

# ON SOLAR NEUTRINO PROBLEM

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ABSTRACT. The current neutrino oscillation theory asserts that neutrinos are massive subatomic particles, and can undergo their flavor oscillation. First in this article, we examine carefully the current oscillation theory, and argue that a massless neutrino oscillation mechanism based on the Weyl equations for neutrinos is also a feasible theory. This massless neutrino oscillation mechanism resolves both the parity problem and the handedness and speed of neutrino problem, which the current massive neutrino oscillation mechanism faces. Second, we propose a neutrino non-oscillation mechanism based on the weakton model of subatomic particles, providing an alternative resolution to the solar neutrino loss problem.

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## 1. INTRODUCTION

Neutrino was first proposed by Wolfgang Pauli in 1930 in order to guarantee the energy and momentum conservation for  $\beta$ -decay. In the current standard model of particle physics, there are three flavors of neutrinos: the electron neutrino  $\nu_e$ , the tau neutrino  $\nu_\tau$  and the mu neutrino  $\nu_\mu$ . The solar neutrino problem is referred to the discrepancy of the number of electron neutrinos arriving from the Sun are between one third and one half of the number predicted by the Standard Solar Model, and was first discovered by R. Davis, D. S. Harmer and K. C. Hoffmann [1] in 1968.

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The current dominant theory to resolve the solar neutrino problem is the neutrino oscillation theory, which are based on three basic assumptions: 1) the neutrinos are massive, and, consequently, are described by the Dirac equations, 2) the three flavors of neutrinos  $\nu_e, \nu_\mu, \nu_\tau$  are not the eigenstates of the Hamiltonian, and 3), instead, the three neutrinos are some linear combinations of three distinct eigenstates of the Hamiltonian. However, the massive neutrino assumption gives rise two serious problems. First, it is in conflict with the known fact that the neutrinos violate the parity symmetry. Second, the handedness of neutrinos implies their velocity being at the speed of light.

To resolve these difficulties encountered by the classical theory, we argue that there is no physical principle that requires that neutrino must have mass to ensure oscillation. The Weyl equations were introduced by H. Weyl in 1929 to describe massless spin- $\frac{1}{2}$  free particles [15], which is now considered as the basic dynamic equations of neutrino [4, 6, 14]; see also [2]. One important property of the Weyl equations is that they violate the parity invariance. Hence by using the Weyl equations, we are able to introduce a massless neutrino oscillation model. With this massless model, we not only deduce the same oscillation mechanism, but also resolve the above two serious problems encountered in the massive neutrino oscillation model.

Despite of the success of neutrino oscillation models and certain level of experimental support, the physical principles behind the neutrino oscillation are still entirely unknown. Recently, the authors developed a phenomenological model of elementary particles, called the weakton model [9]. The  $\nu$  mediator in the weakton model leads to an alternate explanation to the solar neutrino problem. When the solar electron neutrinos collide with anti electron neutrinos in the atmosphere, which are abundant due to the  $\beta$ -decay of neutrons, they can form  $\nu$  mediators, causing the loss of electron neutrinos. Note that  $\nu$  mediator can also have the following elastic scattering

$$\nu + e^- \longrightarrow \nu + e^-.$$

Also  $\nu$  participates only the weak interaction similar to the neutrinos, and consequently possesses similar behavior as neutrinos. Consequently, the new mechanism proposed here does not violate the existing experiments (SNO and KamLAND).

This article is organized as follows. Sections 2–4 examines the solar neutrino problem, the classical oscillation theory, and the MSW effect. Section 5 introduces the massless neutrino oscillation mechanism, and Section 6 introduces a non-oscillation mechanism based on the weakton model.

## 2. DISCREPANCY OF SOLAR NEUTRINOS

The solar neutrino problem is known as that the number of electron neutrinos arriving from the Sun are between one third and one half of the number predicted by the Standard Solar Model. This important discovery was made in 1968 by R. Davis, D. S. Harmer and K. C. Hoffmann [1].

To understand clearly this problem, we begin with a brief introduction to the Standard Solar Model, following [3].

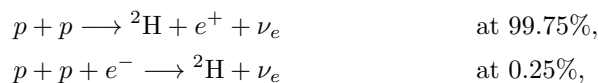
In the nineteenth century, most physicist believed that the source of the Sun's energy was gravity. However, based on this assumption, Rayleigh showed that the maximum possible age of the Sun was substantially shorter than the age of the earth estimated by geologists.

At the end of the nineteenth century, Bacquerel and Curies discovered radioactivity, and they noted that radioactive substances release a large amounts of heat. This suggested that nuclear fission, not gravity, might be the source of the Sun's energy, and it could allow for a much longer lifetime of the Sun. But, the crucial problem for this solar model was that there were no heavier radioactive elements such as uranium or radium present in the Sun, and from the atomic spectrum, it was known that the Sun is made almost entirely of hydrogen.

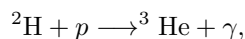
Up to 1920, F. W. Aston gave a series of precise measurements of atomic masses. It was found that four hydrogen atoms are more weight slight than one atom of helium-4. This implied that the fusion of four hydrogens to form a  ${}^4\text{He}$  would be more favorable, and would release a substantial amounts of energy. A. Eddington proposed that the source of the Sun's energy is the nuclear fusion, and in essence he was correct.

In 1938, H. Bethe in collaboration with C. Critchfield had come up with a series of subsequent nuclear reactions, which was known as the proton-proton  $p-p$  chain. The  $p-p$  cycle well describes the reaction processes in the Sun, and consists of the following four steps:

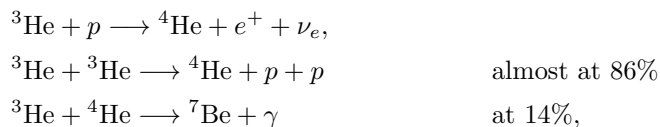
STEP 1: two protons yield a deuteron



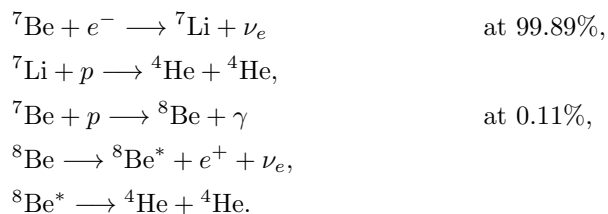
STEP 2: a deuteron and a proton produces a helium-3



STEP 3: helium-3 makes helium-4 or beryllium

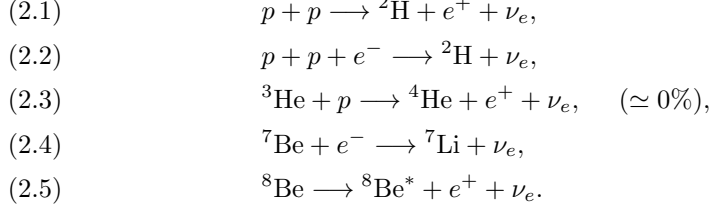


STEP 4: beryllium makes helium -4



In the  $p-p$  chain, it all starts out as hydrogen (proton), and it all ends up as  ${}^4\text{He}$  plus some electrons, positrons, photons and neutrinos. Because neutrinos interact so weakly, they are the unique products in the  $p-p$  reactions reaching the earth's surface.

In the  $p - p$  chain there are five reactions to yield neutrinos:



But the problem is that the detection of the neutrinos have an effect threshold which will lead to a nearly vanishing response to all neutrinos of lower energy. The energy spectras of neutrinos in the five reactions are

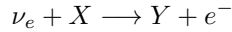
$$\begin{aligned}
 (2.6) \quad & E_m \simeq 0.4\text{MeV} \quad \text{for (2.1),} \\
 & E_m \simeq 1.44\text{MeV} \quad \text{for (2.2),} \\
 & E_m \simeq 18\text{MeV} \quad \text{for (2.3),} \\
 & E_m \simeq 0.9\text{MeV} \quad \text{for (2.4),} \\
 & E_m \simeq 14\text{MeV} \quad \text{for (2.5),}
 \end{aligned}$$

where  $E_m$  is the maximum energy of neutrinos, and the energy flux are

$$\begin{aligned}
 (2.7) \quad & F \simeq 10^{11} / \text{cm}^2 \cdot s \quad \text{for (2.1),} \\
 & F \simeq 10^8 / \text{cm}^2 \cdot s \quad \text{for (2.2)} \\
 & F \simeq 10^2 / \text{cm}^2 \cdot s \quad \text{for (2.3)} \\
 & F \simeq 10^{10} / \text{cm}^2 \cdot s \quad \text{for (2.4),} \\
 & F \simeq 10^6 / \text{cm}^2 \cdot s \quad \text{for (2.5).}
 \end{aligned}$$

#### *The Homestake experiments*

The experimental search for solar neutrinos has been undertaken since 1965 by R. Davis and collaborators in the Homestake goldmine in South Dakota. Since the neutrinos cannot be directly detected by instruments, it is only by the reactions



to detect the outgoing products that counte the neutrinos. The Homestake experiments take



The effective threshold of the reaction (2.8) is

$$E_c = 5.8\text{MeV}.$$

Thus, by (2.6) only these neutrinos from both reactions (2.3) and (2.5) can be observed, which occur at a frequency of 0.015%. Theoretic computation showed that the expected counting rate of solar neutrinos is at

$$(2.9) \quad N_{Th} = (5.8 \pm 0.7) \text{ snu},$$

where *snu* stands for solar neutrino unit:

$$1 \text{ snu} = 10^{-36} \text{ reactions}/({}^{37}\text{Cl atom} \cdot s).$$

In 1968, R. Davis et al [1] reported the experimental results, their measuring rate is

$$(2.10) \quad N_{\text{Exp}} = (2.0 \pm 0.3) \text{ snu.}$$

the experimental value (2.10) is only about one third of the theoretically expected value (2.9). It gave rise to the famous solar neutrino problem.

#### *Super-K Experiment*

In 2001, the Super-Kamiokande collaboration presented its results on solar neutrinos. Unlike the Homestake experiment, Super-*K* uses water as the detector. The process is elastic neutrino-electron scattering:

$$\nu_x + e \longrightarrow \nu_x + e,$$

where  $\nu_x$  is one of the three flavors of neutrinos. This reaction is sensitive to  $\mu$  and  $\tau$  neutrinos as well as  $e$ -neutrinos, but the detection efficiency is 6.5 times greater for  $e$ -neutrinos than for the other two kinds. The outgoing electron is detected by the Cherenkov radiation it emits in water. They observed the rate at

$$r = 45\% \text{ of the expected value.}$$

The Super-Kamiokande detector is located in the Mozumi Mine near Kamioka section of the city of Hida, Japan.

#### *Sudbury Neutrino Observatory (SNO)*

Meanwhile, in the summer of 2001 the SNO collaboration reported their observation results. They obtained

$$r = 35\% \text{ of the predicted value.}$$

The SNO used heavy water ( $^2\text{H}_2\text{O}$ ) instead of ordinary water ( $\text{H}_2\text{O}$ ), and the SNO detection method is based on the following reactions:

$$(2.11) \quad \nu_e + ^2\text{H} \longrightarrow p + p + e^-,$$

$$(2.12) \quad \nu_x + ^2\text{H} \longrightarrow p + n + \nu_x,$$

$$(2.13) \quad \nu_x + e^- \longrightarrow \nu_x + e^-.$$

SNO detects electrons  $e^-$ , but not  $\tau^-$  and  $\mu^-$ , as there is not enough energy in the solar electron-neutrino such that the transformed tau and mu neutrino can excite neutrons in  $^2\text{H}$  to produce either  $\tau^-$  or  $\mu^-$ .

#### *KamLAND*

The loss of reactor electron anti-neutrino  $\bar{\nu}_e$  is verified by the KamLAND experiment.

#### *A potential alternative experiment*

It is known that the following reaction

$$(2.14) \quad \nu_\mu + n \longrightarrow \mu^- + p$$

may occur if the energy of  $\nu_\mu$  satisfies

$$E_{\nu_\mu} > m_\mu c^2 = 106 \text{ MeV.}$$

By the energy spectrum (2.6), the maximum energy of solar neutrinos is about  $14 \sim 18$  MeV, which is much smaller than  $m_\mu c^2$ . Hence, assuming oscillation does occur for solar neutrinos, the reaction

$$\nu_\mu + {}^2\text{H} \longrightarrow \mu^- + p + p$$

does not occur for the transformed  $\nu_\mu$  from solar electron-neutrinos.

However, based on the weakton model, the complete reaction for (2.14) should be

$$\nu_\mu + n + \gamma \longrightarrow \mu^- + p.$$

Consequently, the following reaction

$$(2.15) \quad \nu_\mu + {}^2\text{H} + \gamma \longrightarrow \mu^- + p + p$$

would occur if

$$(2.16) \quad E_{\nu_\mu} + E_\gamma > 106 \text{ MeV}.$$

Hence one may use high energy photons to hit the heavy water to create the situation in (2.16), so that the reaction (2.15) may take place. From (2.15), we can detect the  $\mu^-$  particle to test the neutrino oscillation.

Alternatively, by the  $\mu$ -decay:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu,$$

we may measure the electrons to see if there are more electrons than the normal case to test the existence of mu-neutrinos.

### 3. CLASSICAL THEORY ON NEUTRINO OSCILLATIONS

Before presenting our own view, we recapitulate in this section the classical neutrino oscillation theory.

**3.1. Neutrino Oscillations.** In order to explain the solar neutrino problem, in 1968 B. Pontecorvo [12, 13] introduced the neutrino oscillation mechanism, which amounts to saying that the neutrinos can change their flavors, i.e. an electron neutrino may transform into a muon or a tau neutrino. According to this theory, a large amount of electron neutrinos  $\nu_e$  from the Sun have changed into the  $\nu_\mu$  or  $\nu_\tau$ , leading the discrepancy of solar electron neutrinos. This neutrino oscillation mechanism is based on the following assumptions:

- The neutrinos are massive, and, consequently, are described by the Dirac equations.
- The three types of neutrinos  $\nu_e, \nu_\mu, \nu_\tau$  are not the eigenstates of the Hamiltonian (i.e. the Dirac operator)

$$(3.1) \quad \hat{H} = -i\hbar c(\vec{\alpha} \cdot \nabla) + mc^2\alpha_0.$$

- There are three discrete eigenvalues  $\lambda_j$  of (3.1) with eigenstates:

$$(3.2) \quad \hat{H}\nu_j = \lambda_j\nu_j \quad \text{for } 1 \leq j \leq 3,$$

such that  $\nu_e, \nu_\mu, \nu_\tau$  are some linear combinations of  $\{\nu_j \mid 1 \leq j \leq 3\}$ :

$$(3.3) \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = A \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

where  $A \in SU(3)$  is a third-order complex matrix given by (3.10) below.

**Remark 3.1.** *The formulas (3.1)-(3.3) constitute the current model of neutrino oscillation, which requires the neutrinos being massive. However, the massive neutrino assumption gives rise two serious problems. First, it is in conflict with the known fact that the neutrinos violate the parity symmetry. Second, the handedness of neutrinos implies their velocity being at the speed of light.*

*In fact, by using the Weyl equations as the neutrino oscillation model we can also deduce the same conclusions and solve the two mentioned problems. Moreover, the  $\nu$  mediator introduced by the authors in [9] leads to an alternate explanation to the solar neutrino problem.*

Under the above three hypotheses (3.1)-(3.3), the oscillation between  $\nu_e, \nu_\mu$  and  $\nu_\tau$  are given in the following fashion. For simplicity we only consider two kinds neutrinos  $\nu_e, \nu_\mu$ , i.e.  $\nu_\tau = 0$ . In this case, (3.3) becomes

$$(3.4) \quad \begin{aligned} \nu_1 &= \cos \theta \nu_\mu - \sin \theta \nu_e, \\ \nu_2 &= \sin \theta \nu_\mu + \cos \theta \nu_e. \end{aligned}$$

By the Dirac equations,  $\nu_1$  and  $\nu_2$  satisfy

$$i\hbar \frac{\partial \nu_k}{\partial t} = \lambda_k \nu_k \quad \text{for } k = 1, 2.$$

The solutions of these equations read

$$(3.5) \quad \nu_k = \nu_k(0) e^{-i\lambda_k t/\hbar}, \quad k = 1, 2.$$

Assume that the initial state is at  $\nu_e$ , i.e.

$$\nu_e(0) = 1, \quad \nu_\mu(0) = 0.$$

Then we derive from (3.4) that

$$(3.6) \quad \nu_1(0) = -\sin \theta, \quad \nu_2(0) = \cos \theta.$$

It follows from (3.5) and (3.6) that

$$(3.7) \quad \nu_1 = -\sin \theta e^{-i\lambda_1 t/\hbar}, \quad \nu_2 = \cos \theta e^{-i\lambda_2 t/\hbar}.$$

Inserting (3.7) into (3.4) we deduce that

$$\nu_\mu(t) = \cos \theta \nu_1(t) + \sin \theta \nu_2(t) = \sin \theta \cos \theta (-e^{-i\lambda_1 t/\hbar} + e^{-i\lambda_2 t/\hbar}).$$

Hence, the probability of  $\nu_e$  transforming to  $\nu_\mu$  at time  $t$  is

$$(3.8) \quad P(\nu_e \rightarrow \nu_\mu) = |\nu_\mu(t)|^2 = \left[ \sin 2\theta \sin \left( \frac{\lambda_2 - \lambda_1}{2\hbar} t \right) \right]^2.$$

Also, we derive in the same fashion that

$$\nu_e(t) = \cos \theta \nu_2 - \sin \theta \nu_1 = \cos^2 \theta e^{-i\lambda_1 t/\hbar} + \sin^2 \theta e^{-i\lambda_2 t/\hbar},$$

and the probability of  $\nu_\mu$  to  $\nu_e$  is given by

$$(3.9) \quad P(\nu_\mu \rightarrow \nu_e) = |\nu_e(t)|^2 = \cos^2 \left( \frac{\lambda_2 - \lambda_1}{2\hbar} t \right) + \cos^2 2\theta \sin^2 \left( \frac{\lambda_2 - \lambda_1}{\hbar} t \right)$$

From formulas (3.8) and (3.9), we derive the oscillation between  $\nu_e$  and  $\nu_\mu$ , the energy difference  $\lambda_2 - \lambda_1$ , and the angle  $\theta$ , if the discrepancy probability  $P(\nu_e \rightarrow \nu_\mu)$  is measured.

**3.2. Mixing matrix and neutrino masses.** As mentioned in Remark 3.1, the current neutrino oscillation requires mass matrix  $A$  defined in (3.3). In this subsection we shall discuss these two topics.

*Mixing matrix*

The matrix  $A$  given in (3.3) is called the MNS matrix, which is due to Z. Maki, M. Nakagawa and S. Sakata for their pioneering work in [10]. This can be considered as an analog for leptons as the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks. The MNS matrix is written as

$$(3.10) \quad A = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where  $\delta$  is the phase factor, and

$$c_{ij} = \cos \theta_{ij}, \quad s_{ij} = \sin \theta_{ij},$$

with the values  $\theta_{ij}$  being measured as

$$\theta_{12} \simeq 34^\circ \pm 2^\circ, \quad \theta_{23} \simeq 45^\circ \pm 8^\circ, \quad \theta_{13} \simeq 10^\circ.$$

The matrix  $A$  of (3.10) is a unitary matrix:  $A^\dagger = A^{-1}$ . Therefore, (3.3) can be also rewritten as

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = A^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}.$$

*Neutrino masses*

As masses are much less than kinetic energy  $c|p|$ , by the Einstein triangular relation of energy-momentum

$$E^2 = p^2c^2 + m^2c^4,$$

we obtain an approximate relation:

$$E \simeq |p|c + \frac{1}{2} \frac{m^2c^3}{|p|}.$$

The eigenvalues  $\lambda_k$  of (3.2) and  $E$  satisfy

$$(3.11) \quad \lambda_k = E_k \simeq |p|c + \frac{1}{2} \frac{m_k^2c^3}{|p|} \quad \text{for } k = 1, 2, 3.$$

Then we have

$$(3.12) \quad \lambda_i - \lambda_j = E_i - E_j \simeq \frac{(m_i^2 - m_j^2)}{2E}c^4, \quad E \simeq |p|c.$$

By (3.12), if we can measure the energy difference  $\lambda_i - \lambda_j$ , then we get the mass square difference of  $\nu_i$  and  $\nu_j$ :

$$\Delta_{ij} = m_i^2 - m_j^2.$$

There are three mass square differences for  $\nu_1, \nu_2, \nu_3$ :

$$(3.13) \quad \Delta_{21} = m_2^2 - m_1^2, \quad \Delta_{32} = m_3^2 - m_2^2, \quad \Delta_{31} = m_3^2 - m_1^2,$$

only two of which are independent ( $\Delta_{31} = \Delta_{32} + \Delta_{21}$ ).



Now, we consider the mass relation between  $\nu_e, \nu_\mu, \nu_\tau$  and  $\nu_1, \nu_2, \nu_3$ . Applying the Dirac operator  $\hat{H}$  on both sides of (3.3), by (3.2), we have

$$(3.14) \quad \hat{H} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = A \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

where  $A$  is the MNS matrix (3.10). The energies  $E_e, E_\mu, E_\tau$  of  $\nu_e, \nu_\mu, \nu_\tau$  are given by

$$(3.15) \quad \begin{aligned} E_e &= \int \nu_e^* \hat{H} \nu_e dx = \int A_{1k}^* A_{1j} \nu_k^* \hat{H} \nu_j dx \quad (\hat{H} \nu_j = \lambda_j \nu_j), \\ E_\mu &= \int \nu_\mu^* \hat{H} \nu_\mu dx = \int A_{2k}^* A_{2j} \nu_k^* \hat{H} \nu_j dx, \\ E_\tau &= \int \nu_\tau^* \hat{H} \nu_\tau dx = \int A_{3k}^* A_{3j} \nu_k^* \hat{H} \nu_j dx, \end{aligned}$$

where  $A_{ij}$  are the matrix elements of  $A$ ,  $A_{ij}^*$  are the complex conjugates of  $A_{ij}$ . The masses  $m_e, m_\mu, m_\tau$  of  $\nu_e, \nu_\mu, \nu_\tau$  are as follows

$$(3.16) \quad E_e^2 = p^2 c^2 + m_e^2 c^4, \quad E_\mu^2 = p^2 c^2 + m_\mu^2 c^4, \quad E_\tau^2 = p^2 c^2 + m_\tau^2 c^4.$$

It is very difficult to compute  $E_e, E_\mu, E_\tau$  by (3.15). However, since  $A \in SU(3)$  is norm-preserving:

$$E_e^2 + E_\mu^2 + E_\tau^2 = E_1^2 + E_2^2 + E_3^2,$$

by (3.16) and  $E_k^2 = p^2 c^2 + m_k^2 c^4$ , we deduce that

$$m_e^2 + m_\mu^2 + m_\tau^2 = m_1^2 + m_2^2 + m_3^2,$$

which leads to

$$(3.17) \quad m_e^2 + m_\mu^2 + m_\tau^2 = \Delta_{32} + 2\Delta_{21} + 3m_1^2,$$

where  $\Delta_{32}$  and  $\Delta_{21}$  are as in (3.13).

If neutrinos have masses, then only the mass square differences  $\Delta_{ij}$  in (3.13) can be measured by current experimental methods. Hence, the only mass information of  $\nu_e, \nu_\mu, \nu_\tau$  is given by the relation (3.17).

#### 4. MSW EFFECT

In 1978, L. Wolfenstein [16] first noted that as neutrinos pass through matter there are additional effects due to elastic scattering

$$\nu_e + e \longrightarrow \nu_e + e.$$

This phenomenon was also observed and expanded by S. Mikheyev and A. Smirnov [11], and is now called the MSW effect.

The MSW effect can be reflected in the neutrino oscillation model. We recall the oscillation model without MSW effect expressed as

$$(4.1) \quad \begin{aligned} \nu_k &= \varphi_k(x) e^{-i\lambda_k t/\hbar}, \\ [-i\hbar c(\vec{\alpha} \cdot \nabla) + mc^2 \alpha_0] \varphi_k &= \lambda_k \varphi_k \quad k = 1, 2, 3, \\ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} &= A \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad A \text{ is as in (3.10)}. \end{aligned}$$

To consider the MSW effect, we have to add weak interaction potentials in the Hamiltonian operator  $\hat{H}$  for neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ . The weak potential energy is given as follows [7, 8]:

$$(4.2) \quad V_\nu = g_s^2 \left( \frac{\rho_\nu}{\rho_e} \right)^3 N_w e^{-kr} \left[ \frac{1}{r} - \frac{B_\nu}{\rho} (1 + 2kr) e^{-kr} \right],$$

where  $\rho_\nu, \rho_e$  are the radii of neutrinos and electron,  $g_s$  is the weak charge, and  $N_w$  is the weak charge density. Namely the Hamiltonian with MSW effect for  $(\nu_e, \nu_\mu, \nu_\tau)$  is

$$(4.3) \quad \hat{\mathcal{H}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \hat{H} + V_e & 0 & 0 \\ 0 & \hat{H} + V_\mu & 0 \\ 0 & 0 & \hat{H} + V_\tau \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},$$

where  $\hat{H} = -i\hbar c(\vec{\alpha} \cdot \nabla) + mc^2\alpha_0$ , and  $V$  is as in (4.2).

The equations in (4.1) are also in the form

$$(4.4) \quad \begin{pmatrix} e^{i\lambda_1 t/\hbar} & & \\ & e^{i\lambda_2 t/\hbar} & \\ & & e^{i\lambda_3 t/\hbar} \end{pmatrix} A^\dagger \hat{H} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}.$$

Replacing  $\hat{H}$  by  $\hat{\mathcal{H}}$  in (4.4), we infer from (4.1) that

$$(4.5) \quad A^\dagger \hat{\mathcal{H}} A \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \beta_1 & & \\ & \beta_2 & \\ & & \beta_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

The equation (4.5) is the neutrino oscillation model with the MSW effect, where the eigenvalues  $\beta_k$  and eigenstates  $\nu_k$  ( $1 \leq k \leq 3$ ) are different from that of (4.1). In fact, the MSW effect is just the weak interaction effect.

## 5. MASSLESS NEUTRINO OSCILLATION MODEL

There are several serious problems in the massive neutrino oscillation model (4.1), which we briefly explain as follows.

*Parity problem.* It is known that all weak interaction decays and scatterings involving neutrinos violate the parity symmetry, discovered by Lee and Yang in 1956 and experimentally verified by C. Wu [5, 17]. It means that the neutrinos are parity non-conserved. Hence it requires that under the space reflective transformation

$$(5.1) \quad x \longrightarrow -x,$$

the equations governing neutrinos should violate the reflective invariance. We know that the Dirac equations are covariant under the reflection (5.1), while the Weyl equations are not covariant. Hence, the massive neutrino oscillation model is in conflict with the violation of parity symmetry.

*Handedness and speed of neutrinos.* Experiments showed that all neutrinos possess only the left-handed spin  $J = -\frac{1}{2}$ , and anti-neutrinos possess the right-handed spin  $J = \frac{1}{2}$ . It implies that the velocity of free neutrinos must be at the speed of light, which is a contradiction with massive neutrino assumption.

In fact, the handedness is allowed only for massless particles. Otherwise, there exist two coordinate systems  $A$  and  $B$  satisfying

$$v_A < v_p < v_B,$$

where  $v_A, v_B$  and  $v_p$  are the velocities of  $A, B$  and the particle. When we look at the particle  $\nu$  from  $A$  and  $B$ , the spins would be reversed. Therefore, all massive particles must have both left-handed and right-handed spins.

In addition, all experiments measuring neutrino velocity had found no violation to the speed of light.

*Infinite number of eigenvalues and eigenstates.* The neutrino oscillation theory faces the problem of the existing of infinite number of eigenvalues. In the massive model (4.1), the wave functions are the Dirac spinors

$$\varphi = (\varphi^1, \varphi^2, \varphi^3, \varphi^4)^t.$$

For free neutrinos moving on a straight line,  $\varphi$  depends only on  $z$ . Thus the eigenvalue equations in (4.1) become

$$(5.2) \quad \begin{aligned} -i\hbar c\sigma_3 \frac{d}{dz} \begin{pmatrix} \varphi^3 \\ \varphi^4 \end{pmatrix} + mc^2 \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} &= \lambda \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix}, \\ -i\hbar c\sigma_3 \frac{d}{dz} \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} - mc^2 \begin{pmatrix} \varphi^3 \\ \varphi^4 \end{pmatrix} &= \lambda \begin{pmatrix} \varphi^3 \\ \varphi^4 \end{pmatrix}, \end{aligned}$$

where

$$(5.3) \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The equations (5.2) possess infinite number of eigenvalues

$$(5.4) \quad \lambda = \sqrt{m^2 c^4 + \frac{4\pi^2 n^2 \hbar^2 c^2}{l^2}}, \quad \forall l > 0, n = 0, 1, 2, \dots,$$

and each eigenvalue has two eigenstates

$$(5.5) \quad \begin{aligned} \varphi_1 &= \frac{e^{i2\pi n z/l}}{\sqrt{2}l^{3/2}} \begin{pmatrix} \sqrt{1 + mc^2/\lambda} \\ 0 \\ \sqrt{1 - mc^2/\lambda} \\ 0 \end{pmatrix}, \\ \varphi_2 &= \frac{e^{i2\pi n z/l}}{\sqrt{2}l^{3/2}} \begin{pmatrix} 0 \\ \sqrt{1 + mc^2/\lambda} \\ 0 \\ -\sqrt{1 - mc^2/\lambda} \end{pmatrix}. \end{aligned}$$

The problem is that which eigenvalues and eigenstates in (5.4) and (5.5) are the ones in the neutrino oscillation model (4.1), and why only three of (5.4)-(5.5) stand for the flavors of neutrinos.

The Weyl equations can replace the Dirac equations to describe the neutrino oscillation, which we call massless neutrino oscillation model, expressed as follows

$$(5.6) \quad \begin{aligned} \nu_k &= \varphi_k(x) e^{-i\lambda_k t/\hbar}, \\ i\hbar c(\vec{\sigma} \cdot \nabla)\varphi_k &= \lambda_k \varphi_k \quad \text{for } k = 1, 2, 3, \\ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} &= A \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad A \text{ is as in (3.10),} \end{aligned}$$

where  $\nu_k$  ( $1 \leq k \leq 3$ ) are the two-component Weyl spinors, and  $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$

Based on the massless model (5.6), both problems of parity and handedness of neutrinos have been resolved, and we can derive in the same conclusions as given in (3.8) and (3.9). In this case, the differences  $\lambda_i - \lambda_j$  of eigenvalues in the transition probabilities such as (3.8) and (3.9) stand for the differences of frequencies:

$$(5.7) \quad \lambda_i - \lambda_j = \omega_i - \omega_j,$$

where  $\omega_k$  ( $1 \leq k \leq 3$ ) are the frequencies of  $\nu_k$ .

However, the massless model also faces the problem of infinite number of eigenvalues as mentioned above. The eigenvalue equations in (5.6) for the straight line motion on the  $y$ -axis is written as

$$(5.8) \quad i\hbar c \alpha^2 \frac{d}{dy} \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} = \lambda \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} \quad \text{with} \quad \alpha^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

The eigenvalues of (5.8) are

$$(5.9) \quad \lambda_k = k\hbar c, \quad \forall k > 0,$$

and each eigenvalue of (5.9) has two eigenstates

$$(5.10) \quad \begin{pmatrix} \varphi_1^1 \\ \varphi_1^2 \end{pmatrix} = \begin{pmatrix} \sin ky \\ -\cos ky \end{pmatrix}, \quad \begin{pmatrix} \varphi_2^1 \\ \varphi_2^2 \end{pmatrix} = \begin{pmatrix} \cos ky \\ \sin ky \end{pmatrix}.$$

The eigenvalues of (5.6) at  $x$ -axis and  $z$ -axis are all the same as in (5.9), and the eigenstates at the  $x$  and  $z$  axes are

$$(5.11) \quad \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} = \begin{pmatrix} e^{-ikx} \\ e^{-ikx} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} = \begin{pmatrix} e^{-ikz} \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ e^{ikz} \end{pmatrix}.$$

## 6. NEUTRINO NON-OSCILLATION MECHANISM

Although the massless neutrino oscillation model can solve the parity and the handedness problems appearing in the massive neutrino oscillation mechanism, the problem of infinite numbers of eigenvalues and eigenstates still exists in the model (5.6).

In fact, the weakton model first introduced in [9] can provide an alternative explanation to the solar neutrino problem. Based on the weakton model, there exists a  $\nu$ -mediator, whose weakton constituents are given by

$$(6.1) \quad \nu = \alpha_e \nu_e \bar{\nu}_e + \alpha_\mu \nu_\mu \bar{\nu}_\mu + \alpha_\tau \nu_\tau \bar{\nu}_\tau,$$

where  $\alpha_e^2 + \alpha_\mu^2 + \alpha_\tau^2 = 1$ . The values  $\alpha_e^2, \alpha_\mu^2, \alpha_\tau^2$  represent the ratio of the neutrinos  $\nu_e, \nu_\mu$  and  $\nu_\tau$  in our Universe

In view of (6.1), we see the reaction

$$(6.2) \quad \begin{aligned} \nu_e + \bar{\nu}_e &\longrightarrow \nu & (\nu_e \bar{\nu}_e), \\ \nu_\mu + \bar{\nu}_\mu &\longrightarrow \nu & (\nu_\mu \bar{\nu}_\mu), \\ \nu_\tau + \bar{\nu}_\tau &\longrightarrow \nu & (\nu_\tau \bar{\nu}_\tau), \end{aligned}$$

which are generated by the weak interaction attracting force, as demonstrated in the weak charge potentials

$$(6.3) \quad \Phi_i = g_w e^{-r/r_0} \left[ \frac{1}{r} - \frac{B_i}{\rho_\nu} \left( 1 + \frac{2r}{r_0} \right) e^{-r/r_0} \right] \quad \text{for } 1 \leq i \leq 3,$$

where  $r_0 = 10^{-16}$  cm,  $B_1, B_2, B_3 > 0$  are the weak interaction constants for  $\nu_e, \nu_\mu, \nu_\tau$  respectively, and  $\rho_\nu$  is the neutrino radius.

The formula (6.3) defines attractive radii  $R_i$  for the neutrinos and antineutrinos of the same flavors. Namely, when  $\nu_i$  and  $\bar{\nu}_i$  are in the radius  $R_i$ , the reaction (6.2) may occur:

$$(6.4) \quad \nu_i + \bar{\nu}_i \longrightarrow \nu (\nu_i \bar{\nu}_i) \quad \text{if } \text{dist}(\nu_i, \bar{\nu}_i) < R_i \quad \text{for } 1 \leq i \leq 3,$$

where  $\nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau$ , and  $\text{dist}(\nu_i, \bar{\nu}_i)$  is the distance between  $\nu_i$  and  $\bar{\nu}_i$ . The condition (6.4) implies that the transition probability  $\Gamma_i$  depends on  $R_i$ :

$$(6.5) \quad \Gamma_i = \Gamma_i(R_i) \quad \text{for } 1 \leq i \leq 3.$$

The attracting radius  $R_i$  satisfies that

$$(6.6) \quad \frac{d}{dr} \Phi_i(R_i) = 0.$$

Thus we can give a non-oscillation mechanism of neutrinos to explain the solar neutrino problem. Namely, due to the  $\beta$ -decay, there are large amounts of electronic anti-neutrinos  $\bar{\nu}_e$  around the earth, which generate the reaction (6.3) with  $\nu_e$ , leading to the discrepancy of the solar neutrino.

In addition, there are three axis eigenvalues of the Weyl equations given by (5.10) and (5.11). We believe that they are the three flavors of neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ . Namely, the following three wave functions

$$(6.7) \quad \begin{aligned} \psi_1 &= c_1 \begin{pmatrix} \sin ky \\ -\cos ky \end{pmatrix} + c_2 \begin{pmatrix} \cos ky \\ \sin ky \end{pmatrix}, \\ \psi_2 &= \begin{pmatrix} e^{-ikx} \\ e^{-ikx} \end{pmatrix}, \\ \psi_3 &= c_3 \begin{pmatrix} e^{-ikz} \\ 0 \end{pmatrix} + c_4 \begin{pmatrix} 0 \\ e^{ikz} \end{pmatrix} \end{aligned}$$

represent the three flavors of neutrinos. In fact, massive particles in field equations are distinguished by different masses, and flavors of neutrinos by different axis eigenstates.

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