

# Excited by a quantum field: Does shape matter?<sup>1</sup>

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**Abstract.** The instantaneous transition rate of an arbitrarily accelerated Unruh-DeWitt particle detector on four-dimensional Minkowski space is ill defined without regularisation. We show that Schlicht's regularisation as the zero-size limit of a Lorentz-function spatial profile yields a manifestly well-defined transition rate with physically reasonable asymptotic properties. In the special case of stationary trajectories, including uniform acceleration, we recover the results that have been previously obtained by a regularisation that relies on the stationarity. Finally, we discuss evidence for the conjecture that the zero-size limit of the transition rate is independent of the detector profile.

## 1. Introduction

Starting with the seminal work of Unruh [1], it has now been recognised for 30 years that a uniformly accelerated observer in Minkowski space sees Minkowski vacuum as a thermal state in temperature  $T = a/(2\pi)$ , where  $a$  is the magnitude of the proper acceleration. This result is of interest already in its own right within flat spacetime quantum field theory, and it has been confirmed by a number of methods [1, 2, 3, 4, 5]. For relativists, the result is of particular interest because of its close mathematical similarity to the thermal properties of quantum fields in stationary black hole spacetimes [1, 6].

A conceptually concrete way to address quantum effects in accelerated motion is to analyse a particle detector coupled to the quantum field. For the uniformly accelerated motion, a subtlety in such an analysis arises from the fact that the motion is *stationary*, that is, the orbit of a timelike Killing vector. Because of stationarity, the first-order perturbation theory transition probability over the whole trajectory is infinite, owing to the infinite total proper time. However, this probability can be formally factorised into the product of the total proper time and a finite remainder, and the remainder can by stationarity be interpreted as the transition rate per unit proper time [7, 8, 9, 10]. This regularisation prescription can be extended from stationary trajectories in Minkowski space to curved spacetime [11], both for the Unruh-DeWitt monopole detector [1, 2] and a variety of its generalisations. A recent review can be found in [12].

For nonstationary motions the transition rate need not be constant along the detector's trajectory, and a regularisation that relies on stationarity is no longer available. The first message of this talk is:

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- For an Unruh-DeWitt monopole detector, the instantaneous transition rate is ill-defined without regularisation.

This observation appears to have been first made by Schlicht [13, 14], who showed that a conventional  $i\epsilon$  regularisation yields a Lorentz-noninvariant transition rate for uniformly accelerated motion. We discuss the mathematical reason for this phenomenon and show that the  $i\epsilon$  regularisation leads to a Lorentz-noninvariant transition rate for *every* non-inertial trajectory. The conventional  $i\epsilon$  prescription does therefore not provide a physically acceptable regularisation for the instantaneous transition rate.

Schlicht proposed to regularise the transition rate in arbitrary motion by making the detector spatially extended in its instantaneous rest frame, with a spatial sensitivity profile that has a certain fixed shape but depends on a size parameter, and then letting the size parameter approach zero [13, 14]. He showed that this regularisation yields the expected Planckian spectrum for uniform acceleration, and he analysed a selection of nonstationary trajectories via mainly numerical methods. We show that Schlicht's regularisation yields a well-defined transition rate for every trajectory satisfying certain technical conditions, and we express the result as a manifestly finite integral formula that no longer involves regulators or limits. For the stationary trajectories the result agrees with that obtained in [2, 7, 8], and for nonstationary trajectories we extract asymptotic results that appear physically reasonable. The second message of this talk thus is:

- A spatial sensitivity profile is a viable regulator for the instantaneous transition rate.

The rest of the talk will put some flesh on these messages. The main conclusions rely on a particular choice of the spatial profile function, but in section 6 we present some evidence suggesting that that the zero size limit may be insensitive to the detailed form of the profile. The talk is based on [15], where further detail can be found.

We work in four-dimensional Minkowski spacetime with metric signature  $(-+++)$  and in units in which  $\hbar = c = 1$ . Boldface letters denote spatial three-vectors and sans-serif letters spacetime four-vectors, and a square of a spatial vector (respectively spacetime vector) is understood in the sense of the Euclidean (Minkowskian) scalar product.

## 2. Unruh-DeWitt detector

Consider a pointlike detector that moves in four-dimensional Minkowski space on the world line  $x(\tau)$ , where  $\tau$  is the proper time. We take the detector to have two quantum states, denoted by  $|0\rangle_d$  and  $|1\rangle_d$ , which are eigenstates of the detector internal Hamiltonian  $H_d$  with the respective eigenvalues 0 and  $\omega$ ,  $\omega \neq 0$ . The detector is coupled to the real, massless scalar field  $\phi$  with the interaction Hamiltonian

$$H_{\text{int}} = c\chi(\tau)\mu(\tau)\phi(x(\tau)) , \quad (1)$$

where  $c$  is a coupling constant and  $\mu(\tau)$  is the detector's monopole moment operator, evolving in the Heisenberg picture under  $H_d$ .  $\chi(\tau)$  is a switching function, which specifies how the interaction is switched on and off by an external agent.

Suppose first that the switching function is smooth and has compact support, so that the initial and final states can be described in terms of the uncoupled system. If before the interaction the field is in the Minkowski vacuum  $|0\rangle$  and the detector in the state  $|0\rangle_d$ , the first-order perturbation theory probability of finding the detector in the state  $|1\rangle_d$  after the interaction is [1, 2, 3, 5, 16]

$$P(\omega) = c^2 |{}_d\langle 0|\mu(0)|1\rangle_d|^2 F(\omega) , \quad (2)$$

where the response function  $F(\omega)$  is given by

$$F(\omega) = \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau'' e^{-i\omega(\tau' - \tau'')} \chi(\tau')\chi(\tau'') W(\tau', \tau'') , \quad (3)$$

and the correlation function  $W$  is the pull-back of the Wightman distribution,

$$W(\tau', \tau'') := \langle 0 | \phi(\mathbf{x}(\tau')) \phi(\mathbf{x}(\tau'')) | 0 \rangle . \quad (4)$$

As  $W$  is a well-defined distribution on  $\mathbb{R} \times \mathbb{R}$  [16], the transition probability given by (2)–(4) is well defined.

Suppose then that no friendly neighbourhood external agent is available to switch the interaction off before we observe the detector. We wish to ask: *What is the probability of finding the detector in the state  $|1\rangle_d$  while the interaction is still switched on?* This is arguably the question encountered in a practical measurement where one looks at an ensemble of accelerated detectors (say, atoms or ions) at a given moment of time and counts what fraction of the detectors are in an excited state.

An attempt to answer this question within the detector model (1) would be to introduce in the switching function a sharp cutoff,  $\chi(\tau') \rightarrow \chi(\tau')\Theta(\tau - \tau')$ , where  $\tau$  is the proper time at which the detector is read and  $\Theta$  is the Heaviside function. If we further push the switch-on to the asymptotic past, this would mean making in (2)–(4) the replacement  $\chi(\tau') \rightarrow \Theta(\tau - \tau')$ . Formal manipulations then yield for the  $\tau$ -derivative of the response function the expression

$$\dot{F}_\tau(\omega) = 2 \operatorname{Re} \int_0^\infty ds e^{-i\omega s} W(\tau, \tau - s) . \quad (5)$$

$\dot{F}_\tau(\omega)$  differs from the instantaneous transition rate only by a multiplicative constant that is independent of the trajectory, and we shall from now on suppress this constant.

The problem with these manipulations is that formula (5) is ambiguous, since  $W$  is a distribution with a singularity at the coincidence limit and the integration range has a sharp boundary at this singularity. To see that the problem is significant, suppose we go to a specific Lorentz frame,  $\mathbf{x} = (t, \mathbf{x})$ , replace the Wightman distribution by its conventional  $i\epsilon$  regularisation,

$$\langle 0 | \phi(\mathbf{x}) \phi(\mathbf{x}') | 0 \rangle_\epsilon = \frac{-1}{4\pi^2} \frac{1}{(t - t' - i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2} , \quad \epsilon > 0 , \quad (6)$$

and take the limit  $\epsilon \rightarrow 0_+$  after performing the integral in (5). Assuming that the trajectory is sufficiently differentiable and has suitable falloff properties in the distant past, the result is [15]

$$\begin{aligned} \dot{F}_\tau(\omega) &= -\frac{\omega}{4\pi} + \frac{1}{2\pi^2} \int_0^\infty ds \left( \frac{\cos(\omega s)}{(\Delta\mathbf{x})^2} + \frac{1}{s^2} \right) \\ &\quad - \frac{1}{4\pi^2} \frac{\ddot{t}}{(\dot{t}^2 - 1)^{3/2}} \left[ \dot{t} \sqrt{\dot{t}^2 - 1} + \ln \left( \dot{t} - \sqrt{\dot{t}^2 - 1} \right) \right] , \end{aligned} \quad (7)$$

where  $\Delta\mathbf{x} := \mathbf{x}(\tau) - \mathbf{x}(\tau - s)$ . The last term in (7) vanishes for inertial trajectories but is Lorentz-noninvariant wherever the proper acceleration is nonzero. In the usual distributional setting of integrating against smooth test functions, the functions (6) duly converge to the Lorentz-invariant Wightman distribution as  $\epsilon \rightarrow 0$  [17], but the instantaneous transition rate (5) falls outside this setting because of the sharp switch-off and retains a Lorentz-noninvariant piece even in the limit  $\epsilon \rightarrow 0_+$ .

**Moral:** The instantaneous transition rate (5) is ill-defined as it stands and needs to be regularised.

### 3. Spatial profile

Schlicht [13, 14] proposed to regularise the transition rate (5) by giving the detector a spatial sensitivity profile that is rigid in the detector's instantaneous rest frame. This idea can be motivated by the observation that real material systems (say, atoms or ions) are not pointlike.

Technically, Schlicht's proposal is to replace the field operator in the interaction Hamiltonian (1) by a spatially smeared field operator,

$$\phi(\mathbf{x}(\tau)) \rightarrow \int d^3\xi \epsilon^{-3} f(\boldsymbol{\xi}/\epsilon) \phi(\mathbf{x}(\tau) + \xi^i \mathbf{e}_i(\tau)) , \quad (8)$$

where  $\mathbf{e}_i$  are three unit vectors that together with the velocity  $\dot{\mathbf{x}}$  form an orthonormal tetrad, Fermi-Walker transported along the trajectory. The four quantities  $(\tau, \boldsymbol{\xi}) = (\tau, \xi^1, \xi^2, \xi^3)$  are thus Fermi-Walker coordinates in a neighbourhood of the trajectory [18]. The profile function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  is assumed to be non-negative and to integrate to unity, and  $\epsilon$  is a positive parameter that determines the characteristic size of the smeared detector. When  $f$  is chosen to be the Lorentzian function,

$$f(\boldsymbol{\xi}) = \frac{1}{\pi^2} \frac{1}{(|\boldsymbol{\xi}|^2 + 1)^2} , \quad (9)$$

Schlicht showed that  $W$  in (5) gets replaced by

$$W_\epsilon(\tau, \tau') = \frac{1}{4\pi^2} \frac{1}{[\mathbf{x} - \mathbf{x}' - i\epsilon(\dot{\mathbf{x}} + \dot{\mathbf{x}}')]^2} , \quad (10)$$

where the unprimed and primed quantities are evaluated respectively at  $\tau$  and  $\tau'$ . Note that  $W_\epsilon$  (10) is manifestly Lorentz covariant. Schlicht further showed that the  $\epsilon \rightarrow 0_+$  limit yields the Planckian spectrum for the uniformly accelerated motion, thus agreeing with the regularisation that relies on stationarity [1, 3, 5]. He also examined the  $\epsilon \rightarrow 0_+$  limit for a number of other trajectories, with physically reasonable results.

Schlicht's results have been generalised by P. Langlois [12, 19] to a variety of situations, including Minkowski space in an arbitrary number of dimensions, quotients of Minkowski space under discrete isometry groups, the massive scalar field, the massless Dirac field and certain curved spacetimes. Langlois also observed that an alternative way to arrive at  $W_\epsilon$  (10) is to regularise the mode sum expression for the Wightman function by an exponential frequency cut-off in the detector's instantaneous rest frame, rather than in a fixed Lorentz frame.

#### 4. Lorentz-function profile: Zero-size limit

When the regularised correlation function (10) is substituted in (5), the existence of an  $\epsilon \rightarrow 0_+$  limit is not obvious for an arbitrary trajectory since  $\epsilon$  appears under the integral. However, for trajectories that are sufficiently differentiable and have suitable falloff properties in the distant past, the limit exists and equals [15]

$$\dot{F}_\tau(\omega) = -\frac{\omega}{4\pi} + \frac{1}{2\pi^2} \int_0^\infty ds \left( \frac{\cos(\omega s)}{(\Delta\mathbf{x})^2} + \frac{1}{s^2} \right) . \quad (11)$$

Since the integrand in (11) remains finite at  $s \rightarrow 0_+$  and since  $(\Delta\mathbf{x})^2 \leq -s^2$ , formula (11) is manifestly well-defined.

Formula (11) gives the transition rate as split into its odd and even parts in  $\omega$ . Another useful split is into the inertial part and the noninertial correction, as introduced for stationary trajectories in [7, 8]. This can be accomplished by a suitable addition and a subtraction in the integrand, with the result

$$\dot{F}_\tau(\omega) = -\frac{\omega}{2\pi} \Theta(-\omega) + \frac{1}{2\pi^2} \int_0^\infty ds \cos(\omega s) \left( \frac{1}{(\Delta\mathbf{x})^2} + \frac{1}{s^2} \right) . \quad (12)$$

The first term in (12) is the transition rate of a detector in inertial motion, and the integral term is thus the correction due to acceleration. As the correction is even in  $\omega$ , we see that the acceleration induces excitations and de-excitations with the same probability.

Note that the correction term in (12) is nonvanishing for *every* noninertial trajectory. Note also that inversion of the cosine transform in (12) shows that  $\dot{F}_\tau(\omega)$  fully determines  $(\Delta\mathbf{x})^2$  as a function of  $s$  and  $\tau$ .

From (12) it follows that  $\dot{F}_\tau(\omega)$  has a large  $|\omega|$  expansion that proceeds in inverse powers of  $\omega^2$ , with coefficients given by  $\tau$ -derivatives of  $\mathbf{x}(\tau)$ . In the leading order we obtain

$$\dot{F}_\tau(\omega) = -\frac{\omega}{2\pi}\Theta(-\omega) + \frac{\ddot{\mathbf{x}} \cdot \mathbf{x}^{(3)}}{24\pi^2\omega^2} + O(\omega^{-4}) \quad \text{as } |\omega| \rightarrow \infty, \quad (13)$$

which shows that for a generic trajectory the first correction to the inertial response is of order  $\omega^{-2}$ .

## 5. Examples

A case-by-case analysis of all stationary trajectories shows that the transition rate (11) for them agrees with that obtained with the regularisation that relies on stationarity [7, 8]. In particular, in the special case of uniform acceleration of magnitude  $a$  we have the Planckian spectrum,

$$\dot{F}_\tau(\omega) = \frac{\omega}{2\pi} \frac{1}{e^{2\pi\omega/a} - 1}. \quad (14)$$

As an example of nonstationary motion, consider a detector that moves in a timelike plane with the proper acceleration  $a/(1 + e^{-a\tau})$ , where  $a$  is a positive constant. In the distant past the trajectory is asymptotically inertial, and we obtain the transition rate

$$\dot{F}_\tau(\omega) = -\frac{\omega}{2\pi}\Theta(-\omega) + O(e^{2a\tau}), \quad \tau \rightarrow -\infty, \quad (15)$$

where the  $O$ -term holds uniformly in  $\omega$ . In the distant future the trajectory has asymptotically uniform acceleration of magnitude  $a$ , and we obtain the transition rate

$$\dot{F}_\tau(\omega) = \frac{\omega}{2\pi} \frac{1}{e^{2\pi\omega/a} - 1} + o(1), \quad \tau \rightarrow \infty, \quad (16)$$

where  $o(1)$  stands for a term that goes to zero as  $\tau \rightarrow \infty$ . The first term in (16) is the transition rate (14) in uniform acceleration. The asymptotics thus agrees with what one would expect on physical grounds, both in the future and in the past.

## 6. Does shape matter?

The above results rely on the choice (9) for the profile function. While all sufficiently regular profile functions are known to yield the same  $\epsilon \rightarrow 0_+$  limit for inertial motion [13], it is at present not known to what extent the  $\epsilon \rightarrow 0_+$  limit might depend on the profile function for more general motions.

There is however a modified notion of spatial smearing in which we have been able to establish a result on profile-independence. For positive  $\epsilon$ , the transition rate with this modified smearing reads

$$\dot{F}_\tau^{(\epsilon)}(\omega) := \int_{\boldsymbol{\xi} \neq \boldsymbol{\xi}'} d^3\xi d^3\xi' \epsilon^{-6} f(\boldsymbol{\xi}/\epsilon) f(\boldsymbol{\xi}'/\epsilon) G_\tau(\boldsymbol{\xi}, \boldsymbol{\xi}'; \omega), \quad (17)$$

where

$$G_\tau(\boldsymbol{\xi}, \boldsymbol{\xi}'; \omega) := 2 \operatorname{Re} \int_0^\infty ds e^{-i\omega s} \langle 0 | \phi(\mathbf{x}(\tau) + \xi^i \mathbf{e}_i(\tau)) \phi(\mathbf{x}(\tau - s) + \xi'^j \mathbf{e}_j(\tau - s)) | 0 \rangle. \quad (18)$$

Equations (17) and (18) would follow from (5) with the replacement (8) if it were known that the interchange of the  $ds$  and  $d^3\xi d^3\xi'$  integrals is valid in a sense in which  $G_\tau(\boldsymbol{\xi}, \boldsymbol{\xi}'; \omega)$  (18) contains no distribution with support at  $\boldsymbol{\xi} = \boldsymbol{\xi}'$ . While we do not know whether the interchange can be justified in this sense, we shall take equations (17) and (18) as a *definition* of a detector model in their own right, arguing that this model captures at least some of the effects of the spatial smearing of section 3.

Now, if the trajectory is real analytic and satisfies suitable falloff conditions in the distant past, and if the profile function  $f$  is smooth and has compact support, it can be shown [15] that  $\dot{F}_\tau^{(\epsilon)}(\omega)$  is well defined by (17) and (18) for sufficiently small  $\epsilon$ , and the  $\epsilon \rightarrow 0_+$  limit exists and is given by (11). As this limit agrees with that obtained with the Lorentzian profile function (9) (which is not of compact support), we suspect that the equivalence of the two models of spatial smearing could be established for at least some classes of profile functions.

## 7. Discussion

We have shown that regularising the transition rate of an accelerated Unruh-DeWitt detector on Minkowski space by a spatial profile is a mathematically well-defined procedure and yields physically viable predictions in a number of situations. For the Lorentz-function spatial profile (9) the zero size limit could be computed explicitly, leading to the transition rate (11). For other spatial profiles the results remain to some extent inconclusive but they suggest that the zero-size limit may not be sensitive to the details of the profile.

We re-emphasise that the need for a spatial smearing arose because we chose to address the instantaneous transition rate *while the interaction continues to be switched on*, rather than the total excitation probability after the interaction has been smoothly switched on and off by an external agent. It would be of interest to examine in comparison a pointlike detector whose smooth switching function is allowed to approach the step-function: Might there exist limiting prescriptions that reproduce the effects of spatial smearing?

If the detector is turned on sharply at the finite proper time  $\tau_0$ , the transition rate formula (11) is replaced by [15]

$$\dot{F}_\tau(\omega) = -\frac{\omega}{4\pi} + \frac{1}{2\pi^2} \int_0^{\tau-\tau_0} ds \left( \frac{\cos(\omega s)}{(\Delta x)^2} + \frac{1}{s^2} \right) + \frac{1}{2\pi^2(\tau - \tau_0)}, \quad \tau > \tau_0, \quad (19)$$

which is asymptotically proportional to  $(\tau - \tau_0)^{-1}$  as  $\tau \rightarrow \tau_0$ . The total transition probability, obtained by integrating the transition rate (19), is therefore infinite, owing to the violent switch-on event, regardless how small the coupling constant in the interaction Hamiltonian is. For the stationary trajectories the transition rate (11) of a detector switched on in the asymptotic past is constant in time, and the total transition probability is again infinite, now owing to the infinite amount of time elapsed in the past. In these situations one may therefore have reason to view our results, all of which were obtained within first-order perturbation theory, as suspect. However, in situations where the detector is switched on in the asymptotic past of infinite proper time and the total probability of excitation ( $\omega > 0$ ) is finite, the first-order perturbation theory result should be reliable at least for the excitation rate, although the total probability of de-excitation ( $\omega < 0$ ) then still diverges. This situation occurs for the asymptotically inertial trajectory discussed in section 5, and we expect it to occur whenever the proper acceleration vanishes sufficiently fast in the distant past.

It would be interesting to investigate to what extent our results can be generalised to the variety of situations to which Schlicht's Lorentzian profile detector was generalised in [12, 19]. For example, do the formulas (11) and (19) generalise to spacetime dimensions other than four, and if yes, what is the form of the subtraction term? Does the clean separation of the spectrum into its even and odd parts continue? Further, to what extent can the notion of spatial profile be

employed to regularise the transition rate in a curved spacetime, presumably reproducing known results for stationary trajectories [11] but also allowing nonstationary motion? In particular, might there be a connection with the regularisation prescriptions of the classical self-force problem [20, 21, 22, 23]? Finally, would a nonperturbative treatment be feasible?

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