{O}_X(X)$ such that $g^p = f$.

3. Examples

In Examples 3.2, 3.3 and 3.4, below, we give direct treatments of some special cases of Theorem 2.6. The notion of p-basis (see [10], Section 26) plays a key role in the computations. Let $F \subset L$ be fields of characteristic $p > 0$. A set $\xi_1, \ldots, \xi_n \in L$ of distinct elements is said to be *p-independent over F* if, and only if, the set

$$\Gamma := \{\xi_1^{\nu_1} \cdots \xi_n^{\nu_n} : 0 \leq \nu_i < p,\ 1 \leq i \leq n\}$$

of *p-monomials in ξ* is linearly independent over $L^p(F)$. Let $B \subset L$. If every finite subset of B is p-independent over F, then we say that B is *p-independent over F*. If $B \subset L$ is p-independent over F and $L = L^p(F, B)$, then we call B a *p-basis of L over F*.

A *differential basis* of L over F is a subset B of L with the property that $\{dx : x \in B\}$ forms a basis of the L-vector space $\Omega_{L/F}$, the module of differentials of L over F. Lemma 3.1, below, is a direct consequence of [10], Theorem 26.5: in characteristic $p > 0$, the notions of p-basis and of differential basis coincide.

With the above in mind, suppose that $B \subset L$ is a p-basis of L over L^p, and consider the derivation $d\colon L \to \Omega_{L/L^p}$. A straightforward computation shows that

$$\{dx : x \neq 1 \text{ is a } p\text{−monomial in } B\}$$

is a basis of Ω_{L/L^p}, considered as a vector space over L^p.

Lemma 3.1. *Let $R \subset S$ be integral domains of characteristic $p > 0$, and let $Q(S)$ be the field of fractions of S. Suppose that there is a set $B \subset R$ such that the p-monomials in B generate the R^p-module R and $B \subset Q(S)$ is a p-basis of $Q(S)$ over $Q(S^p)$, then*

$$R^p = S^p \cap R = \ker d \cap R,$$

where $d\colon S \to \Omega_{S/S^p}$ is the derivation from S to the module of differentials of S over S^p.

Proof. The inclusions $R^p \subset S^p \cap R \subset \ker d \cap R$ are clear; hence, it suffices to show that $\ker d \cap R \subset R^p$. Let $f \in R$; then there are $\xi_1, \ldots, \xi_n \in B$ and elements $f_\nu \in R$ for which

$$f = \sum_{\vec{0} \leq \nu < \vec{p}} f_\nu^p \xi^\nu.$$

Thus,

$$df = \sum_{\vec{0} \neq \nu < \vec{p}} f_\nu^p d(\xi^\nu).$$

Since the $d(\xi^\nu)$ are linearly independent over $Q(S^p)$, if $df = 0$ then $f = f_0^p \in R^p$, as desired. $\square$

In the following, we assume that K is algebraically closed.

Example 3.2. Consider the Tate ring $T_n = K\langle \xi_1, \dots, \xi_n \rangle$, and let $x \in \operatorname{Max} T_n$. By the Nullstellensatz, the x-adic completion of T_n is isomorphic to $K[\![\xi_1, \dots, \xi_n]\!]$. For each $f \in K[\![\xi]\!]$, there are uniquely determined $f_\nu \in K[\![\xi]\!]$ such that

$$f(\xi) = \sum_{\vec{0} \leq \nu < \vec{p}} f_\nu(\xi^p)\xi^\nu.$$

Note that if f belongs to T_n, the corresponding f_ν also belong to T_n. From the above equation, it follows that $\{\xi_1, \dots, \xi_n\}$ is a p-basis of $Q(T_n)$ over $Q((T_n)^p)$ as well as of $Q(K[\![\xi]\!])$ over $Q((K[\![\xi]\!])^p)$. Now apply Lemma 3.1.

Example 3.3. Let $a, \varepsilon, \delta \in K$ with $|a| \leq 1$ and $|\varepsilon| \leq |\delta| \leq 1$. Consider the K-annulus

$$\begin{gathered} X := \{x \in K : |\varepsilon| \leq |x - a| \leq |\delta|\} = \operatorname{Sp} A, \\ A := K\left\langle \xi, \tfrac{\xi - a}{\delta}, \tfrac{\varepsilon}{\xi - a} \right\rangle. \end{gathered}$$

Claim. Let $f \in A$ and $\alpha \in X$; then there are $f_0, \dots, f_{p-1} \in A^p$ such that

$$f = \sum_{i=0}^{p-1} f_i \cdot (\xi - \alpha)^i.$$

Proof. Indeed, we may write

$$\begin{aligned} f &= \textstyle\sum_{\nu \geq 0} c_\nu \left(\frac{\xi - a}{\delta}\right)^\nu + \sum_{\nu > 0} e_\nu \left(\frac{\varepsilon}{\xi - a}\right)^\nu \\ &= \sum_{i=0}^{p-1} g_i \left(\left(\tfrac{\xi - a}{\delta}\right)^p\right) \cdot \left(\tfrac{\xi - a}{\delta}\right)^i + \sum_{i=0}^{p-1} h_i \left(\left(\tfrac{\varepsilon}{\xi - 1}\right)^p\right) \cdot \left(\tfrac{\varepsilon}{\xi - a}\right)^i \end{aligned}$$

for some $g_i, h_i \in T_1$. Now, observe that $g_i \left(\left(\frac{\xi - a}{\delta}\right)^p\right), h_i \left(\left(\frac{\varepsilon}{\xi - 1}\right)^p\right) \in A^p$, the $\left(\frac{\xi - a}{\delta}\right)^i$ are polynomials, and

$$\frac{\varepsilon}{\xi - a} = \varepsilon \cdot \frac{\xi^{p-1} + a\xi^{p-2} + \cdots + a^{p-1}}{(\xi - a)^p},$$

which establishes the claim. $\square$

Thus the p-monomials in $\{\xi-\alpha\}$ generate the $Q(A^p)$-module $Q(A)$. Put $\mathfrak{m} := (\xi-\alpha)A$ and let $\widehat{A}_{\mathfrak{m}}$ be the $\mathfrak{m}$-adic completion of $A_{\mathfrak{m}}$; then $\widehat{A}_{\mathfrak{m}} = K[\![\xi-\alpha]\!]$. Clearly, $\{\xi-\alpha\}$ is a p-basis of $Q(\widehat{A}_{\mathfrak{m}})$ over $Q((\widehat{A}_{\mathfrak{m}})^p)$. Let $d_{\mathfrak{m}}$ be the derivation from $\widehat{A}_{\mathfrak{m}}$ to $\Omega_{\widehat{A}_{\mathfrak{m}}/(\widehat{A}_{\mathfrak{m}})^p}$. Applying Lemma 3.1, we see that for any $f \in A$, $d_{\mathfrak{m}}(f) = 0$ if, and only if, $f \in A^p$.

Example 3.4. For $a \in K$ and $r \in |K|$, $r > 0$, define

$$\begin{aligned} B^-(a,r) &:= \{x \in K : |x-a| < r\} \text{ and} \\ B^+(a,r) &:= \{x \in K : |x-a| \le r\}, \end{aligned}$$

the "open" and "closed" discs, respectively, of radius r about a.

Let $X = \operatorname{Sp} A$ be a non-empty, Zariski-connected, open affinoid subvariety of the unit disc $\operatorname{Sp} T_1(K) \simeq B^+(0,1)$ in the affine line over K. By [1], Theorem 9.7.2.2, X is a *standard set;* i.e., a set of the form

$$X = B^+(a_0, r_0) \setminus \bigcup_{i=1}^{n} B^-(a_i, r_i),$$

where $a_0, \ldots, a_n \in B^+(0,1)$, $r_0, \ldots, r_n \in |K|$, $0 < r_i \le 1$, and the discs $B^-(a_i, r_i)$, $1 \le i \le n$, are pairwise disjoint and are each contained in $B^+(a_0, r_0)$.

Fix $\varepsilon_i \in K$ with $|\varepsilon_i| = r_i$, $0 \le i \le n$, and consider the affinoid algebra

$$A := K\left\langle \xi, \frac{\xi - a_0}{\varepsilon_0}, \frac{\varepsilon_1}{\xi - a_1}, \ldots, \frac{\varepsilon_n}{\xi - a_n} \right\rangle.$$

By [1], Section 7.2.3, $X = \operatorname{Sp} A$.

Claim. Let $f \in A$ and $\alpha \in X$; then there are $f_0, \ldots, f_{p-1} \in A^p$ such that

$$f = \sum_{i=0}^{p-1} f_i \cdot (\xi - \alpha)^i.$$

Proof. By the affinoid Mittag-Leffler Theorem of [4], Proposition 2.2.6, there are $f_0, \ldots, f_n \in T_1$ such that

$$f = f_0\left(\frac{\xi - a_0}{\varepsilon_0}\right) + \sum_{i=1}^{n} f_i\left(\frac{\varepsilon_i}{\xi - a_i}\right).$$

To complete the proof of the claim, proceed exactly as in Example 3.3. □

Continuing as in Example 3.3, the claim shows that the p-monomials in $\{\xi-\alpha\}$ generate the $Q(A^p)$-module $Q(A)$. Put $\mathfrak{m} := (\xi-\alpha)A$ and let $\widehat{A}_{\mathfrak{m}}$ be the $\mathfrak{m}$-adic completion of $A_{\mathfrak{m}}$; then $\widehat{A}_{\mathfrak{m}} = K[\![\xi-\alpha]\!]$. Clearly,

$\{\xi - \alpha\}$ is a p-basis of $Q(\widehat{A}_{\mathfrak{m}})$ over $Q((\widehat{A}_{\mathfrak{m}})^p)$. Let $d_{\mathfrak{m}}$ be the derivation from $\widehat{A}_{\mathfrak{m}}$ to $\Omega_{\widehat{A}_{\mathfrak{m}}/(\widehat{A}_{\mathfrak{m}})^p}$. Applying Lemma 3.1, we see that for any $f \in A$, $d_{\mathfrak{m}}(f) = 0$ if, and only if, $f \in A^p$.

A *connected affinoid subset* F *of the projective line* $\mathbb{P}$ *over* K is the complement of a finite union of open discs (see [4], Section 2.1.) By [4], Example 3.3.5, F may be identified with a standard set, which yields Theorem 2.6 for the case $X = F$.

Remark 3.5. Example 3.4 employs the affinoid Mittag-Leffler Theorem of [4], Proposition 2.2.6, the proof of which relies on the assumption that the coefficient field K is algebraically closed. The extension of that theorem to the case of more general coefficient fields will be treated in a forthcoming paper. Closely related results can be found in [3], Section 3.

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