The problem of fitting of the zero-range model of the tectonic plate under a localized boundary stress

L. Petrova[1],B. Pavlov [1,2].

[1] V.A.Fock Institute for Physics, St. Petersburg University, St.Petersburg, Russia

[2] Department of Mathematics, the University of Auckland, New Zealand,

Abstract

We suggest a method of fitting of the zero-range model of the tectonic plate under the boundary stress based on comparison of the theoretical formulae for the corresponding eigenfunctions/eigenvalues with the results of instrumental measurements of frequency and the shape and the shape of the relevant seismo-gravitational modes in the remoted zone. Comparison of the data of the instrumental measurements of the variation of frequency of seismo-gravitational oscillations with results of theoretical analysis of the fitted model permits to localize the stressed zone on the boundary of the tectonic plate and estimate the risk of a powerful earthquake at this zone.

Key-words *Thin plate, Zero-range potential, Operator extension*

1 Dynamics of the system of tectonic plates and the motivation of the zero-range model.

The lithosphere of Earth consists of 13 tectonic plates which jigsaw fit each other, contacting at some boundary zones. The plates are isolated from underlying solid structures within the Earth mantle by the liquid layer of astenosphere which is formed, due to high pressure and temperature, in the interval of depth between 100 - 200 km. Fig. 1 below [1] shows a complex

[1]Fig.1 is borrowed from the book of S. Aplonov, [3], and is included in our text on the permission of the Publishing House of the St. Petersburg University.

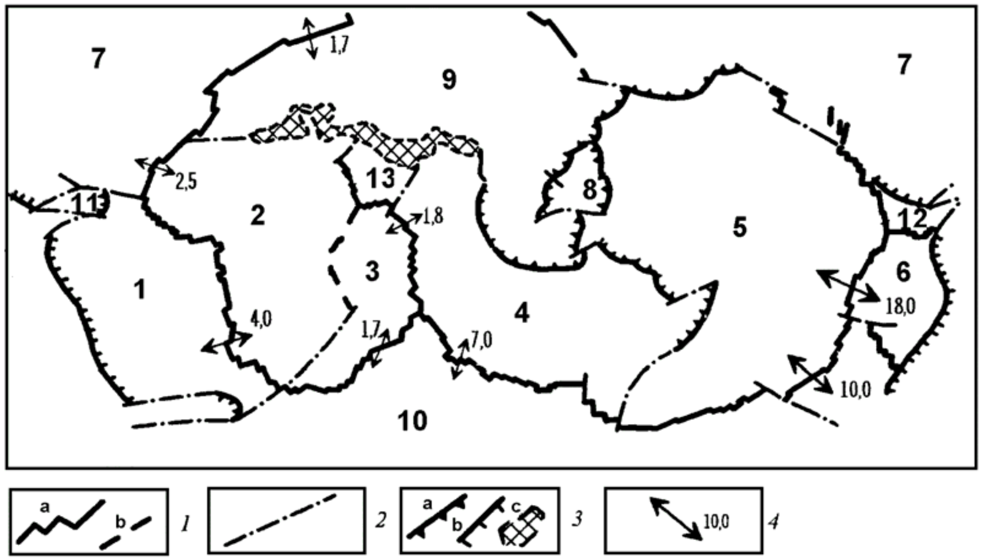

Figure 1:

Boundaries of tectonic plates: 1- divergent boundaries (a - oceanic ridges, b- continental rifts), 2- transforming boundaries, 3-convergent boundaries (a- insular, b- active continental outskirts, c - collisions of plates). Directions and velocities of movement of plates (cm/ year).

form of boundaries between tectonic plates and their movements in different directions. The plates are enumerated in the following order : 1. South-American plate, 2. African plate , 3. Somali plate, 4. Indian plate, 5. Pacific plate, 6. Naska plate, 7. North-American plate, 8. Phillippines plate, 9. Euro-Asian plate, 10. Antarctic plate, 11. Caribean plate, 12. Cocos plate, 13. Arabian plate. In [23] observations of oscillatory processes on tectonic plates - seismo-gravitational oscillations (SGO)- are described. SGO are scattered in the liquid layer of the astenosphere and the energy of the oscillations is damped in it due to viscosity. From the point of view of elasticity, the loose materials filling the gaps between neighboring tectonic plates are essentially different from the materials composing the plates. This suggests the idea that the tectonic plates are decoupled from each other everywhere on the boundary, except few relatively small zones of contact. The tectonic plates, in a certain range of frequencies $0.2 - 5h^{-1}$ are also elastically disconnected from the underlying layers of the astenosphere.

For measurements of of parameters SGO special devices were constructed, with high sensitivity to the variations of amplitudes and frequencies of SGO within the extended interval of periods 0.5 -5 hours. Observations with these devices reveal a wide spectrum of SGO and correlations of their short-time (24-hour) variations with short-time variations of the angular speed of Earth.

Direct calculations of eigenfrequencies of the bending modes of rectangular thin plate 4 000 $\times$ 8 000 km, were done, see [19], based on bi-harmonic model, with Neumann boundary conditions on the boundary, under assumption that the density of the material is 3380 kg/m^3 and the speeds of longitudinal and transversal waves are ν_s = 8080 m/sec and ν_p = 4470 m/sec respectively. It appeared that periods of the lower eigen-modes $f_{1,1} - f_{5,5}$ sit in the interval 0.21 h - 5.2 h and their total number and distribution looks similar, see Fig. 2, to SGO described in the paper [23], and other relevant literature, despite of the fact that in [19] the trivial Neumann boundary conditions were used, and the trivial (rectangular) geometry of the plate.

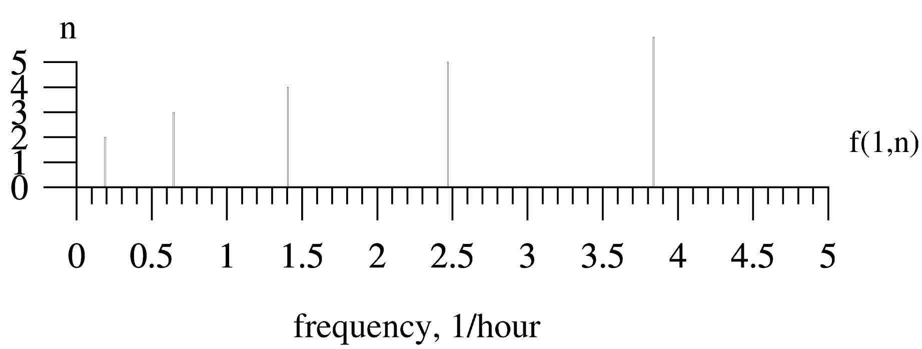

Figure 2:

All experimental data described above indicate presence of active energy in the system of tectonic plates, relevant to the inhomogeneity of the lithosphere. Some essential features of the dynamics of the frequencies of SGO may be explained based on processes involving redistribution of this energy in the astenosphere.

We consider here the scenario of arising of stress in contact zones of the plates based on variations of the angular speed of earth. Due to the liquid friction in the astenosphere, the smaller tectonic plates react immediately on these variations, and larger plates lag behind. Generically,with a certain correction depending on the shape of the plates, this is causing collision of neighboring plates and additional stress in active zones on the west boundaries of large plates with smaller plates, when the speed grows, or on the east boundaries, when the speed decreases. Once the tectonic plates collide, causing a local strass in the relatively small active zones near the contact sites of the plates. This stress may be either *discharged* via forming cracks near the zone of contacts, splitting the zone into independently moving fragments, or, if the stress is not discharged, and applied for extended period, it may cause *accumulation of a considerable amount of elastic energy*, which eventually results in a powerful earthquake. The accumulation of the elastic energy, due to standard variational principle [8], causes the *increment of eigenfrequencies* of the tectonic plates. It is natural to expect that extended, long-time variations of the angular speed of earth, caused by the major change of the moment of inertia of earth, say, due to non-balanced displacement of huge masses on the surface of the planet, may cause accumulation of a considerable amount of elastic energy at the active zones of contacts of the plates and the corresponding growth of the frequencies of SGO. Hence growth of the frequencies of the SGO may be considered as a precursor of strong earthquakes. Our ability to extract a useful information from the observations on SGO is limited by our understanding of the mechanism arising of the boundary stress on the tectonic plates.

Note, that in numerous observation done on SGO in Leningrad (St. Petersburg) intense short pulses were recorded. They had approximately sinusoidal form with period 30 min. to 1 hour, continued for 6-10 hours and then disappeared. In several case they were followed by powerful earthquakes in 2-4 days. This process was noticed first in [13] and was given the name of “pulsations”. In [13] a conjecture was formulated about the connection between the pulsation, and the subsequent strong earthquakes occurred in Phillipines, Spitak and San-Francisco, suggesting to consider the pulsation as another precursor of a strong earthquake.

Essential information on SGO is obtained from the observation of variations of the frequencies and the shape of the corresponding modes in the remoted zone. The size of the zones of contact ≈ 100 km is comparable with the vertical size of the plates, i.e. it is negligible - “point-wise”- when compared with the wavelength of corresponding seismo-gravitational standing waves (SGW), calculated as $\Lambda_{s,p} = v_{s,p}\, T$ based on the period T of the oscillation and the speed $v_{s,p}$ of the seismic waves.

Summary of the results obtained in [19] resulted in conjecture that *SGO may be interpreted as bending eigen-modes* of the relatively thin tectonic plate. Hence we can model it by the bi-harmonic boundary problem with appropriate boundary conditions, see next section.

The boundary of a real tectonic plate is not smooth, however the details of the local geometry of the boundary may be neglected compared with the wavelength of typical standing waves on the plates.

Based on the above analysis, a solvable zero-range model for the tectonic plate under the point-wise boundary stress was suggested in [25]. The role of the parameter of the model is played by some finite matrix M, see next section. We assume, that the matrix M defines the type of the stress, depending on mutual positions of the neighboring plates at the zone of the contact. We assume that these positions remain essentially unchanged during extended period, so the matrix M characterizes the type of contact for all earthquakes arising from the given zone of contact. This permits to *fit* the model (select the matrix M) based on analysis analysis of instrumental observation of SGO and/or pulsations. Once *fitted*, the constructed model would allow to calculate in explicit form the increment of the eigenvalues and the variations of the shape of the eigenfunctions of the stressed (perturbed) plate depending on the type and the magnitude of the local stress. We assume that the corresponding fitting is done for all active zones on the boundary of some tectonic plate, and the shifts of the eigenfrequencies and the shape of the eigen-modes is done for each zone of contact.

Generically earthquake hits only one active zone at a time. Then comparing the observed shift of the eigenfrequencies and the variations of the shape of the mode in the remote zone, where Saint-Venant principle is applicable, with the results of computing based on the model, we will be able to localize the excited active zone, causing the observed changes of the eigen-frequencies and the shapes of the eigen-modes.

Mathematically the zero-range model of the point-wise stressed isolated tectonic plate and a similar zero-range model of the point-wise stressed tectonic plate submerged into environment formed by other plates, intermediate layers and astenosphere, differ by the type of basic equations, but have a lot in common. In particular, the number of free parameters in both models is the same. We interpreted these parameters in the spirit of Saint-Venant principle as essential parameters describing the shape of the wave-process in the remote zone, see [25]. In this paper we obtain the first order approximation for the perturbed eigenfrequencies and eigenfunctions based on a *modified* Krein formula, based on [1] for the point-wise stressed thin plate described by the bi-harmonic equation. The explicit representation of the perturbed eigen-modes permits to *fit* the zero-range model suggested in [25] based on results of instrumental measurements of SGO. We also conjecture that the properly fitted solvable model of the stressed tectonic plate may help to enlighten the nature of the pulsations.

2 Zero-range model for the point-wise stress

We may base our approach on the standard mathematical model of the tectonic plate in form of a thin elastic plate, thickness h, with free edge, or on the 3-d Lame equations for displacements. We consider both options, describing in the next two subsections specific details of both models. Then we develop the common part of the theory, for both models simultaneously.

2.1 Thin plate model for the isolated tectonic plate under the localized boundary stress

Denoting by D the "the bending stiffness ", connected with Young modulus E, the thickness h of the plate and the Poisson coefficient σ by the formula $D = Eh^2[12(1-\sigma^2)]^{-1}$, we represent, following [12], the corresponding dynamical equation for the normal displacement u as

$$\rho \frac{\partial^2 u}{\partial s^2} = D\Delta^2 u.$$

We consider time-periodic solutions $u(\omega s,\, x)$of the equation and separate the time reducing the dynamical problem to the spectral problem with the spectral parameter

$$\lambda = \rho D^{-1}\ \omega^2 \tag{1}$$

for a bi-harmonic operator on a compact 2-d domain Ω - the tectonic plate - with a smooth boundary $\partial\Omega$:

$$Au = \Delta^2 u = \lambda u,\ u \in W_2^4(\Omega),$$

and free boundary condition involving the tangential and normal derivatives of the displacement u and the tension Δu:

$$\left[\frac{\partial \Delta u}{\partial n} + (1-\sigma)\frac{\partial^3 u}{\partial n \partial t^2}\right]\Big|_{\partial\Omega} = 0,$$

$$\left[\Delta u - (1-\sigma)\frac{\partial^2 u}{\partial t^2}\right]\Big|_{\partial\Omega} = 0. \tag{2}$$

Here n, t are the normal and the tangent directions on the boundary.

The bi-harmonic operator A is selfadjoint in the Hilbert space $L_2(\Omega) := \mathcal{H}$. The eigenfunctions of A are smooth and form an orthogonal basis in $L_2(\Omega) = \mathcal{H}$. We consider the restriction A_0 of A onto $D(A_0)$ all smooth functions vanishing near the boundary point $a \in \partial\Omega$. The restriction is a symmetric, but it is not selfadjoint, because the range of it $(A - \lambda I)\, D(A_0)$, for complex Λ has a nontrivial complement in $L_2\Omega =$ spanned by the Green function $G(x, a, \bar\lambda) := g_0(x, \bar\lambda)$ and it's tangential derivatives $\frac{\partial G(x,a,\bar\lambda)}{\partial t} := g_1(x, \bar\lambda)$, $\frac{\partial^2 G(x,a,\bar\lambda)}{\partial t^2} := g_2(x, \bar\lambda)$ of the first and second order, at the point a. The orthogonal complement N_λ of the range is called the"deficiency subspace", and elements of it - "deficiency elements":

$$N_\lambda := \mathcal{H} \ominus (A_0 - \lambda I) D_0 = \bigvee_{s=0}^{2} g_s(*, \bar\lambda).$$

The deficiency elements have at the boundary point a singularities of different types (see for instance [14], where much more general problem is considered) :

$$g_0(n,t) \approx (n^2+t^2)\ln(n^2+t^2),\ \ g_1(n,t) \approx t\ln(n^2+t^2),\ \ g_2(n,t) \approx \ln(n^2+t^2),$$

hence they are linearly independent and form a basis in the deficiency subspace. The deficiency subspace at the spectral point $\bar\lambda$is

$$N_{\bar\lambda} := \mathcal{H} \ominus (A_0 - \bar\lambda I) D_0 = \bigvee_{s=0}^{2} g_s *, \lambda.$$

The dimensions of the deficiency subspaces $(3,3)$ constitute the "deficiency index". Hereafter we select $\lambda = i$ and attempt to construct a self-adjoint extension of A_0, which will play a role of a zero-range model of the tectonic plate under the boundary strain.

Note that in case of lame equations the deficiency index is also $(3,3)$, on a smooth boundary. The role of deficiency elements is played by the columns of the Green matrix. The boundary of the tectonic plates may be assumed smooth, for the long waves, since their shape is not affected by the details of the local geometry.

2.2 Construction of the self-adjoint extension

Extend A_0 from D_0 onto $D(A_0^+) = D_0 + N_i + N_-i$ as an "adjoint operator" A_0^+ by setting $\left(A_0^+ + \pm iI\right) g = 0$ for $g \in N_{\pm i}$. This operator not selfadjoint, even it is not symmetric, so that the boundary form

$$\langle A_0^+ u,\, v\rangle - \langle u,\, A_0^+ v\rangle = \mathcal{J}(u,v) \tag{3}$$

does not vanish, generally, for $u, v \in D(A_0^+)$. One can re-write (3) in more convenient form with using new symplectic coordinates with respect to new basis in N

$$W_s^+ = \frac{1}{2}\left[g_s + \frac{A+iI}{A-iI} g_s\right] = \frac{A}{A-iI} g_s$$
$$W_s^- = \frac{1}{2i}\left[s_s - \frac{A+iI}{A-iI} g_s\right] = -\frac{I}{A-iI} g_s$$

Due to $A_0^+ g_s + i g_s = 0,\ [A_0^+ - iI]\frac{A+iI}{A-iI} g_s = 0$ we have,

$$A_0^+ W_s^+ = W_s^-,\ A_0^+ W_s^- = -W_s^+. \tag{4}$$

Following [21] we will use the representation of elements from the domain of the adjoint operator by the expansion on the new basis:

$$u = u_0 + \sum_s \xi_+^s W_s^+ + \xi_-^s W_s^- = u_0 + \frac{A}{A-iI}\sum_s \xi_+^s g_s - \frac{I}{A-iI}\sum_s \xi_-^s g_s :=$$
$$= u_0 + \frac{A}{A-iI}\vec{\xi}_+ - \frac{I}{A-iI}\vec{\xi}_-. \tag{5}$$

Note that due to (4)

$$A^+ \frac{A}{A-iI}\,\vec{\xi}_+ = -\frac{I}{A-iI}\vec{\xi}_+,\ A^+ \frac{-I}{A-iI}\,\vec{\xi}_- = -\frac{A}{A-iI}\vec{\xi}_-.$$

Note that the boundary form

$$\langle A_0^+ u, v\rangle - \langle u, A_0^+ v\rangle := \mathcal{J}(u,v)$$

of elements u, v,

$$u = u_0 + \frac{A}{A-iI}\vec{\xi}_+^u - \frac{I}{A-iI}\vec{\xi}_-^u := u_0 + n^u,\ u_0 \in D(A_0)\, n^u \in N, \qquad (6)$$

$$v = v_0 + \frac{A}{A-iI}\vec{\xi}_+^v - \frac{I}{A-iI}\vec{\xi}_-^v := v_0 + n^v,\ u_0 \in D(A_0)\, n^v \in N$$

depends only on components n^u, n^v of them in the defect N. Then the boundary form is represented as:

$$\langle A_0^+ u, v\rangle - \langle u, A_0^+ v\rangle := \mathcal{J}(u,v) = \langle \vec{\xi}_+^u, \vec{\xi}_-^v\rangle - \langle \vec{\xi}_-^u, \vec{\xi}_+^v\rangle \qquad (7)$$

with Euclidean dot-product for vectors $\vec{\xi}_\pm \in N_i$. Note that the representation of the boundary form in terms of abstract boundary values $\vec{\xi}_\pm$ contains only integral characteristics of the elements from the domain of the operators considered, hence is stable with respect of minor local perturbations of geometry of the plates. This permits to substitute, for practical calculations, the real sophisticated boundaries of the plates by the smoothened boundaries, obtained via elimination of minor geometrical details, compared with the length of standing waves of SGO, 1000 -4000 km.

The boundary form vanishes on the Lagrangian plane defined in $D(A_0^+)$ defined by the "boundary condition" with an hermitian operator $M : N_i \to M_i$:

$$\vec{\xi}_+ = M\vec{\xi}_-. \qquad (8)$$

This boundary condition defines a self-adjoint operator A_M as a restriction of A_0^+ onto the Lagrangiam plane $\mathcal{T}_M \in D(A_0^+)$ defined by the boundary condition (8). The resolvent of A_M defined by the boundary conditions is represented, at regular points of A_M, by the Krein formula, see [2, 21]:

$$(A_M - \lambda I)^{-1} = \frac{I}{A-\lambda I} - \frac{A+iI}{A-\lambda I} P_+ M \frac{I}{I + P_+ \frac{I+\lambda A}{A-\lambda I} P_+ M} P_+ \frac{A-iI}{A-\lambda I} \qquad (9)$$

2.3 Compensation of singularities in Krein formula and calculation of the perturbed spectral data

Singularities of the resolvent $(A_M - \lambda I)^{-1}$ coincide with the spectrum of A_M. But both terms in the right side of (9) also have singularities on the spectrum of the non-perturbed operator A. *The singularities of the first and second term the eigenvalues of A compensate each other.* We are able to derive this statement via straightforward calculation, similar to [1]. Moreover, in course of this calculation we can recover both the eigenvalues of the perturbed operator A_M and the corresponding eigenfunctions. Similar statement, as a lemma on compensation of singularities of the corresponding Weyl-Titchmarsh function, was discovered in [6] for 1-d solvable model of the quantum network in form of a quantum graph. Later, in [16] and in [15], similar statements were proven for Dirichlet-to-Neumann maps of quantum networks. We derive here this statement for the resolvent of the selfadjoint extension based on ideas proposed in [17].

We will observe the effect of compensation of singularities on a certain spectral interval $\Delta_0 = [\lambda_0 - \delta,\, \lambda_0 + \delta]$ centered at the *resonance eigenvalue* λ_0 of the non-perturbed plate, assuming that the perturbation, defined by the matrix M is relatively small, in a certain sense.

Assuming that there is a single eigenvalue λ_0 of A on the interval Δ_0, with the eigenfunction φ_0, we use the following representations, separating the polar terms from smooth operator functions K_i, K_{-1}, K on Δ_0

$$\frac{A + iI}{A - \lambda I} = (\lambda_0 + i)\frac{\varphi_0\rangle\langle\varphi_0}{\lambda_0 - \lambda} + K_{-i},$$

$$\frac{A - iI}{A - \lambda I} = (\lambda_0 - i)\frac{\varphi_0\rangle\langle\varphi_0}{\lambda_0 - \lambda} + K_{i},$$

$$P_+ \frac{I + \lambda A}{A - \lambda I} P_+ = (1 + \lambda_0^2)\frac{P_+\varphi_0\rangle\,\langle P_+\varphi_0}{\lambda_0 - \lambda} + K(\lambda), \tag{10}$$

with a smooth matrix-function $K(\lambda) = K_0 + o(|\lambda - \lambda_0|)$, with $K_0 = K(\lambda_0)$ and $\parallel o(|\lambda - \lambda_0|) \parallel \leq C_0\delta$.

Definition 2.1 *We say that the matrix M is relatively small, if $[I + K(\lambda)M]^{-1}$ exists and is bounded on Δ_0.*

This condition is obviously fulfilled if

$$\parallel [I + K(\lambda_0)M]^{-1} \parallel \; C_0\delta << 1. \tag{11}$$

To calculate the second term in the right side of the Krein formula (9) we have to compute the inverse of the denominator, that is to solve the equation

$$\left[I + P_+ \frac{I+\lambda A}{A-\lambda I} P_+ M\right] u = g \tag{12}$$

Though the standard analytic perturbation technique is still not applicable to this equation under the above conditions 2.1 or (11), we are able to construct the inverse based on finite dimensionality (one-dimensionality) of the polar term.

$$u = [I+KM]^{-1}g - (1+\lambda_0^2)\frac{[I+KM]^{-1}P_+\varphi_0\rangle\langle P_+\varphi_0 M[I+KM]^{-1}g\rangle}{\lambda_0 - \lambda + \langle P_+\varphi_0 M[I+KM]^{-1}\varphi_0\rangle}. \tag{13}$$

Based on the last formula we are able, see [7, 1] to observe the compensation of singularities in the above Krein formula (9) and calculate the polar term of the resolvent at the single eigenvalue of the operator A_M on the interval Δ_0:

Theorem 2.1 *If the perturbation is relatively small, as required in (2.1), then there exist a single eigenvalue λ_M of the perturbed operator A_M on the interval Δ_0 which is found as zero of the denominator in (13)*

$$\lambda_0 - \lambda + (1+\lambda_0^2)\langle P_+\varphi_0 M[I+KM]^{-1}\varphi_0\rangle := \mathbf{d}_M(\lambda),\, \mathbf{d}_M(\lambda_M) = 0 \tag{14}$$

and the corresponding eigenfunction

$$\varphi_M = \varphi_0 - (\lambda_0 - i)K_{-i}M[I+KM]^{-1}P_+\varphi_0, \tag{15}$$

computed at the zero λ_M. The polar term of the resolvent of the perturbed operator at the eigenvalue is represented as:

$$\frac{\varphi_0 - (\lambda_0 - i)K_{-i}M[I+KM]^{-1}P_+\varphi_0\rangle\langle\varphi_0 - (\lambda_0 - i)K_{-i}M[I+KM]^{-1}P_+\varphi_0}{\mathbf{d}_M(\lambda)}$$

Proof of this statement can be obtained similarly to relevant statement in [1]. We postpone it to further publication. If the stronger condition (11) is fulfilled, then the approximate eigenvalue and the corresponding approximate eigenfunction of A_M can be obtained via replacement $K, K_{\pm i}$ in (14,15) by $K(\lambda_0), K_{\pm i}(\lambda_0)$:

$$\lambda_M \approx \lambda_0 + (1+\lambda_0^2)\langle P_+\varphi_0 M[I+K(\lambda_0)M]^{-1}K(\lambda_0)\varphi_0\rangle,$$

$$\varphi_M \approx \varphi_0 - (\lambda_0 - i) K_{-i}(\lambda_0) M [I + K(\lambda_0) M]^{-1} P_+ \varphi_0. \tag{16}$$

Remark Analysis of the multi-point boundary condition which corresponds to several strains applied at the points $a_1, a_2, \dots a_m$ on the boundary of the plate Ω differs from the above analysis of the single-point case only in the first step. In multi-point strain we have to construct of elements $\{g_0^r(x, a_r, i),\ g_1^r(x, a_r, i),\ g_2^r(x, a_r, i)\}_{r=1}^m$ a basis in the larger deficiency subspace N_i, dim $N_i = 3m$. Due to presence of singularities of different types at different points, the deficiency elements are linearly independent.

3 Concluding remarks on fitting of the model

The pair of data (16) may be used in two different ways: either for calculation of the shift of the frequency of SGO and the corresponding perturbation of the eigenfunction, under the point-wise stress characterized by the matrix M, or, vice versa, for recovering of the data on the location, type and intensity of the stress from instrumental observations.

Indeed, if the geological structure of the tectonic plates at the active zones, encoded in matrices M_s attached to the active zones a_s where the collisions occur, are known, then, theoretically, we are able to calculate the eigenfunctions and the eigenfrequencies of the plates, taking into account the stress caused by collisions. We are also able, theoretically, to construct the deficiency elements for all active zones. Then the self-adjoint extension of the bi-harmonic operator on the plate, with the point-wise boundary stress, can be constructed, with the matrices M_s, corresponding to given collision points. The obtained theoretical results can be compared with the corresponding results of the instrumental measurements. This permits to recover the matrices M_s, which correspond to the stressed points $a_1,\ a_2,\ \dots$.

Assume that the structure of the plates at the collision point a remains unchanged, but the tension is increasing linearly with time as τ: $M(\tau) = m\tau$, with some matrix coefficient $m : N \to N$. Then the formulae (15,14), for small τ, define the derivatives of λ_M, φ_M with respect to τ at the moment $\tau = 0$:

$$\frac{\partial \varphi_M}{\partial \tau}\Big|_{\tau=0} = -(\lambda_0 - i) K_{-i} m_0 P_+ \varphi_0(x_s), \quad \frac{\partial \lambda_M}{\partial \tau}\Big|_{\tau=0} = (1 + \lambda_0^2) \langle P_+ \varphi_0, m\ P_+ \varphi_0 \rangle.$$

Comparing this result with ratios

$$\frac{\varphi_{m\tau}(x_s) - \varphi_0}{\tau}(x_s)\Big|_{\tau=0}, \quad \frac{\lambda_{m\tau} - \lambda_0}{\tau}\Big|_{\tau=0},$$

measured experimentally for the amplitude and frequency of SGO, we are able to find fit m, and calculate the increment of M $\delta M = \tau m$.

Practical experience in analytic perturbations shows, that minor perturbations affect rather the eigenvalues, that the eigenfunctions of the spectral problem. Based on this observation we can estimate the speed of accumulation of elastic energy $\mathcal{E}$, under the point-wise boundary stress depending on the speed of the shift of the eigenvalues (eigenfrequencies) of SGO and initial distribution of the elastic energy on the modes φ_0^s defined by the corresponding Fourier coefficients $\langle u, \varphi_0^s \rangle$:

$$\frac{d\mathcal{E}}{d\tau} \approx \sum_s \frac{d\lambda_M^s}{d\tau} |\langle u, \varphi_0^s \rangle|^2 = \sum_s (1 + (\lambda^s)_0^2) \langle P_+ \varphi_0^s, m \ P_+ \varphi_0^s \rangle \ |\langle u, \varphi_0^s \rangle|^2,$$

with the summation extended only on the eigen-modes which correspond to variing eigenvalues. If there is only one active zone a_s involved at a time, then only one matrix M_s is taken into account, so that the risk of the powerful earthquake may be estimated based on the magnitude of $\delta_s = \tau m_s$.

If there are several active zones at the points $a_1, a_2, \dots a_m$ on the boundary of the plate, then the corresponding matrices $M_1, M_2, \dots M_m$ can be fitted based on observations of SGO in the remote zone during preceding earthquakes which occurred at $a_1, a_2, \dots a_m$. Variations of the frequencies and the shape of SGO may arise from the stress at any active zone, but usually only one active zone is involved at a time. Once the matrices $M_1, M_2, \dots$ are known, then comparison of the perturbation of the frequencies and the shapes of SGM (or, probably, pulsations) at the given groups of points in the remote zone with results of previous measurements at these groups permits to identify the active zone where the stress is applied. We guess that the above model gives a chance to introduce a useful system into the scope of the experimental data on the seismo-gravitational oscillations in remote zone and use them for estimating of risk and localization of the powerful earthquakes. This opens an alternative to the statistical methods, see [30], of estimation of risk of powerful earthquakes.

Fitting of the proposed model in reality requires both extended computing and a major experimental data base. Because the lengths of standing waves, 1000 - 4000 km, dominate the size of active zones, one may assume, that the straightforward computing with averaged and smothered data for Young modulus and geometric characteristics of the plates will permit to obtain a realistic approximation of the deficiency elements, with singularities at the active zones and combine the eigenfunctions of the plates, which correspond to SGO.

More accurate theory requires taking into account more realistic boundary conditions and the exchange of energy with the liquid underlay and neighboring plates. The choice of realistic boundary conditions has to be done based on experimental data interpreted within an appropriate extension of the scheme proposed with Lame and hydro-dynamical equations involved.

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