

Credit Spreads, Optimal Capital Structure, and Implied Volatility with Endogenous Default and Jump Risk*

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Abstract

We propose a two-sided jump model for credit risk by extending the Leland-Toft endogenous default model based on the geometric Brownian motion. The model shows that jump risk and endogenous default can have significant impacts on credit spreads, optimal capital structure, and implied volatility of equity options: (1) The jump and endogenous default can produce a variety of non-zero credit spreads, including upward, humped, and downward shapes; interesting enough, the model can even produce, consistent with empirical findings, upward credit spreads for speculative grade bonds. (2) The jump risk leads to much lower optimal debt/equity ratio; in fact, with jump risk, highly risky firms tend to have very little debt. (3) The two-sided jumps lead to a variety of shapes for the implied volatility of equity options, even for long maturity options; and although in general credit spreads and implied volatility tend to move in the same direction under exogenous default models, but this may not be true in presence of endogenous default and jumps. In terms of mathematical contribution, we give a proof of a version of the “smooth fitting” principle for the jump model, justifying a conjecture first suggested by Leland and Toft under the Brownian model.

1 Introduction

Of great interest in both corporate finance and asset pricing is credit risk due to the possibility of default. In corporate finance the optimal capital structure for a firm may be selected by considering the trade-off between tax credits from coupon payments to debtholders and potential

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financial costs related to default. Credit risk also leads to yield spreads between defaultable and risk-free bonds. Furthermore, credit risk may affect firms' equity values, which in turn will contribute to implied volatility smiles in equity options.

There are basically two approaches to model credit risk, the structural approach and the reduced form approach. Reduced-form models aim at providing a simple framework to fit a variety of credit spreads by abstracting from the firm-value process and postulating default as a single jump time¹. Starting from Black and Scholes (1973) and Merton (1974), the structural approach aims at providing an intuitive understanding of credit risk by specifying a firm value process and modeling equity and defaultable bonds as contingent claims on the firm value. An important class of the structural models are first passage time models, which specify the default as the first time the firm value falls below a barrier level. Depending on whether the barrier is a decision variable or not, the default can be classified as endogenous or exogenous. For first-passage time models with exogenous default, see, e.g., Black and Cox (1976), Longstaff and Schwartz (1995), and Collin-Dufresne and Goldstein (2001); and for endogenous default models, see, e.g. Leland (1994), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), and Ju and Ou-Yang (2005). There are also links between the two approaches, if one incorporates jumps or different information sets into structure models; see Duffie and Lando (2001) and Jarrow and Protter (2004).

Before presenting the main contribution of the current paper, we shall give a brief review of empirical facts most relevant to our study. For further background on credit risk, see Bielecki and Rutkowski (2002), Das (1995), Duffie and Singleton (2003), Kijima (2002), Lando (2004), Schönbucher (2003).

1.1 Related Empirical Facts

We try to build a model to incorporate some stylized facts from three areas, credit spreads, optimal capital structure, and implied volatility. First, for credit spreads, related stylized facts are: (1) It is well recognized that credit spreads do not converge to zero even for very short maturity bonds, and there are many models addressing this issue². (2) The credit spreads can have a variety of shapes, including upward, humped, and downward shapes; as the firm's finan-

¹For reduced-form models, see, for example, Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), Duffie and Singleton (1999), Collin-Dufresne et al. (2004), Madan and Unal (1998).

²For instance, Duffie and Lindo (2001) and Huang and Huang (2003) suggest that incomplete accounting information or liquidity may lead to non-zero credit spreads. Leland (2004) supports the explanation of jumps by pointing out that including jumps “may solve the underestimation of both default probabilities and yield spreads.”

cial situation deteriorates, the credit spreads tend to change from upward shapes, to humped shapes, and may even to downward shapes when facing severe financial distresses; see, for example, Jones et al. (1984), Sarig and Warga (1989), He et al. (2000). (3) For speculative grade bonds, in addition to humped and downward shapes, credit spreads can even be upwards; see Helwege and Turner (1999), He et al. (2000). (4) Credit spreads tend to be negatively correlated with risk-free rates (Longstaff and Schwartz, 1995, Duffee, 1998), and credit spreads tend to be mean reverting (Schmid, 2004, pp. 180-182). (5) A comprehensive empirical study in Eom et al. (2004) suggests that the Leland-Toft model overpredict credit spreads for long maturity bonds and underpredict credit spreads for short maturity bonds (due to the problem of non-zero credit spreads). All of the above mentioned empirical facts related to credit spreads will be incorporated in our model.

Second, we consider empirical facts related to capital structure, which is one of the most important areas in corporate finance, going back to the celebrated MM theorem in Modigliani and Miller (1958). In particular, many factors may affect the capital structure, such as taxes, bankruptcy costs, agency costs and conflicts between debtholders, equity holders, and managers, asymmetric information, corporate takeover and corporate control, and interactions between investment and production decisions³.

It is too ambitious to try to build a single model to address all these issues. Instead, in this paper we shall focus on a neoclassical view investigating the optimal capital structure as a trade-off between taxes and bankruptcy costs. Regarding this trade-off, empirical evidences seem to suggest that high volatility (Bradley et al., 1984, Friend and Lang, 1988, Kim and Sorensen, 1986) and low recovery values upon default (Bradley et al., 1984, Long and Malitz, 1985, Kim and Sorensen, 1986, Titman and Wessels, 1988) generally lead to low debt/equity ratios. However, it is quite interesting to observe that high tech firms, such as Internet and biotechnology companies, which tend to have high volatility, large jump risk and low recovery values upon default, almost have no debt, despite the fact that tax credits would be given for coupon payments to the debtholders. Large (perhaps even unrealistic) diffusion volatility parameters are needed for pure diffusion-type endogenous models to generate very low debt/equity ratio for these high tech firms. In this paper we show that jumps can lead to significantly lower debt/equity ratios, resulting in very low ratios for high tech companies.

Third, we move on to study the connection between implied volatility in equity options and

³For surveys, see Harris and Raviv (1991), Ravid (1988), Bradley et al. (1984), Masulis (1988), Brealey and Myers (2001), and a special issue of "Journal of Economic Perspectives" partly devoted to the subject (Miller, 1988, Stiglitz, 1988, Ross, 1988, Bhattacharya, 1988, Modigliani, 1988).

credit spreads for corresponding defaultable bonds. Discussion of this connection arguably went back at least to the leverage effect suggested in Black (1976). Recent empirical studies in Toft and Prucyk (1997), Hull et al. (2003), Cremers et al. (2005a) seem to indicate a significant connection between implied volatility and credit spreads. What we point out in this paper is that, from a theoretical viewpoint, although in general there may be a positive connection between implied volatility in equity options and credit spreads in related defaultable bonds with exogenous default and jumps, the situation may be reversed for short maturity credit spreads with endogenous default and jumps. Therefore, we should be careful with the difference between exogenous and endogenous defaults when we study the connection between implied volatility and credit spreads.

1.2 Contribution of the Current Paper

Furthering the jump models in Hilberink and Rogers (2002), we extend the Leland-Toft endogenous default model (Leland, 1994, Leland and Toft 1996) to a two-sided jump model, in which the jump sizes have a double exponential distribution (Kou, 2002, Kou and Wang 2003). The model shows that jump risk and endogenous default can have significant impacts on credit spreads, optimal capital structure, and implied volatility of equity options. More precisely, we show: (1) The jump and endogenous default can produce a variety of non-zero credit spreads, including upward, humped, and downward shapes; interesting enough, the model can even produce, consistent with empirical findings, upward credit spreads for speculative grade bonds. See Section 3.2. for details. (2) The jump risk leads to much lower optimal debt/equity ratios, comparing to the diffusion model without jumps; in fact, with jump risk, highly risky firms tend to have very little debt. This helps to explain why Internet and biotech firms have almost no debt. See Section 3.1. for details. (3) The two-sided jumps lead to a variety of shapes for the implied volatility of equity options, even for long maturity options; and although in general credit spreads and implied volatility tend to move in the same direction, but this may not be true in presence of endogenous default and jumps. In fact, higher diffusion volatility may lead to a lower endogenous default barrier, resulting in smaller credit spreads for bonds with short maturities because the default is more likely to be caused by jumps for short maturity bonds. See Section 3.3 for details.

In terms of mathematical contribution, we give a proof of a version of the “smooth fitting” principle for the jump model, justifying a conjecture first suggested by Leland and Toft (1996) in the setting of the Brownian model. See Theorem 1 and Remark B.1 in the Appendix B.

1.3 Related Literature

Kijima and Suzuki (2001) and Zhou (2001) introduce jump diffusions to exogenous default models in Merton (1974) and Black and Cox (1976), respectively. Here we consider endogenous default along with optimal capital structure, under a different jump diffusion model. We also discuss the connection with implied volatility. Fouque et al. (2004) extend the exogenous default model of Black and Cox (1976) to include stochastic volatility.

A closely related paper is Hilberink and Rogers (2002), which extends the Leland-Toft model to Lévy processes with one-sided jumps and focuses on the study of capital structure⁴. Here we consider a jump diffusion model with two-sided jumps. In addition to optimal capital structure, we also discuss broader issues such as various credit spread shapes, and links between credit spreads and implied volatility. Two-sided jumps can generate more flexible implied volatility smiles, such as convex (not necessarily monotone) implied volatility smiles.

Linetsky (2005) extends the firm model in Black and Scholes (1973) to include one jump to default, with the exogenous default intensity being a negative power of the stock price. Carr and Linetsky (2005) generalize the CEV model to include one jump to default, with the exogenous default intensity an affine function of the CEV variance. Both papers reach some similar conclusions about credit spreads and implied volatility as in the current paper. Here we use endogenous default under a different model, and we also study optimal capital structure, various shapes of credit spreads, and the impact of endogenous default on implied volatility.

Structure models with jumps tend to have similar impacts on credit spreads as those by models with incomplete accounting information (Duffie and Lando, 2001), or unobservable default barriers (Giesecke, 2001, 2004). This is mainly because there are intrinsic connections between reduced-form models and structural models by changing information sets; see, for example, Collin-Dufresne et al. (2003), Çetin et al. (2004), Jarrow and Protter (2004), and Guo et al. (2005). The main differences between the current paper and these papers are that we also discuss endogenous default, optimal capital structure, and related implied volatility in equity options.

There are several other papers using the double exponential jump diffusion model to study credit risk. Mainly focusing on various empirical aspects of exogenous default, Huang and Huang (2003) and Cremers et al. (2005b) use the double exponential jump diffusion model to extend the exogenous default model in Black and Cox (1976). Here we look at modeling aspects

⁴Various discussions and representations on the optimal capital structure (but not explicit calculation) for general Lévy processes are also given in Boyarchenko (2000), Le Courtois and Quittard-Pinon (2004).

of endogenous default (by extending Leland and Toft, 1996). Dao (2005) also studies endogenous default with the double exponential jump diffusion model, but emphasizes on behavior finance aspects and studies other jump size distributions, while we investigate various shapes of credit spreads, analytical solutions of endogenous default boundaries, and implied volatility. As a result, all these studies complement each other nicely without many overlaps.

In summary, what differentiate the current paper from the related literature is that we put optimal capital structure, credit spreads, and implied volatility into a unified framework with both endogenous default and jumps.

2 Basic Setting of the Model

2.1 Asset Model

To generalize the Leland and Toft (1996) model to include jumps, we will use a double exponential jump diffusion model (Kou, 2002, Kou and Wang, 2003) for the firm asset. Essentially, it replaces the jump size distribution in the Merton's (1976) normal jump diffusion model to a double exponential distribution. Besides giving heavier tails, a main advantage of the double exponential distribution is that it leads to analytical solution for the debt and equity values, while it is not the case for the normal jump diffusion model.

Since we view equity and debts as contingent claims on the asset, it is enough to specify dynamics under a risk-neutral probability measure P , which can be determined by using the rational expectations argument (Lucas, 1978) with a HARA type of utility function for the representative agent, so that the equilibrium price of an asset is given by the expectation, under this risk-neutral measure P , of the discounted asset payoff. For a detailed justification of the rational expectations equilibrium argument⁵ see Kou (2002) and Naik and Lee (1990).

More precisely, we shall assume that under such a risk-neutral measure P , the asset value of the firm⁶ V_t follows a double exponential jump diffusion process

$$\frac{dV_t}{V_{t-}} = (r - \delta)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(Z_i - 1)\right) - \lambda\xi dt,$$

whose solution is given by

$$V_t = V_0 \exp\left\{\left(r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi\right)t + \sigma W_t\right\} \prod_{i=1}^{N_t} Z_i,$$

⁵Instead of using the rational expectations equilibrium argument, alternative approaches to find the pricing measures for credit derivatives are given in Giesecke and Goldberg (2005) and Bielecki, Jeanblanc, and Rutkowski (2004, 2005)

⁶One can think V_t as the value of an otherwise identical un-leveraged firm.

with r being the constant risk-free interest rate, δ the total payout rate to the firm's investors (including both bond and equity holders), and the mean percentage jump size ξ given by

$$\xi = E[Z - 1] = E[e^Y - 1] = \frac{p_u \eta_u}{\eta_u - 1} + \frac{p_d \eta_d}{\eta_d + 1} - 1.$$

Here W_t is a standard Brownian motion under P , $\{N_t, t \geq 0\}$ is a Poisson process with rate λ , the Z_i 's are i.i.d. random variables and $Y_i := \ln(Z_i)$ has a double-exponential distribution with density by

$$f_Y(y) = p_u \eta_u e^{-\eta_u y} \mathbf{1}_{\{y \geq 0\}} + p_d \eta_d e^{\eta_d y} \mathbf{1}_{\{y < 0\}}, \quad \eta_u > 1, \eta_d > 0.$$

Note that under the above risk-neutral measure V_t is a martingale after proper discounting: $V_t = E \left[e^{-(r-\delta)(T-t)} V_T | \mathcal{F}_t \right]$, where \mathcal{F}_t is the information up to time t .

2.2 Debt Issuing and Coupon Payments

The setting of debt issuing and coupon payment follows Leland and Toft (1996). In the time interval $(t, t + dt)$, the firm issues new debt with par value pdt , and maturity profile φ , where $\varphi(t) = me^{-mt}$, i.e. the maturity of a specific bond is chosen randomly according to an exponentially distributed r.v. with mean $1/m$. At time interval $(t, t + dt)$, the amount of due on their maturity is

$$\left(\int_{-\infty}^t p\varphi(t-u) du \right) dt = \left(\int_{-\infty}^t pme^{-m(t-u)} du \right) dt = pdt$$

and the firm retires all of them. One nice thing from the assumption of exponential maturity profile is that the par value of all of the debt maturing in $(t, t + dt)$ is equal to the par value of the newly issued debt. Thus, the par value of all pending debt is constant, equal to $P = p \int_0^{+\infty} e^{-ms} ds = p/m$. Before maturity, bondholders will receive coupons at rate ρ until default.

At each moment the firm faces two cash outflows and two cash inflows. The two cash outflows are after-tax coupon payment $(1 - \kappa)\rho P dt$ and due principal pdt , where κ is the tax rate; the two cash inflows are $b_t dt$ from selling new debts, where b_t is the price of the total newly issued bonds, and the total payout from the asset $\delta V_t dt$. If the total cash inflow $(\delta V_t + b_t) dt$ is greater than the cash outflow $((1 - \kappa)\rho P + p) dt$, we assume that the difference of the two goes to the equity holders as dividends; otherwise, additional equity will be issued to fulfill the due liability. Note that the difference $((1 - \kappa)\rho P + p) - (\delta V_t + b_t) dt$ is an infinitesimal quantity. Thus, such a financing strategy is feasible as long as the total equity value remains positive before bankruptcy. Therefore, we need to impose the limited liability constraint on the dynamic of the firm's equity. We will discuss this constraint later in Section 3.1.

2.3 Default Payments

As in first passage models, we assume that the default occurs at time $\tau = \inf\{t \geq 0 : V_t \leq V_B\}$. Upon default, the firm loses $(1 - \alpha)$ of V_τ , due to reorganization of the firm, and the debtholders as a whole get the rest of the value left, αV_τ , after reorganization.

To find the total debt value, total equity value, and optimal leverage ratio, it is not necessary to spell out the recovery value for individual bonds with different maturities. However, to model credit spreads we need to specify how the remaining asset of the firm αV_τ is distributed among bonds with different maturities. Three standard assumptions are recovery at a fraction of par value, of market values, and of corresponding treasury bonds; see Lando (2004), Duffie and Singleton (2000), Jarrow and Turnbull (1995). Here we shall use the assumption of recovery at a fractional treasury (Jarrow and Turnbull, 1995). We use this assumption because it makes analytical calculation easier. More precisely, upon default the payoff of a bond with unit face value and maturity $T > \tau$ is given by

$$ce^{-r(T-\tau)}, \quad 0 \leq c \leq 1.$$

To determine c , we shall match the total payment to bondholder with the total remaining asset αV_τ . By the memoryless property of exponential distribution, the conditional distribution of a bond's maturity does not change given its maturity is beyond τ . Thus, we have

$$P \int_{\tau}^{+\infty} ce^{-r(T-\tau)} \cdot me^{-m(T-\tau)} dT = \alpha V_\tau,$$

yielding

$$c = \frac{m + r}{m} \frac{\alpha V_\tau}{P}.$$

To make sure that $0 \leq c \leq 1$, we shall impose that

$$\frac{m + r}{m} \frac{\alpha V_B}{P} \leq 1. \tag{1}$$

We will see that this assumption is satisfied by the optimal default barrier; see (5).

We should emphasize that this recovery assumption is irrelevant if one is only interested in understanding debt values as a whole, but is relevant when we consider credit spreads, which in turn depend on cash flows to bonds with different maturities. In particular, our results about optimal default boundary, optimal leverage level, and implied volatility will not be affected by the recovery assumption.

2.4 Debt, Equity, and Market Value of the Firm

At time 0 the price of a bond with face value 1 and maturity T when the firm asset $V_0 = V$ is given by

$$\begin{aligned}
B(V; V_B, T) &= E \left[e^{-rT} \mathbf{1}_{\{\tau > T\}} + e^{-r\tau} \cdot ce^{-r(T-\tau)} \mathbf{1}_{\{\tau \leq T\}} \right] + E \left[\int_0^{\tau \wedge T} \rho e^{-rs} ds \right] \\
&= e^{-rT} P[\tau \geq T] + \frac{\alpha}{P} \frac{m+r}{m} E[V_\tau e^{-rT} \mathbf{1}_{\{\tau \leq T\}}] + \frac{\rho}{r} (1 - E[e^{-r(\tau \wedge T)}]) \\
&= \left(1 - \frac{\rho}{r}\right) e^{-rT} P[\tau \geq T] + \frac{\alpha}{P} \frac{m+r}{m} E[V_\tau e^{-rT} \mathbf{1}_{\{\tau \leq T\}}] + \frac{\rho}{r} (1 - E[e^{-r\tau} \mathbf{1}_{\{\tau \leq T\}}]).
\end{aligned}$$

In our endogenous model, both P (which is related to the optimal debt/equity ratio) and the endogenous default barrier V_B will be decision variables.

According to the Modigliani-Miller theorem (Brealey and Myers, 1991), the total market value of the firm is the asset value plus tax benefits and minus the bankruptcy costs. More precisely, the market value of the firm at time 0 is

$$v(V; V_B) = V + E \left[\int_0^\tau \kappa \rho P e^{-rt} dt \right] - (1 - \alpha) E[V_\tau e^{-r\tau}].$$

The total equity value equals to the total value of the firm less the total value of bonds,

$$S(V; V_B) = v(V; V_B) - D(V; V_B),$$

where the total debt $D(V; V_B)$ is equal to

$$D(V; V_B) = P \int_0^{+\infty} m e^{-mT} B(V; V_B, T) dT.$$

We should point out that in this static model (thanks to the exponential maturity profile), the total firm value, total debt and equity values are all Markovian, and are independent of the time horizon.

2.5 Preliminary Results for Debt and Equity Values

To compute the total debt and equity values, one needs to compute the distribution of the default time τ and the joint distributions of V_τ and τ . The analytical solutions for these distributions depend on the roots of the following equation (which is essentially a four-degree polynomial)

$$G(x) = r + \beta, \quad \beta > 0, \quad G(x) := -(r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi)x + \frac{1}{2}\sigma^2 x^2 + \lambda \left(\frac{pd\eta_d}{\eta_d - x} + \frac{pu\eta_u}{\eta_u + x} - 1 \right).$$

Lemma 2.1 in Kou and Wang (2003) implies that the above equation has exactly four real roots. Denote the four roots to the equation above by $\gamma_{1,\beta}, \gamma_{2,\beta}, -\gamma_{3,\beta}, -\gamma_{4,\beta}$, with

$$0 < \gamma_{1,\beta} < \eta_d < \gamma_{2,\beta} < \infty, \quad 0 < \gamma_{3,\beta} < \eta_u < \gamma_{4,\beta} < \infty.$$

A simple analytical solutions for all these roots are given in the appendix of Kou, Petrella, and Wang (2005).

Lemma 1. *The value of total debt at time 0 is*

$$\begin{aligned} D(V; V_B) &= P \int_0^{+\infty} m e^{-mT} B(V; V_B, T) dT \\ &= \frac{P(\rho + m)}{r + m} \left\{ 1 - d_{1,m} \left(\frac{V_B}{V} \right)^{\gamma_{1,m}} - d_{2,m} \left(\frac{V_B}{V} \right)^{\gamma_{2,m}} \right\} + \alpha V_B \left\{ c_{1,m} \left(\frac{V_B}{V} \right)^{\gamma_{1,m}} + c_{2,m} \left(\frac{V_B}{V} \right)^{\gamma_{2,m}} \right\}. \end{aligned}$$

The Laplace transform of a bond price, $B(V; V_B, T)$, is given by

$$\begin{aligned} \int_0^{+\infty} e^{-\beta T} B(V; V_B, T) dT &= \frac{\rho + \beta}{\beta(r + \beta)} \left\{ 1 - d_{1,\beta} \left(\frac{V_B}{V} \right)^{\gamma_{1,\beta}} - d_{2,\beta} \left(\frac{V_B}{V} \right)^{\gamma_{2,\beta}} \right\} \\ &\quad + \frac{\alpha(m + r)}{mP(\beta + r)} V \left\{ c_{1,\beta} \left(\frac{V_B}{V} \right)^{\gamma_{1,\beta} + 1} + c_{2,\beta} \left(\frac{V_B}{V} \right)^{\gamma_{2,\beta} + 1} \right\}, \end{aligned}$$

where

$$c_{1,\beta} = \frac{\eta_d - \gamma_{1,\beta}}{\gamma_{2,\beta} - \gamma_{1,\beta}} \frac{\gamma_{2,\beta} + 1}{\eta_d + 1}, \quad c_{2,\beta} = \frac{\gamma_{2,\beta} - \eta_d}{\gamma_{2,\beta} - \gamma_{1,\beta}} \frac{\gamma_{1,\beta} + 1}{\eta_d + 1}, \quad d_{1,\beta} = \frac{\eta_d - \gamma_{1,\beta}}{\gamma_{2,\beta} - \gamma_{1,\beta}} \frac{\gamma_{2,\beta}}{\eta_d}, \quad d_{2,\beta} = \frac{\gamma_{2,\beta} - \eta_d}{\gamma_{2,\beta} - \gamma_{1,\beta}} \frac{\gamma_{1,\beta}}{\eta_d}.$$

The total market value of the firm is given by

$$\begin{aligned} v(V; V_B) &= V + \frac{P\kappa\rho}{r} \left\{ 1 - d_{1,0} \left(\frac{V_B}{V} \right)^{\gamma_{1,0}} - d_{2,0} \left(\frac{V_B}{V} \right)^{\gamma_{2,0}} \right\} - (1 - \alpha) V_B \left\{ c_{1,0} \left(\frac{V_B}{V} \right)^{\gamma_{1,0}} + c_{2,0} \left(\frac{V_B}{V} \right)^{\gamma_{2,0}} \right\}, \end{aligned}$$

and the total equity of the firm is given by $S(V; V_B) = v(V; V_B) - D(V; V_B)$.

A short proof of this lemma, basically following from the calculation of the first passage times in Kou and Wang (2003), will be given in Appendix A. Various versions of the lemma and some similar results are also given in Huang and Huang (2003) and Dao (2005). The formulae above have interesting interpretations. For example, in the formula for the total debt value, note that $\frac{P(\rho+m)}{r+m}$ is the present value of the debt with face value P and maturity profile $\phi(t) = m e^{-mt}$ in absence of bankruptcy. The term $1 - d_{1,m} (V_B/V)^{\gamma_{1,m}} - d_{2,m} (V_B/V)^{\gamma_{2,m}}$ in the first sum is the present value of \$1 contingent on future bankruptcy; the second sum is what the bondholders can get from the bankruptcy procedure. Note that due to jumps, the remaining asset after bankruptcy is not αV_B any more. A similar interpenetration holds for the equity value.

3 Main Results

We shall study three issues, the optimal capital structure and endogenous default barrier, credit spreads, and implied volatility generated by the model. Both theoretical and numerical results will be reported in this section. Table 1 summarizes the parameters to be used in the numerical investigation. In addition, we set the number of shares of stocks is 100, and we assume that one year is equal to 252 trading days.

Basic parameters: $\sigma = 0.2, \kappa = 35\%, r = 8\%, \alpha = 0.5, \rho = 8.162\%, \delta = 6\%, V_0 = 100, 1/m = 5.$
Case A: Pure Brownian case, i.e. the jump rate $\lambda = 0.$
Case B: Large but infrequent jumps, $1/\eta_u = 1/3, 1/\eta_d = 1/2, p_u = 0.5, \lambda = 0.2$
Case C: Moderate jumps, $1/\eta_u = 1/8, 1/\eta_d = 1/6, p_u = 0.25, \lambda = 1.$

Table 1: Basic parameters for numerical illustration. The riskless interest rate $r = 8\%$ is closed to the average historical treasury rate during 1973-1998, and the coupon rate $\rho = 8.162\%$ is the par coupon rate for risk free bonds with semi-annual coupon payments when the continuously compounded interest rate is 8% according to Huang and Huang (2003), who also set $\delta = 6\%$. The diffusion volatility $\sigma = 0.2$, corporate tax rate is 35%, and the recovery fraction $\alpha = 0.5$ are chosen in consistent with Leland and Toft (1996).

3.1 Optimal Capital Structure and Endogenous Default

In this subsection, we consider the problem of optimal capital structure and optimal endogenous default barrier, i.e. the choices of optimal debt level P and bankrupt trigger V_B . After presenting results for the optimal P and V_B , we shall point out jump risk can significantly reduce the optimal debt/equity ratio. In particular, the model implies that for a firm with high jump risk and few tangible assets (hence low recovery rate), such as Internet and biotech companies, the optimal level P is close to zero.

3.1.1 The Solution of a Two-Stage Optimization Problem

Deciding optimal P and choosing optimal V_B are two entangled problems, and cannot be separated easily. For example, when a firm chooses P to maximize the total firm value at time 0, the decision on P obviously depends on the bankruptcy trigger level V_B . Similarly, after the debt being issued, the equity holders will choose an optimal V_B , which of course depends on how much debt the firm has. Here we shall choose P and V_B according to a two-stage optimization procedure as in Leland (1994) and Leland and Toft (1996).

More precisely, for a fixed P , the equity holders find the optimal default barrier by maximizing the equity value, subject to the limited liability constraint. It is the equity value that is maximized, because after the debt being issued it is the equity holders who control the firm. Mathematically, the equity holders find the best V_B by solving

$$\max_{V_B \leq V} S(V; V_B) \quad (2)$$

subject to the limited liability constraint

$$S(V'; V_B) \geq 0, \text{ for all } V' \geq V_B \geq 0. \quad (3)$$

The limited liability constraint is imposed so that the equity value is always nonnegative for any future firm asset value V' , as long as the asset value V' is above the bankruptcy level V_B . This is the first stage optimization problem.

Clearly the optimal $V_B^* \equiv V_B^*(P)$ to be chosen by the equity holders will depend on P . At time 0, the firm will conduct the second stage optimization to maximize the firm value, in anticipation of what the equity holders will do later. More precisely, the second stage optimization is that

$$\max_P v(V; V_B^*(P)). \quad (4)$$

The two-stage optimization problem partly arises due to the conflict of interests between debt and equity holders. It is obvious that choosing the leverage level P and bankrupt trigger V_B simultaneously will lead to a better market value of the firm. Leland (1998) uses the difference of the two to explain agency costs due to conflicts between the equity and debt holders. Obviously the two-stage optimization is only a rough approximation to complicated decision and negotiation between debt and equity holders⁷. Here is the result for the two-stage optimization.

Theorem 1. (a) *The first stage optimization: Given the debt level P , the optimal barrier level V_B^* solving (2) with the constraint in (3) is given by*

$$V_B^* = \epsilon P, \quad \epsilon := \frac{\frac{\rho+m}{r+m}(d_{1,m}\gamma_{1,m} + d_{2,m}\gamma_{2,m}) - \frac{\kappa\rho}{r}(d_{1,0}\gamma_{1,0} + d_{2,0}\gamma_{2,0})}{(1-\alpha)(c_{1,0}\gamma_{1,0} + c_{2,0}\gamma_{2,0}) + \alpha(c_{1,m}\gamma_{1,m} + c_{2,m}\gamma_{2,m}) + 1}.$$

(b) *The second stage optimization: After plugging the optimal V_B^* into the second stage optimization, we have that $v(V; \epsilon P)$ is a concave function of P , which implies that we can find a unique optimal debt level P for the problem (4).*

⁷An alternative approach is to model the strategic behavior by the various shareholders of the firm; see Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997).

The proof, which is one of the main results in the paper, will be given in Appendix B. A mathematical contribution of the paper is that the proof also gives a rigorous justification of a smooth-paste principle; more precisely, V_B^* is the solution of $\frac{\partial S(V; V_B^*)}{\partial V}|_{V=V_B^*} = 0$. Even in the pure Brownian model, Leland and Toft (1996) did not prove the result, mainly because the proof needs the local convexity at V_B , i.e. $\frac{\partial^2 S(V; V_B)}{\partial V^2}|_{V=V_B} \geq 0$. Instead, Leland and Toft (1996, footnote 9) verified the above local convexity numerically, and made a conjecture that the smooth-pasting principle should hold. Hilberink and Rogers (2002) gave a numerical verification of the local convexity, and conjectured that the smoothing-pasting principle should hold for a one-sided jump model. Here we are able to prove the smooth-pasting principle by first proving the local convexity. See Remark B.1 at the end of Appendix B for details.

It is easy to see that V_B^* satisfies (1) because

$$\epsilon < \frac{\frac{\rho+m}{r+m}(d_{1,m}\gamma_{1,m} + d_{2,m}\gamma_{2,m})}{\alpha(c_{1,m}\gamma_{1,m} + c_{2,m}\gamma_{2,m} + 1)} < \frac{1}{\alpha} \frac{\rho + m}{r + m}, \quad (5)$$

which follows from the definitions of $d_{i,m}$ and $c_{i,m}$ as

$$d_{1,m}\gamma_{1,m} + d_{2,m}\gamma_{2,m} = \frac{\gamma_{1,m}\gamma_{2,m}}{\eta_d}, \quad c_{1,m}\gamma_{1,m} + c_{2,m}\gamma_{2,m} + 1 = \frac{(\gamma_{1,m} + 1)(\gamma_{2,m} + 1)}{\eta_d + 1}, \quad \frac{\gamma_{1,m}}{\eta_d} < \frac{\gamma_{1,m} + 1}{\eta_d + 1}.$$

3.1.2 The Impact of Jumps on Optimal Capital Structure

Table 2 shows the effect of various parameters on the optimal leverage level. Consistent with our intuition, the table shows that the optimal leverage ratio is an increasing function of the fractional remaining asset α and average maturity profile $1/m$, and is a decreasing function of jump rate λ and diffusion volatility σ . More importantly, the table shows that the jump risk leads to much lower leverage ratios. In particular with infrequent large jumps the optimal leverage ratio is close to zero, even if there is only one jump every year ($\lambda = 1$), and recovery rate $\alpha = 25\%$ with maturity profile $1/m$ not more than 2 years. Internet and biotech companies typically have low α (as they do not have many “tangible” assets) and short maturity profile (as they do not have long operating history to secure long term debt, even if they want to issue debt). Therefore, jump risk can lead to a much lower debt/equity ratio, even making it close to zero.

3.2 Flexible Credit Spreads

In this subsection we try to incorporate four stylized facts related to credit spreads as outlined in Section 1.1, namely (1) non-zero credit spreads for very short maturity bonds; (2) flexible

		$m^{-1} = 0.5$		$m^{-1} = 1$		$m^{-1} = 2$		$m^{-1} = 5$		
		$\sigma = 0.2$	$\sigma = 0.4$	$\sigma = 0.2$	$\sigma = 0.4$	$\sigma = 0.2$	$\sigma = 0.4$	$\sigma = 0.2$	$\sigma = 0.4$	
Case B	$\alpha = 5\%$	$\lambda = 0$	7.14%	1.11%	11.19%	2.37%	17.56%	5.04%	30.67%	12.91%
		$\lambda = 0.5$	0.67%	0.19%	1.63%	0.60%	3.94%	1.86%	11.70%	7.32%
		$\lambda = 1$	0.12%	0.04%	0.42%	0.19%	1.45%	0.84%	6.54%	4.79%
		$\lambda = 2$	0.01%	0.001%	0.05%	0.003%	0.34%	0.24%	3.16%	2.64%
	$\alpha = 25\%$	$\lambda = 0$	13.78%	3.21%	18.31%	5.28%	25.08%	9.13%	38.41%	19.07%
		$\lambda = 0.5$	2.49%	0.88%	4.35%	1.88%	8.07%	4.28%	18.66%	12.63%
		$\lambda = 1$	0.66%	0.28%	1.5%	0.77%	3.67%	2.29%	11.89%	9.22%
		$\lambda = 2$	0.067%	0.04%	0.28%	0.18%	1.18%	0.86%	6.87%	5.96%
	$\alpha = 50\%$	$\lambda = 0$	25.44%	9.22%	30.30%	12.63%	37.29%	18.27%	50.52%	31.19%
		$\lambda = 0.5$	8.88%	4.13%	12.39%	6.65%	18.50%	11.56%	33.33%	25.52%
		$\lambda = 1$	3.71%	1.97%	6.09%	3.74%	10.95%	7.79%	25.25%	21.25%
		$\lambda = 2$	0.87%	0.53%	2.02%	1.42%	5.22%	4.18%	18.45%	16.90%
Case C	$\alpha = 5\%$	$\lambda = 0$	7.14%	1.11%	11.19%	2.37%	17.56%	5.04%	30.67%	12.91%
		$\lambda = 0.5$	4.80%	0.87%	7.96%	1.94%	13.22%	4.33%	24.88%	11.73%
		$\lambda = 1$	3.45%	0.69%	6.01%	1.61%	10.48%	3.75%	21.04%	10.73%
		$\lambda = 2$	1.95%	0.44%	3.71%	1.12%	7.10%	2.87%	16.09%	9.13%
	$\alpha = 25\%$	$\lambda = 0$	13.78%	3.21%	18.31%	5.28%	25.08%	9.13%	38.41%	19.07%
		$\lambda = 0.5$	10.09%	2.64%	13.96%	4.51%	19.98%	8.10%	32.51%	17.73%
		$\lambda = 1$	7.75%	2.20%	11.11%	3.88%	16.54%	7.22%	28.40%	16.56%
		$\lambda = 2$	4.92%	1.54%	7.53%	2.92%	12.07%	5.84%	22.92%	14.63%
	$\alpha = 50\%$	$\lambda = 0$	25.44%	9.22%	30.30%	12.63%	37.29%	18.27%	50.52%	31.19%
		$\lambda = 0.5$	20.58%	8.11%	25.16%	11.38%	31.92%	16.90%	45.24%	29.88%
		$\lambda = 1$	17.14%	7.17%	21.44%	10.29%	27.96%	15.69%	41.28%	28.70%
		$\lambda = 2$	12.50%	5.67%	16.35%	8.51%	22.48%	13.65%	35.82%	26.69%

Table 2: Effects of various parameters on the optimal leverage level. The basic parameters are given Table 1. $\lambda = 0$ is the pure Brownian case (case A). Note that comparing to the pure Brownian case the jump risk reduces the optimal leverage ratio significantly, many times even making the ratio close to zero.

credit spreads including upward, humped, and downward shapes; (3) upwards credit spreads even for speculative grade bonds; (4) the problem of overprediction of credit spreads for long maturity bonds in the Leland-Toft model.

By analogy to the case of discrete coupons, in our case of continuous coupon rate we shall define the yield to maturity, $\nu(T)$, of a defaultable bond with maturity T and coupon rate ρ as the one satisfies

$$B(V; V_B, T) = e^{-\nu(T)T} + \int_0^T \rho e^{-\nu(T)s} ds = e^{-\nu(T)T} + \frac{\rho}{\nu}(1 - e^{-\nu(T)T})$$

and the credit spread is defined as $\nu(T) - r$. The bond price $B(V; V_B, T)$ can be computed by using Theorem 1 and numerical Laplace inversion algorithms, such as the Euler inversion algorithm in Abate and Whitt (1992).

3.2.1 Non-Zero Credit Spreads and A Connection with Reduced-Form Models

For very short maturity bonds, the analytical solution of credit spreads is available in the next theorem, and credit spreads are shown to be strictly positive.

Theorem 2. *We have*

$$\lim_{T \rightarrow 0} \nu(T) = \lambda p_d \left(\frac{V_B}{V} \right)^{\eta_d} \left[1 - \frac{\alpha V_B}{P} \frac{m+r}{m} \frac{\eta_d}{\eta_d+1} \right].$$

In particular, the above limit for the short maturity credit spread is strictly positive, as long as there is a downside jump risk (i.e. $\lambda p_d > 0$).

The proof will be given in Appendix C. As we mentioned in Section 1.2, there are well recognized connections linking reduced-form models and structural models. This is perhaps one of the main reasons that adding jumps can produce flexible shapes of non-zero credit spreads, just as in many reduced-form models. In our particular setting, this link can be expressed explicitly. More precisely, for our jump diffusion model

$$P[\tau \leq t + \Delta t | \tau > t] = \lambda p_d \left(\frac{V_B}{V_t} \right)^{\eta_d} \Delta t + o(\Delta t);$$

while the intensity h_t in reduced-form model satisfies

$$P[\tau \leq t + \Delta t | \tau > t] = h_t \Delta t + o(\Delta t).$$

Therefore, to the first order approximation, the model behaves like a reduced-form model with $h_t = \lambda p_d (V_B/V_t)^{\eta_d}$, despite that the model also has a predictable component (i.e. the diffusion component).

3.2.2 Upward, Humped, and Downward Credit Spreads

Although adding jump is a natural extension of Leland and Toft (1996) model, just by adding jumps the model already has the capacity of producing flexible credit spreads, including upward, humped, and downwards shapes⁸. Normally, the shape is upward. As the firm's financial situation deteriorates, it becomes humped and even downward in face of immediate financial distress. Figure 1 illustrates that our model can reproduce this phenomenon.

⁸Interesting enough, all three kinds of shapes may not only preveal for corporate bonds, but also for investment grade sovereign bonds. For example, Schmid (20004, p. 277) shows that the credit spreads between Italian bonds (with S&P rating AA) and Geman bonds (with AAA rating which may be effectively treated as "risk-free") can have both upward and humped shapes; while the spreads between Greek bonds (with A- rating) in general have downward shapes.

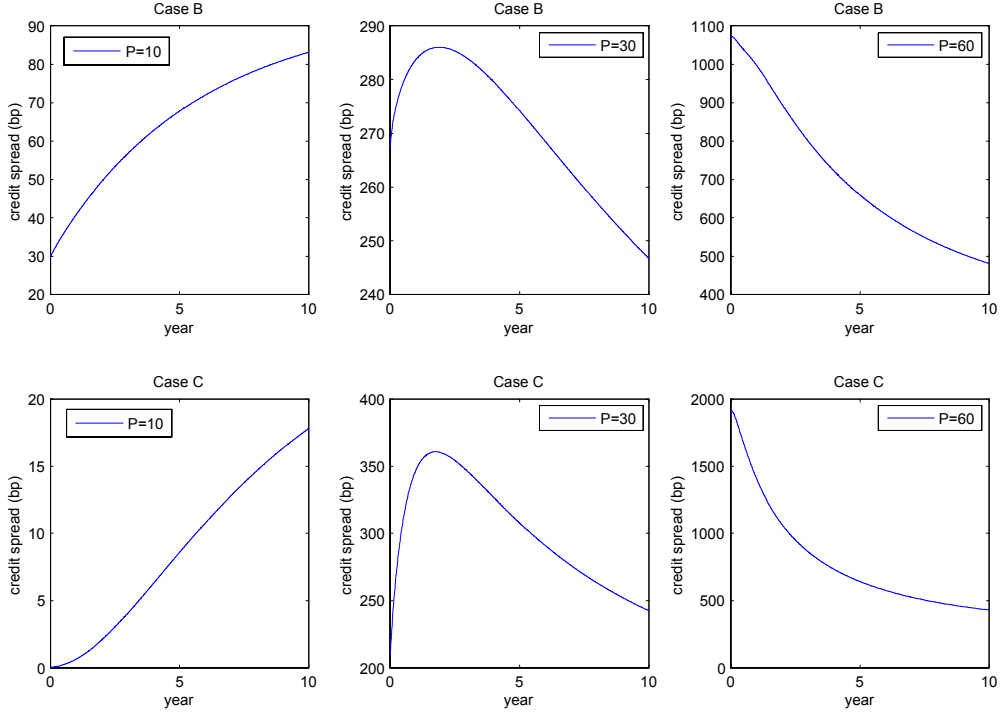


Figure 1: Various shapes of credit spreads. The parameters used are interest rate $r = 8\%$, coupon rate $\rho = 1\%$, pay ratio $\delta = 1\%$, volatility $\sigma = 10\%$, corporate tax rate $\kappa = 35\%$, bankrupt loss fraction $\alpha = 50\%$, average maturity $m^{-1} = 0.5$ years. In the first panel, the jump parameters are specified in Case B, while in the second panel the jump sizes are given in Case C with jump rates $\lambda = 2$.

The shape of credit spreads for speculative bonds is a somewhat controversial subject. Traditional empirical studies based on aggregated data for speculative bonds suggested downward and humped shapes (e.g. Sarig and Warga, 1989, Fons, 1994). Helwege and Turner (1999) argued that possible maturity bias (as healthier firms are able to issue longer maturity bonds) may affect the aggregated empirical studies; instead, they suggested that even speculative bonds may have upward shapes during normal times. However, He et al. (2000) later argued that empirically humped and downward shapes should be seen for speculative bonds, even after adjusting for the maturity bias.

For theoretical models, it is desirable to have all three shapes for speculative bonds. We have seen that the model leads to humped and downwards credit spreads for firms with large

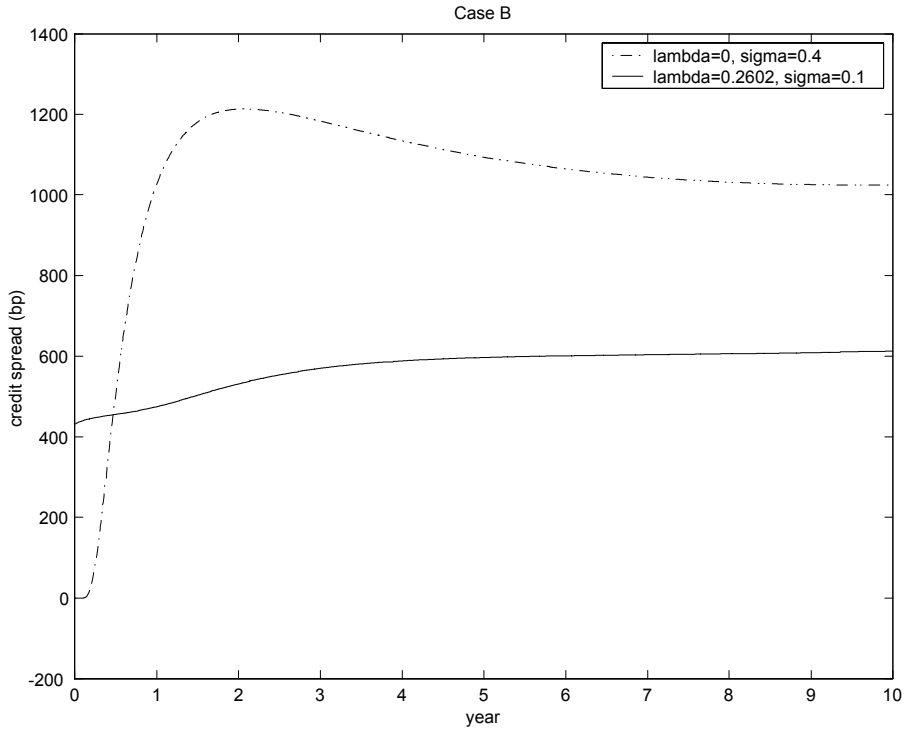


Figure 2: Upward credit spread curves for speculative bonds. The parameters used are the leverage level $P/V = 90\%$, total volatility $\sigma = 40\%$, average bonds maturity $m^{-1} = 5$ years, and all the other parameters are the same as Case B and basic parameters in Table 1. This is consistent with a plot in Collin-Dufresne and Goldstein (2001) with an additional feature of non-zero credit spreads.

leverage levels. Figure 2 shows that we can generate upward shaped curves even for the leverage level $P/V = 90\%$ and total volatility⁹ $\sigma_{total} = 40\%$. As we can see in pure Brownian motion with $\lambda = 0$ (the dash line), the credit spread curve is humped with a zero credit spread as the maturity goes to zero. But with jumps, the credit spread shape becomes upward with nonzero values for short maturity bonds. Collin-Dufresne and Goldstein (2001) generated a similar upward shape credit spread curve for speculative bonds (c.f. Figure 3 in their paper) using the Brownian motion with a stochastic exogenous default barrier; however, being a diffusion model,

⁹The total variance is denoted by $\sigma_{total}^2 = \sigma_{diff}^2 + \sigma_{jump}^2$, where the diffusion volatility σ_{diff} is equal to σ in the description of our model, and

$$\sigma_{jump}^2 = \frac{1}{t} Var\left(\sum_{i=1}^{N(t)} (Z_i - 1)\right) = \lambda \left\{ \left[\frac{p_u \eta_u}{\eta_u - 2} + \frac{p_d \eta_d}{\eta_d + 2} \right] - \left[\frac{p_u \eta_u}{\eta_u - 1} + \frac{p_d \eta_d}{\eta_d + 1} \right]^2 \right\}, \eta_u > 2.$$

the model leads to zero credit spreads as the maturity goes to zero.

3.2.3 Effects of Various Parameters on Credit Spreads

Figure 3 shows that in our model credit spreads decrease as the interest rate increases. This is consistent with the negative correlation between credit spreads and risk-free rate found in empirical studies mentioned in Section 1.2. Furthermore, if the risk-free rate has mean reverting, then the credit spreads will also likely be mean reverting, as they are inversely related in the model.

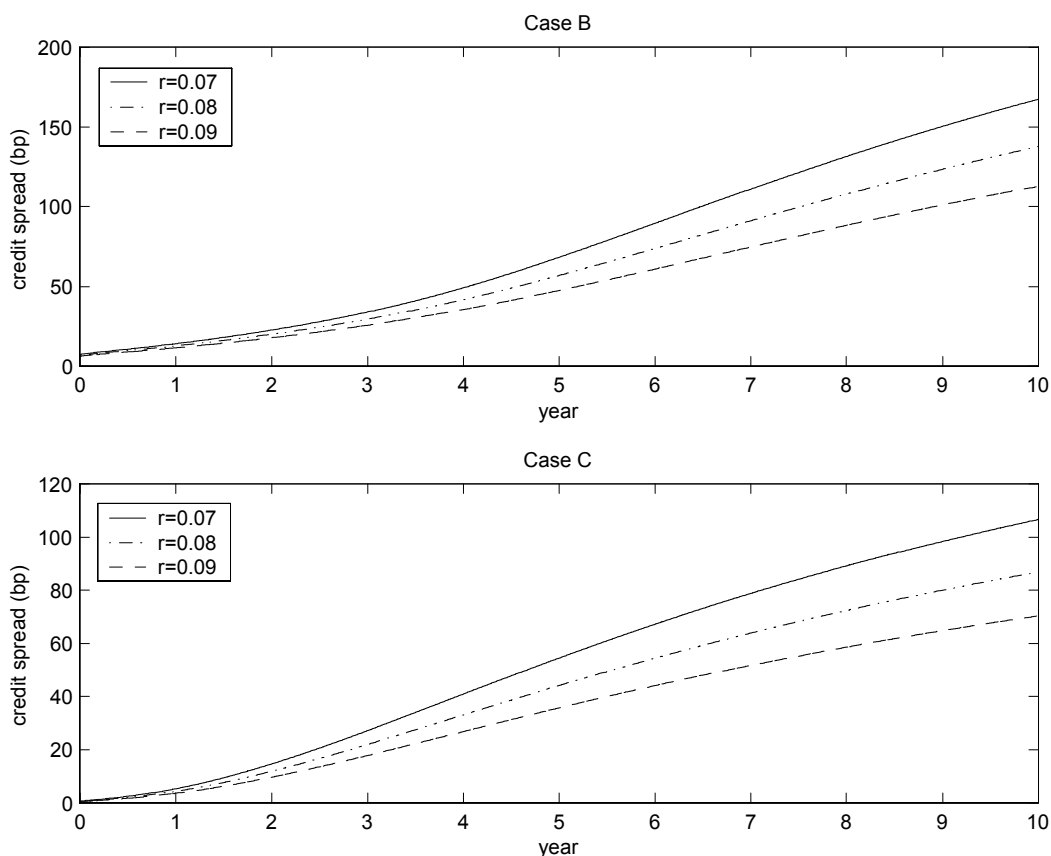


Figure 3: The effect of the risk-free rate on credit spreads. The parameters used are the leverage level $P/V = 30\%$, average bonds maturity $m^{-1} = 5$ years, and all the other parameters are the same as those in Table 1.

Figure 4 illustrates the effects of various parameters on credit spreads. In particular, credit spreads decrease in α , increase in λ , and decrease in average maturity $1/m$. All of these are

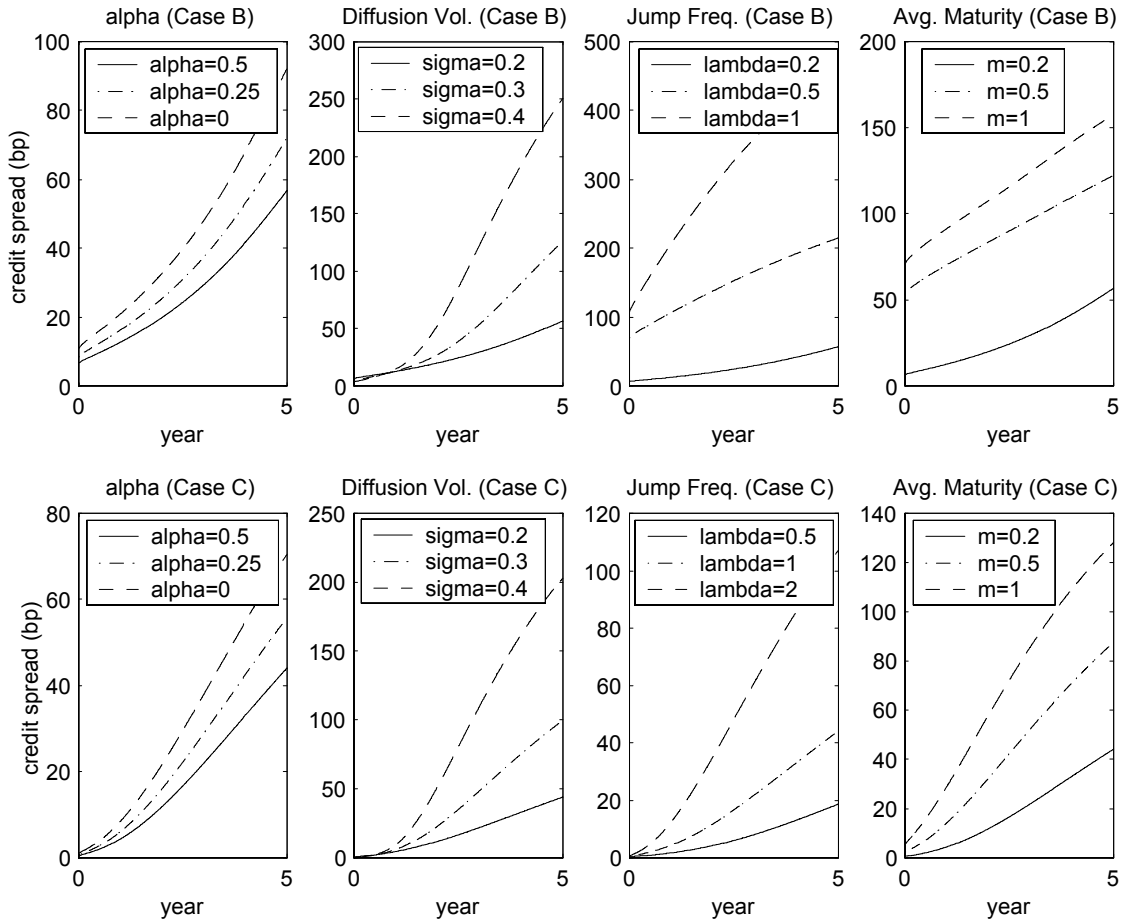


Figure 4: The effect of various parameters on credit spreads. The defaulting parameters used are the leverage level $P/V = 30\%$, and all the other parameters are the same as those in Table 1. Note the non-monotonicity in Case B in terms of σ .

consistent with our intuition. However, it is interesting to point out that, in Case B, for short maturity bonds the credit spreads is actually an increasing function of diffusion volatility σ . To explain this, note that the endogenous optimal bankrupt barriers are $V_B = 21.6947$ ($\sigma = 0.2$), $V_B = 19.5422$ ($\sigma = 0.3$), $V_B = 17.3502$ ($\sigma = 0.4$), respectively. For short maturity bonds, the defaults will be caused mainly by jumps rather than by the diffusion part. Therefore, for short maturity bonds the credit spreads decrease in σ . But for long maturity bonds, the diffusion part of the process plays a more important role to determine credit spreads; the more diffusion volatility is, the more credit spreads are.

In Figure 5, we fix the total volatility of the asset process to see the impacts of jump

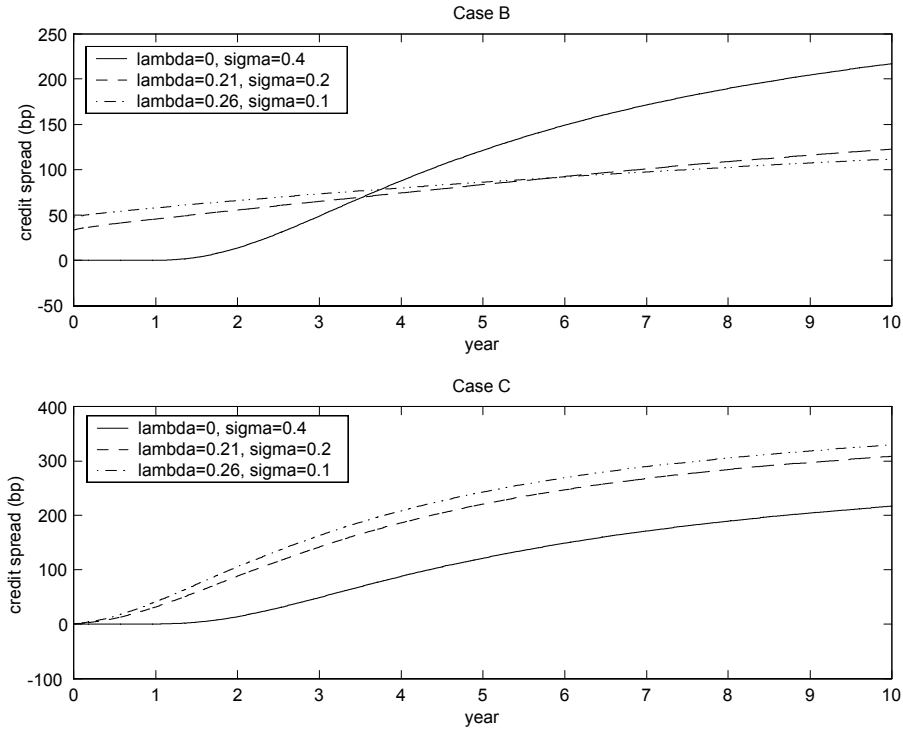


Figure 5: Credit spreads: Jump volatility vs. diffusion volatility. The defaulting parameters used are the leverage level $P/V = 30\%$, $\sigma = 0.40$, and all the other parameters are the same as those in Table 1. The plot for Case B seems to be consistent with the empirical finding in Eom et al. (2004).

and diffusion parts to credit spreads. Eom et al. (2004), found that empirically the Leland-Toft model tend to overpredict credit spreads for long maturity bonds and underpredict credit spreads for short maturity bonds (due to the problem of nonzero credit spreads for diffusion models). Figure 5 seems to suggest that large infrequent jumps, as in Case B, lead to results that are more consistent with this empirical finding, as in that case jumps significantly reduce credit spreads for long maturity bonds while lift up credit spreads for short maturity bonds.

3.3 Volatility Smile

In this subsection, we study the connection between credit spreads and implied volatility. The connection was suggested first by Black (1976); for recent empirical studies of the connection see, e.g., Toft and Prucyk (1997), Hull et al. (2004), Cremers (2005a), Carr and Linetsky (2005). The interesting points in this subsection are: (1) We should carefully distinguish exogenous and

endogenous defaults when we study the possible connection between credit spreads and implied volatility; see Fig. 8. (2) Default and jumps together can generate significant volatility smiles even for long maturity equity options; see Figure 9.

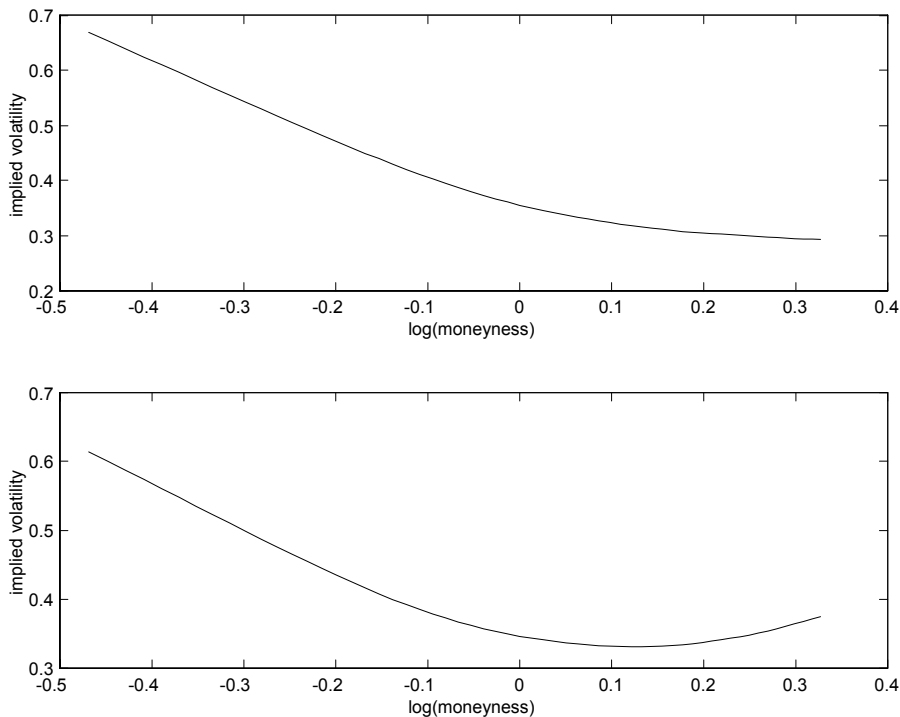


Figure 6: One sided jumps vs. two sided jumps. The first panel is the case of one-sided jumps while the second panel two-sided jumps. The parameters used here are the same basic parameters in Table 1, and the leverage level $P = 30\%$, the option maturity $T = 0.25$. The jump parameters follow Case C, except in the first panel (the one-side jump case) where we set $p_u = 0$.

In our investigation, we use 10,000 simulation sample runs to study European call options with 60 different strike prices, $K_i = S_0 - 0.02 \cdot (i - 30), i = 1, \dots, 60$, covering the cases of deep-in-the-money, at-the-money, deep-out-of-the money¹⁰. In all the plots, the “moneyness” is defined to be the ratio the strike price over stock price. Figures 6 shows that two-sided jumps can generate more flexible implied volatility curves, compared to one-sided jumps which tend to generate monotone curves.

¹⁰To get implied volatilities from the Black-Scholes formula, we need to compute the dividend rate of the underlying stock. Since the average dividend rate d of the underlying stock over $[0, T]$ must satisfy $S_0 = e^{-(r-d)T} E[S_T]$, where S_0 and S_T are the stock prices at time 0 and T respectively, we use the average dividend $d = r + \log(S_0/E[S_T])/T$ in the Monte Carlo simulation to compute the implied volatility.

3.3.1 Connection between Implied Volatility and Credit Spreads

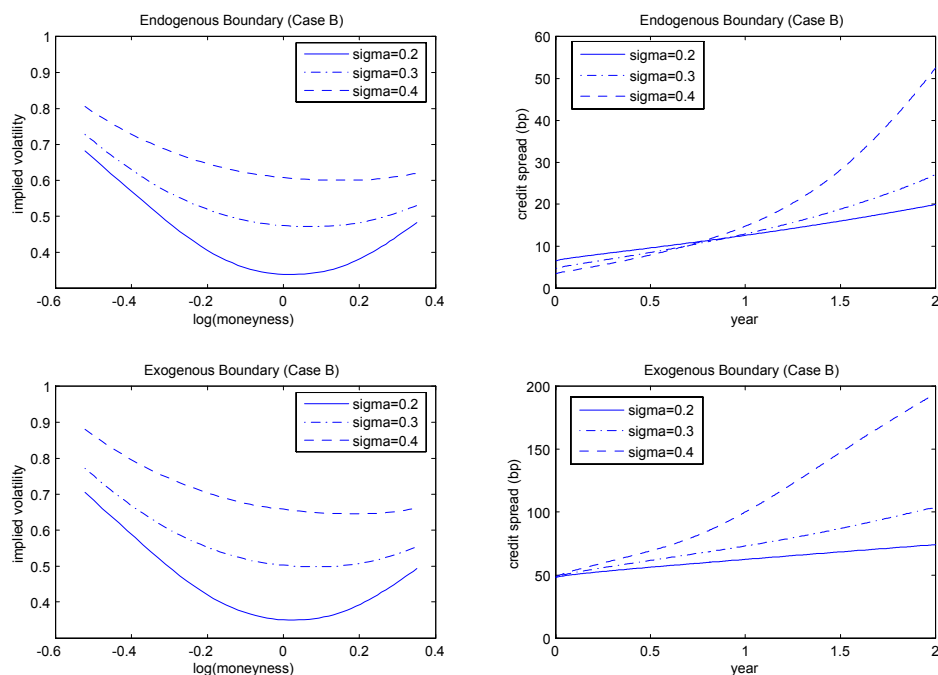


Figure 7: Exogenous default vs. endogenous default. We use leverage level $P = 30\%$, the option maturity $T = 0.25$ year, and the rest of parameters as in Case B in Table 1. Note the non-monotonicity of the credit spreads under endogenous default.

Figure 7 shows a clear difference between exogenous and endogenous defaults in terms of the connection between implied volatility and credit spreads. In particular, for short maturity bonds, under exogenous default, both credit spreads and implied volatility are increasing functions of the diffusion volatility. However, this is not true under endogenous default, because the optimal default barrier tends to be lower with higher diffusion volatility; in fact, $V_B = 21.6947$ ($\sigma = 0.2$), $V_B = 19.5422$ ($\sigma = 0.3$), $V_B = 17.3502$ ($\sigma = 0.4$), respectively. For short maturity bonds, the defaults will be caused mainly by jumps rather than by the diffusion part, resulting in lower credit spreads for higher diffusion volatility in the case of short maturity bonds. On the other hand, the implied volatility seems to be an increasing function of the diffusion volatility with exogenous default.

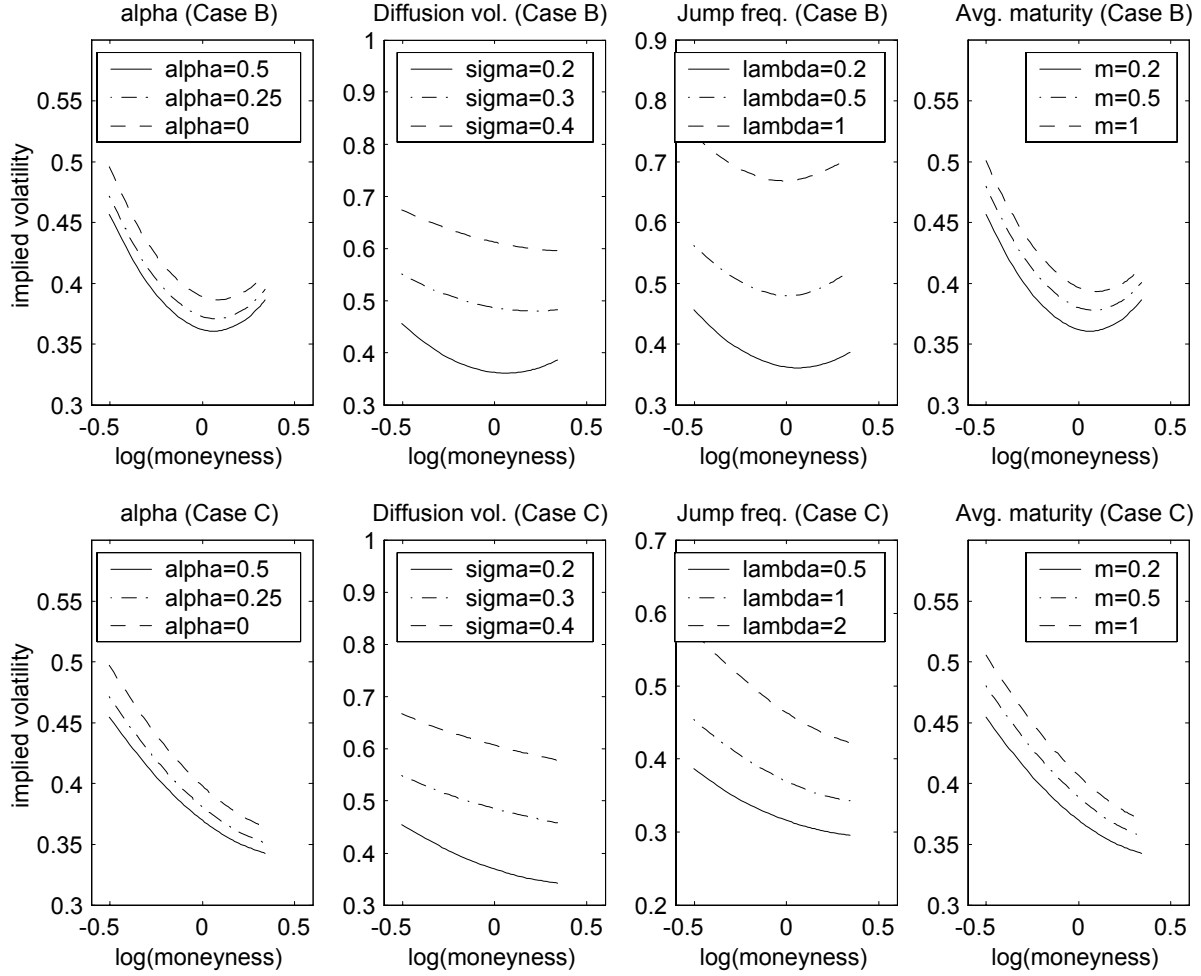


Figure 8: Effects of various parameters on implied volatility. We use $P = 30\%$, call options maturity $T = 1$, and all other parameters as in Table 1.

3.3.2 Effects of Various Parameters on Implied Volatility

Figure 8 illustrates the effects of various parameters on the implied volatility, which seems to increase in σ , λ , and to decrease in average maturity of the bond profile $1/m$, and to decrease in α ; all of these make intuitively sense. Both σ and λ seem to have significant impacts on implied volatility, while α and $1/m$ do not seem to have similar significance.

Figure 9 aims at comparing the impacts of jump volatility and diffusion volatility by fixing the total volatility. It is interesting to observe that even for very long maturity options, such as $T = 8$ years, the implied volatility smile is still significant, due to both default risk and jump

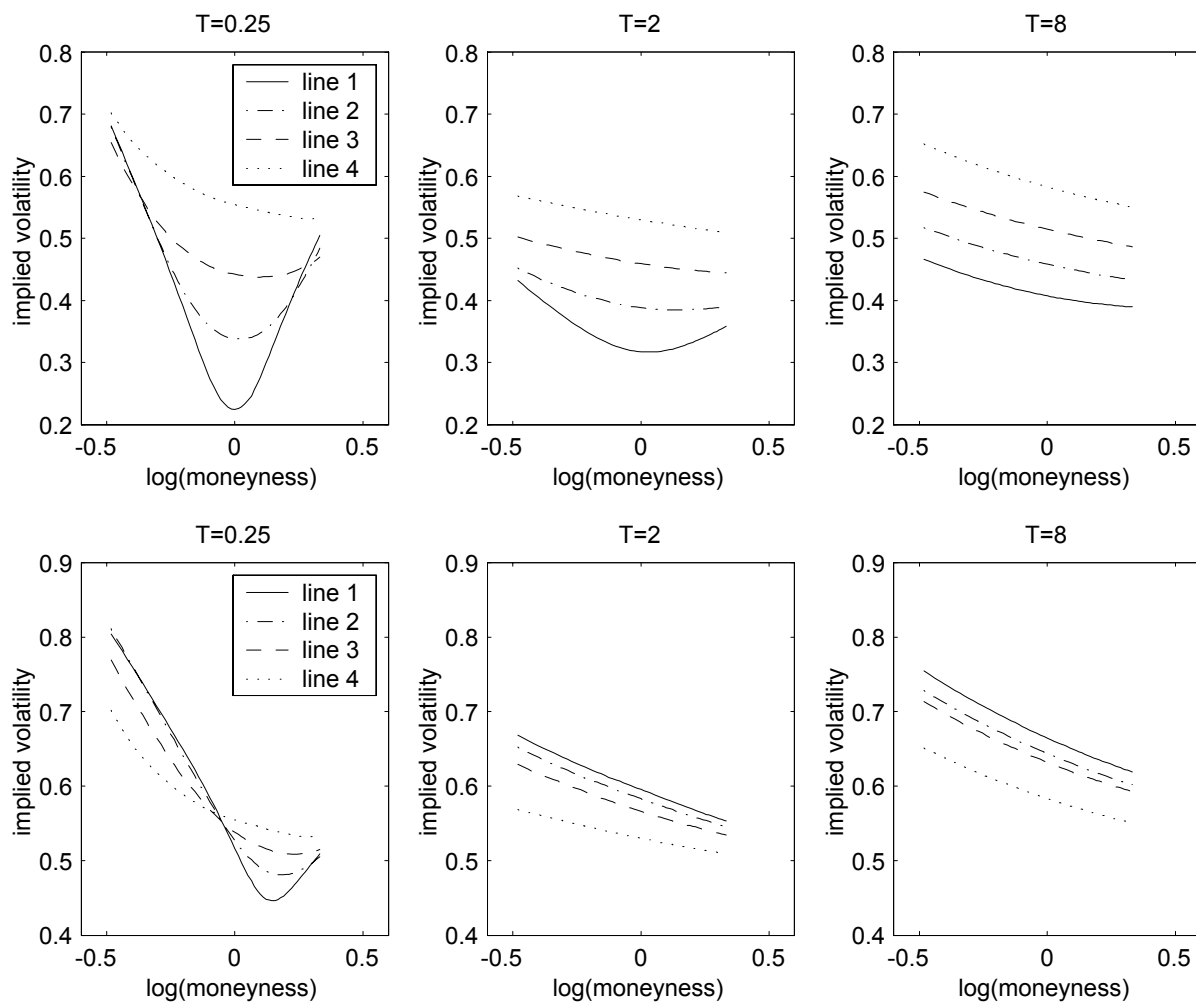


Figure 9: Implied volatility as jump volatility vs diffusion volatility with the total volatility being fixed as 40%. The first row is for Case B, with line 1 for $\sigma = 0.1, \lambda = 0.26$, line 2 for $\sigma = 0.2, \lambda = 0.20$, line 3 for $\sigma = 0.3, \lambda = 0.12$, line 4 for $\sigma = 0.4, \lambda = 0$. The second row is for Case C, with line 1 for $\sigma = 0.1, \lambda = 4.47$, line 2 for $\sigma = 0.2, \lambda = 3.57$, line 3 for $\sigma = 0.3, \lambda = 2.08$, line 4 for $\sigma = 0.4, \lambda = 0$. All the other parameters follow Table 1. Note that the implied volatility is still significant even for $T = 8$ years.

risk. This should be compared with Lévy process models without default risk, in which case the implied volatility smile tend to disappear for long maturity options, as the jump impacts are washed out in long terms; see Cont and Tankov (2003). However, the combination of jump and default seems to prolong the effect of implied volatility significantly.

4 Conclusion

We have demonstrated the significant impacts of jump risk and endogenous default on credit spreads, on optimal capital structure, and on implied volatility of equity options. The jump and endogenous default can produce a variety of non-zero credit spreads, upward, downward, and humped shapes, consistent with empirical findings of investment grade and speculative grade bonds. The jump risk leads to much lower optimal debt/equity ratios, helping to explain why some high tech companies (such as biotech and Internet companies) have almost no debt. The two-sided jumps lead to a variety of shapes for the implied volatility of equity options even for long maturity options; and although in general credit spreads and implied volatility tend to move in the same direction for exogenous default, but this may not be true in presence of endogenous default and jumps.

There are several possible directions for future research. First, it will be of interest to study convertible bonds with jump risk, as convertible bonds provide a natural link between credit spreads and equity options. Second, the model in the current paper is only a one dimensional model. Extensions of the model to higher dimensions, so that one can study correlated default events and pricing of basket credit default swaps (CDS) and collateralized debt obligations (CDO), will be very useful. References of models for these products can be found in, e.g., Sirbu and Shreve (2005), Medova and Smith (2004), and Hurd and Kuznetsov (2005a, 2005b).

A Proof of Lemma 1

For any $\beta > 0$, consider the Laplace transform of the bond price

$$\begin{aligned} & \int_0^{+\infty} e^{-\beta T} B(V; V_B, T) dT \\ = & \left(1 - \frac{\rho}{r}\right) \int_0^{+\infty} e^{-(r+\beta)T} P[\tau \geq T] dT + \frac{\alpha}{P} \frac{m+r}{m} \int_0^{+\infty} e^{-(\beta+r)T} E[V_\tau \mathbf{1}_{\{\tau \leq T\}}] dT \\ & + \frac{\rho}{r\beta} - \frac{\rho}{r} \int_0^{+\infty} e^{-\beta T} E[e^{-r\tau} \mathbf{1}_{\{\tau \leq T\}}] dT. \end{aligned}$$

By Fubini's theorem, we can see that the three integral terms inside are

$$\begin{aligned} & \int_0^{+\infty} e^{-(r+\beta)T} P[\tau \geq T] dT = E\left[\int_0^\tau e^{-(r+\beta)T} dT\right] = \frac{1}{r+\beta} [1 - Ee^{-(r+\beta)\tau}], \\ & \int_0^{+\infty} e^{-(\beta+r)T} E[V_\tau \mathbf{1}_{\{\tau \leq T\}}] dT = E[V_\tau \int_\tau^{+\infty} e^{-(\beta+r)T} dT] = \frac{1}{\beta+r} E[V_\tau e^{-(\beta+r)\tau}], \\ & \int_0^{+\infty} e^{-\beta T} E[e^{-r\tau} \mathbf{1}_{\{\tau \leq T\}}] dT = E[e^{-r\tau} \int_0^{+\infty} e^{-\beta T} \mathbf{1}_{\{\tau \leq T\}} dT] = \frac{1}{\beta} E[e^{-(r+\beta)\tau}]. \end{aligned}$$

In summary, we know that

$$\int_0^{+\infty} e^{-\beta T} B(V; V_B, T) dT = \frac{\rho + \beta}{\beta(r + \beta)} [1 - E[e^{-(r+\beta)\tau}]] + \frac{\alpha(m + r)}{mP(\beta + r)} E[V_\tau e^{-(\beta+r)\tau}].$$

Define $X_t = \ln(V_t/V)$, which means $X_t = (r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi)t + \sigma W_t + \sum_{i=1}^{N_t} Y_i$, and consider $\tilde{X}_t = -X_t$, $\tilde{\tau} \equiv \inf\{t \geq 0 : \tilde{X}_t \geq -\ln(\frac{V_B}{V})\}$. It is easy to see that $\tilde{\tau} = \tau$, so that we only need the exact forms of $E[e^{-(\beta+r)\tilde{\tau} + X_{\tilde{\tau}}}]$ and $E[e^{-(r+\beta)\tilde{\tau}}]$, which are all given in Kou and Wang (2003). In particular, with the notations of $G(\cdot)$, $\gamma_{1,\beta}$, $\gamma_{2,\beta}$, $-\gamma_{3,\beta}$, $-\gamma_{4,\beta}$ and $c_{1,\beta}$, $c_{2,\beta}$, $d_{1,\beta}$, $d_{2,\beta}$, we have

$$\begin{aligned} \int_0^{+\infty} e^{-\beta T} B(V; V_B, T) dT &= \frac{\rho + \beta}{\beta(r + \beta)} - \frac{\rho + \beta}{\beta(r + \beta)} E[e^{-(r+\beta)\tilde{\tau}}] + \frac{\alpha(m + r)}{mP(\beta + r)} V E[e^{-(\beta+r)\tilde{\tau} - \tilde{X}_{\tilde{\tau}}}] \\ &= \frac{\rho + \beta}{\beta(r + \beta)} - \frac{\rho + \beta}{\beta(r + \beta)} \left\{ d_{1,\beta} e^{\gamma_{1,\beta} \ln(V_B/V)} + d_{2,\beta} e^{\gamma_{2,\beta} (\ln(V_B/V))} \right\} \\ &\quad + \frac{\alpha(m + r)}{mP(\beta + r)} V e^{\ln(V_B/V)} \left\{ c_{1,\beta} e^{\gamma_{1,\beta} \ln(V_B/V)} + c_{2,\beta} e^{\gamma_{2,\beta} (\ln(V_B/V))} \right\}, \end{aligned}$$

from which the conclusion follows. The debt value follows readily by letting $\beta = m$. Next,

$$v(V; V_B) = V + \frac{P\kappa\rho}{r} \{1 - E[e^{-r\tilde{\tau}}]\} - (1 - \alpha) V E[e^{-r\tilde{\tau} - \tilde{X}_{\tilde{\tau}}}],$$

from which the result follows. Q. E. D.

B Proof of Theorem 1 and the Local Convexity

Lemma B.1. Consider the function $f(x) = Ax^{\alpha_1} + Bx^{\beta_1} - Cx^{\alpha_2} - Dx^{\beta_2}$, $0 \leq x \leq 1$. Note that $f(1) = A + B - C - D$. In the case of $0 \leq \alpha_1 \leq \alpha_2 \leq \beta_1 \leq \beta_2$, if $A + B \geq C + D$ and $A \geq C$ then $f(x) \geq 0$ for all $0 \leq x \leq 1$.

Proof. Simply note that

$$f(x) \geq Ax^{\alpha_2} + Bx^{\beta_2} - Cx^{\alpha_2} - Dx^{\beta_2} = x^{\alpha_2} \left\{ (A - C) - (D - B)x^{\beta_2 - \alpha_2} \right\} \geq 0.$$

Lemma B.2. We have

$$\epsilon \geq \frac{\frac{\rho+m}{r+m} d_{1,m} \gamma_{1,m} - \kappa \frac{\rho}{r} d_{1,0} \gamma_{1,0}}{\alpha c_{1,m} (\gamma_{1,m} + 1) + (1 - \alpha) c_{1,0} (\gamma_{1,0} + 1)}. \quad (6)$$

Proof. By the definition of $d_{1,m}$, $d_{2,m}$, $d_{1,0}$, $d_{2,0}$ and $c_{1,m}$, $c_{2,m}$, $c_{1,0}$, $c_{2,0}$, and the fact that $\gamma_{1,m} > \gamma_{1,0}$ and $\gamma_{2,m} > \gamma_{2,0}$, we have

$$d_{2,m} \gamma_{2,m} c_{1,0} (\gamma_{1,0} + 1) - d_{1,m} \gamma_{1,m} c_{2,0} (\gamma_{2,0} + 1)$$

$$\begin{aligned}
&= \frac{\gamma_{1,m}\gamma_{2,m}(\gamma_{1,0}+1)(\gamma_{2,0}+1)}{\eta_d(\eta_d+1)(\gamma_{2,m}-\gamma_{1,m})(\gamma_{2,0}-\gamma_{1,0})} [(\eta_d-\gamma_{1,0})(\gamma_{2,m}-\eta_d) - (\eta_d-\gamma_{1,m})(\gamma_{2,0}-\eta_d)] \geq 0; \\
&\quad d_{1,0}\gamma_{1,0}c_{2,m}(\gamma_{2,m}+1) - d_{2,0}\gamma_{2,0}c_{1,m}(\gamma_{1,m}+1) \\
&= \frac{\gamma_{1,0}\gamma_{2,0}(\gamma_{1,m}+1)(\gamma_{2,m}+1)}{\eta_d(\eta_d+1)(\gamma_{2,m}-\gamma_{1,m})(\gamma_{2,0}-\gamma_{1,0})} [(\eta_d-\gamma_{1,m})(\gamma_{2,0}-\eta_d) - (\eta_d-\gamma_{1,0})(\gamma_{2,m}-\eta_d)] \leq 0.
\end{aligned}$$

These inequalities along with the fact that

$$d_{2,m}\gamma_{2,m}c_{1,m}(\gamma_{1,m}+1) = d_{1,m}\gamma_{1,m}c_{2,m}(\gamma_{2,m}+1), \quad d_{2,0}\gamma_{2,0}c_{1,0}(\gamma_{1,0}+1) = d_{1,0}\gamma_{1,0}c_{2,0}(\gamma_{2,0}+1),$$

yield

$$\frac{\frac{\rho+m}{r+m}d_{2,m}\gamma_{2,m} - \frac{\kappa\rho}{r}d_{2,0}\gamma_{2,0}}{(1-\alpha)c_{2,0}(\gamma_{2,0}+1) + \alpha c_{2,m}(\gamma_{2,m}+1)} \geq \frac{\frac{\rho+m}{r+m}d_{1,m}\gamma_{1,m} - \frac{\kappa\rho}{r}d_{1,0}\gamma_{1,0}}{(1-\alpha)c_{1,0}(\gamma_{1,0}+1) + \alpha c_{1,m}(\gamma_{1,m}+1)},$$

from which the conclusion follows as $a/b > c/d$ if and only if $\frac{a+b}{c+d} > \frac{c}{d}$. Q.E.D.

Lemma B.3. For any $V \geq V_B \geq \epsilon P$, we have $H \leq 0$, where

$$\begin{aligned}
H &:= \frac{(\rho+m)P}{(r+m)V_B} \left\{ d_{1,m}\gamma_{1,m} \left(\frac{V_B}{V}\right)^{\gamma_{1,m}} + d_{2,m}\gamma_{2,m} \left(\frac{V_B}{V}\right)^{\gamma_{2,m}} \right\} \\
&\quad - \frac{P\kappa\rho}{rV_B} \left\{ d_{1,0}\gamma_{1,0} \left(\frac{V_B}{V}\right)^{\gamma_{1,0}} + d_{2,0}\gamma_{2,0} \left(\frac{V_B}{V}\right)^{\gamma_{2,0}} \right\} \\
&\quad - (1-\alpha) \left\{ c_{1,0}(\gamma_{1,0}+1) \left(\frac{V_B}{V}\right)^{\gamma_{1,0}} + c_{2,0}(\gamma_{2,0}+1) \left(\frac{V_B}{V}\right)^{\gamma_{2,0}} \right\} \\
&\quad - \alpha \left\{ c_{1,m}(\gamma_{1,m}+1) \left(\frac{V_B}{V}\right)^{\gamma_{1,m}} + c_{2,m}(\gamma_{2,m}+1) \left(\frac{V_B}{V}\right)^{\gamma_{2,m}} \right\}.
\end{aligned}$$

Proof. Note that

$$H \leq C_2 \left(\frac{V_B}{V}\right)^{\gamma_{1,m}} + D_2 \left(\frac{V_B}{V}\right)^{\gamma_{2,m}} - A_2 \left(\frac{V_B}{V}\right)^{\gamma_{1,0}} - B_2(\gamma_{2,0}+1) \left(\frac{V_B}{V}\right)^{\gamma_{2,0}},$$

where

$$\begin{aligned}
A_2 &= \frac{P\kappa\rho}{rV_B}d_{1,0}\gamma_{1,0} + \alpha c_{1,m}(\gamma_{1,m}+1) + (1-\alpha)c_{1,0}(\gamma_{1,0}+1), \\
B_2 &= \frac{P\kappa\rho}{rV_B}d_{2,0}\gamma_{2,0} + \alpha c_{2,m}(\gamma_{2,m}+1) + (1-\alpha)c_{2,0}(\gamma_{2,0}+1), \\
C_2 &= \frac{(\rho+m)P}{(r+m)V_B}d_{1,m}\gamma_{1,m}, \quad D_2 = \frac{(\rho+m)P}{(r+m)V_B}d_{2,m}\gamma_{2,m}.
\end{aligned}$$

Since $0 < \gamma_{1,0} \leq \gamma_{1,m} < \gamma_{2,0} < \gamma_{2,m}$, by Lemma B.1 we only need to show $A_2 + B_2 \geq C_2 + D_2$ and $A_2 \geq C_2$. To do this, note that, since $c_{1,m} + c_{2,m} = 1$ and $c_{1,0} + c_{2,0} = 1$, we have

$$\epsilon P = \frac{C_2 + D_2 - \frac{\kappa\rho P}{rV_B}(d_{1,0}\gamma_{1,0} + d_{2,0}\gamma_{2,0})}{A_2 + B_2 - \frac{\kappa\rho P}{rV_B}(d_{1,0}\gamma_{1,0} + d_{2,0}\gamma_{2,0})} V_B.$$

The fact $V_B \geq \epsilon P$ implies that $A_2 + B_2 \geq C_2 + D_2$. By (6),

$$\frac{V_B}{P} \geq \epsilon \geq \frac{\frac{\rho+m}{r+m}d_{1,m}\gamma_{1,m} - \kappa\frac{\rho}{r}d_{1,0}\gamma_{1,0}}{\alpha c_{1,m}(\gamma_{1,m} + 1) + (1 - \alpha)c_{1,0}(\gamma_{1,0} + 1)}.$$

Therefore, $A_2 \geq C_2$, and the conclusion follows. Q. E. D.

Now we are in a position to prove that the optimal $V_B^* = \epsilon P$. The proof is based on four facts:

Fact (i): The optimal V_B must satisfy $V_B \geq \epsilon P$. To show this, note that for all $V' > V_B$, $0 \leq S(V'; V_B)$, which is equivalent to say that V_B must satisfy the constraints that for all $0 < x (= V_B/V') < 1$,

$$\begin{aligned} & \frac{V_B}{x} + \frac{P\kappa\rho}{r} \{1 - (d_{1,0}x^{\gamma_{1,0}} + d_{2,0}x^{\gamma_{2,0}})\} - \frac{(\rho+m)P}{r+m} \{1 - (d_{1,m}x^{\gamma_{1,m}} + d_{2,m}x^{\gamma_{2,m}})\} \\ & - (1 - \alpha)V_B \{c_{1,0}x^{\gamma_{1,0}} + c_{2,0}x^{\gamma_{2,0}}\} - \alpha V_B \{c_{1,m}x^{\gamma_{1,m}} + c_{2,m}x^{\gamma_{2,m}}\} \geq 0. \end{aligned}$$

Rearranging the terms, we have for all $0 < x < 1$,

$$V_B \geq \frac{\frac{(\rho+m)P}{r+m} \{1 - (d_{1,m}x^{\gamma_{1,m}} + d_{2,m}x^{\gamma_{2,m}})\} - \frac{P\kappa\rho}{r} \{1 - (d_{1,0}x^{\gamma_{1,0}} + d_{2,0}x^{\gamma_{2,0}})\}}{\frac{1}{x} - (1 - \alpha) \{c_{1,0}x^{\gamma_{1,0}} + c_{2,0}x^{\gamma_{2,0}}\} - \alpha \{c_{1,m}x^{\gamma_{1,m}} + c_{2,m}x^{\gamma_{2,m}}\}}.$$

In particular,

$$\begin{aligned} V_B & \geq \lim_{x \rightarrow 1} \frac{\frac{(\rho+m)P}{r+m} \{1 - (d_{1,m}x^{\gamma_{1,m}} + d_{2,m}x^{\gamma_{2,m}})\} - \frac{P\kappa\rho}{r} \{1 - (d_{1,0}x^{\gamma_{1,0}} + d_{2,0}x^{\gamma_{2,0}})\}}{\frac{1}{x} - (1 - \alpha) \{c_{1,0}x^{\gamma_{1,0}} + c_{2,0}x^{\gamma_{2,0}}\} - \alpha \{c_{1,m}x^{\gamma_{1,m}} + c_{2,m}x^{\gamma_{2,m}}\}} \\ & = P \cdot \frac{\frac{\rho+m}{r+m}(d_{1,m}\gamma_{1,m} + d_{2,m}\gamma_{2,m}) - \frac{\kappa\rho}{r}(d_{1,0}\gamma_{1,0} + d_{2,0}\gamma_{2,0})}{(1 - \alpha)(c_{1,0}\gamma_{1,0} + c_{2,0}\gamma_{2,0}) + \alpha(c_{1,m}\gamma_{1,m} + c_{2,m}\gamma_{2,m}) + 1} = \epsilon P, \end{aligned}$$

thanks to the L'Hospital rule.

Fact (ii): The solution of $\frac{\partial S(V; V_B)}{\partial V}|_{V=V_B} = 0$ is given by $V_B = \epsilon P$. Indeed, we have

$$\begin{aligned} \frac{\partial}{\partial V} S(V; V_B) & = 1 + \frac{P\kappa\rho}{r} \frac{1}{V} \left\{ d_{1,0}\gamma_{1,0} \left(\frac{V_B}{V}\right)^{\gamma_{1,0}} + d_{2,0}\gamma_{2,0} \left(\frac{V_B}{V}\right)^{\gamma_{2,0}} \right\} \\ & \quad - \frac{(\rho+m)P}{r+m} \frac{1}{V} \left\{ d_{1,m}\gamma_{1,m} \left(\frac{V_B}{V}\right)^{\gamma_{1,m}} + d_{2,m}\gamma_{2,m} \left(\frac{V_B}{V}\right)^{\gamma_{2,m}} \right\} \\ & \quad + \alpha \frac{V_B}{V} \left\{ c_{1,m}\gamma_{1,m} \left(\frac{V_B}{V}\right)^{\gamma_{1,m}} + c_{2,m}\gamma_{2,m} \left(\frac{V_B}{V}\right)^{\gamma_{2,m}} \right\} \\ & \quad + (1 - \alpha) \frac{V_B}{V} \left\{ c_{1,0}\gamma_{1,0} \left(\frac{V_B}{V}\right)^{\gamma_{1,0}} + c_{2,0}\gamma_{2,0} \left(\frac{V_B}{V}\right)^{\gamma_{2,0}} \right\}. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial S(V; V_B)}{\partial V}|_{V=V_B} & = 1 + \frac{1}{V_B} \frac{P\kappa\rho}{r} \{\gamma_{1,0}d_{1,0} + \gamma_{2,0}d_{2,0}\} + (1 - \alpha) \{c_{1,0}\gamma_{1,0} + c_{2,0}\gamma_{2,0}\} \\ & \quad - \frac{1}{V_B} \frac{(\rho+m)P}{r+m} \{d_{1,m}\gamma_{1,m} + d_{2,m}\gamma_{2,m}\} + \alpha \{c_{1,m}\gamma_{1,m} + c_{2,m}\gamma_{2,m}\}, \end{aligned}$$

which shows (ii).

Fact (iii): For all $V \geq V_B \geq \epsilon P$, we have $\frac{\partial S(V; V_B)}{\partial V} \geq 0$. To show this, note that

$$\begin{aligned} \frac{\partial}{\partial V} S(V; V_B) &= -\frac{V_B}{V} H + 1 - \alpha \frac{V_B}{V} \left\{ c_{1,m} \left(\frac{V_B}{V} \right)^{\gamma_{1,m}} + c_{2,m} \left(\frac{V_B}{V} \right)^{\gamma_{2,m}} \right\} \\ &\quad - (1 - \alpha) \frac{V_B}{V} \left\{ c_{1,0} \left(\frac{V_B}{V} \right)^{\gamma_{1,0}} + c_{2,0} \left(\frac{V_B}{V} \right)^{\gamma_{2,0}} \right\} \geq 0, \end{aligned}$$

via Lemma B.3 and the facts that $c_{1,m} + c_{2,m} = 1$ and $c_{1,0} + c_{2,0} = 1$.

Fact (iv): We have $S(V; y_1) \geq S(V; y_2)$, if $\epsilon P \leq y_1 \leq y_2 \leq V$. Indeed, for any fixed V , we have $\frac{\partial}{\partial V_B} S(V; V_B) = H \leq 0$ for all $0 \leq V_B/V \leq 1$.

With the above four facts, we can show that ϵP is the optimal solution. Indeed, first, ϵP satisfies the constraints that $S(V'; \epsilon P) \geq 0$ for all $V' \geq \epsilon P$, because $S(\epsilon P, \epsilon P) = 0$ and S is nondecreasing in V by (iii); second, any $V_B \in (\epsilon P, V]$ cannot be better, as by (iv) $S(V; \epsilon P) \geq S(V; V_B)$; and any V_B less than ϵP is ruled out by (i).

For the second stage optimization problem, plugging $V_B = \epsilon P$ into $v(V, V_B)$, we have

$$\begin{aligned} v(V; \epsilon P) &= V + \frac{P\kappa\rho}{r} \left\{ 1 - d_{1,0} \left(\frac{\epsilon P}{V} \right)^{\gamma_{1,0}} - d_{2,0} \left(\frac{\epsilon P}{V} \right)^{\gamma_{2,0}} \right\} \\ &\quad - (1 - \alpha) V \left\{ c_{1,0} \left(\frac{\epsilon P}{V} \right)^{\gamma_{1,0}+1} + c_{2,0} \left(\frac{\epsilon P}{V} \right)^{\gamma_{2,0}+1} \right\} \\ &= V \left\{ 1 + \frac{\kappa\rho}{r} \left(\frac{P}{V} \right) - B_3 \left(\frac{P}{V} \right)^{\gamma_{1,0}+1} - C_3 \left(\frac{P}{V} \right)^{\gamma_{2,0}+1} \right\}, \end{aligned}$$

$$B_3 := (1 - \alpha)c_{1,0}\epsilon^{\gamma_{1,0}+1} + \frac{\kappa\rho}{r}d_{1,0}\epsilon^{\gamma_{1,0}}, \quad C_3 := (1 - \alpha)c_{2,0}\epsilon^{\gamma_{2,0}+1} + \frac{\kappa\rho}{r}d_{2,0}\epsilon^{\gamma_{2,0}}.$$

Since $B_3 > 0$, $C_3 > 0$, the function $v(V; \epsilon P)$ is concave in P . Q. E. D.

Remark B.1: In the above proof the results (ii) and (iii) actually imply a local convexity at the optimal V_B . More precisely, for all $V \geq V_B \geq \epsilon P$, we have the local convexity

$$\frac{\partial^2 S(V; V_B)}{\partial V^2} \Big|_{V=V_B} \geq 0.$$

We can show this by contradiction. Suppose not. By (ii), we have

$$0 > \frac{\partial^2 S(V; V_B)}{\partial V^2} \Big|_{V=V_B} = \lim_{V' \downarrow V_B} \frac{\frac{\partial S(V; V_B)}{\partial V} - \frac{\partial S(V; V_B)}{\partial V} \Big|_{V=V_B}}{V' - V_B} = \lim_{V' \downarrow V_B} \frac{\frac{\partial S(V; V_B)}{\partial V}}{V' - V_B}.$$

Thus, there must be $\tilde{V} > V_B \geq \epsilon P$ such that $\frac{\partial S(\tilde{V}; V_B)}{\partial \tilde{V}} < 0$, which contradicts to the result (ii). The local convexity has been conjectured in Leland and Toft (1996, footnote 9) for the Brownian model, and in Hilberink and Rogers (2002) for a one-sided jump model; both papers verified

it numerically. Here we are able to give a proof for the local convexity for the two-sided jump model, mainly because we prove the local convexity indirectly by using Laplace transforms, while it is difficult to verify the convexity directly without using the Laplace transform even in the case of Brownian motion, due to the difficulty in study the monotonicity of functions involving various normal distribution functions.

C Proof of Theorem 2

Note, first,

$$1 = \lim_{T \rightarrow 0} \frac{1}{T} E \left[\int_0^T e^{-rs} ds \right] \geq \limsup_{T \rightarrow 0} \frac{1}{T} E \left[\int_0^{\tau \wedge T} e^{-rs} ds \right] \geq \liminf_{T \rightarrow 0} E \left[\frac{1}{T} \int_0^T e^{-rs} ds \cdot \mathbf{1}_{\{\tau \geq T\}} \right] = 1,$$

by the dominated convergence theorem. Thus,

$$1 = \lim_{T \rightarrow 0} \frac{1}{T} E \left[\int_0^{\tau \wedge T} e^{-rs} ds \right].$$

Second, by the conditional memoryless property of the overshoot distribution

$$E[V_\tau | \tau \leq T] = V \cdot E \left[\exp \left(\log \left(\frac{V_\tau}{V} \right) \right) | \tau \leq T \right] = V \cdot \frac{V_B}{V} \frac{\eta_d}{\eta_d + 1} + o(T),$$

as the probability of the default caused by the diffusion is $o(T)$. Third,

$$P[\tau \leq T] = \lambda p_d T \left(\frac{V_B}{V} \right)^{\eta_d} + o(T).$$

In summary, we have,

$$\begin{aligned} & B(V, 0; V_B, T) \\ &= e^{-rT} P[\tau > T] + \frac{\alpha}{P} \frac{m+r}{m} E[V_\tau \cdot \mathbf{1}_{\{\tau \leq T\}}] + \rho T + o(T) \\ &= (1-rT) \left(1 - \lambda p_d T \left(\frac{V_B}{V} \right)^{\eta_d} \right) + \frac{\alpha V}{P} \frac{m+r}{m} E[E[V_\tau | \tau \leq T] \cdot \lambda p_d T \left(\frac{V_B}{V} \right)^{\eta_d}] + \rho T + o(T) \\ &= 1 - \left[r + \lambda p_d \left(\frac{V_B}{V} \right)^{\eta_d} \right] T + \frac{\alpha V}{P} \frac{m+r}{m} \frac{\eta_d}{\eta_d + 1} \cdot \lambda p_d T \left(\frac{V_B}{V} \right)^{\eta_d + 1} + \rho T + o(T) \end{aligned}$$

Thus, the L'Hospital's rule leads to

$$\nu(0) = \lim_{T \rightarrow 0} \frac{1 - B(V, 0; V_B, T)}{T} + \rho = r + \lambda p_d \left(\frac{V_B}{V} \right)^{\eta_d} \left[1 - \frac{\alpha V_B}{P} \frac{m+r}{m} \frac{\eta_d}{\eta_d + 1} \right],$$

from which the proof is terminated. Q. E. D.

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