Mastering the Master Space

Davide Forcella[1*]**, Amihay Hanany**[2†]**, Yang-Hui He**[3‡]**, Alberto Zaffaroni**[4§]

[1] *International School for Advanced Studies (SISSA/ISAS) & INFN- Sezione di Trieste, via Beirut 2, I-34014, Trieste, Italy*
    PH-TH Division, CERN CH-1211 Geneva 23, Switzerland

[2] *Department of Physics, Technion, Israel Institute of Technology, Haifa 32000, Israel*
    Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.

[3] *Collegium Mertonense in Academia Oxoniensis, Oxford, OX1 4JD, U.K.*
    Mathematical Institute, Oxford University, 24-29 St. Giles', Oxford, OX1 3LB, U.K.
    Rudolf Peierls Centre for Theoretical Physics, Oxford University, 1 Keble Road, OX1 3NP, U.K.

[4] *Università di Milano-Bicocca and INFN, sezione di Milano-Bicocca, Piazza della Scienza, 3; I-20126 Milano, Italy*

Abstract

Supersymmetric gauge theories have an important but perhaps under-appreciated notion of a master space, which controls the full moduli space. For world-volume theories of D-branes probing a Calabi-Yau singularity $\mathcal{X}$ the situation is particularly illustrative. In the case of one physical brane, the master space $\mathcal{F}^\flat$ is the space of F-terms and a particular quotient thereof is $\mathcal{X}$ itself. We study various properties of $\mathcal{F}^\flat$ which encode such physical quantities as Higgsing, BPS spectra, hidden global symmetries, etc. Using the plethystic program we also discuss what happens at higher number N of branes. This letter is a summary and some extensions of the key points of a longer companion paper hep-th/0801.1585.

*forcella@sissa.it

†a.hanany@imperial.ac.uk, hanany@physics.technion.ac.il

‡hey@maths.ox.ac.uk

§alberto.zaffaroni@mib.infn.it

1 Introduction

The vacuum of a quantum field theory is of vital physical significance. For $\mathcal{N} = 1$ supersymmetric gauge theories in four dimensions, the vacuum is obtained by the space of solutions $D^\flat$ of **D-terms**, coming from the gauge and matter content and the solution space $\mathcal{F}^\flat$ of **F-terms**, coming from the critical points of the superpotential. This vacuum moduli space $\mathcal{M}$ is typically a high dimensional object of subtle structure and consists of many branches, such as mesonic versus baryonic, and Higgs versus Coulomb, etc. Conceptually, $\mathcal{M}$ is a quotient of $\mathcal{F}^\flat$ by the gauge symmetries prescribed by $D^\flat$. In this short summary of a companion paper [1] we would like to emphasize the role played by $\mathcal{F}^\flat$ and present it as a critical object in the study of supersymmetric gauge theories.

The study of $\mathcal{F}^\flat$ for a generic supersymmetric gauge theory is an important and a long term project. Here, as a starter, we focus on a special class of supersymmetric gauge theories where, in the context of string theory, the $\mathcal{N} = 1$ gauge theory arises as the four dimensional world-volume theory of a stack of N coincident D3-branes transverse to a Calabi-Yau threefold singularity $\mathcal{X}$. We look at the spectrum of chiral (BPS) operators in such theories and divide them into two types of gauge invariant operators which are typically called mesons (trace invariants) and baryons (determinant invariants). Correspondingly there are two types of moduli spaces, mesonic and baryonic, respectively, along which operators of the corresponding type admit a vacuum expectation value. The mesonic branch is referred to in the literature as the N-th symmetrized product[1] of $\mathcal{X}$. We do not focus on this point but give more attention on aspects of the baryonic branch and how it combines with the mesonic moduli space into a bigger space.

For $N = 1$, a single D3-brane, the situation is particularly interesting: the mesonic branch is simply $\mathcal{X}$; there are no gauge groups in the IR and $\mathcal{M} \simeq \mathcal{F}^\flat$. $\mathcal{F}^\flat$ is called, in accord with the standard mathematical parlance, the **master space** since its quotient is a moduli space [3] and turns out to have some remarkable properties. This forms a convenient starting point. Moreover, for simplicity, $\mathcal{X}$ is taken to be toric so that at least three $U(1)$ isometries are at hand. For $N > 1$, a great deal can be learned via the plethystic program [4, 5, 6, 7, 8] despite the increasing subtlety in the structure of $\mathcal{M}$.

This letter summarizes the key results of [1] and uses the language of (computational) algebraic geometry (cf. [9]); hence it outlines the requisite terminology where necessary.

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[1] Cf. [2] for a consistency analysis of this identification.

2 Theme and Variations in $\mathcal{F}^\flat$

For $\mathcal{X}$ an affine toric Calabi-Yau threefold and a single D3-brane, the gauge theory is a $U(1)^g$ quiver theory with g nodes, E bi-fundamental fields and a number V of terms in the superpotential[2]. Decoupling the Abelian factors in the IR, we are left with the space of F-flatness $\mathcal{F}^\flat$ which is the principal object of our investigation. The first two important properties of $\mathcal{F}^\flat$ are

1. $\mathcal{F}^\flat$ *is a toric variety of complex dimension* $g+2$. This is so because as mentioned in the introduction, $\mathcal{X} \simeq \mathcal{F}^\flat//U(1)^g$, and an overall $U(1)$ decouples; thus $3 = \dim(\mathcal{F}^\flat) - (g - 1)$. It is toric since it is acted upon by exactly $g + 2$ $\mathbb{C}^*$-actions corresponding to the classical global symmetries of the gauge theory: one R and two flavor, coming from the isometries of the toric threefold $\mathcal{X}$, as well as $g - 1$ baryonic, IR relic symmetries of the non-trivial $U(1)$ factors, some of which are anomalous. Specifically, we can define $\mathcal{F}^\flat$ as an affine algebraic variety in $\mathbb{C}[x_1, \ldots, x_E]$ with appropriate $U(1)$ charges (weights) to the variables x_i under the $g + 2$ dimensional toric action.

2. The moduli space of gauge theories is well-known to have many branches; this is reflected by the fact that $\mathcal{F}^\flat$ *is typically a reducible algebraic variety.* Either directly or using methods of toric ideals, we can perform **primary decomposition** [11] thereupon to extract the irreducible pieces. We find that it contains a top-dimensional irreducible component of the same dimension and degree, as well as many smaller dimensional irreducible linear pieces, realised as coordinate hyperplanes. The top component is usually dubbed the **coherent component**, which we denote as $^{\mathrm{Irr}}\mathcal{F}^\flat$. An interesting aspect of it is being a Calabi-Yau manifold of dimension $g + 2$.

Now, one of the most fundamental quantities which characterizes an algebraic variety X is the **Hilbert series**, which is the generating function for the number $\dim(X_i)$ of independent polynomials at a given degree on X:

$$H(t;\ X) = \sum_{i=0}^{\infty} \dim(X_i)t^i = (1 - t)^{-\dim(X)}P(t) \ . \tag{2.1}$$

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[2]Recently it is realised that the most conducive way of thinking of toric quiver gauge theories is via the language of **dimer models/brane tilings** [10]. This graphical method combines the matter content (quiver diagram) and the interaction terms (superpotential) into a single object: a periodic tiling of the 2-dimensional plane. It follows, for example, that $V - E + g = 0$.

In the above, $P(t)$ turns out to be a polynomial with integer coefficients. We can readily refine (2.1) by having a list $\underline{t}$ of dummy variables in which case the number of polynomials of a given multi-degree would be counted. We can conveniently use [11] to compute the Hilbert series, or, alternatively, use the Molien formula as given below in (2.3).

The Hilbert series turns out to be of fundamental importance to supersymmetric gauge theories. Physically, the Hilbert series is the key to the **Plethystic program** [4, 6, 7]: the dummy variables are naturally identified with *chemical potentials* associated to the multi-degrees which are combinations of $U(1)$-charges and the object of counting is the spectrum of mesonic and baryonic BPS operators. In (2.1), for example, t can be taken to be the chemical potential[3] for the R-charge. Plethystic exponentiation of the refined Hilbert series then counts all chiral BPS operators.

Upon studying $\mathcal{F}^\flat$ for a wealth of illustrative examples, we discover that the master space $\mathcal{F}^\flat$ and especially its coherent component $^{\mathrm{Irr}}\mathcal{F}^\flat$ enjoy many remarkable properties [1]:

1. **Symplectic quotient description of $^{\mathrm{Irr}}\mathcal{F}^\flat$:** The traditional approach in understanding the moduli space of toric gauge theories is to use Witten's gauged linear sigma model (GLSM) where the spacetime fields are parameterized in terms of the latter [13]. We find an algebraic parameterization of the space of solutions of F-terms in terms of c new fields p_α, as $x_i = \prod_{\alpha=1}^{c} p_\alpha^{P_{i\alpha}}$ where $P_{i\alpha}$ turns out to be a matrix with 0 and 1 entries. The fields p_α are charged under $c - g - 2$ $U(1)$ gauge groups. In mathematical language, this is the symplectic quotient description of the *coherent component* of our affine toric variety

$$^{\mathrm{Irr}}\mathcal{F}^\flat = \mathbb{C}^c // (\mathbb{C}^*)^{c-g-2} \ , \tag{2.2}$$

with the action specified by a c by $c - g - 2$ matrix of charges Q. The GLSM fields can be associated with points in the toric diagram of $\mathcal{X}$ with multiplicities [13].

The GLSM description allows us to compute the Hilbert series using a localisation formula based on the Molien integral, which projects onto $(\mathbb{C}^*)^{c-g-2}$ invariants [7]:

$$H(\underline{t}; {}^{\mathrm{Irr}}\mathcal{F}^\flat) = \oint_{|z_i|=1} \prod_{i=1}^{c-g-2} \frac{dz_i}{z_i} \prod_{\alpha=1}^{c} \frac{1}{1 - y_\alpha \, \underline{z}^{\underline{q}^\alpha}} \ , \tag{2.3}$$

where $\underline{q}^\alpha$ is the vector of $U(1)^{c-g-2}$ charges of the fields p_α, given by the α-th column of Q, and $y_\alpha \equiv y_\alpha(\underline{t})$ is a monomial in $\underline{t}$ specifying the $U(1)$ global charges of p_α.

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[3]To be precise t is identified with the "fugacity" of the R-charge and $w = -\log t$ is identified as the chemical potential but we will call t the chemical potential by abuse of notation.
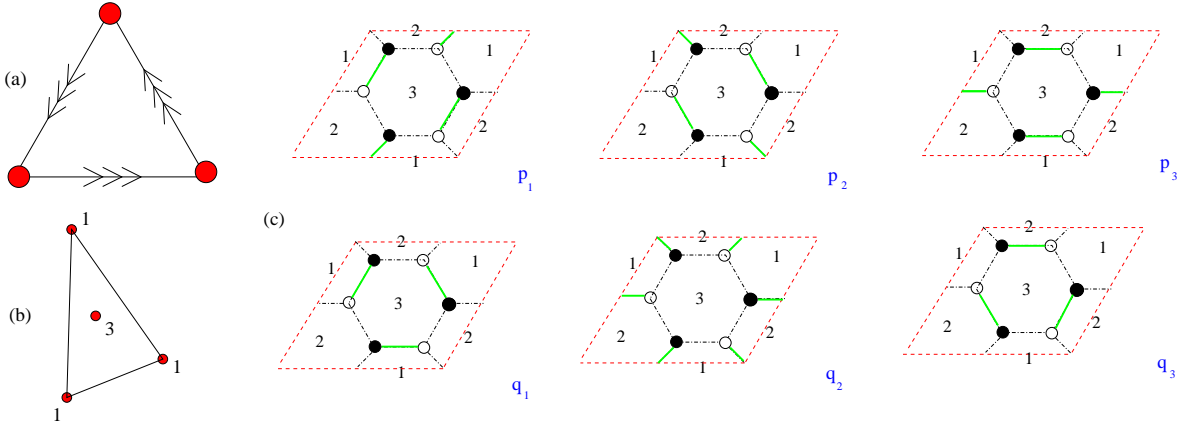
Figure 1: (a) The quiver diagram for $dP_0 \equiv \mathbb{C}^3/\mathbb{Z}_3$ with gauge group $U(N)^3$ and nine fields with superpotential $W = \epsilon_{\alpha\beta\gamma} X_{12}^{(\alpha)} X_{23}^{(\beta)} X_{31}^{(\gamma)}$; (b) The toric diagram, with the labeled multiplicity of GLSM fields; (c) The perfect matchings for the corresponding dimer model, with p_i the external matchings and q_i, the internal. There is only one linear relation $-p_1 - p_2 - p_3 + q_1 + q_2 + q_3 = 0$, giving the GLSM description of $^{\mathrm{Irr}}\mathcal{F}_{\mathbb{C}^3/\mathbb{Z}_3}^\flat$ as $\mathbb{C}^6//[-1, -1, -1, 1, 1, 1]$.

2. **Dimer Model:** The GLSM fields p_α are now elegantly understood to be perfect matchings in the associated dimer model [10] and the charge-matrix Q to be given by the linear relations among these perfect matchings [1]. We illustrate with the example for $\mathcal{X} = dP_0 \equiv \mathbb{C}^3/\mathbb{Z}_3$ as given in Figure 1. In accord with the dimer language, we see that perfect matching generate the coherent component.

3. **Surgery:** The Hilbert series for the various irreducible pieces in the primary decomposition of $\mathcal{F}^\flat$ obey surgery rules [12] according to the intersection of the pieces.

4. **Calabi-Yau property:** The coherent component $^{\mathrm{Irr}}\mathcal{F}^\flat$ is affine Calabi-Yau[4] of dimension $g + 2$. This easily follows from the dimer description as shown in [1].

5. **Palindromic Hilbert series:** An intriguing property of the Hilbert Series for $^{\mathrm{Irr}}\mathcal{F}^\flat$ is its symmetry. The numerator $P(t)$ of $H(t; {}^{\mathrm{Irr}}\mathcal{F}^\flat)$, which we recall to be an integer polynomial of degree, say, n, has a palindromic symmetry for its coefficients $a_{j=0,\ldots,n}$, viz., $P(t)$ is invariant under the exchange $a_j \leftrightarrow a_{n-j}$. This is a consequence of the Stanley theorem [14] and the fact $^{\mathrm{Irr}}\mathcal{F}^\flat$ is toric Calabi-Yau.

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[4]Recall that the symplectic quotient (2.2) is Calabi-Yau iff the vector of charges, given by the columns of Q, each sums to zero.

6. **Invariance under Seiberg duality:** We can perform the decomposition analysis to various toric/Seiberg dual phases for the same geometry [15]. We find a suggestive property and conjecture that for Seiberg dual phases, the coherent components of the master space are isomorphic as expected from the fact that Seiberg dual theories should have the same moduli space. However, the linear components differ, suggesting that some of the smaller-dimension pieces may be lifted by quantum corrections.

7. **Linear Components and Flows:** Physically, we can interpret the coherent component $^{\mathrm{Irr}}\mathcal{F}^{\flat}$ as the Higgs branch and the linear components, as the Coulomb branch of the moduli space. An archetypal theory which exhibits the branch structure of moduli spaces is the $\mathbb{C}^2/\mathbb{Z}_2$ orbifold theory, which has $\mathcal{N} = 2$ supersymmetry. We find that the acquisition of vacuum expectation values (VEV) of the fields parameterizing the linear components induces flows in the gauge theory.

   Indeed, a gauge theory coming from a toric singularity $\mathcal{X}$ can be Higgsed to another; in the toric diagram, this is seen as the deletion of nodes [13, 16], or, geometrically, as the partial resolution of $\mathcal{X}$. Many examples of this phenomen are demonstrated in [1].

   The affine cone F_0 over the zeroth Hirzebruch surface, for example, has a toric diagram which contains that of $\mathbb{C}^2/\mathbb{Z}_2$. We find that, upon primary decomposition of the master space $\mathcal{F}^{\flat}_{F_0}$ the linear pieces are coordinate hyperplanes and giving VEV's to the variables therein gives $\mathbb{C}^2/\mathbb{Z}_2$. We can also find chains of toric theories flowing by the successive acquisition of VEV's (i.e., deletion of nodes in the toric diagram) of fields parameterizing the linear components of the master spaces, e.g., $dP_3 \rightarrow dP_2 \rightarrow F_0 \rightarrow \mathbb{C}^2/\mathbb{Z}_2$, where dP_n is the affine cone over the n-th del Pezzo surface.

8. **Hidden Global Symmetries:** The moduli space of a field theory may possess symmetries beyond gauge or explicit global symmetries, which develop as the theory flows to the IR. We call them **hidden global symmetries** since they are not manifest in the UV Lagrangian.

   For D-brane gauge theories, the UV symmetries of the Lagrangian are generically Abelian: the three isometries of $\mathcal{X}$ are visible in the UV as flavor symmetries. The Abelian gauge factors become weakly coupled in the IR and give rise to (possibly anomalous) baryonic symmetries. In [17], additional symmetry structure of $\mathcal{X}$ was investigated by grouping the fields according to representations of non-Abelian groups and the terms in the superpotential to invariants.

Such hidden global symmetries are surprisingly manifest in the Hilbert series of the master space. As shown in the final Table, the full symmetry of $^{\mathrm{Irr}}\mathcal{F}^\flat$ is in many cases non-Abelian. The lesson we learn is that *the terms in the refined Hilbert series of $\mathcal{F}^\flat$ can be arranged according to the representations of the hidden global symmetry of the gauge theory.*

Let us illustrate with $dP_0 = \mathbb{C}^3/\mathbb{Z}_3$. The quiver is shown in Figure 1. The master space is irreducible and given by $\mathcal{F}^\flat \simeq {}^{\mathrm{Irr}}\mathcal{F}^\flat = \mathbb{C}^6//[-1, -1, -1, 1, 1, 1]$. We immediately see, by grouping the three $+1$ and three -1 in the charge matrix, that the symmetry of $\mathcal{F}^\flat$ is $U(1)_R \times SU(3)_M \times SU(3)_H$. Here, $U(1)_R \times SU(3)_M$ is an obvious (mesonic) symmetry of the Lagrangian and corresponds to the symmetry of $\mathbb{C}^3/\mathbb{Z}_3$. On the other hand, $SU(3)_H$ is a hidden symmetry enhancing the two anomalous baryonic symmetries.

As re-writing of the refined Hilbert series reveals that it organizes and decomposes according to the representation of the full group $U(1)_R \times SU(3)_M \times SU(3)_H$:

$$
\begin{aligned}
g_1(t; \mathbb{C}^3/\mathbb{Z}_3) &= \left(1 - [0, 1, 1, 0]t^2 + ([1, 1, 0, 0] + [0, 0, 1, 1])t^3 - [1, 0, 0, 1]t^4 + t^6\right) PE\left[[1, 0, 0, 1]t\right] \\
&= \sum_{n=0}^{\infty} [n, 0, 0, n]\, t^n ,
\end{aligned}
$$
(2.4)

where $[m, n, p, q]$ denotes the character of the product representation with weights $[m, n]$ under $SU(3)_M$ and $[p, q]$ under $SU(3)_H$. The nine fundamental fields, whose plethystic exponential we are taking in (2.4), transform as $[1, 0, 0, 1]$.

Therefore, we see that the hidden global symmetry of the theory is encoded in the generating function which counts the BPS spectrum, viz., the refined Hilbert series organizes according to the representations of the global symmetry. This phenomenon persists for a host of illustrative and non-trivial geometries [1]. In particular, for $dP_{n=1,\ldots,8}$, the symmetry is the exceptional Lie group E_n as conjectured in [17]. We present some of the results in the table at the end of this letter.

It is an interesting question to understand when the symmetry for one brane extends to a general number N of branes and thus to a hidden symmetry of the theory for some or all values of N. This is shown in a set of examples in [1]. The structure of the master space for an arbitrary number of branes also becomes subtle: The moduli space is given as a quotient of the space of F-terms by the non-Abelian factors of the gauge group since the Abelian $U(1)$ factors decouple in the IR. A further quotient by $U(1)^{g-1}$ leads to the mesonic moduli space which for N-branes is conjectured to be

the symmetrized product of the Calabi-Yau singularity $\mathcal{X}$. The master space thus has dimension $3N + g - 1$.

One of the salient features of the Plethystic program is that it addresses many properties of arbitrary number of branes with ease and without explicit knowledge of the actual space: the generating function for one brane $g_1(t) = H(t;{}^{\mathrm{Irr}}\mathcal{F}^\flat)$ determines the generating function for arbitrary N. One can explictly check that the $SU(4)_H$ symmetry for the conifold persist only up to $N = 2$ (since it mixes mesonic and baryonic symmetries and it enters in conflict with the plethystic exponential which is performed in the sector with definite baryonic charge) while the hidden symmetries of $\mathbb{C}^3/\mathbb{Z}_3$ and F_0 extend to arbitrary N. We summarize some of the above discussions with Table 1.

References

[1] D. Forcella, A. Hanany, Y.-H. He and A. Zaffaroni, "The Master Space of $\mathcal{N} = 1$ Gauge Theories," arXiv:0801.1585 [hep-th].

[2] D. Berenstein, "Reverse geometric engineering of singularities," JHEP **0204**, 052 (2002) [arXiv:hep-th/0201093].

[3] C. Okonek, A. Teleman, "Master Spaces and the Coupling Principle: From Geometric Invariant Theory to Gauge Theory", Commun. Math. Phys. 205, 437 – 458 (1999); and cf. references therein.

[4] S. Benvenuti, B. Feng, A. Hanany and Y. H. He, "Counting BPS operators in gauge theories: Quivers, syzygies and plethystics," arXiv:hep-th/0608050.
B. Feng, A. Hanany and Y. H. He, "Counting gauge invariants: The plethystic program," JHEP **0703**, 090 (2007) [arXiv:hep-th/0701063].

[5] A. Butti, D. Forcella and A. Zaffaroni, "Counting BPS baryonic operators in CFTs with Sasaki-Einstein duals," JHEP **0706**, 069 (2007) [arXiv:hep-th/0611229].

[6] D. Forcella, A. Hanany and A. Zaffaroni, "Baryonic generating functions," arXiv:hep-th/0701236.

[7] A. Butti, D. Forcella, A. Hanany, D. Vegh and A. Zaffaroni, "Counting Chiral Operators in Quiver Gauge Theories," arXiv:0705.2771 [hep-th].

[8] D. Forcella, "BPS Partition Functions for Quiver Gauge Theories: Counting Fermionic Operators," arXiv:0705.2989 [hep-th].

[9] J. Gray, Y. H. He, V. Jejjala and B. D. Nelson, "Exploring the vacuum geometry of N = 1 gauge theories," Nucl. Phys. B **750**, 1 (2006) [arXiv:hep-th/0604208].

[10] A. Hanany and K. D. Kennaway, "Dimer models and toric diagrams," arXiv:hep-th/0503149. S. Franco, A. Hanany, K. D. Kennaway, D. Vegh and B. Wecht, "Brane dimers and quiver gauge theories," JHEP **0601**, 096 (2006) [arXiv:hep-th/0504110].
K. D. Kennaway, "Brane Tilings," Int. J. Mod. Phys. A **22**, 2977 (2007) [arXiv:0706.1660 [hep-th]], cf. also refs therein.

[11] D. Grayson and M. Stillman, "Macaulay 2, a software system for research in algebraic geometry," Available at http://www.math.uiuc.edu/Macaulay2/

[12] A. Hanany and C. Romelsberger, "Counting BPS operators in the chiral ring of N = 2 supersymmetric gauge theories or N = 2 braine surgery," arXiv:hep-th/0611346.

[13] D. R. Morrison and M. R. Plesser, "Non-spherical horizons. I," Adv. Theor. Math. Phys. **3**, 1 (1999) [arXiv:hep-th/9810201].
C. Beasley, B. R. Greene, C. I. Lazaroiu and M. R. Plesser, "D3-branes on partial resolutions of abelian quotient singularities of Calabi-Yau threefolds," Nucl. Phys. B **566**, 599 (2000) [arXiv:hep-th/9907186].
B. Feng, A. Hanany and Y. H. He, "D-brane gauge theories from toric singularities and toric duality," Nucl. Phys. B **595**, 165 (2001) [arXiv:hep-th/0003085].

[14] R. Stanley, "Hilbert functions of graded algebras," Adv. Math. 28 (1978), 57-83.

[15] B. Feng, A. Hanany, Y. H. He and A. M. Uranga, "Toric duality as Seiberg duality and brane diamonds," JHEP **0112**, 035 (2001) [arXiv:hep-th/0109063].

[16] B. Feng, S. Franco, A. Hanany and Y. H. He, "Unhiggsing the del Pezzo," JHEP **0308**, 058 (2003) [arXiv:hep-th/0209228].

[17] S. Franco, A. Hanany and P. Kazakopoulos, "Hidden exceptional global symmetries in 4d CFTs," JHEP **0407**, 060 (2004) [arXiv:hep-th/0404065].

$\mathcal{X}$	$\dim(\mathcal{F}^\flat)$	$^{\mathrm{Irr}}\mathcal{F}^\flat$	$H(t;\ {}^{\mathrm{Irr}}\mathcal{F}^\flat)$	Global Symmetry
$\mathbb{C}^3$	3	$\mathbb{C}^3$	$(1-t)^{-3}$	$U(3)$
$\mathcal{C}$	4	$\mathbb{C}^4$	$(1-t)^{-4}$	$U(1)_R \times SU(4)_H$
$(\mathbb{C}^2/\mathbb{Z}_2) \times \mathbb{C}$	4	$\mathcal{C} \times \mathbb{C}$	$\frac{1+t}{(1-t)^4}$	$U(1)_R \times SU(2)_R \times U(1)_B \times SU(2)_H$
$\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$	6	$-$	$\frac{1+6t+6t^2+t^3}{(1-t)^6}$	$U(1)_R \times U(1)^2 \times SU(2)_H^3$
SPP	5	$\mathcal{C} \times \mathbb{C}^2$	$\frac{1+t}{(1-t)^5}$	$U(1)_R \times U(1)_M \times SU(2)_H^3$
dP_0	5	$\simeq \mathcal{F}^\flat$	$\frac{1+4t+t^2}{(1-t)^5}$	$U(1)_R \times SU(3)_M \times SU(3)_H$
F_0	6	$\mathcal{C} \times \mathcal{C}$	$\frac{(1+t)^2}{(1-t)^6}$	$U(1)_R \times U(1)_B \times SU(2)_M^2 \times SU(2)_H^2$
dP_1	6	$-$	$\frac{1+4t+7t^2+4t^3+t^4}{(1-t)^6(1+t)^2}$	$U(1)_R \times SU(2)_M \times U(1)^3 \times SU(2)_H$
dP_2	7	$-$	$\frac{1+2t+5t^2+2t^3+t^4}{(1-t)^7(1+t)^2}$	$U(1)_R \times SU(2)_H \times U(1)^5$
dP_3	8	$-$	$\frac{1+4t^2+t^4}{(1-t)^8(1+t)^2}$	$(SU(2) \times SU(3))_H \times U(1)^5$

Table 1: The toric Calabi-Yau threefold $\mathcal{X}$ is exemplified by the list in the left-most column, where $\mathcal{C}$ is the conifold, SPP, the suspended pinched points, F_0, the cone over $\mathbb{P}^1 \times \mathbb{P}^1$ and dP_n the cone over the n-th del Pezzo surface. For these we tabulate the dimension of the single-brane master space $\mathcal{F}^\flat$. The top component thereof, $^{\mathrm{Irr}}\mathcal{F}^\flat$ is always of the same dimension and is Calabi-Yau; we present, where possible, what this space is explicitly, as well as its Hilbert series. We also record the global symmetry of the respective theories: the subscript R denotes R-symmetry, M denotes the symmetry of the mesonic branch, B is the baryonic charge, and H denotes the hidden global symmetry. Note that the rank of the global symmetry group is equal to the dimension of $\mathcal{F}^\flat$.