On a Relevant Aspect in Difference Equations

By

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Abstract: In this short note, we obtain the solution of the most general linear system of difference. First this was solved in [1], however here we solve it in a much more easy way.

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Consider the infinite set of nonlinear equation

$$a_{j+1} = \sum_{k=0}^{j} \alpha_k^{j+1}\, a_k + \sum_{i=1}^{j-1}\sum_{k=1}^{j-1} \Upsilon_{l,k}^{j+1}\, a_l a_k + \ b_{j+1} \qquad for\ j > 0$$

Introducing

$$^{1}a_j = \ ^{2}a_j + \ ^{2}Q_j$$

it turns out $^{1}a_j = \ ^{2}q_j$ for $0 \leq i \leq 2$ because $^{1}a_{j+1}$ only has nonlinear terms for $j \geq 2$.

If we develop a_j as a funcion of the initial condition a_0 we have

$$^{2}Q_{j+1} = \left(\sum_{r=1}^{j+1} \sum_{(l_1,l_2,\dots,l_r)^{j+1} \in E_r^{j+1}} \nabla_{(l_1,\dots,l_r)^{j+1}}^{j+1} \right) q_o$$

$$+ \sum_{i=1}^{i} \left(\sum_{r=1}^{j+1-2} \sum_{(l_1,\dots,l_r)^{j+1-2} \in E_r^{j+}} {}^{1}\nabla^{j+1}{}_{(l_1,\dots,l_r)} \right) {}^{1}b_i \} + {}^{1}b_{j+1}$$

Where E_r^1 is the set of all $r -$ components vectors $(l_1, \dots, l_r)$ such that

$$\sum_{i=1}^{r} l_i = l \quad \text{whith} \qquad l_1 = \text{integer}$$

and

$$\nabla_{(l_1,l_2,\dots,l_r)^e}^{i} \cong {}^{\mathrm{i}}\alpha_{j-e_1}^{j}\, {}^{\mathrm{i}}\alpha_{j-l_1-l_2}^{j-l_1}\, {}^{\mathrm{i}}\alpha_{j-l_1-l_2-l_2}^{j-l_1-l_2} \sim {}^{\mathrm{i}}\alpha_{j-l}^{j-\sum_{i=1}^{r-1} l_1}$$

Introduction

The subject of difference equations is a very important topic from the theoretical point of view and mainly for applications. One good reference is the book by Poole [2].

We remember the reader that if we wish to solve the most general linear problem in difference equations, them we have to solve the following one, namely:

$$x_m = a_{m-1}^m x_{m-1} + a_{m-2}^m x_{m-2} + \cdots + a_1^m x_1 + a_e^m x_e + b_m^m$$

This has been solved by induction principle by the authors many years ego. However this approuch was difficoult.

We give a view of the older method. Now here we are solving it in a easy way.

By the way, we would like to say that many specialists in the subject and some paragraphs in advance and common books, say that this general problem is unsolvable. Moreover, many of these specialists critizaise it, since in the computer the memory blows up for some complicate problems.

Our old formula was applied succesfully in several subjects in particular to mathematical models in biothecnology.

Now, we have found a now form which is easy that the old one.

In the equation (1) if we revert the order, that is to say

$$x_m = a_o^n x_o + a_1^m x_1 \ldots \ldots + a_{m-2}^m x_{m-2} + a_{m-1}^m x_{m-1} + b^m$$

and now take the first n terms from $n = 0$ in the form

$$x_1 = a_0^1 x_0 \qquad\qquad +b^1$$

$$x_2 = a_0^2 x_0 + a_1^2 x_1 \qquad\qquad +b^2$$

$$x_3 = a_0^3 x_0 + a_1^3 x_1 + a_2^3 x_2 \qquad\qquad +b^3$$

$$x_{m-2} = a_0^{m-2} x_0 + a_1^{m-2} x_1 + a_2^{m-2} x_2 + \cdots \qquad\qquad + b^{m-2}$$

$$x_{m-1} = a_0^{m-1} x_0 + a_1^{m-1} x_1 + a_2^{m-1} x_2 + \cdots a_{m-2}^{m-1} x_{m_2} \qquad\qquad + b^{m-1}$$

$$x_m = a_0^m x_0 + a_1^m x_1 + a_2^m x_2 + \cdots + a_{m-1}^m x_{m-1} \qquad\qquad + b^m$$

Now if you take and define the matrix mxm

$$A_m = \begin{pmatrix} a_0^1 \\ a_0^2 a_1^2 \\ a_0^3 a_1^3 a_2^3 \\ \\ \\ a_0^m a_1^m \ldots\ldots\ldots\ldots\ldots\ldots a_{m-1}^m \end{pmatrix}$$

and the column vector

$$\begin{pmatrix} b^1 \\ b^2 \\ \vdots \\ \vdots \\ \vdots \\ b^m \\ \\ \end{pmatrix}$$

Then the previous system by linear algebra may reduces to

$$\begin{pmatrix} x_1 \\ x_1 \\ \vdots \\ \vdots \\ x_m \end{pmatrix} = A \begin{pmatrix} x_0 \\ x_1 \\ \\ \\ x_{m-1} \end{pmatrix} + b$$

Introducing

$$\bar{x}(r,s) = \begin{pmatrix} x_r \\ \vdots \\ x_s \end{pmatrix}$$

$s > r$, of lenght $s - r$, the one of the system is transformd to

$$(\bar{x}(1,m) - b) = A\big(\bar{x}(0,m-1)\big) + b$$

From here we have

$$(\bar{x}(1,m) - b) \ = A\bar{x}(0,m-1)$$

Now A Is triangular and if the determinant is diferent of zero, or

$$\prod_{j=1}^{m} a_{j-1}^{j}$$

then it is compute and is well define the inverse of A. We call A^{-1} its inverse. Then multiply from the left we have

$$A^{-1}(\bar{x}(1,m) - b) = \bar{x}(0,m-1)$$

or

$$-A^{-1}b = \bar{x}(0,m-1) - A^{-1}\bar{x}(1,m)$$

$$-A^{-1}b = \begin{pmatrix} x_0 \\ x_{m-1} \end{pmatrix} - A^{-1}\begin{pmatrix} x_1 \\ x_m \end{pmatrix}$$

$$-A^{-1}b = \begin{pmatrix} x_0 - A_{11}^{-1}x_1 \\ x_1 - A_{22}^{-1}x_2 \\ \end{pmatrix}$$

This given x_0 all the other x^1s may be easy determinated. Therefore this is a solution more general but simple.

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