

# Fire sales, indirect contagion and systemic stress-testing

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## Abstract

We present a framework for modeling the phenomenon of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. Asset losses triggered by macro-shocks may interact with portfolio constraints, resulting in liquidation of assets, which in turn affects market prices, leading to contagion of losses when portfolios are marked to market. If mark-to-market losses are large, this may in turn lead to a new round of fire sales.

In contrast to balance sheet contagion mechanisms based on direct linkages, this price-mediated contagion is transmitted through common asset holdings, which we quantify through liquidity-weighted overlaps across portfolios. Exposure to price-mediated contagion leads to the concept of indirect exposure to an asset class, as a consequence of which the risk of a portfolio depends on the matrix of asset holdings of other large and leveraged portfolios with similar assets.

Our model provides an operational systemic stress testing method for quantifying the exposure of the financial system to these effects. Using data from the European Banking Authority, we apply this method to examine the exposure of the EU banking system to price-mediated contagion. Our results indicate that, even with optimistic estimates of market depth, moderately large macro-shocks may trigger fire sales which then lead to substantial losses across bank portfolios, modifying the outcome of bank stress tests. Moreover, we show that price-mediated contagion leads to a heterogeneous cross-sectional loss distribution across banks, which cannot be replicated simply by applying a macro-shock to bank portfolios in absence of fire sales. We propose a bank-level indicator, based on the analysis of liquidity-weighted overlaps across bank portfolios, which is shown to be strongly correlated with bank losses due to fire sales and may be used to quantify the contribution of a financial institution to price-mediated contagion.

Unlike models based on 'leverage targeting', which assume symmetric reactions to gains or losses, our approach is based on the asymmetric interaction of portfolio losses with one-sided constraints such as leverage or capital requirements, makes a distinction between insolvency and illiquidity and leads to substantially different loss estimates in stress scenarios.

**Keywords:** Financial stability, stress testing, liquidity, systemic stress testing, macroprudential regulation, macroprudential stress testing, endogenous risk, systemic risk, price-mediated contagion, indirect exposures.

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# 1 Introduction

## 1.1 The need for macroprudential stress tests

The financial crisis of 2007-2008 has underlined the need for better monitoring of systemic risk and led to the emergence of a “macroprudential” approach to financial regulation [Hanson et al., 2011]. Yet, as pointed out by the Basel Committee on Banking Supervision’s recent report, stress tests conducted by bank supervisors, which remain their primary monitoring tool, still lack a genuine macroprudential component [Basel Committee on Banking Supervision, 2015]. The report identifies the key missing ingredients as “endogenous reactions and feedback effects to initial stress”. As noted by ECB Vice-President Vitor Constâncio [Constancio, 2016], in the current approach to bank stress tests “no bank reaction is considered. It would be far more realistic to assume that market participants could react to adverse conditions, rather than assuming passive bank behaviour throughout the entire stress test period. Bank behaviour or reaction could take the form of deleveraging, straight capital increases or working out of non-performing loans.”

There is ample empirical evidence that such deleveraging occurred on a large scale in 2008, leading to cross-asset contagion and amplification of losses in the financial system [Kashyap et al., 2008, Brunnermeier and Pedersen, 2009, Khandani and Lo, 2011, Manconi et al., 2012, Cont and Wagalath, 2016]. The well-documented occurrence of fire sales during market downturns [Ellul et al., 2011, Coval and Stafford, 2007, Shleifer and Vishny, 2011, Jotikasthira et al., 2012] is not a coincidence: portfolio constraints -capital, leverage or liquidity constraints- that financial institutions are subject to forces them to deleverage when these constraints are breached as a result of losses, leading to fire sales of assets [Kyle and Xiong, 2001, Cont and Wagalath, 2013]. Similar large-scale deleveraging is also foreseeable in future stress scenarios, and foreseen by financial institutions themselves: “*If we are unable to raise needed funds in the capital markets (including through offerings of equity and regulatory capital securities), we may need to liquidate unencumbered assets to meet our liabilities. In a time of reduced liquidity, we may be unable to sell some of our assets, or we may need to sell assets at depressed prices, which in either case could adversely affect our results of operations and financial condition*” [Credit Suisse, 2015].

Fire sales generate endogenous risk [Shin, 2010] and can also act as channel of loss contagion across asset classes and across financial institutions holding these assets [Khandani and Lo, 2011, Cont and Wagalath, 2016, Caccioli et al., 2014]. Unlike direct contagion through counterparty exposures [Cont et al., 2013], fire-sales spillovers are mediated by prices and thus defy limits on counterparty exposures and other institutional ring-fencing measures.

It is therefore important for supervisors to include in macro-stress testing frameworks used for assessing bank capital adequacy the impact of fire sales and the deleveraging of portfolios in stress scenarios. This is especially relevant given that, post-crisis, supervisory stress tests have set the binding constraint for bank capital adequacy.

Fire sales and the resulting destabilizing feedback effects have been extensively studied in the literature [Kyle and Xiong, 2001, Shleifer and Vishny, 2011] from a conceptual viewpoint. The challenge is to develop a quantitative framework versatile enough to be taken to empirical data and used in an operational macro-stress testing framework to quantify the endogenous risk and spillover effects arising from fire sales.

The goal of the present work is to address this challenge, by proposing a modeling framework for quantifying the exposure of the financial system to the endogenous losses and feedback effects resulting from fire sales in a macro-stress scenario. We provide a detailed discussion of the model, its use for the design of *systemic stress tests*, and the results obtained by applying the methodology to EU bank portfolios.

Previous attempts by regulators to account indirectly for the impact of fire sales in bank stress tests include the use of (exogenously specified) liquidation costs or increasing the severity of shocks in single-bank stress tests to account for possible loss amplification due to feedback from fire sales. These adjustments may mimic the severity of potential losses which may result

from fire sales but fail to capture key cross-sectional features of fire-sales spillovers, such as the contagion across asset classes and the heterogeneous distribution of fire-sales losses across financial institutions and across asset classes of varying liquidity.

More recently, [Greenwood et al., 2015, Duarte and Eisenbach, 2015] have proposed a stress testing approach incorporating the impact on asset prices of deleveraging in bank portfolios, based on the assumption of *leverage-targeting* [Adrian and Shin, 2010] i.e. that, in response to a shock, financial institutions rebalance their portfolios to maintain a constant leverage, which leads to a linear deleveraging rule in reaction to market shocks. This approach was used to analyze fire sales spillovers in the EU banking system by [Greenwood et al., 2015] and in the US banking system by [Duarte and Eisenbach, 2015]. Both studies find evidence of potentially large exposures of the banking system to contagion via fire sales. This approach has been explored by supervisors as a possible method for incorporating fire-sales in macro stress tests [Henry et al., 2013, Cappiello et al., 2015]. There is some empirical evidence that in the medium term large financial institutions maintain fairly stable levels of leverage [Adrian and Shin, 2010] but it is not clear why the same institutions would enforce such leverage targets in the short term, especially in stress scenarios where this could entail high liquidation costs.

We propose a different approach for modeling fire sales, based on the premise that deleveraging by financial institutions occurs in reaction to losses in their portfolios [Kyle and Xiong, 2001, Cont and Wagalath, 2013]. This deleveraging may be the result of investor redemptions for funds, as evidenced in [Ellul et al., 2011, Coval and Stafford, 2007, Shleifer and Vishny, 2011, Jotikasthira et al., 2012]; but for regulated financial institutions such as banks, large scale deleveraging is mainly driven by portfolio constraints -capital, leverage or liquidity constraints- which may be breached when large losses occur. We have focused for simplicity on leverage constraints, but the model is easily extendable to multiple constraints on portfolios. Given the one-sided nature of these constraints, such institutions react asymmetrically to large losses and large gains [Ang et al., 2006]. The asymmetry and the threshold nature of deleveraging differentiates our approach from models based on leverage targeting, leading to quite different outcomes.

Deleveraging by financial institutions impacts market prices and, when portfolios are marked to market, leads to further losses which may in turn trigger further deleveraging. We quantify this impact and the resulting endogenous risk, paying attention to the estimation of market impact parameters; the magnitude of these parameters, and their heterogeneity across asset classes, is shown to greatly influence the results of the stress tests.

We use these ingredients to design a systemic stress testing framework for banking systems. Application of the method to the EU banking system shows that this approach may lead to outcomes which are substantially different from single-bank stress tests. We emphasize in particular the concept of *indirect exposure*, which we show to be relevant for bank stress testing.

## 1.2 Summary and main findings

We present a framework for quantitative modeling of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. Asset sales may be triggered in reaction to external shocks to asset values when portfolios are subject to capital or leverage constraints; the market impact of these asset sales then leads to contagion of mark-to-market losses to other portfolios, which may in turn be led to deleverage if their constraints are breached. In contrast to balance sheet contagion which arises through direct bilateral exposures, this *price-mediated contagion* occurs through common asset holdings, even in absence of direct linkages between financial institutions. The resulting feedback loop may lead to loss amplification, *systemic risk* and large-scale instability of the financial system. Our model provides a method for quantifying the exposure of the financial system to these effects, which we apply to data on European banks. This leads to several interesting findings.

- **Existence of a tipping point:** we show the existence of a critical macro-shock level beyond which fire sales trigger large-scale contagion and lead to systemic risk. The level of this critical shock depends on the institutions' leverage, as well as the concentration and commonality of their asset holdings. In the European banking system, this critical shock size is found to correspond to large, but not extreme, losses in asset values.
- **Magnitude and heterogeneity of losses due to fire sales:** we find that fire sales contribute significantly to system-wide losses in stress scenarios, accounting for more than 35% of the total losses and between 20 to 40% of system bank equity. These results are significant enough to modify the outcome of bank stress tests.

Moreover, while the total system-wide loss can always be replicated in a stress test without fire sales by applying a larger shock to assets, the heterogeneous cross-sectional distribution of losses across banks cannot be reproduced in absence of fire sales by simply applying a larger macro-shock.

- **Importance of gain-loss asymmetry and the threshold nature of fire-sales:** We argue that fire sales arise when portfolio constraints such as leverage, liquidity or capital ratios are breached as a result of large portfolio losses. The *one-sided* nature of these constraints leads to an asymmetric reaction of banks to gains and losses, which differentiates our model from the 'leverage targeting' model used in some previous studies of fire sales [Adrian and Shin, 2010, Duarte and Eisenbach, 2015, Greenwood et al., 2015]. Comparison with stress tests based on leverage targeting shows substantial differences in outcomes: leverage targeting models overestimate the magnitude of fire sales, especially at smaller shock levels, but underestimate the acceleration (convexity) of fire sales with increasing shock size present in the threshold model.
- **Distinction between insolvency and illiquidity:** unlike previous models of contagion which have mainly focused on modeling insolvency, our model distinguishes between failure due to insolvency and failure due to illiquidity.
- **Indirect exposures:** As a result of fire-sales spillovers, a portfolio's exposure to an asset class in a stress scenario may be larger than its notional exposure. This naturally leads to the notion of *indirect exposure*, which is scenario-dependent, and can be quantified using our model. One striking finding is that a bank may have a non-zero (indirect) exposure to an asset class it does not hold. We find evidence that EU banks have substantial indirect exposures to housing prices in EU countries where they do not have direct mortgage holdings.
- **Sensitivity to market depth across asset classes:** The level of fire sales losses in stress tests is found to be sensitive to the assumptions on market depth across asset classes. Calibration to data on market prices, trading volumes and turnover reveals a significant dispersion in market depth across asset classes. We show that ignoring this heterogeneity of market depth across asset classes leads to a considerable bias in the estimation of fire sales losses.
- **Second-round effects are significant:** fire sales may potentially lead to a feedback loop which generates market losses and further fire sales across other financial institutions. Ignoring these feedback effects and the corresponding second- and higher-round deleveraging may lead to a significant underestimation of system-wide losses.

These findings have many implications for risk management in financial institutions and for the monitoring of systemic risk in the financial sector. In particular, they underline the need for a systemic approach to stress testing and the necessity of macroprudential tools for tackling the

risks resulting from price-mediated contagion. We believe the model presented here provides a useful tool for monitoring of system-wide and bank-level exposure to price-mediated contagion.

Based on these findings, we design an indicator, the Indirect Contagion Index (ICI), which may be computed from bank portfolio holdings and measures a bank’s exposure to price mediated contagion. Importantly, the computation of this indicator does not require running a stress test and provides a simpler proxy for monitoring exposure to fire sales contagion.

### 1.3 Outline

The paper is structured as follows. Section 2 introduces our modeling framework and outlines a method for systemic stress testing in presence of fire sales. Section 3 describes the application of this stress testing approach to the European banking system using data from the European Banking Authority (EBA) and details our empirical findings. In Section 5, we introduce the concept of *indirect exposure* resulting from fire sales spillovers and illustrate its magnitude of indirect exposures in the European banking system. Section 4 introduces the *Indirect Contagion Index* and shows how this indicator provides useful information on a bank’s exposure to price-mediated contagion based on cross-sectional data on portfolio holdings. Section 6 discusses implications of our findings for systemic stress testing, risk management in institutions and macroprudential policy.

## 2 Modeling spillover effects from fire-sales

### 2.1 Balance sheets and portfolio constraints

We consider a stylized model of the financial sector, with multiple financial institutions whose portfolios may contain holdings across multiple asset classes. We will sometimes refer to these institutions as ‘banks’ although, as we will explain below, it is relevant to include non-banks in the scope of the model. Each institution holds two types of assets:

1. Illiquid assets: These are portfolio holdings which are either not easily marketable or would be subject to a deep discount if they are to be sold during a stress scenario; we assume that they are not subject to fire sales. This category includes non-securitised loans, commercial and residential mortgage exposures and retail exposures<sup>1</sup>.
2. Securities: These are positions in marketable securities which may be liquidated over a short time scale if necessary. Sovereign bonds, corporate bonds and derivatives exposures are included in this category. Each asset class  $\mu$  in this category is characterized by a market depth parameter  $D_\mu$ , whose estimation we will describe in detail below. The market liquidity of assets within this group can vary strongly and our model accounts for this heterogeneity.

Financial institutions are labeled by  $i = 1..N$ , security (types) by  $\mu = 1..M$ , illiquid asset classes by  $\kappa = 1..K$ . Securities holdings (in Euros) of institution  $i$  are denoted  $(\Pi^{i,\mu}, \mu = 1..M)$  and holdings in illiquid assets are denoted  $(\Theta^{i,\kappa}, \kappa = 1..K)$ .

The capital (equity) of financial institution  $i$  is denoted  $C^i$  and  $I_i = \sum_{\kappa=1}^K \Theta^{i,\kappa}$  is the total value of illiquid assets. The leverage ratio of the institution is then given by

$$\lambda^i(\Pi, C, I) = \frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i}{C^i} = \frac{\sum_{\mu=1}^M \Pi^{i,\mu} + \sum_{\kappa=1}^K \Theta^{i,\kappa}}{C^i} \leq \lambda_{\max}, \quad (1)$$

where the upper bound  $\lambda_{\max}$  corresponds to a regulatory leverage constraint, as for instance required by Basel 3. We consider here for simplicity that the leverage ratio will be the binding

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<sup>1</sup>This category further includes assets that do not suffer any direct or indirect losses, are not available for deleveraging, but still contribute to balance sheet size and leverage (e.g. intangible assets).

constraint for financial institutions' capital, but the model may be easily adapted to include more than one constraint: for instance, if we introduce regulatory risk weights  $w_\mu$  for each asset class, the constraint on the ratio of capital to risk weighted assets may be expressed as

$$\frac{\sum_{\mu=1}^M w_\mu \Pi^{i,\mu} + \sum_{\kappa=1}^K w_\kappa \Theta^{i,\mu}}{C^i} \leq R_{\max}$$

Other portfolio constraints, such as liquidity ratios, lead to similar one-sided linear inequalities.

The state of the financial system is summarized by the matrices of holdings  $\Pi$ ,  $\Theta$  and the capital levels  $C$ . We will now describe how this system evolves when subject to macroeconomic stress.

## 2.2 Stress scenarios

We consider stress scenarios described through the percentage loss  $\epsilon_\kappa \in [0, 100\%]$  in values for each asset class  $\kappa$ , as in the approach used in regulatory bank stress tests. For simplicity, and to emphasize contagion across asset classes, we will consider shocks to illiquid assets in the examples below, but the model perfectly accommodates stress scenarios with heterogeneous shocks across all asset classes. The initial loss of bank  $i$  in the stress scenario  $\epsilon = (\epsilon_\kappa, \kappa = 1..K)$  is given by

$$\langle \Theta^i, \epsilon \rangle = \sum_{\kappa=1}^K \epsilon_\kappa \Theta^{i,\kappa}. \quad (2)$$

This results in a loss (i.e. a reduction) in the value of illiquid holdings, reducing it to

$$I_i(\epsilon) = I_i - \langle \Theta^i, \epsilon \rangle$$

and a corresponding decrease in the equity:

$$C_0^i(\epsilon) = (C^i - \langle \Theta^i, \epsilon \rangle)_+. \quad (3)$$

The leverage of bank  $i$  then increases to

$$\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) = \frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon)}{C_0^i(\epsilon)} = \frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i - \langle \Theta^i, \epsilon \rangle}{(C^i - \langle \Theta^i, \epsilon \rangle)_+} \quad (4)$$

If this value exceeds the leverage constraint  $\lambda_{\max}$  then the institution needs to deleverage, i.e. sell some assets. This occurs if the loss level exceeds the threshold

$$\langle \Theta^i, \epsilon \rangle \geq \frac{C^i (\lambda_{\max} - \lambda^i(\Pi, C, I))}{\lambda_{\max} - 1}, \quad (5)$$

which is proportional to the capital buffer of  $i$  or its distance from the leverage constraint prior to the shock. The amount of loss an institution can absorb before being led to deleverage is equal to its capital buffer in excess of regulatory requirements.

In the case where the shock is applied to a single asset class  $\kappa$ , institution  $i$  is led to deleverage when the (percentage) loss  $\epsilon_\kappa$  exceeds the level

$$\epsilon_{i,\kappa}^*(\Pi, C, I) = \frac{C^i (\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1) \Theta^{i,\kappa}}. \quad (6)$$

This threshold is clearly different across institutions, depending on their initial leverage, capital buffer and holdings in the illiquid asset subject to losses. The model hence explicitly accounts for the heterogeneity in the individual bank's resilience to losses.

Hence, any stress scenario  $\epsilon$  which falls outside of the (convex) set

$$\mathcal{S}(\Pi, C, \Theta) = \left\{ \epsilon \in [0, 1]^K \left| \forall i = 1..N, \langle \Theta^i, \epsilon \rangle \leq \frac{C^i(\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1)} \right. \right\}. \quad (7)$$

will lead to deleveraging by one or more financial institutions. The convex set  $\mathcal{S}(\Pi, C, \Theta)$  corresponds to a 'safety zone' of stress scenarios which all banks can withstand; the larger this safety zone, the more resilient is the network to losses.

We assume that no major deleveraging occurs inside this zone. This is different from 'leverage targeting' models such as [Greenwood et al., 2015], where portfolios react (symmetrically) to arbitrarily small shocks.

The size of the safety zone depends on the capital levels  $C_i$  of the financial institutions but also on their capital buffers in excess of requirements. If the vector  $\epsilon$  of shocks to assets lies outside the safety zone, then one or more institutions are led to deleverage their portfolios. We now describe how this deleveraging is modeled.

### 2.3 Deleveraging

If the magnitude of losses in asset values is such that the leverage constraint is breached for institution  $i$ , it deleverages a proportion  $\Gamma^i$  of its portfolio in order to restore its leverage ratio to a leverage target  $\lambda_b \leq \lambda_{\max}$ . This leads to the following equation for  $\Gamma^i$ :

$$\frac{(1 - \Gamma^i) \sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon)}{C_0^i} = \lambda_b \quad (8)$$

Solving for  $\Gamma^i$  yields

$$\Gamma^i(\Pi, C) = \left( \frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon) - \lambda_b C_0^i}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1 \right) \mathbf{1}_{\frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon)}{C_0^i} > \lambda_{\max}}. \quad (9)$$

Thus, in response to an external shock  $\epsilon$ , institution  $i$  needs to deleverage a fraction

$$\Gamma^i(\Pi_0, C_0) = \left( \frac{(\lambda_b - 1) \langle \Theta^i, \epsilon \rangle - C_0^i(\lambda_b - \lambda^i(\Pi, C, I(\epsilon)))}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1 \right) \mathbf{1}_{\lambda^i(\Pi, C_0, I(\epsilon)) > \lambda_{\max}}. \quad (10)$$

of its marketable assets, where  $\Pi_0, C_0, I(\epsilon)$  are given by (3). Unlike the state variables  $\Pi, C$  which evolve as deleveraging occurs,  $\Theta$  and  $\epsilon$ , which represent respectively the holdings in illiquid assets and the initial shocks to these assets, are static parameters.

As in previous studies on fire sales [Greenwood et al., 2015, Duarte and Eisenbach, 2015] we assume that banks delever their marketable assets proportionally. This proportional deleveraging assumption is supported by empirical studies on assets sales of large financial institutions [Getmansky et al., 2016]. An alternative would be to assume that banks determine their liquidation policy using an optimization criterion, such as maximization of liquidation value [Braouezec and Wagalath, 2016].

Figure 1 (circles) displays the dependence of the deleveraging ratio (10) on the shock level  $\epsilon$  for a portfolio holding a single class of illiquid asset: this ratio is zero for shocks lower than the threshold  $\epsilon_i^*$  corresponding to the breach of the leverage constraint, then increases linearly afterwards. There is a discontinuity at the onset of deleveraging, which depends on the size of the capital buffer which the bank intends to rebuild by deleveraging. In the case where the leverage constraint is saturated after deleveraging i.e.  $\lambda_b = \lambda_{\max}$ , the dependence on the shock size is continuous and convex (solid line). In the example shown in Figure 1 we have assumed  $\lambda_b = 0.95\lambda_{\max}$  which corresponds to a safety buffer 5% below the leverage constraint.



Previous studies on fire sales have assumed instead a *linear* dependence of the volume of deleveraging with respect to the shock size (dotted line): this is the leverage targeting model, which we will discuss further in Section 2.6.

Unlike the leverage targeting model, our model gives rise to deleveraging only if the shock level exceeds a (bank-specific) threshold  $\epsilon_i^*$ . Once this threshold is reached, the volume of deleveraging increases linearly with  $(\epsilon - \epsilon_i^*)_+$  until no more marketable assets are available for sale (i.e.  $\Gamma^i = 1$ ). At this point, although the institution is still solvent, it may become illiquid. The model thus leads to a natural distinction between failures due to insolvency, which may occur if the initial loss in asset values is large enough, and failures due to illiquidity, which may occur further down the road if the liquidation of marketable assets fails to raise enough liquidity.

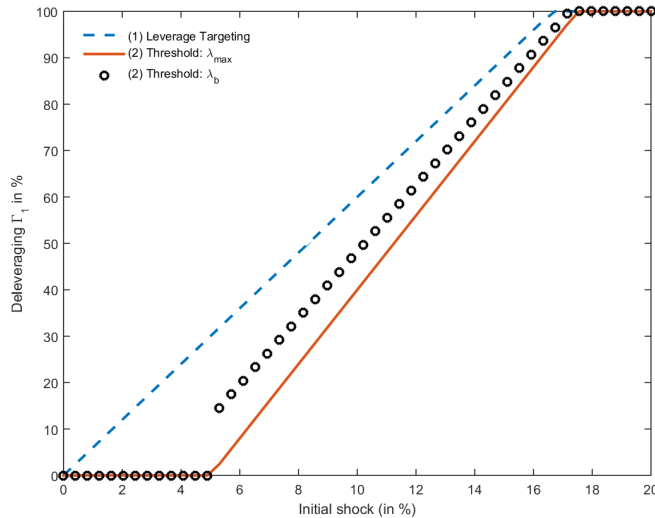


Figure 1: Volume of asset sales ( % of marketable assets) as a function of the percentage loss in value of illiquid assets for a portfolio with 20% in illiquid assets and initial leverage of 25, and a leverage constraint of 33. Leverage targeting leads to a linear response (dashed line), whereas our assumption of deleveraging to comply with a leverage constraint leads to no asset sales for shocks smaller than a threshold and a linear increase above the threshold (solid line). Finally, if we assume that the bank deleverages to restore a non-zero capital buffer we obtain the discontinuous response function (circles).

Summing across all institutions  $j = 1..N$  in the network yields the total volume of asset sales (in monetary units) in the stress scenario  $\epsilon$ :

$$q^\mu(\epsilon, \Pi, C) = \sum_{j=1}^N \Gamma^j(\Pi, C_0(\epsilon)) \Pi^{j,\mu}$$

for the asset class  $\mu$ . In the case  $\lambda_b = \lambda_{\max}$ , the functions  $\Gamma^j(\Pi, C_0(\epsilon))$  are convex with respect to  $\epsilon$  over a large range of values i.e. in the region  $\max_i \Gamma^i \leq 1$ . In this range, the aggregate volume of asset sales  $q^\mu(\epsilon, \Pi, C)$  exhibits a convex dependence in  $\epsilon$ , which leads to a 'multiplier effect': as more severe stress scenarios (larger  $\epsilon$ ) are considered, the marginal response in terms of deleveraging also increases. An example, based on data from the EU banking system (see next section) is shown in Figure 2.

## 2.4 Market impact and price-mediated contagion

If this volume of deleveraging represents a sizeable fraction of the market depth, it may have a non-negligible impact on the market price of these assets and lead to a price decline, whose

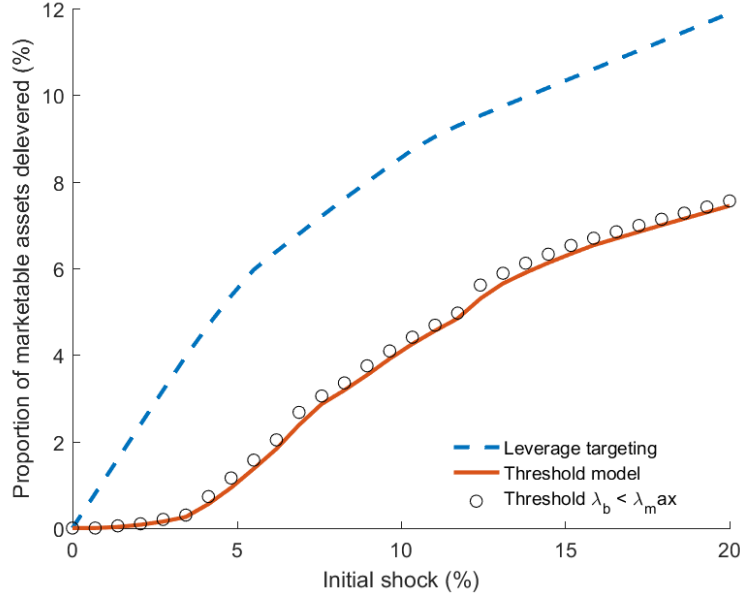


Figure 2: Volume of asset sales ( % of marketable assets) across EU banks in reaction to a scenario the Spanish housing sector depreciates by  $\epsilon$  (horizontal axis). The Basel 3 leverage constraint  $\lambda_{\max} = 33$  is used in this example. The solid red line corresponds to  $\lambda_{\max} = \lambda_b$ , the circles correspond to  $\lambda_b = 0.95\lambda_{\max}$ .

magnitude  $\Delta S^\mu$  is an increasing function of  $q^\mu$ :

$$\frac{\Delta S^\mu}{S^\mu} = -\Psi_\mu(q^\mu) \quad (11)$$

where  $\Psi_\mu : \mathbb{R} \rightarrow [0, 1]$  is a continuous, increasing and concave function with  $\Psi_\mu(0) = 0$ , which we call the *market impact function* for asset class  $\mu$ .  $\Psi_\mu$  may be thought of as an inverse demand function. Assuming  $\Psi_\mu$  is a smooth function, linearizing for small volumes yields

$$\Psi_\mu(q) \simeq \frac{q}{D_\mu} \quad \text{where} \quad D_\mu = \frac{1}{\Psi'_\mu(0)}$$

is a measure of *market depth* for asset class  $\mu$ . Naturally this quantity depends on the asset  $\mu$ , as the same dollar liquidation volume  $q$  will have a different price impact depending on the asset class. A simple one-parameter specification often used in practice is  $\Psi_\mu(q) = \psi(q/D_\mu)$  where  $\psi(\cdot)$  is an increasing function with  $\psi'(0) = 1$ . We discuss parametric specifications of  $\Psi_\mu$  and their estimation in Section 3.2.

We can now describe the processes which occur at (the  $k$ -th round of) deleveraging. Denote by  $\Pi_{k-1}, C_{k-1}, S_{k-1}$  respectively the holdings in marketable assets, the equity and the (vector of) asset prices after  $k-1$  rounds of deleveraging. At round  $k$ :

1. Each institution  $j$  deleverages by selling a proportion  $\Gamma^j(\Pi_{k-1}, C_{k-1})$  of its marketable assets, leading to an aggregate amount  $q_k^\mu = \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu}$  of sales in asset class  $\mu$ .
2. The market impact of asset sales results in a decline in market prices, moving the market price to

$$S_k^\mu = S_{k-1}^\mu (1 - \Psi_\mu(q_k^\mu)). \quad (12)$$

3. This decline in price changes the market value of holdings in asset class  $\mu$  to

$$\Pi_k^{i,\mu} = \underbrace{(1 - \Gamma^i(\Pi_{k-1}, C_{k-1}))}_{\text{Remainder after deleveraging by } i} \underbrace{\Pi_{k-1}^{i,\mu}}_{\text{Previous value}} \underbrace{\left(1 - \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu}\right)\right)}_{\text{Price impact on remaining holdings}}. \quad (13)$$

This generates two types of losses for portfolio  $i$ . First, the price moves due to the market impact of fire sales, which leads to a mark-to-market loss given by

$$\begin{aligned} M^i(\Pi_{k-1}, C_{k-1}) &= (1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \left( \Pi_{k-1}^{i,\mu} - \Pi_k^{i,\mu}(\Pi_{k-1}, C_{k-1}) \right) \\ &= (1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i,\mu} \Psi_\mu \left( \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu} \right). \end{aligned} \quad (14)$$

A second source of loss, not accounted for in previous studies, stems from the fact that assets are not liquidated at the current market price but at a discount: this ‘implementation shortfall’, as it is called in the literature on optimal trade execution [Almgren and Chriss, 2000]) corresponds to the difference between the market price at the time of sale and the volume-weighted average price (VWAP) during liquidation. This VWAP lies somewhere between the pre and post-fire sales prices; we model it as a weighted average with weights  $(1 - \alpha, \alpha)$ ,  $\alpha \in [0, 1]$  of the pre- and post-fire sales prices. This leads to an implementation shortfall:

$$\begin{aligned} R^i(\Pi_{k-1}, C_{k-1}) &= \Gamma^i(\Pi_{k-1}, C_{k-1}) \sum_{\mu=1}^M \left[ \Pi_{k-1}^{i,\mu} - \left( \alpha \Pi_k^{i,\mu}(\Pi_{k-1}, C_{k-1}) + (1 - \alpha) \Pi_{k-1}^{i,\mu} \right) \right] \\ &= \alpha \Gamma^i(\Pi_{k-1}, C_{k-1}) \sum_{\mu=1}^M \Pi_{k-1}^{i,\mu} \Psi_\mu \left( \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu} \right). \end{aligned} \quad (15)$$

In the empirical examples below, we will use  $\alpha = \frac{1}{2}$ , which corresponds to a VWAP midway between the pre- and post fire sales prices. In [Cont and Schaanning, 2016] it is shown that  $\alpha \geq \frac{1}{2}$  is a plausible assumption, also from a modeling perspective.

Summing (15) with (14) yields the total loss of portfolio  $i$  at the  $k$ -th round of deleveraging:

$$\begin{aligned} L^i(\Pi_{k-1}, C_{k-1}) &= M^i(\Pi_{k-1}, C_{k-1}) + R^i(\Pi_{k-1}, C_{k-1}) \\ &= (1 - (1 - \alpha) \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i,\mu} \Psi_\mu \left( \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu} \right). \end{aligned} \quad (16)$$

This loss reduces the equity of institution  $i$  by the same amount:

$$C_k^i = (C_{k-1}^i - L^i(\Pi_{k-1}, C_{k-1}))_+ \quad (17)$$

Linearizing the market impact function  $\Psi_\mu$  yields

$$L^i(\Pi_{k-1}, C_{k-1}) \approx (1 - (1 - \alpha) \Gamma^i) \underbrace{\sum_{j=1}^M \sum_{\mu=1}^N \frac{\Pi_{k-1}^{i,\mu} \Pi_{k-1}^{j,\mu}}{D^\mu}}_{\Omega_{ij}(\Pi_{k-1})} \Gamma^j = (1 - (1 - \alpha) \Gamma^i) \sum_{j=1}^M \Omega_{ij}(\Pi_{k-1}) \Gamma^j, \quad (18)$$

which shows that the magnitude of fire-sale spillovers from institution  $i$  to institution  $j$  is proportional to the *liquidity weighted overlap*  $\Omega_{ij}$  between portfolios  $i$  and  $j$  [Cont and Wagalath, 2013]:

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^M \frac{\Pi^{i,\mu} \Pi^{j,\mu}}{D_{\mu}}. \quad (19)$$

The matrix of portfolio overlaps

$$\Omega(\Pi) = \Pi D^{-1} \Pi^{\top}, \quad (20)$$

where  $D$  is the diagonal matrix of market depths  $D_{\mu}$ , can be viewed as a weighted adjacency matrix of the underlying network linking portfolios through their common exposures. We will further analyze the properties of this matrix in Section 3.

In summary, an initial loss in asset values may trigger a feedback loop, schematically represented in Figure 3, in which, at each iteration, deleveraging of portfolio leads to fire sales, leading to price declines and mark-to-market losses which may in turn trigger further fire sales. The state variables representing the matrix  $\Pi$  of portfolio holdings in marketable assets and the equity levels  $C = (C_i, i = 1..N)$  are initialized as described in equations (3)-(4) (for  $k = 0$ ) and updated at each round of deleveraging

$$(\Pi_k, C_k) = f(\Pi_{k-1}, C_{k-1}), \quad (21)$$

where  $\Pi_k$  is defined in (13) and

$$C_k^i = (C_{k-1}^i - L^i(\Pi_{k-1}, C_{k-1}))_+ \quad (22)$$

where the loss  $L^i$  is defined in (16).

## 2.5 Feedback loops, insolvency and illiquidity

The iteration described above continues in principle as long as at least one institution is in breach of its leverage/ capital constraint after losses due to deleveraging are accounted for. However, in the (realistic) situation where we assume that institutions build a non-zero buffer beyond the minimal capital requirements (i.e.  $\lambda_b < \lambda_{\max}$  in the notation of Section 2.3 ), this *fire sales cascade* terminates after a finite number  $T$  of iterations [Cont and Schaanning, 2016]. As we will discuss below, this is not the case in leverage targeting models, which lead to infinite fire sales cascades.

Along the way, some institutions may become insolvent: this occurs if at any point in the iterations the loss  $L^i(\Pi_k, C_k)$  exceeds the capital  $C_k$ . Then institution  $i$  becomes insolvent and does not play any further role in subsequent rounds. Another type of failure which may occur along the cascade is failure due to *illiquidity*: this occurs when an institution has sold all of its marketable assets and is left with no further liquid assets. This may occur even though the institution is still solvent.

This distinction between failure due to insolvency and failure due to illiquidity is highly relevant in practice. In fact, one can note that this was precisely the scenario that occurred in the failure of Bear Stearns and Lehman Brothers.<sup>2</sup> In contrast to most default risk models and previous studies on fire sales, our model distinguishes between these two causes of failure and highlights the fact that institutions can fail even when they have positive equity.

In contrast to models of default contagion, contagion of losses across institutions occurs not just at default but actually *before* the default of an institution, and its scope is not limited to counterparties. Deleveraging by distressed institutions, which is precisely aimed at *preventing* their default, is in fact what triggers this contagion.

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<sup>2</sup>See letter by the then SEC chairman Christopher Cox to the Basel Committee on Banking Supervision <https://www.sec.gov/news/press/2008/2008-48.htm>.

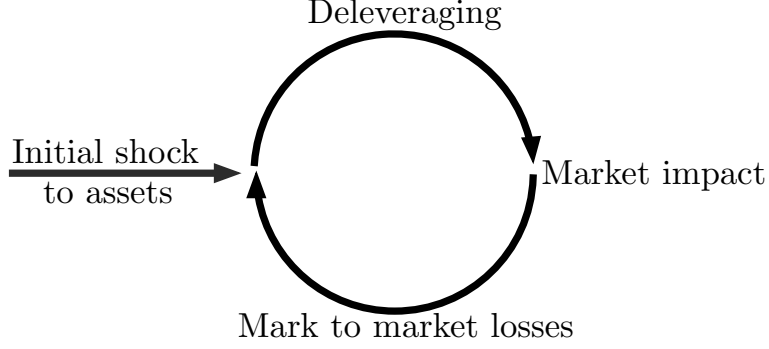


Figure 3: An initial loss in asset values may generate a feedback loop which may lead to multiple rounds of deleveraging and further declines in asset values.

Denoting by  $T$  the length of the cascade, the fire sales loss for bank  $i$  triggered by the stress scenario  $\epsilon$  is given by

$$FLoss(i, \epsilon) = \sum_{k=1}^T L^i(\Pi_{k-1}, C_{k-1}). \quad (23)$$

and the total system-wide fire sales loss in this scenario is

$$SLoss(\epsilon) = \sum_{i=1}^N FLoss(i, \epsilon). \quad (24)$$

Note that the fire sales loss (23) does not include the initial loss which triggers the deleveraging: in absence of deleveraging and price-mediated contagion  $FLoss(i) = SLoss = 0$ .

Table 1 summarizes the notations of the model and references the equations where they are defined.

Variable	Notation	Defined in
Financial institutions	$i, j = 1..N$	-
Asset class: illiquid assets	$\kappa = 1..K$	Section 2.1
Asset class: marketable assets	$\mu = 1..M$	Section 2.1
Number of iterations (rounds)	$k$	-
<i>State variables (in EUR)</i>		
Marketable assets	$\Pi^{i,\mu}$	Section 2.1
Capital	$C^i$	Section 2.1
<i>Parameters</i>		
Illiquid asset (in EUR)	$\Theta^{i,\kappa}$	Section 2.1
Initial shock (in %)	$\epsilon_\kappa$	(4)
Market depth for asset class $\mu$	$D_\mu$	(13)
<i>Key quantities</i>		
Deleveraging proportion at round $k$	$\Gamma_k^i$	(9)
Leverage of institution $i$	$\lambda^i$	(1)
Fire sales loss ( $k$ -th round)	$L^i(\Pi_{k-1}, C_{k-1})$	(16)
Fire sales loss for bank $i$ (all rounds)	$FLoss(i, \epsilon)$	(23)
System-wide fire sales loss	$SLoss(\epsilon)$	(24)

Table 1: Overview of model notations.

## 2.6 Comparison with "leverage targeting"

Recent empirical studies on fire sales spillovers [Duarte and Eisenbach, 2015, Greenwood et al., 2015] have explored a different mechanism for fire sales, based on the idea that banks maintain a 'leverage target', which leads them to rebalance their portfolios in a procyclical manner following changes in asset values. An oft-cited argument to support this model is the empirical correlations between quarterly changes in asset size and debt size for banks [Adrian and Shin, 2010, Adrian and Shin, 2014].

While both models incorporate the idea of market impact of deleveraging and the resulting endogenous portfolio losses and contagion effects, they differ in some important ways:

1. *Gain-loss asymmetry*: in the leverage targeting model, banks respond in a symmetric way to gains and losses, deleveraging in the face of losses but also increasing leverage by buying assets when asset values increase. This may be a reasonable description of the medium-term (e.g. quarterly) dynamics of bank or fund portfolios [Coval and Stafford, 2007], but not necessarily a realistic description for the short-term reaction of portfolios under stress. When performing stress tests, we are not concerned with the gradual rebalancing of portfolios which may be conducted in an orderly manner, with little market impact, but primarily concerned with fire sales, i.e. rapid deleveraging triggered when large losses saturate one-sided portfolio constraints. Due to the one-sided nature of these (leverage, capital, liquidity) constraints, fire sales occur in reaction to large losses, not gains. We believe the asymmetric model is a better description of the reaction of bank portfolios in stress scenarios, and consistent with empirical evidence that market participants do not react symmetrically to gains and losses [Ang et al., 2006].
2. *The threshold nature of fire sales*: in the leverage targeting model, bank (de)leveraging in response to arbitrarily small changes in asset values, regardless of their capital or liquidity buffers, generates fire sales losses even for small market stress levels. In our model, deleveraging only occurs when losses are large enough to trigger portfolio constraints: for shocks below this critical level, there is no deleveraging. By assuming that all institutions constantly respond to arbitrarily small changes in asset values, the leverage targeting model overestimates the magnitude of deleveraging in response to small shocks. This is illustrated in Figure 2 which compares the overall deleveraging across EU banks in response to a depreciation in the Spanish housing market.
3. *Dependence of deleveraging on magnitude of losses*: Leverage targeting implies a volume of deleveraging linear in the size of the portfolio loss; since the volume of deleveraging is capped at 100% of assets, this leads to a *concave* dependence of the volume of asset sales on the shock size. By contrast, as shown in Section 2.3 and illustrated in the example of Figure 2, in our model the volume of deleveraging has a *convex* dependence on the loss size for small to moderate shocks: deleveraging accelerates as we increase the shock size to more extreme levels, leading to a 'multiplier effect', absent in the leverage targeting model.
4. *Finite length of fire sales cascades*: The assumption of leverage targeting leads to an infinite sequence of iterations, which never cease since at each round further mark-to-market losses are generated endogenously, which leads to a deviation from the target leverage which generates new asset sales or purchases. In stress tests, one then needs to choose an ad-hoc number of iterations to compute the loss.

By contrast, as shown in [Cont and Schaanning, 2016], in a threshold model with a capital buffer  $\lambda_b < \lambda_{\max}$ , the fire sales cascade always terminates after a finite number of iterations, typically 5 to 10 rounds in most empirical examples, as shown in the next section.

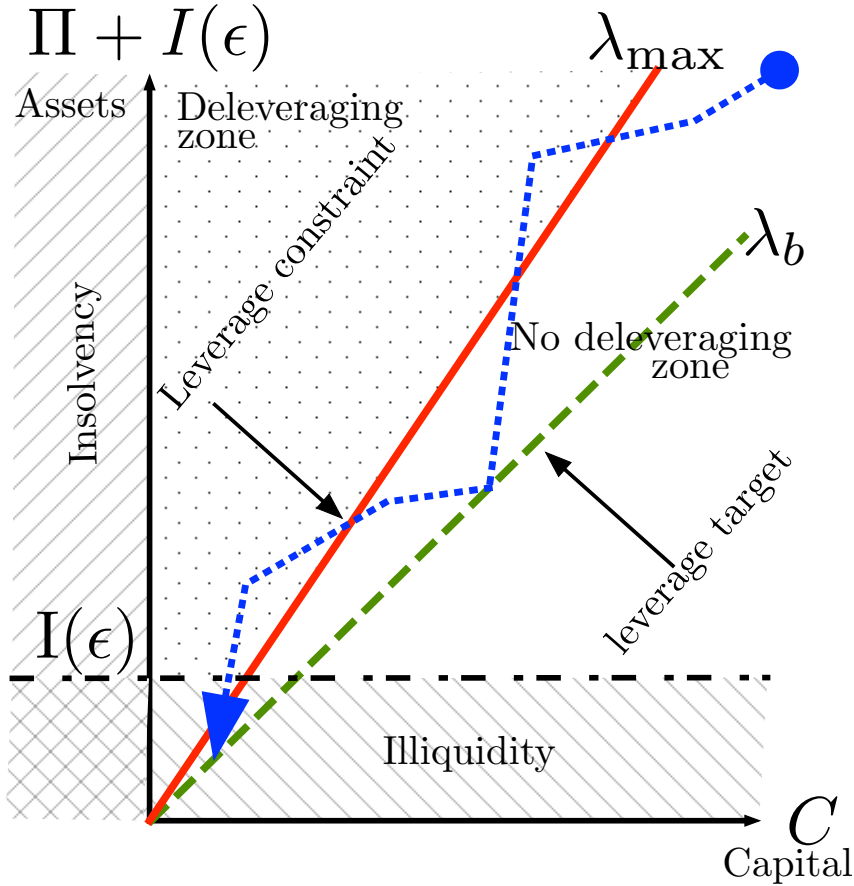


Figure 4: Evolution of asset values (vertical axis) and capital (horizontal axis) in a fire sales cascade. The solid red line corresponds to the leverage constraint, the dashed green line corresponds to a target leverage corresponding to an excess capital buffer, and the dotted blue line is the path of a sample portfolio. Losses in asset values depreciate the equity, moving the portfolio closer to the origin; when the portfolio crosses the red line corresponding to the leverage constraint, deleveraging occurs: the institution tries to reconstitute a buffer by returning to the target leverage (dotted line). The market impact of these asset sales lead to further losses and displace the state to the left; if it crosses the red line again, a new round of deleveraging follows etc. An institution becomes insolvent when it reaches the boundary  $C = 0$  (vertical axis) and illiquid when it reaches the boundary  $\Pi = 0$  (horizontal dashed line).

The quantitative consequences of these differences are explored in more detail in the next section, where we compare the results of stress tests performed using the two approaches, and in the companion paper [Cont and Schaanning, 2016].

### 3 A systemic stress test of the European banking system

We now describe how the model may be used to perform a systemic stress test, in order to quantify the exposure of the the banking system to fire sales spillovers, and apply the framework to data on the European banking system.

#### 3.1 Data

Our empirical study is based on data from the European Banking Authority (EBA), which provides information, collected in 2011 and 2016, on notional exposures of 90 European banks

across 135 asset classes.<sup>3</sup> Holdings are given by asset class and geographic region. Asset classes are specified in Table 2. [Greenwood et al., 2015] assumed all assets to be available for liquidation; we only consider a subset to be marketable i.e. available for liquidation at short notice. We identify four classes of marketable assets (“securities”), which may be liquidated in a stress scenario; the other asset classes are classified as illiquid assets.<sup>4</sup> Assets are further labeled by 37 geographic regions, which correspond to the 27 countries of the EU (i.e. without Croatia at the time) plus the United States (US), Norway (NO), Iceland (IS), Liechtenstein (LI), Japan (JP), Asia (A1), Other non-EEA non-emerging countries (E3), Eastern Europe non-EEA (E5), Middle and South America (M1) and Rest of the world (R5). Hence, with the four marketable asset classes and 37 geographic regions, the matrix of marketable assets  $\Pi$  is given by a  $90 \times 148$  matrix. The illiquid asset holdings are given by a  $90 \times 75$  matrix  $\Theta$ . This corresponds to 74 asset classes for commercial and residential mortgage exposures respectively in the 37 regions and a 75th entry consisting of all remaining illiquid asset holdings.<sup>5</sup>

<b>Illiquid assets</b>
Residential mortgage exposures
Commercial real estate exposure
Retail exposures: Revolving credits, SME, other
Indirect sovereign exposures in the trading book
Defaulted exposures
Residual exposure (cf. Appendix Table 9)
<b>Securities / marketable assets</b>
Corporate bonds
Sovereign debt
Direct sovereign exposures in derivatives
Institutional client exposures: interbank, CCPs,...

Table 2: Asset classes used for the stress test.

### 3.2 Market impact and market depth

A key assumption in models of fire sales spillovers concerns the impact of asset liquidations on market prices. This may be summarized in the choice of a market impact function  $\Psi_\mu$ , which defines the correspondence between the liquidation size  $q$  (in monetary units) and the relative price change for each asset class  $\mu$ :  $\frac{\Delta S_\mu}{S_\mu} = -\Psi_\mu(q)$ . An adequate choice for  $\Psi_\mu$  should be increasing, concave, satisfy  $\Psi_\mu(0) = 0$  and lead to non-negative prices.

Common specifications are the linear model [Kyle, 1985, Bertsimas and Lo, 1998], [Almgren and Chriss, 2000, Obizhaeva, 2012, Cont et al., 2014]

$$\Psi_\mu(q) = \frac{q}{D_\mu} \quad \text{with} \quad D_\mu = c \frac{ADV_\mu}{\sigma_\mu} \quad (25)$$

where  $ADV_\mu$  is the average daily trading volume (in EUR),  $\sigma_\mu$  the daily volatility (in %) of the asset,  $c$  a coefficient close to 0.5, estimated from transactions data, and the square root model

<sup>3</sup>This data set was also used in the study by [Greenwood et al., 2015], and facilitates comparison with the literature.

<sup>4</sup>Table 9 in the Appendix, we provide the data identifiers that allow correspondence with the EBA dataset.

<sup>5</sup>Our representation differs slightly from [Greenwood et al., 2015] who considered 42 asset classes consisting of the 37 sovereign exposures by geographic region and five further classes, aggregated across all geographical regions: “commercial real estate”, “mortgages”, “corporate loans”, “small and medium enterprise loans” and “retail revolving credit lines”. This leads to a less granular model compared to ours as we distinguish assets both by type and country.



[Bouchaud, 2010]

$$\Psi_\mu(q) = c \sigma_\mu \sqrt{\frac{q}{ADV_\mu}} \quad (26)$$

We note both the trading volume and the volatility are associated with a liquidation horizon  $\tau$ , taken in most studies to be daily by default. If we assume the liquidation horizon  $\tau$  to be longer than a day, then the market depth parameter needs to be adjusted. In the linear impact model, the adjustment is:

$$D_\mu(\tau) = c \frac{ADV_\mu \tau}{\sigma_\mu \sqrt{\tau}} = c \frac{ADV_\mu}{\sigma_\mu} \times \sqrt{\tau}. \quad (27)$$

This adjustment is important, and corresponds to the intuitive observation that liquidating the same portfolio over a longer horizon reduces impact. The liquidation horizon  $\tau$  may be interpreted as the time window the banks dispose of to comply with portfolio constraints. In the case study below we will use  $\tau = 20$  days.

By contrast, in the square root model, the impact of a transaction is invariant to a change in the liquidation horizon since the denominator and the numerator in (26) scale in the same way. This leads to the counterintuitive (and, we believe, incorrect) conclusion that impact is insensitive to the rate of liquidation. For this reason, we refrain from using the square-root model in the sequel.

In a linear impact model, asset classes are differentiated according to their market depth  $D_\mu$ . [Greenwood et al., 2015] assume a uniform depth  $D_\mu = 10^{13}$  (EUR) for all asset classes. This homogeneity assumption is not supported by empirical studies on market impact [Bouchaud, 2010, Obizhaeva, 2012, Cont et al., 2014] which indicate that market impact varies widely across assets. Ignoring this heterogeneity may lead to biased results, overestimating losses in more liquid asset classes while underestimating losses in less liquid asset classes.

[Duarte and Eisenbach, 2015] use haircuts and repo rates for determining the liquidity of different asset classes; this does introduce some heterogeneity across asset classes but the relation between these quantities and market impact is not clear. For instance, haircuts may simply reflect the volatility of an asset, rather than its liquidity or market depth.

We use a direct, data-driven approach to the modeling of market impact. To estimate the market depth parameters for each asset class using (27), we

- estimate volatility parameters  $\sigma_\mu$  using daily returns of S&P sector indices<sup>6</sup>.
- obtain average daily volume estimates  $ADV_\mu$  from annual volume data provided by the US Treasury and various central banks (Appendix A.1).

As noted above, we use  $\tau = 20$  which corresponds to a liquidation horizon of 4 weeks, a fairly lenient assumption. Following the empirical findings in [Obizhaeva, 2012, Cont et al., 2014], we use  $c = 0.4$  which corresponds to a one standard deviation price move for 40% of average daily trading volume ([Obizhaeva, 2012]).

ADV for US corporate bonds was obtained from SIFMA (see Appendix A.1). For European bonds, we simply use the same sovereign-to-corporate ratio of ADV as observed in the US ( $567.81bn/269.8bn \approx 0.48$ ) to estimate the ADV of the corporate and “institutional” asset classes for European corporate bonds.

For some asset classes, data on trading volume are unavailable (or difficult to obtain). We work around this issue by estimating, based on OECD data, the following regression model for the relation between average daily volume (ADV) and outstanding notional

$$\log ADV_\mu := c_1 \log(N_\mu) + c_0 + \varepsilon_\mu \quad (28)$$

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<sup>6</sup><http://us.spindices.com/indices/fixed-income/sp-eurozone-sovereign-bond-index>  
<http://us.spindices.com/index-family/us-treasury-and-us-agency/all>  
<http://us.spindices.com/indices/fixed-income/sp-500-investment-grade-corporate-bond-index>

Asset class (sovereign unless specified)	ADV (Bn EUR)	Market depth $D_\mu$ ( $10^{12}$ EUR)	Impact of 10 Bn
US	567.8	428.8	0.00466
US (corp.)	269.8	70.2	0.02848
IT	28.82	21.8	0.09166
ES	28.0	21.1	0.09470
DE	24.7	18.7	0.10684
GB	24	18.1	0.11038
FR	10	7.58	0.26316
GR	8.17	6.18	0.32258
BE	6	5.53	0.36036
SE	3.92	2.96	1.67113
PT	2.27	1.71	1.15602

Table 3: Average daily trading volume and estimated market depth over  $\tau = 20$  days, for the largest holdings in the EBA dataset. The impact in basis points are also given for a liquidation of 10 Bn EUR over  $\tau = 20$  days.

where  $N_\mu$  denotes outstanding notional, and using it to estimate volume for the remaining asset classes. Table 4 shows the results of this regression analysis.

Coefficient	US treas	US corp	UK	DE	ES
$c_1$ :	0.53***	0.64***	0.56***	0.48	0.94***
std. dev.	0.13	0.15	0.05	0.30	0.14
$c_0$ :	0.63	-1.1**	-0.35**	0.4	-1.35***
std. dev.	0.55	0.55	0.14	0.99	0.37
adj. $R^2$	0.39	0.60	0.93	0.15	0.76
$n$	19	19	11	10	15

Table 4: Logarithmic regression of bond trading volume on outstanding notional (Equation 28).

Figure 5 shows the distribution of market depth estimates for all asset classes in the EBA data set, on a logarithmic scale. The histogram reveals considerable heterogeneity in the cross-sectional distribution of market depth, with 4 orders of magnitude separating the most liquid from the least liquid assets. Table 3 provides values for average daily volumes (ADV) and market depths for the asset classes representing the largest holdings. These values are used as base values; we will later perform a sensitivity analysis of our results with respect to changes in these values.

**Extrapolation to large volumes** When applied to large transaction volumes, linear or square-root impact models may lead to negative prices. In [Greenwood et al., 2015] this was addressed by capping the loss at 100%:  $\Psi_\mu(q) = \min\left\{1, \frac{q}{D_\mu}\right\}$ . [Cifuentes et al., 2005] use an exponential specification

$$\Psi_\mu(q) = 1 - \exp\left(-\frac{q}{D_\mu}\right) \quad (29)$$

which also ensures that prices remain non-negative but gives a concave impact. Nevertheless, prices can get arbitrarily close to zero in both of these models. However, it is realistic to assume that long before the price level reaches zero, arbitrageurs will step in to purchase assets subject to fire sales at a discount [Shleifer and Vishny, 1992]. To capture this effect we introduce a price floor  $B_\mu > 0$  and consider a two-parameter level-dependent price impact function:

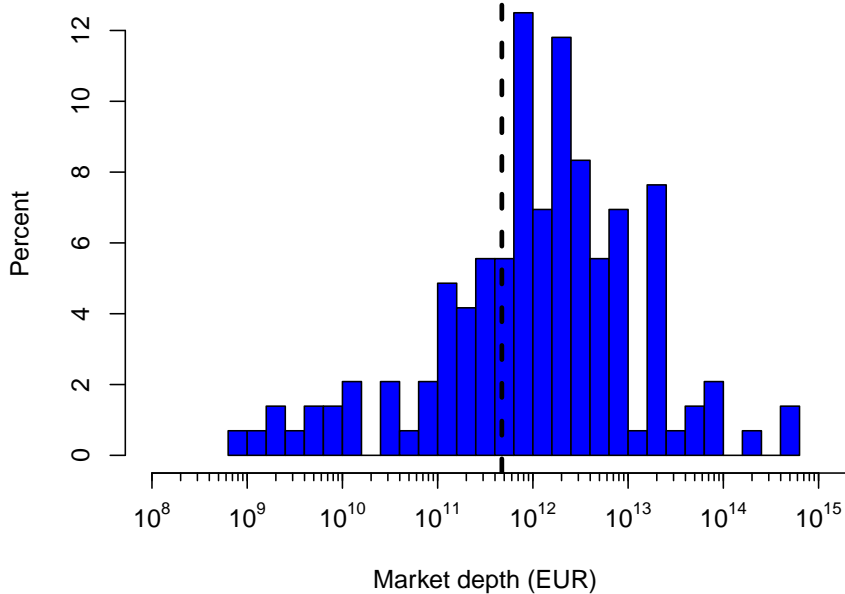


Figure 5: Histogram of estimated market depths for the 148 asset classes in the EBA dataset. The dashed vertical line indicates the holdings-weighted average depth given by (31).

$$\Psi_{\mu}(q, S) := \left(1 - \frac{B_{\mu}}{S}\right) \left(1 - \exp\left(-\frac{q}{\delta_{\mu}}\right)\right). \quad (30)$$

$B_{\mu}$  determines how far the price can fall in a fire sales scenario. Note that this is a lower bound for the price and in a given stress test the price may not actually fall to this level. In the empirical examples below we set  $B_{\mu}$  at 50 % of the market price levels. Choosing

$$\delta_{\mu} = \left(1 - \frac{B_{\mu}}{S_0^{\mu}}\right) D_{\mu}$$

makes the specification (30) compatible with the linear specification (25) for small volumes.

### 3.3 Portfolio overlaps

As shown in Eq. (18), the transmission of fire sales losses from portfolio  $i$  to  $j$  depends on the liquidity weighted overlap between portfolio  $i$  and  $j$ :

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^M \frac{\Pi^{i,\mu} \Pi^{j,\mu}}{D_{\mu}}.$$

The matrix  $\Omega$  of liquidity-weighted overlaps thus plays an important role in the transmission of fire sales losses, which may be viewed as a contagion process on a network of financial institutions in which the link from  $i$  to  $j$  is weighted according to the liquidity weighted overlap  $\Omega_{ij}$ . We call this network the *indirect contagion network*. Similar network structures were explored by [Braverman and Minca, 2014, Guo et al., 2016] for mutual funds. Figure 6 displays the indirect contagion network for European banks as implied by EBA data collected in 2011. The nodes correspond to different banks, with node size proportional to balance sheet size, and edges

correspond to non-zero portfolio overlaps, with edge widths proportional to the logarithm of the liquidity weighted overlap  $\Omega_{ij}$ .

Figure 7 (left) shows the distribution of  $\Omega_{ij}$  in this networks. The peak at zero reflects the fact that many pairs of banks have no common asset holding (so, zero overlap) i.e. the network is sparse. On the other hand, the values are dispersed over five orders of magnitude, which illustrates the heterogeneity of the network.

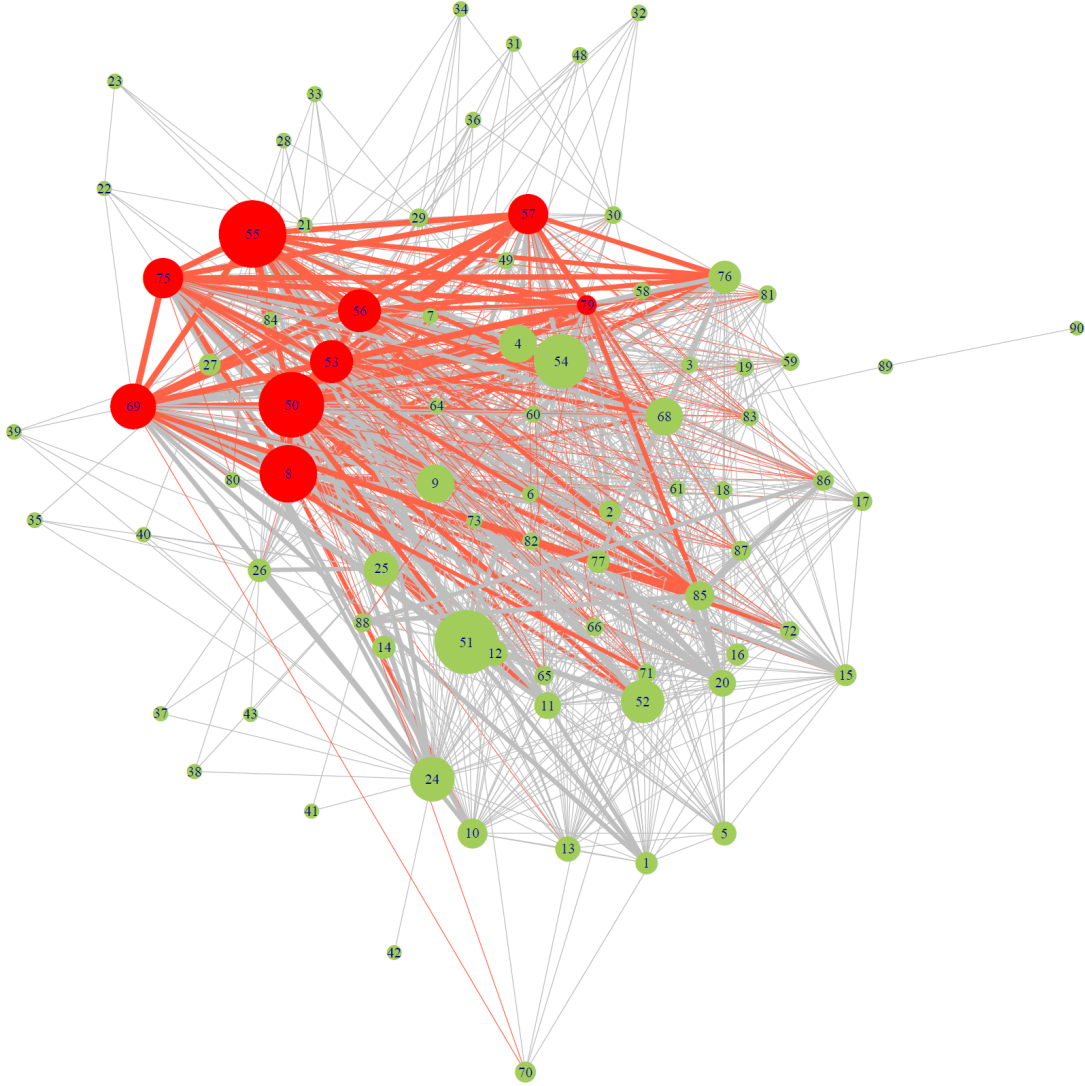


Figure 6: The core of the European *indirect* contagion network: Node sizes are proportional to balance sheet size. Labelled nodes in the first principal component (Figure 20) are coloured in red, as are edges emanating from these. Edge widths are proportional to the liquidity weighted overlap.

The overlap matrix  $\Omega$  also gives a glimpse of the nature of 'second-round' contagion effects in this network. The element  $(i, j)$  of the matrix  $\Omega^2$  may be interpreted as the (maximal) indirect contagion from  $i$  to  $j$  channeled through other nodes:

$$\Omega^2(i, j) = \sum_k \Omega_{ik} \Omega_{kj}.$$

Figure 8 shows the histogram of these 'second-order overlaps'. Unlike Figure 7, where we see a large fraction of zero overlaps, here we see that all coefficients are strictly positive: this means

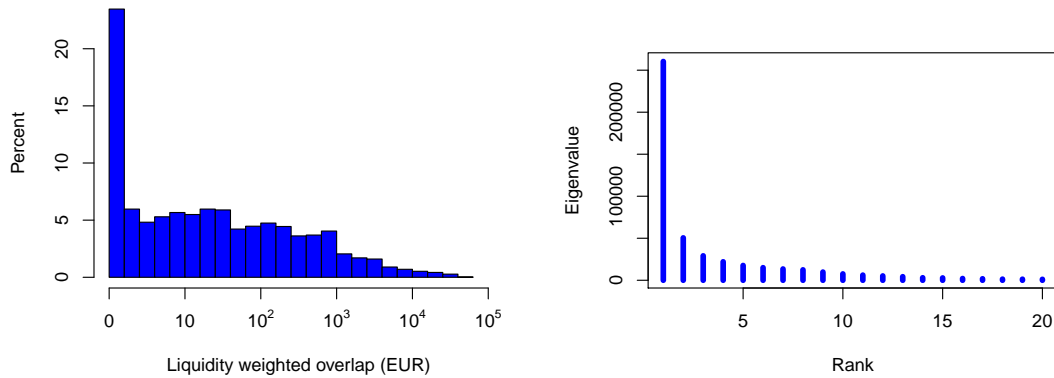


Figure 7: Left: histogram of liquidity weighted portfolio overlaps across EU banks (2011). Right: Ranked eigenvalues of the matrix  $\Omega$  of liquidity-weighted portfolio overlaps.

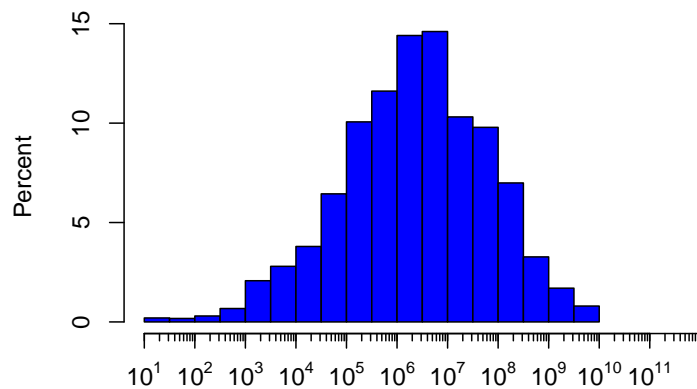


Figure 8: Histogram of 'second-order overlaps' (coefficients of the matrix  $\Omega^2$ ).

that, although many pairs of portfolios have zero overlaps, second-round effects may potentially cause spillovers from *any* institution to any other! This observation shows that second-round effects should not be ruled out a priori and may greatly increase the scope of contagion in the network.

A quantitative way of examining the heterogeneity of this network is to compute the eigenvalues and eigenvectors of  $\Omega$ . The right-hand plot of Figure 7 shows the ranked eigenvalues of  $\Omega$ . The first few eigenvalues clearly dominate the others by an order of magnitude. This means that the overlap in the portfolio network can be characterised quite well by a low-dimensional factor model for portfolio holdings. We will discuss this point further in Section 4.

### 3.4 Stress scenarios

Stress scenarios are defined in terms of percentage shocks across asset classes, given by a vector  $\epsilon = (\epsilon_\kappa, \kappa = 1..K) \in [0, 1]^K$  where  $\epsilon_\kappa$  is the percentage loss (stress level) applied to asset class  $\kappa$ .

We have chosen four scenarios in the examples below mainly to illustrate the properties of the model:

1. Residential and commercial real estate losses in Spain (Region: ES);
2. Residential real estate losses in Northern and Western Europe (Regions: GB, BE, NO, SE);
3. Residential and commercial real estate losses in Southern Europe (Regions: IT, GR, ES, PT);
4. Commercial real estate losses in Eastern Europe (Regions: CZ, EE, HU, LV, LT, PL, RO, SK, RU, BG, E3 (= other non-EEA non-emerging countries));

In each scenario, we increase the shock to the asset classes concerned gradually from 0% to 20%; the other asset classes undergo no initial loss.

In these examples we have chosen stress scenarios where initial losses affect only illiquid assets, to emphasize cross-asset contagion, but scenarios can be extended to include initial shocks to marketable assets.

### 3.5 Systemic stress test of European banks: results

We now report the results of our systemic stress test for the European banking system.

To perform the stress test, we use a leverage constraint  $\lambda_{\max} = 33$ , which corresponds to the Basel 3 leverage constraint. An alternative would be to use as constraint the ratio of capital to risk-weighted assets (RWA), or both. Given the limitations of our data set, we have opted for the former, but the same exercise may be conducted with both constraints. We note that for EU banks it is mostly the case that leverage, not the ratio of capital to RWA, is the binding constraint.

As discussed in Section 2.3, we assume a target leverage slightly below the constraint  $\lambda_b = 0.95\lambda_{\max} = 31.3$  in order to have a buffer. We have conducted the stress test for various values  $0.9\lambda_{\max} \leq \lambda_b \leq \lambda_{\max}$ ; results are similar across this range for brevity we only report them for  $\lambda_b = 0.95\lambda_{\max}$ . In most examples, fire sales die out after a few rounds; in all cases we have limited the iterations to 20 rounds and report losses for each round and the total losses for all rounds.

**Magnitude of fire sales losses.** Figure 9 shows the contribution of fire sales losses to total system-wide loss in the stress test in Scenario 1. We observe that when stress levels exceed 4-5%, fire sales losses cannot be neglected and may account for up to 70% of the total loss. This is definitely an argument for including fire sales and deleveraging effect in a stress test.

Figure 10 represents the magnitude of fire sales losses  $S\text{Loss}(\epsilon)$  as a percentage of bank equity for the four different stress scenarios. What is striking is the existence of a *tipping point* which separates a stability zone corresponding to small shocks from a systemic risk zone, where the magnitude of losses represents a considerable fraction of bank equity. The existence of such tipping points is characteristic of the collective behavior of networks whose components are subject to thresholds [Granovetter, 1978].

Another striking feature of Figure 10 is that, while the 4 scenarios lead to different outcomes in our model, they lead to similar outcomes in the leverage-targeting model (represented by the dashed lines). This is a systematic difference between the two models, which we will discuss further (see also [Cont and Schaanning, 2016]).

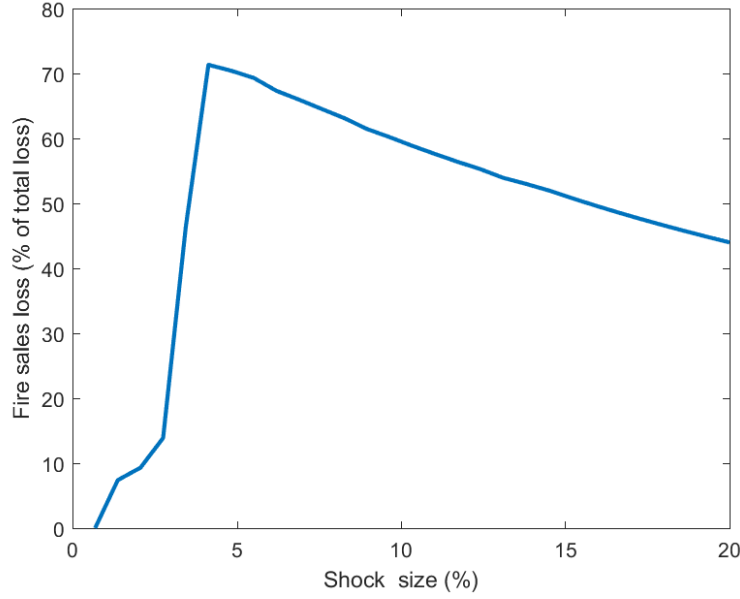


Figure 9: Contribution of fire sales losses to total system-wide loss, as a function of the initial stress level.

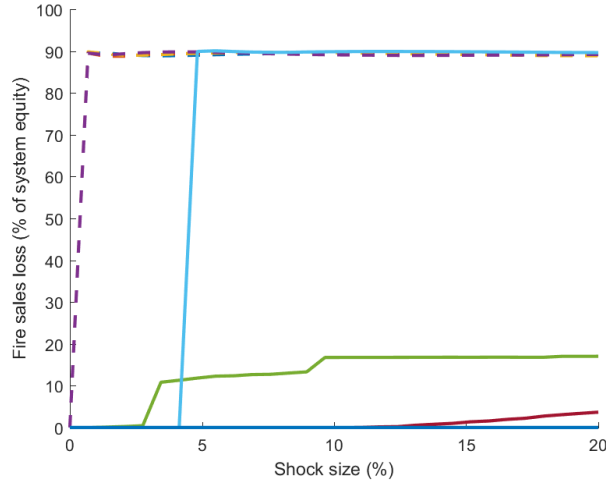


Figure 10: Fire sales losses (as % of bank equity) in the four stress scenarios as a function of the initial shock size. The fourth scenario does not generate fire sales in the threshold model. The losses in the leverage targeting model are displayed in dashed lines.

These results are based on market depth parameters estimated as described in Section 3.2, which may be different in a market stress scenario. This may also be seen in terms of dependence on the liquidation horizon: as observed from (27), changing the liquidation horizon by a factor  $\tau$  adjusts the depth parameter by a factor  $\sqrt{\tau}$ . The sensitivity of results to both of these may be studied by adjusting the market depth parameters by a scaling factor; the value 1 corresponds to the values estimated in Section 3.2; values  $> 1$  correspond to an increase in liquidation horizon, values  $< 1$  correspond to a decrease in market depth.

Figure 11 analyzes the dependence of system-wide fire sales losses  $S\text{Loss}(\epsilon)$  on this scaling factor. We observe the sensitivity of the result on the market depth level, which underlines the importance of a proper estimation procedure for these parameters. We can distinguish three regimes: (i) no fire sales (losses) for small shocks  $\epsilon < \min_i \epsilon^{i*}$ , (ii) significant fire sales losses

on the order of 10-20% of bank equity for large stress levels in presence of large market depth, (iii) a region of *high systemic risk*, where fire sales losses deplete a large proportion of bank equity. The boundary between the regimes may be attained either by increasing the stress level  $\epsilon$  or by decreasing the market depth. In both cases, there is a tipping point above/below which large-scale deleveraging is triggered.

Figure 12 displays a quite different picture for the leverage targeting model: here we observe a very high level of losses (more than 40% of bank equity) at any level of market depth and shock level. Moreover, fire sales losses are essentially independent of the initial shock size. This shows that losses in the leverage targeting model are not driven by the initial shock size but by the rebalancing generated by the leverage targeting rule. These results also indicate that any study on the magnitude of fire sales should include a rigorous sensitivity analysis with respect to the market depth parameters.

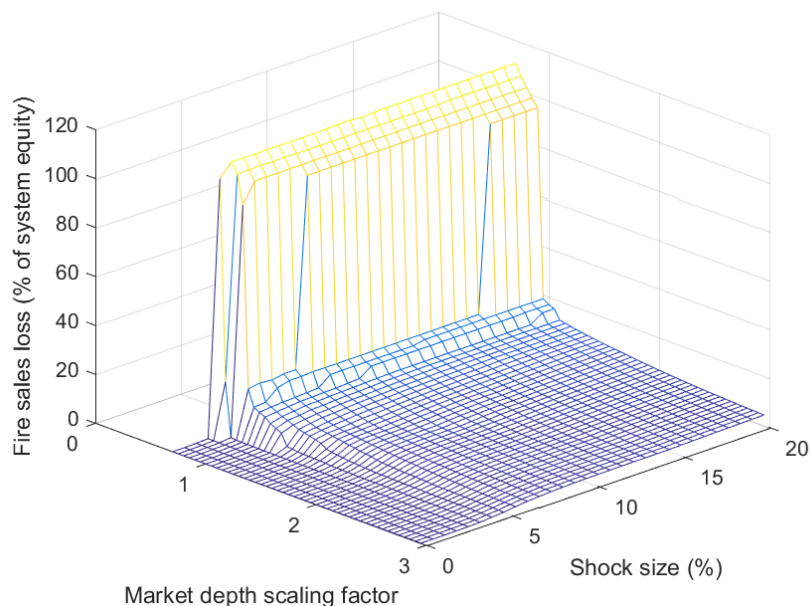


Figure 11: The fire sales losses as a function of the initial shock and the estimated market depth for the threshold model. We can clearly distinguish three regions: (i) no fire sales; (ii) fire sales contagion with losses between 10 - 20% of system equity, (iii) region of high systemic risk: all banks default (i.e. absent regulatory intervention large scale contagion is to be expected). This shows that the market depth has a highly significant impact on the estimated losses and the extent of contagion.

**Distribution of fire sales losses across banks.** We now turn to a more detailed examination of losses induced by fire sales, at the level of individual institutions.

Figure 13 compares bank-level fire sales losses with losses under the leverage targeting model, using the same market depth estimates. Fire sales losses are observed to be much larger for some banks in the leverage targeting model in comparison with the threshold model; note the logarithmic scale in this plot. This figure shows that fire sales losses are not only larger on a system-wide level in the leverage targeting model (as seen in 12), but are also higher at the individual bank level. Moreover, our model distinguishes the three scenarios by severity, while fire sales losses are very similar in the three scenarios for the leverage targeting model.

**Impact of heterogeneity in market depth** [Greenwood et al., 2015] assume the same market depth parameter for all asset classes, whereas our estimation, shown in Figure 5, yields a



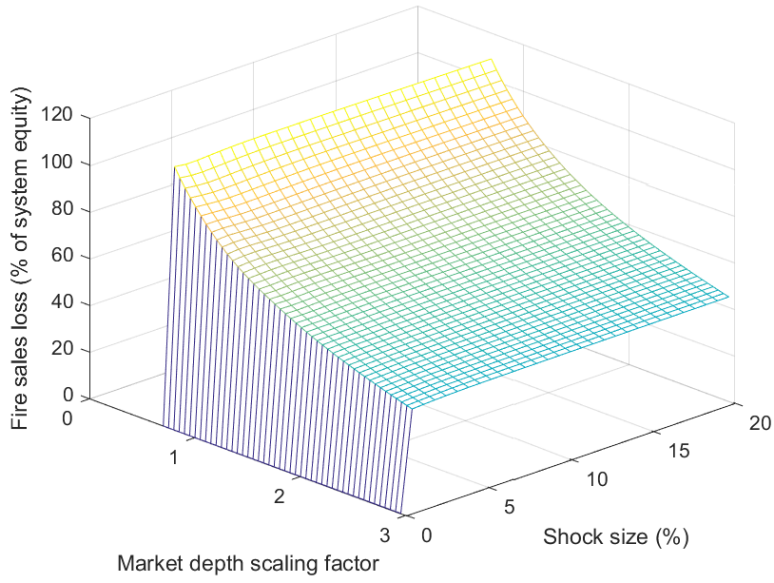


Figure 12: The fire sales losses as a function of the initial shock and the estimated market depth for the leverage targeting model. The leverage targeting model predicts large-scale losses for all combinations of shock sizes and market depths.

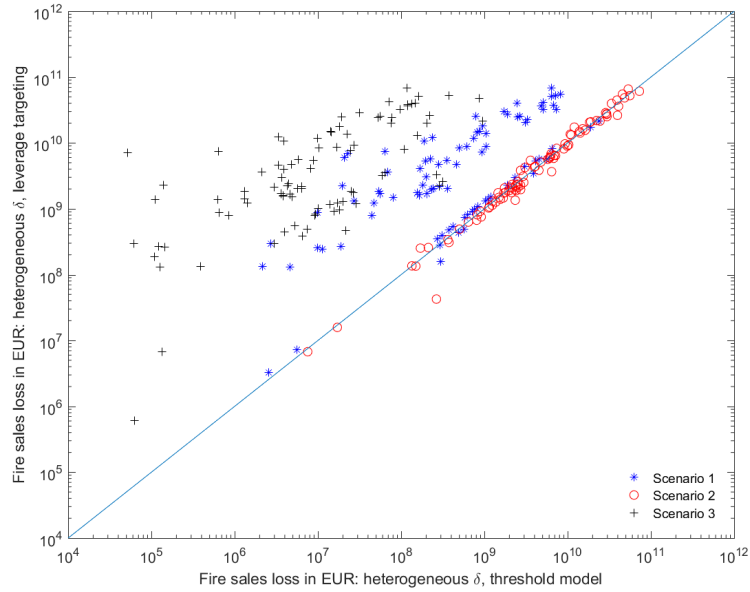


Figure 13: Bank-level fire sales losses, compared with losses under the leverage targeting model, using the same market depth estimates (logarithmic scale). Loss estimates are higher in the leverage targeting model by several orders of magnitude.

heterogeneous distribution of market depth across asset classes. To investigate the influence of this heterogeneity, we compare bank-level losses with losses in a model with uniform market depth  $\bar{D}$  computed as a holdings-weighted average:

$$\sum_{i,\mu} \frac{\Pi_{i,\mu}}{\bar{D}} = \sum_{i,\mu} \frac{\Pi_{i,\mu}}{D_\mu}. \quad (31)$$

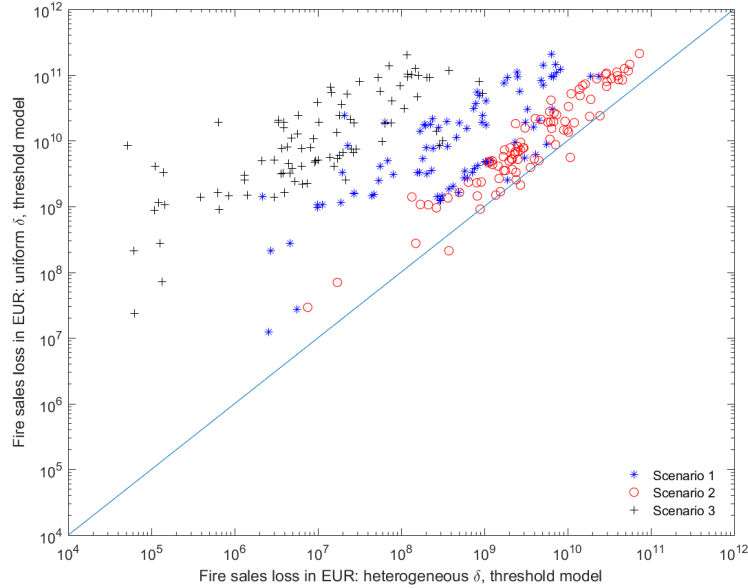


Figure 14: Bank-level fire sales losses computed using heterogeneous market depth parameters vs uniform market depth. The use of a uniform market depth for all assets increases fire sales loss estimates considerably.

The result, displayed in Figure 14 clearly shows a huge impact of heterogeneity in market depth when estimating fire sales losses: clearly, institutions are differentiated according to the liquidity of their holdings, sometimes by 3 orders of magnitude. This clearly shows that bank stress tests with fire sales require a careful estimation of market impact/ market depth parameters for each asset class.

Figure 15 shows that the same experiment in the leverage targeting model leads to quite different results: heterogeneity of market depth does not seem to have much impact in this case. The reason is that the leverage targeting model overestimates fire sales losses, which are so large that they trigger the insolvency of many large leveraged institutions in the first rounds of deleveraging. These defaulted institutions then cease to further contribute to the loss estimates, which then do not have any further dependence on market depth parameters.

For completeness, we also compare in Figure 16 our results with the ones obtained in the leverage targeting model using a uniform market impact parameter  $\bar{D}$  for all assets, as in [Greenwood et al., 2015]. The models agree in the 'meltdown' region where all banks default but for intermediate stress levels the difference between loss estimates in the two models is substantial, up to several orders of magnitude (note the logarithmic scales in Figure 16). As discussed above, the main difference in the loss estimates arises from the deleveraging rule (leverage targeting vs one-sided leverage constraint) but the difference in market depth parameters accentuates the difference.

**Bank failures: illiquidity and insolvency.** Most theoretical models of financial contagion have focused exclusively either on insolvency or on illiquidity as the cause of bank failure, while bank stress tests have traditionally focused exclusively on solvency of banks. As discussed in Section 2.5, our model allows for both possibilities: a bank may become insolvent due to asset losses, or become illiquid when all marketable assets have been sold. The model thus allows to examine which is the principal mode of failure in the stress test.

Note that this distinction is not available in [Greenwood et al., 2015, Duarte and Eisenbach, 2015]: their assumption that all assets are available for liquidation entails that the only situation where

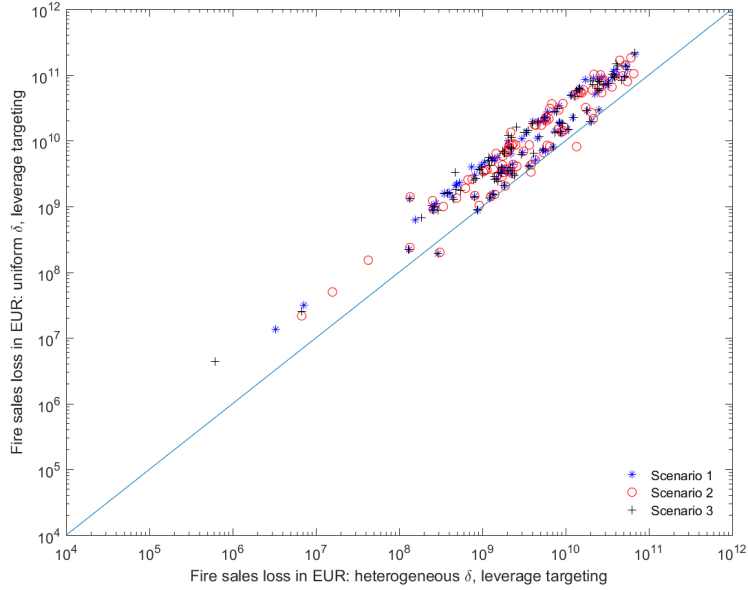


Figure 15: Impact of heterogeneity in market depth in the leverage targeting model.

an institution becomes illiquid is when there are no more assets available for liquidation, which implies that it is also insolvent.

We proceed now to analyse the number of failures respectively due to illiquidity and insolvency as a function of the initial stress level. The left (resp. right) panel in Figure 17 displays the number of banks which become insolvent (resp. illiquid) at after a given number of rounds of deleveraging, as a function of the initial stress level. We observe that insolvency is far from being the only mode of failure: depending on stress levels, 10 to 20 banks are expected to default due to illiquidity, yet remain solvent.

Figure 18 shows respectively the number of failures due to insolvency (left) and those due to illiquidity (right) as a function of the market depth scaling factor and the initial shock size for a given initial shock of 15%. We can see that, while the number of insolvent banks increases with the stress level, the number of failures due to illiquidity is non-monotone: it is maximal in the range of moderate shocks (between 5% and 10%) but then decreases as the shock level increases and insolvency becomes the main mode of default.

**Second and higher round effects.** We end this section by discussing the impact of the number of rounds of the deleveraging. Although in theory deleveraging may continue for many rounds (and, in the leverage targeting model, for an infinite number of rounds), previous empirical studies have mostly focused on a single round of deleveraging. Findings in [Greenwood et al., 2015] and [Duarte and Eisenbach, 2015] seem to suggest that simulating fire sales for several rounds compared to one or two rounds does not lead to significantly different results - the main effects of fire sales seem to be captured by one iteration. We nuance these findings and emphasize several effects which only become visible at second or higher rounds.

First, as noted in Section 3.3, even though the matrix  $\Omega$  of portfolio overlaps is sparse, the matrix  $\Omega^2$  of second-order overlaps is dense, which implies that second round spillovers can potentially lead to contagion from *any* institution to any other, quite unlike what happens at the first round. By comparing the distribution of liquidity weighted overlaps in Figure 7 to second-round overlaps in Figure 8, we see that some shocks need *at least* two rounds to propagate from one bank to another.

Second, as observed in Figure 17, many bank failures occur only at higher rounds, especially

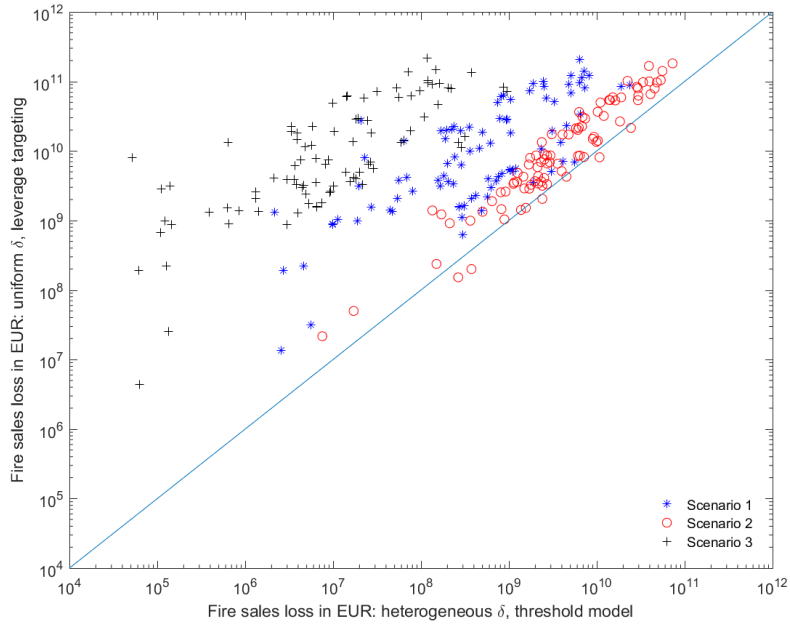


Figure 16: Bank-level fire sales losses in the leverage targeting model with uniform market depth differ by several orders of magnitude from those in the threshold model using heterogeneous market depths by several orders of magnitude.

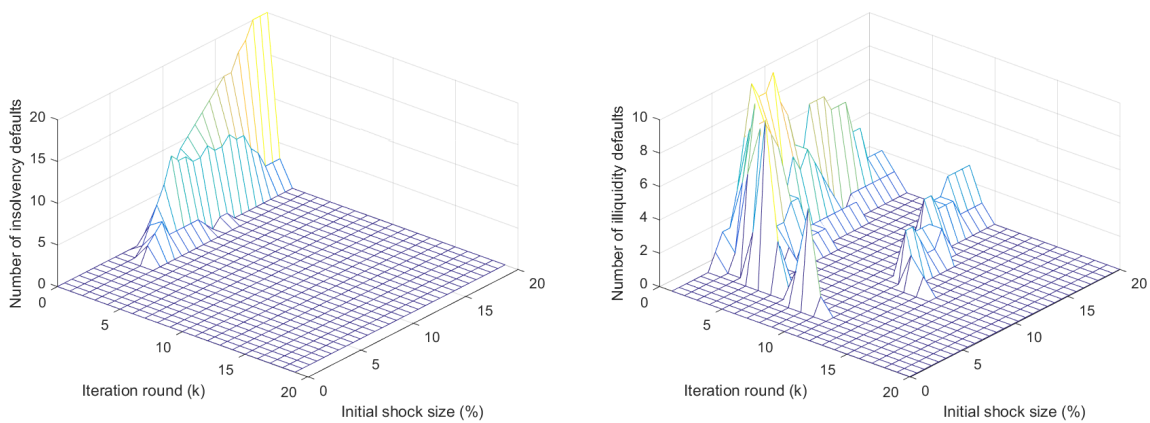


Figure 17: Left: Number of insolvent banks as a function of number of rounds of deleveraging and initial stress level. Right: Number of illiquid banks as a function of number of rounds of deleveraging and initial stress level.

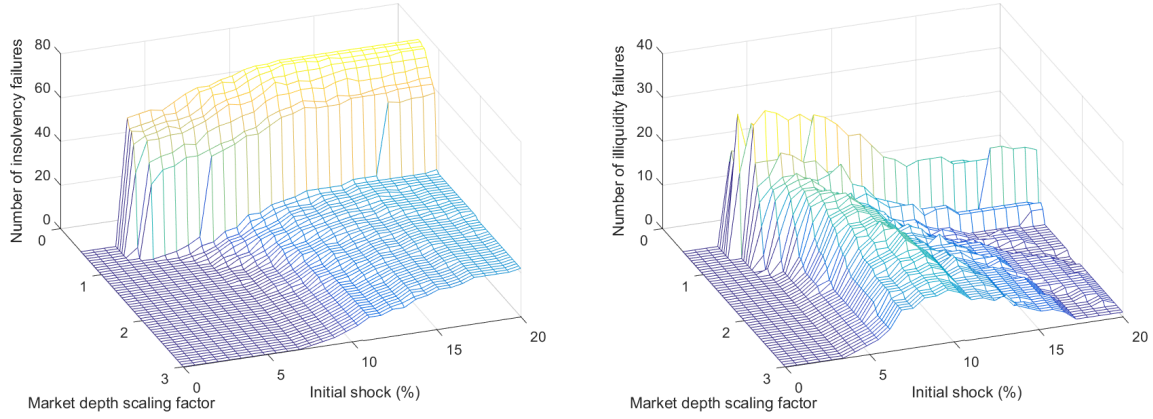


Figure 18: Number of bank failures due to insolvency (left) and illiquidity (right), as a function of the initial stress level and market depth.

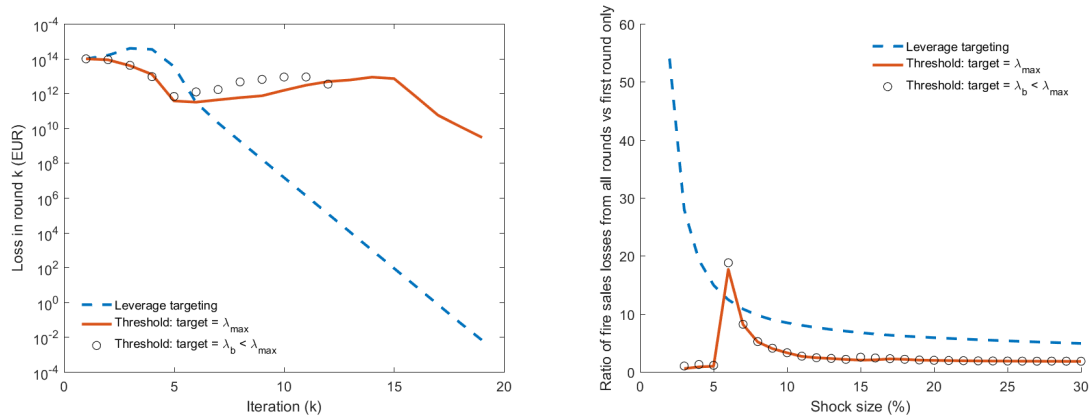


Figure 19: Left: Fire sales losses as a function of the iteration step at the estimated market depth for the leverage targeting model (dashed), and the threshold models with  $\lambda_b = \lambda_{\max}$  (full) and  $\lambda_b < \lambda_{\max}$  (circles). Right: Ratio of total loss to first round loss, as a function of initial stress level.

failures due to illiquidity. Only examining a single round of the feedback loops thus leads to an underestimation of the number and severity of bank failures. The left panel in Figure 19 shows the fire sales loss at each round  $k$  on a log scale for the estimated market depth.

The right panel in Figure 19 shows the ratio of total loss (20 rounds) to the first round loss, as a function of the initial stress level. The non-monotone feature of this dependence shows that fire-sales contagion is most important for moderate stress levels, which trigger deleveraging but are not extreme enough to generate insolvency at the first round. In this case, second and higher rounds considerably change the outcome in terms of total loss level and number of bank failures.

The results are even starker for the leverage targeting model: subsequent rounds lead to an increase by a factor 10 to 50 of estimated fire sales losses, especially when the initial shock is small. This makes the “number of rounds” an important implicit parameter in the leverage targeting model.

## 4 Monitoring exposure to price-mediated contagion: the Indirect Contagion Index

The results above indicate that the contribution of fire sales to system-wide losses is mainly driven by liquidity weighted portfolio overlaps (19). As observed in Section 3.3, the principal component analysis (eigenvalue decomposition) of this matrix reveals that its first two eigenvectors capture the main structure of the network of portfolio overlaps.

We use this insight to design a simple indicator, based on institutional portfolio holdings, which quantifies the exposure of a bank to price-mediated contagion without resorting to a large scale stress-test.

The idea is to use an eigenvalue decomposition of the matrix of portfolio overlaps to identify the main institutions which contribute to price-mediated contagion, by computing their eigenvector centrality in the network defined by portfolio overlaps. To compute this, we proceed as follows:

1. Collect portfolio holdings  $\Pi^{i,\mu}$  by asset class for each financial institution in the network, at the granularity level corresponding to bank stress tests.
2. Estimate a market depth parameter  $D_\mu$  for each asset class.
3. Compute the liquidity weighted overlaps (19) of portfolios.
4. Compute the principal (largest) eigenvalue and the corresponding eigenvector  $U = (U_i, i = 1 \dots N)$  of the matrix of liquidity-weighted overlaps (20).
5. The component  $U_i$  of the principal eigenvector  $U$  then provides a measure of centrality of the node  $i$  in the network whose links are weighted by the overlap matrix (20).

We define the **Indirect Contagion Index (ICI)** of a financial institution as its component in the normalized principal eigenvector  $U_i$  of the matrix (20) of liquidity weighted portfolio overlaps:

$$ICI(i) = U_i \tag{32}$$

Figure 20 shows the principal eigenvector of  $\Omega$  respectively (i.e. the left singular vectors of  $\Pi\sqrt{D^{-1}}$ ). The Indirect Contagion Index can be directly read from Figure 20. The banks with highest  $ICI$  are labelled. All banks that are identified are either Global Systemically Important Banks (GSIBs) or Domestic Systemically Important Banks (DSIB). It is furthermore important to note that the banks are not ranked according to their size. Rather, they are ranked by the amount of contagion they could trigger, should they engage in a fire sale.

The (logarithmic) cross-sectional regression of fire sales losses on the Indirect Contagion Index (Figure 21 left and Table 5) reveals a significant positive relation between  $ICI$  and fire sales losses, with an  $R^2 = 0.69$  in the systemic risk region. This relation is quite useful since the computation of the  $ICI$  does not require simulation or stress testing and is purely based on portfolio holdings data.

The right-hand side graph of Figure 21 shows the  $R^2$  of the regression of the fire sales losses on the components of the first eigenvector as a function of the initial shock size. Both plots show that the first eigenvector clearly contains valuable information on how fire sales losses may spread during a deleveraging cascade. The correlation and regression  $R^2$  both tend to increase with larger shock size.

These results show that the Indirect Contagion Index may be used as a quantitative indicator of 'interconnectedness' in the assessment methodology for Global Systemically Important Banks [Basel Committee on Banking Supervision, 2016]. The information contained in this index complements those currently used for GSIB identification and identifies banks which may contribute to the indirect contagion channel.

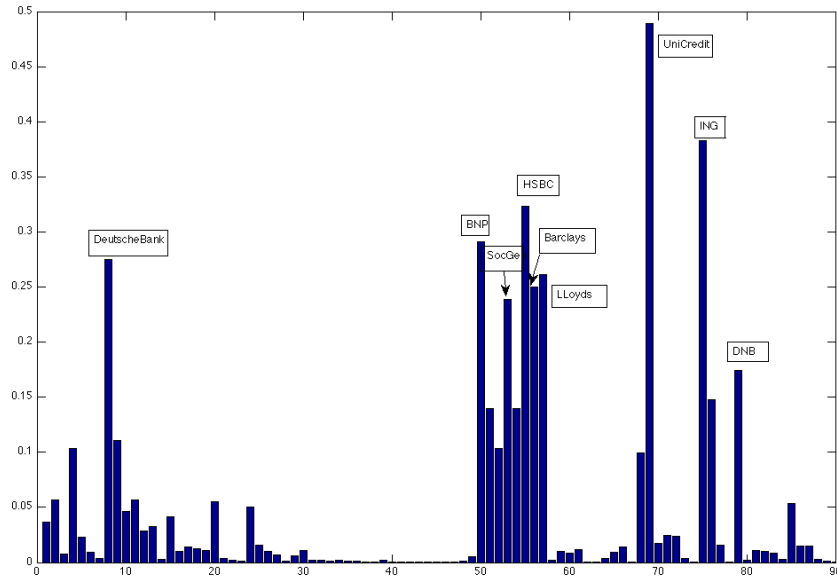


Figure 20: The principal eigenvector of the the matrix of liquidity-weighted portfolio overlaps is concentrated on a few banks, all of which are observed to be GSIBs (DNB is a DSIB only).  $ICI_i$  corresponds to the value of bank  $i$  in this graph. The eigenvector is normalised to have norm 1, such that this provides a relative ranking of the banks.

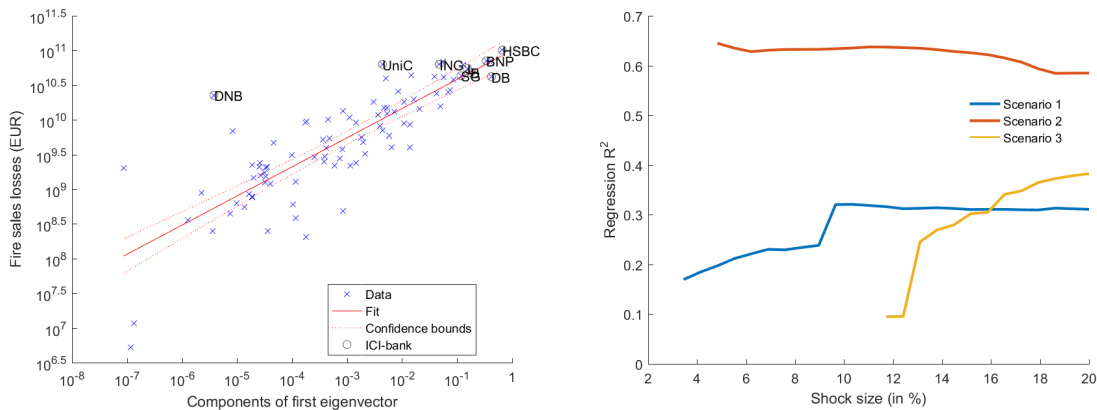


Figure 21: Left: Logarithmic regression of fire sales losses on Indirect Contagion Index (32). Right:  $R^2$  from regressing the fire sales losses on  $ICI_i$  as a function of the initial shock size.

## 5 Indirect exposures

### 5.1 Notional vs effective exposures

The starting point in portfolio risk analysis is the notion of exposure to an asset class, usually quantified by the notional volume of holdings, in monetary units, in that asset class. In absence of contagion, losses in a stress scenario for this asset class will be a linear function of the percentage shock to the asset value, the proportionality coefficient being this notional exposure.

However, the endogenous risk arising from fire-sales may amplify this initial loss in a nonlinear way, leading to a total loss that is higher than the one given by the notional exposure, leading to an *effective exposure* higher than the notional one. Furthermore, spillover effects from fire-sales may further increase this loss through indirect contagion. The end result is that the (marginal)

	Model 1: $L^i = \beta v_i + \varepsilon_i$	Model 2: $\log(L^i) = b \log(v_i) + \varepsilon_i$
Slope	1.58 10 <sup>11</sup> *** (1.50 10 <sup>10</sup> )	0.42*** (0.029)
Intercept	9.85 10 <sup>9</sup> *** (1.59 10 <sup>9</sup> )	5.00*** (0.104)
n	90	90
R <sup>2</sup>	0.56	0.69

Table 5: Regression of bank losses on the Indirect Contagion Index.

ratio of a portfolio’s loss in an asset class to shocks affecting this asset class may be *higher* than its notional exposure. We capture this effect by defining the notion of *indirect exposure* and quantifying it using our model.

Denoting as above by  $\Theta^{i,\kappa}$  the holdings of institution  $i$  in the (illiquid) asset class  $\kappa$ , consider a stress scenario in which the asset  $\kappa$  depreciates by  $\epsilon_\kappa$ . Then institution  $i$  has a direct loss  $\epsilon_\kappa \Theta^{i,\kappa}$  and, as long as the shock size is small, no deleveraging occurs and the latter also represents the total loss, which increases linearly with  $\epsilon_\kappa$ .

However, as stress levels increase, feedback effects and price-mediated contagion may amplify the initial losses and result in a loss  $Loss(i, \epsilon_\kappa) = \epsilon_\kappa \Theta^{i,\kappa} + FLoss(i, \epsilon) > \epsilon_\kappa \Theta^{i,\kappa}$ . The *effective exposure* to the asset class  $\kappa$  accounts for these additional losses and is defined as

$$E^{i,\kappa}(\epsilon_\kappa) := \frac{Loss(i, \epsilon_\kappa)}{\epsilon_\kappa} = \underbrace{\Theta^{i,\kappa}}_{\text{Notional exposure}} + \underbrace{\frac{FLoss(i, \epsilon_\kappa)}{\epsilon_\kappa}}_{\text{Indirect exposure}}, \quad (33)$$

Unlike the notional exposure, the *indirect exposure* depends on the shock size  $\epsilon_\kappa$  and, more importantly, on the configuration –size, leverage– of *other* financial institutions with common asset holdings. Clearly, unlike the first term in (33), the second term cannot be computed by examining the portfolio of  $i$  alone and depends on the entire network of overlapping portfolios which contribute to fire sales losses. Given the discussion in the previous section, this should not come as a surprise but it clearly departs from the common assumption that notional exposures of a portfolio are sufficient to quantify its risk. Here, the risk of a portfolio cannot be quantified in isolation, but depends on the network of overlapping portfolios and the constraints that these portfolios face.

Interestingly, a financial institution may even have a non-zero (indirect) exposure to an asset class which it does not hold in its portfolio i.e. the second term in (33) may be non-zero even in the first term is zero. Consider for example a bank A which does not hold any subprime asset-backed securities, but has common holdings with a bank B with large holdings in subprime ABS. A loss in subprime ABS entails no direct loss to A but, if large enough, may force B to deleverage by selling its marketable assets, also held by A, leading to mark-to-market losses for A. This means that A will have an indirect exposure to large losses in subprime ABS, an asset it does not hold! In this example, deleveraging by B only occurs if subprime losses are significant, so A has no exposure to a *small* depreciation in subprime ABS: the indirect exposure of A is clearly scenario-dependent. However, the threshold beyond which spillover from B to A occurs depends on the leverage, size and composition of B’s portfolio, which is unknown to A. This example, far from being a curiosity, is in fact illustrative of the mechanism which led to amplification and contagion of losses from the subprime asset class to the entire global financial system in 2007-2008 [Longstaff, 2010].

## 5.2 Indirect exposures: empirical evidence

To assess how large such indirect exposures may be in the European banking system, we use the results of the previous section to examine the magnitude of indirect exposures of European



banks to residential and commercial mortgages. A first observation is that most European banks tend to have very few mortgages loans outside of their own country, so direct exposure to foreign residential and commercial mortgage is zero in most cases, except for a few multinational banks. Given the large volume of mortgages on bank portfolios, it is generally assumed that domestic housing prices are the main risk factors for commercial bank portfolios. However, as we shall see now, European banks also have substantial indirect exposures to real estate prices in *other* European countries where they do not issue mortgages.

Figure 22 displays the (total) loss for two banks in our sample, HSBC and Santander, in the scenario where Spanish real estate depreciates (Scenario 1). Santander, a major Spanish bank, holds a lot of Spanish mortgages on its portfolio, representing a notional exposure of 82.7 Bn EUR, which corresponds to the slope of the loss for shocks less than 3%, where no deleveraging occurs. HSBC, on the other hand, holds very few Spanish mortgages (less than 0.5 Bn EUR) so its notional exposure to this asset class is much lower.

As Spanish real-estate losses exceed 3.5%, some Spanish banks start deleveraging and indirect losses lead to a sharp increase in the loss level. For a 5% shock level, the total loss for Santander lies somewhat over 25 Bn EUR, which corresponds to an indirect exposure exceeding 500 Bn EUR! In this regime, the indirect exposure is clearly the main source of losses. This deleveraging by Spanish banks then affects HSBC through price-mediated contagion: as the shock level increases from 2.75% to 3.45%, the fire sales loss for HSBC jumps from 36 M to 2.8 Bn EUR. For a 4% shock level, HSBC's indirect exposure to Spanish real estate is close to 70 Bn EUR, which corresponds to 2.8 Bn EUR of losses due to indirect contagion. Note that in the case of HSBC, the indirect loss corresponds to 140 times its notional exposure to this asset class! This example suggests that in stress scenarios which are sufficiently severe to trigger fire sales, actual losses can be much higher than what is suggested by notional exposures. The difference corresponds to indirect exposures.

The right-hand graph Figure 22 displays the magnitude of indirect exposures corresponding to these losses. We see that the indirect exposure of Santander, which mainly stems from its own deleveraging and deleveraging by other Spanish banks with similar assets, sharply increases when the shock size reaches the threshold of 3%, then peaks around 4%: as the stress level increases beyond a certain level, some banks become insolvent and cease to contribute to fire sales spillovers.

As illustrated in the right panel in Figure 22, these indirect exposures clearly depend on the severity of the stress scenario considered, the bank's initial exposures, and the exposure and constraints of other portfolios holding similar marketable assets. An implication is that one cannot mimic the impact of indirect contagion by simply applying a higher stress level for all banks, as is implicitly done in current supervisory stress tests.

Table 6 reports indirect exposures to Spanish real estate for several other European banks. Comparing with their notional exposures, we observe that non-Spanish banks have a substantial indirect exposure to Spanish real-estate which is not revealed by inspecting their notional exposures to this asset class, which are between ten and a hundred times smaller. This indirect exposure is due to the overlap in marketable assets with banks that are directly exposed to the Spanish housing market.

These observations are not particular to the Spanish real estate sector. Other examples, which we do not report here for the sake of brevity, show that in the European banking system a stress scenario affecting the commercial or residential real-estate sector of a given country may generate significant losses arising from indirect exposures for foreign banks.

The concept of indirect exposures may also be defined at the country level. Figure 23 shows the indirect exposures of European banks (aggregated by country) to the Spanish housing market. Shocks below 1% do not appear to trigger fire sales, and do not generate indirect exposures.

As the stress level is increased, fire sales are triggered and spillover losses materialise; when the initial shock size exceeds 3%, contagion increases fire sales losses by 2 - 3 orders of magnitude.

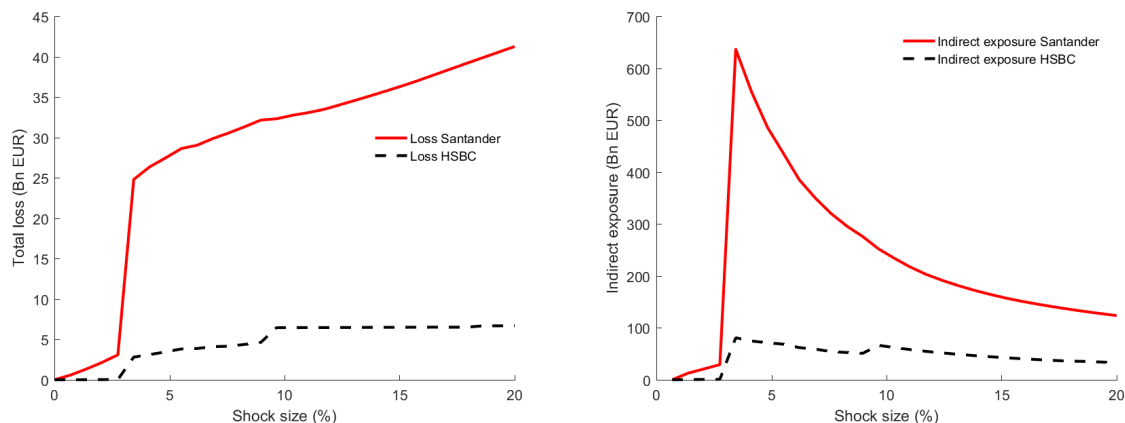


Figure 22: Left: Losses arising from a depreciation in Spanish real estate, for HSBC and Santander, as a function of the depreciation level (horizontal axis). Right: The indirect exposures corresponding to these losses.

Bank	Santander	BBVA	Deutsche Bank	Commerz Bank	HSBC	RBS	Nordea
Notional exposure to Spanish real-estate	82.7	82.5	8.88	5.01	0.452	2.68	0.003
Indirect exposure to 2% shock	29.2	27.2	1.60	0.73	0.84	2.43	0.050
Indirect exposure to 5.5% shock	385	280	64.0	23.9	61.9	58.7	6.40
Indirect exposure to 10% shock	217	169	60.9	22.6	58.1	45.8	7.21
Capital	42.0	24.9	37.6	26.7	86.90	59.0	19.1

Table 6: Estimated direct and indirect exposures (Bn EUR) of selected banks to Spanish commercial and residential real estate.

What is striking is that these thresholds – 1%, 3% – are not large, suggesting that indirect exposures and losses arising from price-mediated contagion cannot be ignored in stress tests. The left panel of Figure 23 reveals that, after Spanish banks, British banks have the largest indirect exposure to the Spanish housing market. Table 7 reports estimated indirect exposures for different stress levels for the countries highlighted in Figure 23.

Overall, the direct exposure of European Banks to the Spanish residential and commercial real estate sector is 740 bn EUR. When averaged both across all banks and all shock sizes (between 0 and 20%), the indirect exposures amount in this example to 184% of the direct exposures, which is non-negligible. To be clear, in the event of a meltdown of the Spanish housing sector, non-Spanish European banks will, on average, be affected for up to 184% of their actual holdings in Spanish mortgages.

This clearly means that housing prices in one European country can strongly affect banks of other European countries, an issue which can only be tackled by macroprudential policies applied at a Europe-wide level.<sup>7</sup>

These indirect exposure figures, as all other results obtained in the stress tests, are sensitive to assumptions on market depth, or, alternatively, on the liquidation horizon. Figure 24 shows the estimated indirect exposure of the UK banking system to the Spanish housing market as a function of the stress level and the market depth scaling factor. Reading the graph from front to back, we see that a contraction of market liquidity can greatly exacerbate fire sales losses:

<sup>7</sup>On this point, we note that Brexit does not act as a barrier for price-mediated contagion.

there is a tipping point at which indirect exposures increases from 10 Bn to 100 Bn.

Indirect exposures and the losses arising from them provide a more palpable explanation of how an initial loss of about 500 Bn USD in the US subprime sector was able to balloon into trillions of dollars of losses across multiple asset classes during the financial crisis.

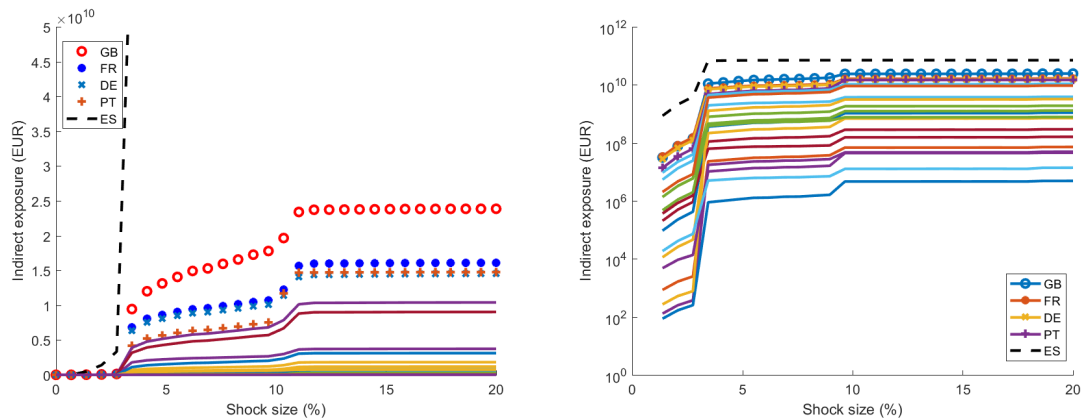


Figure 23: Indirect exposures of European banks, aggregated by country, to the Spanish housing market.

Country	Direct exposure (Bn EUR)	Indirect exposure (Bn EUR) Stress level: 1%	Indirect exposure (Bn EUR) Stress level: 2%	Indirect exposure (Bn EUR) Stress level: 3.5%
ES	22.8	0	101.3	1662.4
GB	7.9	0	3.7	293.2
FR	15.9	0	4.1	196.4
DE	2.4	0	3.2	183.9
PT	697.0	0	1.6	127.4

Table 7: Indirect exposures to Spanish mortgages of EU banks by country in billions of Euros. For shocks below 1%, no fire sales occur and indirect exposures are zero. Shocks around 2% create significant indirect exposures within Spain, but negligible ones for other European countries. Initial shocks larger than 3.5% trigger large scale contagion that may result in cross-border spillovers. This is reflected by significant indirect exposures on the order of 100Bn EUR for other European countries.

### 5.3 Relevance of indirect contagion for bank stress tests

Given the magnitude of indirect contagion, it is not surprising that accounting for indirect losses can modify the outcome of bank stress tests. To show this more clearly, we compare the outcomes of a bank-level stress test, where the (direct) loss resulting from initial stress is compared to its capital, with that of a systemic stress test, where we also account for fire-sales spillover which may follow from the initial loss.

Figure 25 plots, as a function of the shock size, the number of banks (out of 90 banks) in the sample which have enough capital to sustain the initial losses but become insolvent after a single round of fire-sales. For moderate stress levels (5-10%) we identify up to 10 banks (in scenario 1), reported in Table 8, which pass the bank-level stress test but fail the systemic stress test.

Figure 26 shows the number of banks that fail the bank-level stress test but fail the systemic stress test as a function of the initial stress level and the market depth scaling factor. We see that the discrepancy between the outcomes of the systemic and single-bank stress test is precisely in

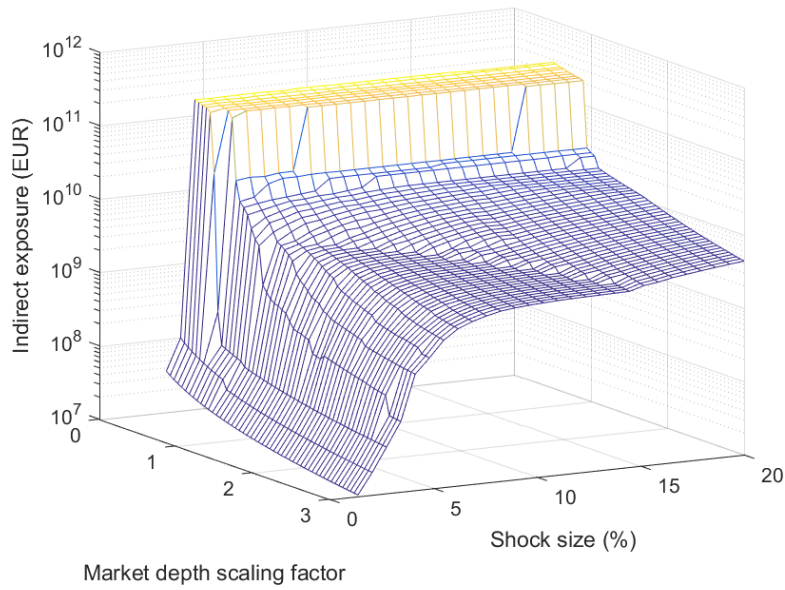


Figure 24: The indirect exposure (in EUR) of the UK banking system to the Spanish residential and commercial mortgage market as a function of the stress level and market depth. Even with optimistic market depth estimates, when the initial shock is above 3.5%, indirect exposures are non-negligible.

the regions corresponding to moderate to large stress levels typically used in supervisory stress tests.

These results illustrate that the contagious losses from fire sales can be of a sufficiently large magnitude to change the outcome of bank stress tests.

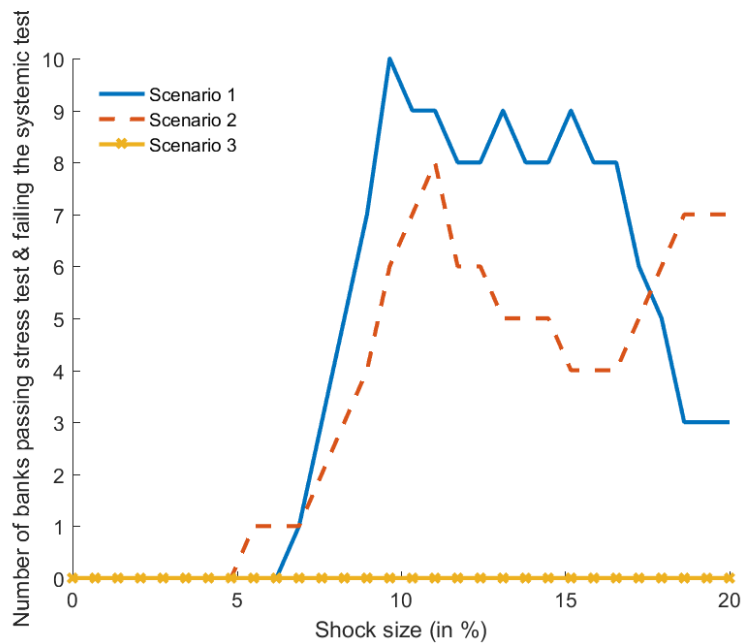


Figure 25: Number of banks with capital sufficient to withstand the initial stress, but failing to withstand the losses due to indirect contagion after a single round of deleveraging.

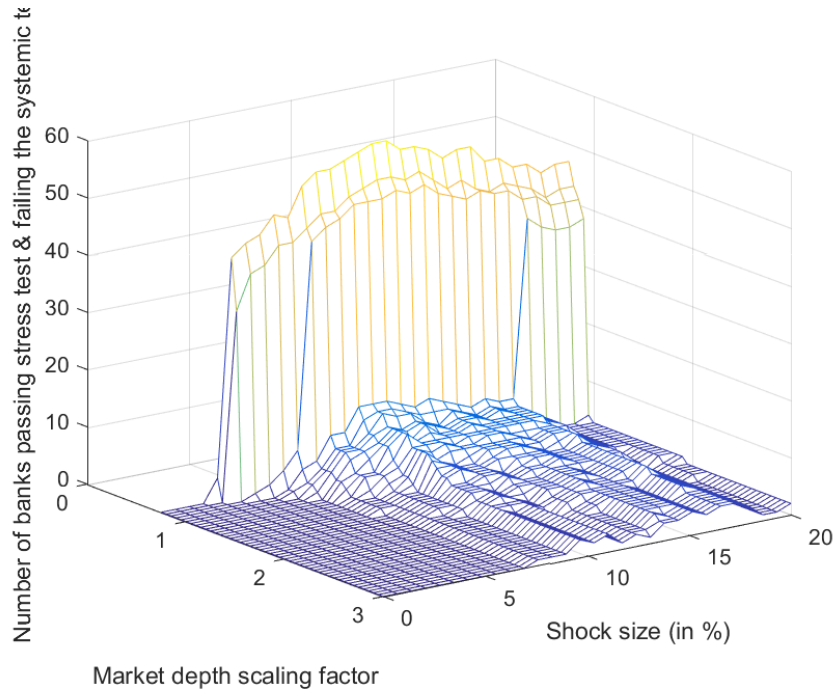


Figure 26: Number of banks with capital sufficient to withstand the initial stress, but which fail to withstand losses due to indirect contagion as a function of the initial shock.

#### 5.4 Why indirect exposures cannot be reproduced in single bank stress tests

An argument sometimes advanced to avoid moving to a systemic stress testing framework is that, while fire sales effects are important and contribute to amplifying losses, their effect can be mimicked in bank-level stress tests by applying more severe shocks, without simulating fire sales in a detailed manner. Let us now examine the validity of this claim using the above results.

First, we note that the total loss across all banks in the systemic stress test can be obviously reproduced in a macro stress-test without fire sales by simply scaling the initial shock size by an adequate factor. However, in absence of fire sales, bank losses are proportional to the banks notional exposure to the asset class subject to stress, whereas the loss in the systemic stress test is proportional to the *effective exposure*, so bank-level losses will not be reproduced correctly under this adjustment. Figure 27 compares bank-level losses in the systemic stress test with fire sales (blue asterisks) with losses of the same banks (depicted with X symbols) in a stress test without fire sales, but with severity of the shock chosen so that the total system-wide loss is equal in the two stress tests.

As is clearly visible from this figure, the distribution of losses across banks is fundamentally different in these two cases. No amount of scaling at the level of macro-shocks defining the stress scenario can adjust for the cross-sectional heterogeneity of indirect exposures. Thus, even if the stress level is adjusted to make the total loss in the banking system equal in both stress tests, the allocation of losses across banks is essentially different once fire sales are accounted for. This is a strong argument for including a proper model of fire sales into any macroprudential stress testing framework.

Scenario	Banks failing systemic stress test after 2 rounds of fire sales
1	BFA-BANKIA Caja de ahorros y pensiones de Barcelona Banco de Sabadell Banco popular Espa�ol Caixa d'Estivalis de Catalunya, Tarragona i Manresa Caixa de Aforros de Galicia, Vigo, Ourense e Pontevedra Groupe BMN Bankinter Caja de Ahorros y M.P. de Zaragoza, Aragon y Rioja Banco Pastor
2	Lloyds Banking Group Unicredit ING DnB NOR Nordea Bank SEB Svenska Handelsbanken Swedbank

Table 8: EU banks that pass the single-bank stress test for a stress level of 10% but fail the systemic stress test after 2 rounds of fire sales.

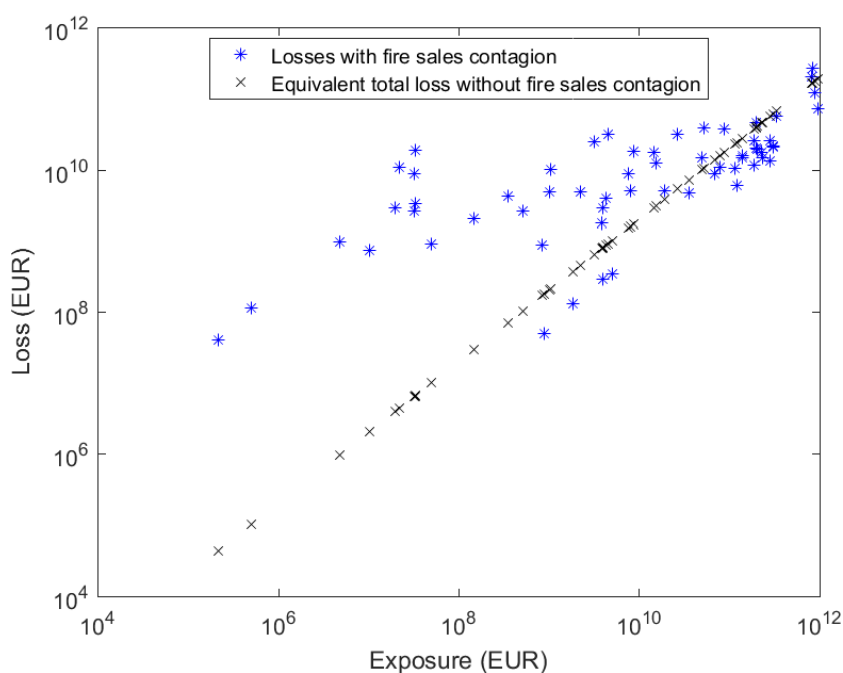


Figure 27: Bank-level losses in a systemic stress test with fire sales (asterisks) compared with losses in a stress test without fire sales (X). (EBA data, 2011)

## 6 Implications for macroprudential stress-testing and regulation

We have presented a stress-testing framework for quantifying the endogenous risk exposure of the banking system to fire sales and feedback effects which may arise from macro-shocks to the

financial system. Our findings provide quantitative evidence for the importance of *endogenous risk* [Danielsson et al., 2004, Shin, 2010, Cont and Wagalath, 2016] in the financial system and have implications for systemic stress testing, macroprudential policy and risk management in financial institutions:

1. *Need for a systemic stress-testing framework for capital adequacy*

Supervisory stress tests, especially in the US, have become the binding constraint for bank capital adequacy. Theoretical studies [Danielsson et al., 2004, Shin, 2010, Pedersen, 2009, Cont and Wagalath, 2013] have repeatedly pointed out the importance of *endogenous risk* for financial stability. Fire sales are arguably an extreme example of endogenous risk and pro-cyclical behavior, yet they are not systematically integrated into bank stress testing methodologies.

Our quantitative findings join a list of previous studies in pointing out the importance and magnitude of fire sales and price-mediated contagion as a risk amplification mechanism for systemic risk. Even under benign assumptions on market liquidity, the magnitude of exposures arising from this channel is too large to ignore; losses arising from this channel can dominate other types of risk exposures in certain risk scenarios. More importantly, we have shown that risk exposures of financial institutions arising from fire sales cannot be replicated in single-institution stress tests, even after scaling to extreme stress levels. This pleads against a widespread approach which consists in simply applying a (constant) discount to asset values in stress test to account for liquidation costs: we have argued that this discount is endogenous and strongly depends on the degree of leverage and concentration of asset holdings across financial institutions.

These observations, which are based on public data and may be readily replicated by regulators, plead for a systemic approach to bank stress testing which accounts for fire sales contagion in a proper way. We have presented the building blocks of an operational framework for estimating such endogenous effects and incorporating them into a macroprudential stress testing framework.

2. *Bank-level exposure to price-mediated contagion may be quantified using the Indirect Contagion Index:*

Conducting a system-wide stress test can be difficult and costly. We have identified a simple proxy, the Indirect Contagion Index (Section 4), for assessing the vulnerability of banks to fire sales, in absence of a system-wide stress test. We have argued that this indicator may be used alongside other measures of interconnectedness currently used for identifying globally systemically important banks (GSIBs) [Basel Committee on Banking Supervision, 2016].

3. *Implications for the interaction between banks and non-banks:*

Price-mediated contagion is not limited to banks; any institution exposed to fire sales risk and having common asset holdings with banks may become a source of price-mediated contagion for the banking sector. There is ample empirical evidence of fire sales by asset managers [Coval and Stafford, 2007, Jotikasthira et al., 2012] and insurance companies [Ellul et al., 2011].

Regulators have been concerned with monitoring the interaction between the banking sector and non bank financial institutions ('shadow banks'); current attempts to monitor these exposures have focused on balance sheet linkages between shadow banks and the regular banking system measured solely in terms of direct exposures and liabilities [Grillet-Aubert et al., 2016]. Given that a large fraction of financial assets are held by non-banks –large asset managers, pensions funds, and insurance companies – the scope for indirect contagion from 'shadow banks' to the banking sector through the fire sales channel exists and its proper assessment calls for a system-wide stress test for quantifying these

inter-sector exposures. Extending the present model to include non-banks would require a careful analysis of mechanisms which would lead such institutions to shed assets: redemption risk for asset managers and asset-liability mismatch for pension funds are plausible avenues to consider.

4. *Indirect exposures as tool for risk management:*

As pointed out by [Ellul et al., 2014] "forward-looking institutions that rationally internalize the probability of fire sales are incentivized to adopt a more prudent investment strategy during normal times which leads to a safer portfolio entering the crisis." One of the obstacles to internalizing the risk of fire sales is that its proper assessment requires some knowledge of the concentration of asset holdings across financial institutions, which is typically only available to regulators.

One of the by-products of our stress-testing approach is the ability to compute the *indirect exposures* of an institution to various asset class (Section 5). Communicating to an institution the magnitude of its indirect exposures to various asset classes, as evaluated in the systemic stress test, allows the institution to have a better assessment of its risk and provides incentives to reduce such exposures.

5. *Need for transnational coordination on stress-testing and macroprudential policy:*

As revealed by our analysis of common asset holdings in Section 3.3, the European indirect contagion network is completely connected and banks in different countries may have strongly overlapping portfolios, which lead to substantial indirect exposures across EU countries, as shown in Table 6. Unlike direct exposures, which may be limited through various capital restrictions, portfolio overlaps and indirect exposures are difficult to put limits on: any such restrictions would amount to limiting international diversification of bank portfolios. Any meaningful systemic stress test should therefore account for the magnitude of these cross-country indirect exposures, cannot be conducted at the level of a single country and consequently calls for transnational coordination of macroprudential policies.

6. *The role of mark-to-market accounting rules:*

Previous studies [Allen and Carletti, 2008, Ellul et al., 2014] have focused on mark-to-market accounting as the channel of transmission of losses in fire sales contagion. [Allen and Carletti, 2008] present arguments against the use of mark-to-market accounting for determining solvency during crisis periods; indeed, such temporary suspensions were used in 2009 by US banks. [Jotikasthira et al., 2015] use the example of the insurance sector to caution against this recommendation.

Our model offers a perspective on this debate: having distinguished between marketable ('Level I') assets and illiquid assets, we can note that a suspension of mark-to-market accounting rules is only likely to affect the latter. Suspension of mark-to-market accounting for illiquid assets may indeed affect the first step in our iteration, in which losses to illiquid assets trigger the initial deleveraging. But once deleveraging by a set of institutions takes place, the subsequent losses are not accounting losses, but market losses in marketable securities whose magnitude is not affected by accounting conventions. Thus, temporary suspension of market accounting rules may reduce the scope of institutions affected by an initial stress to some illiquid asset class, but once fire sales affect liquid 'Level I' securities, all institutions holding them will be affected by market losses.



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## A Appendix

### A.1 Sources for market data

US treasuries: <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/SF-US-SF-Trading-Volume-SIFMA.xls>

US corporate bonds: <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/Corporate-US-Corporate-Trading-Volume-SIFMA.xls>

DE treasuries: <http://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-marke>

UK treasuries: <http://www.dmo.gov.uk/index.aspx?page=Gilts/Turnover>

ES treasuries: <http://www.tesoro.es/sites/default/files/estadisticas/18.pdf>

FR treasuries: [http://www.aft.gouv.fr/rubriques/trading-volume\\_109.html](http://www.aft.gouv.fr/rubriques/trading-volume_109.html)

IT treasuries: [http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti\\_en/debito\\_publico/presentazioni\\_studi\\_relazioni/3\\_3\\_2000\\_13\\_43\\_The-Italian-Treasury-.pdf](http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti_en/debito_publico/presentazioni_studi_relazioni/3_3_2000_13_43_The-Italian-Treasury-.pdf)

BE treasuries: [http://www.debtagency.be/fr\\_products\\_olo\\_volume.htm](http://www.debtagency.be/fr_products_olo_volume.htm)

SE treasuries: [http://www.riksbank.se/Documents/Rapporter/Finansmarknaden/2014/rap\\_finansm\\_140829\\_eng.pdf](http://www.riksbank.se/Documents/Rapporter/Finansmarknaden/2014/rap_finansm_140829_eng.pdf)

PT treasuries: [http://www.igcp.pt/fotos/editor2/2015/Estatisticas/12\\_Transacoes\\_medias\\_diarias\\_OT\\_e\\_BT\\_Dez15\\_1.pdf](http://www.igcp.pt/fotos/editor2/2015/Estatisticas/12_Transacoes_medias_diarias_OT_e_BT_Dez15_1.pdf)

GR treasuries: <http://www.bankofgreece.gr/Pages/en/Markets/HDAT/statistics.aspx>

### A.2 EBA: data identifiers and residual exposures

Model variable	EBA dataset identifier
<b>Assets</b>	
<i>Illiquid assets</i> $\Theta$	
Residential mortgage exposures ( $\epsilon_\kappa \geq 0$ )	33013
Commercial real estate exposures ( $\epsilon_\kappa \geq 0$ )	33018
Retail: Revolving exposures ( $\epsilon_\kappa \equiv 0$ )	33015
Retail: SME exposures ( $\epsilon_\kappa \equiv 0$ )	33016
Retail: other exposures ( $\epsilon_\kappa \equiv 0$ )	33017
Indirect sovereign exp. in the trading book ( $\epsilon_\kappa \equiv 0$ )	34017
Defaulted exposures ( $\epsilon_\kappa \equiv 0$ )	33020
Remaining exposures* ( $\epsilon_\kappa \equiv 0$ )	-
<i>Securities II</i>	
Institutional client exposures	33010
Corporate exposures	33011
Sovereign exposures	34013 , 34014 , 34015
Direct sovereign exposures in derivatives	34016
<b>Liabilities</b>	
Tier 1 capital	30014
Debt	-

Table 9: Mapping of EBA data to model variables.

**Correcting for data inconsistencies.** The EBA data provides information on notional exposures of each bank to 135 asset classes.<sup>8</sup> Bank BE005 records a zero value for total exposures

<sup>8</sup>For more details on the regulatory definition of “exposure” cf. the EBA methodological note: [https://www.eba.europa.eu/documents/10180/15932/EBA-ST-2011-004-Detailed-Methodological-Note\\_1.pdf](https://www.eba.europa.eu/documents/10180/15932/EBA-ST-2011-004-Detailed-Methodological-Note_1.pdf) as well as the identifier 33021 in the dataset.

despite positive exposures in the individual asset classes. As a proxy for this bank, we use information from the “Total assets after the effects of mandatory restructuring plans”<sup>9</sup>, which is usually quite close to the values of “total exposures” across the dataset.

Another consistency issue is that the individual exposures in the dataset do not always sum up to the total exposure figure. Indeed, EBA explains in a footnote of the “total exposure” data that: “*Total exposures is the total EAD according to the CRD definition based on which the bank computes RWA for credit risk. Total exposures, in addition to the exposures broken down by regulatory portfolios in this table [corresponding to the four asset classes in the portfolio II above] include EAD for securitisation transactions, counterparty credit risk, sovereigns, guaranteed by sovereigns, public sector entities and central banks.*” Due to this, the sum of balance sheet items deviates from the “total exposures” information recorded in the dataset. In order to correct for this deviation we add a “remaining exposures” item to the illiquid assets category. The average size of the negative correction terms is 5.9% of the corresponding balance sheet sizes. Double counting is thus a minor issue. The average size of the positive correction terms is 13.4%. This average is inflated by five small banks that are outliers and have correction terms above 50%. Excluding these outliers, the average of the correction terms reduces to 9%. The average size of the correction term over the entire data set is 8.9%. On average, we thus underestimate the balance sheet sizes of the banks. This may only bias results in the sense of underestimating the impact of fire sales. We adjust for this with the “other illiquid assets” category, as they play no role in the fire sale cascade.

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<sup>9</sup>This information is recorded under identifier 30029 in the EBA dataset.