# New string symmetries

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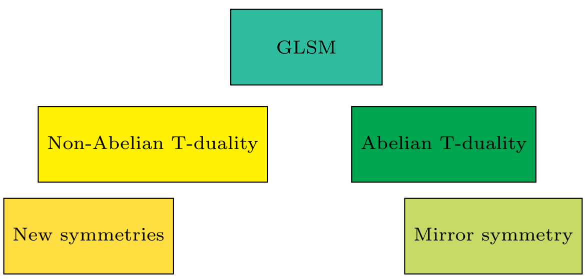
*We study novel non-Abelian dualities between gauged linear sigma models, which constitute an ultraviolet description of string theory. These dualities can lead to new physical symmetries between different geometries.*[b] [c]

## Introduction

Dualities and symmetries relate different corners of string theory. In particular, Mirror Symmetry relates type IIA and type IIB string theories on mirror Calabi-Yau (CY) manifolds. On the other hand, T-duality constitutes a symmetry of string theory with one extra dimension on a circle with radius $R$, under the exchange $R \to \alpha'/R$. The process of considering a compact dimension is called a compactification. Mirror symmetry exchanges string modes that wrap the extra dimension with modes that posses linear momentum [4]. Additionally, mirror symmetry exchanges Kähler deformations with complex deformations on mirror manifolds. The former constitute parameters that set the sizes of the extra dimensions, whereas the latter constitute parameters that set their shape. Mirror symmetry is related to T-duality. It has been proved that mirror symmetry is a consequence of T-Duality in Gauged Linear Sigma Models (GLSMs) [3]. The GLSMs in 2D with $(2,2)$ supersymmetry (SUSY) are a powerful tool to describe strings propagating in a Kähler manifold (extra dimensions) [2]. SUSY exchanges bosons and fermions, permuting the spin statistics but leaving the action of the theory invariant. Under the renormalization group, i.e. the energy flow, the GLSMs have infrared fixed points with conformal invariance. These are non linear sigma models (NLSMs) which constitute world-sheet string theories. This means that GLSMs constitute an ultraviolet (UV) description of string theory.

We want to identify distinct geometries of string theory related by Abelian and non-Abelian T-dualities on GLSMs. We have developed a procedure to implement T-dualities in models with global symmetries [1]. It reproduces the standard Mirror Symmetry for the cases of Abelian global symmetries [3]. It also leads to geometrical generalizations. The following diagram shows the outline of our work:




We are considering GLSMs with global symmetries, and implementing Non-Abelian T-dualities in order to obtain new symmetries of string theory, relating apparently distinct geometries.

There are different motivations to explore new symmetries of GLSMs. On one hand, Non-abelian GLSMs [5] provide – for the extra dimensions – more general ambient spaces than toric varieties. CY manifolds in these spaces are related to determinantal, Grassmanian, Pfaffian and more general non-complete intersection varieties [5]. A proposal for non-Abelian GLSMs mirror pairs have been presented in [6], which could be connected to non-Abelian T-dualities. It is also estimated that the class of non-complete intersections CYs [7] is much bigger than the class of complete intersections CYs.

## Dualization of gauged linear sigma models

Our work consists of implementing T-dualities in sigma models with global symmetries, which could be Abelian and non-Abelian. The key observation is that a dualization procedure leads to a single Lagrangian that upon integration over two different subsets of fields conducts to dual field theories. We start with a reduction to 2d of 4d $U(1)$ gauge field theory with $\mathcal{N}=1$ SUSY. This dimensional reduction gives rise to a supersymmetric theory with two left and two right supersymmetries, i.e. $\mathcal{N}=2$ (2,2) 2d theory.

The supersymmetry can be implemented by defining superspace coordinates $x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}$. The first are the space-time coordinates with indices $\mu = 0,3$ and the second and third are anticommuting Grassman numbers with indices $\alpha = 1,2$, $\dot\alpha = 1,2$. The fermion indices can be also expressed as $\pm$ with $(\theta^1,\theta^2) = (\theta^-,\theta^+)$ and $(\theta_1,\theta_2) = (\theta_-,\theta_+)$. One can construct generators that transform the fields $(Q_\alpha, \bar Q_{\dot\alpha})$, and covariant derivates that act on the fields, given by $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial x^\mu}$ and its conjugate $\bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\sigma^\mu_{\alpha\dot\alpha}\theta^\alpha\frac{\partial}{\partial x^\mu}$. We employ the language of superfields in $\mathcal{N}=1$ SUSY in 4D. These superfields have as components the standard fields, which are related by SUSY transformations. There are different representations, and among them there are chiral superfields (csf) $\bar D_{\dot\alpha}\Phi = 0$, antichiral superfields (acsf) $D_\alpha\bar\Phi = 0$ and vector superfields (vsf) $V^\dagger = V$. Twisted-csf satisfy $D_+X = \bar D_-X = 0$ and the twisted-acsf satisfy

$\bar{D}_+\bar{X} = D_-\bar{X} = 0$.

The Lagrangian of a GLSM with gauge group $U(1)$ with vsf $V_0$ and $N$ csfs $\Phi_i$ with charges $Q_i$ can be written as [2]

$$L_0 = \int d^4\theta\Big(\sum_{i=1}^N \bar{\Phi}_i e^{2Q_iV_0}\Phi_i - \tfrac{1}{2e^2}\bar{\Sigma}_0\Sigma_0\Big) - \tfrac{1}{2}\int d^2\tilde{\theta} t\Sigma_0 + \text{c.c.}, \quad (1)$$

where $t = r - i\theta$. The parameters of $L_0$ are the $U(1)$ gauge coupling $e$, the Fayet-Iliopoulos(FI) term $r$ and the Theta angle $\theta$.

We implement Abelian T-dualities in the GLSM in a distinct way to [4]. For mirror symmetry, the results coincide because the duality follows from the existence of a global symmetry. For two chiral fields with equal charge,the GLSM target-space is $\mathbb{CP}^1$ and the mirror dual manifold is the $A_1$-Toda variety.

Consider a particular case of (1), i.e., a GLSM with Abelian gauge group $U(1)$, $N$ chiral superfields $\Phi_{k,i}$ with charges $Q_k$, $\sum_k n_k = N$, $i = 1,\dots,n_k$ where $n_k$ is the total number of chiral superfields with charge $Q_k$. There is a non-Abelian global symmetry $G = U(n_1)\times\ldots\times U(n_k)$, and a subset of it can be gauged to obtain the vsf $V$. By adding a Lagrange multiplier $\Psi$ one gets a master Lagrangian

$$L_2 = \int d^4\theta\Big(\sum_k \bar{\Phi}_{k,i}(e^{2Q_kV_0+V})_{ij}\Phi_{k,j} + \Psi\Sigma + \bar{\Psi}\bar{\Sigma}\Big) - \int d^4\theta\tfrac{1}{2e^2}\bar{\Sigma}_0\Sigma_0 + \tfrac{1}{2}\Big(-\int d^2\tilde{\theta}t\Sigma_0 + \text{c.c.}\Big). \quad (2)$$

The original action is recovered by integrating out $\Psi$. Upon integration of the gauged vector superfield one obtains the dual model, leading to the following equations of motion: $(\bar{\Phi}e^{2QV_0}e^V T_a\Phi) = X_a + \bar{X}_a$. The relevant definition is $X_a = D_+\bar{D}_-\Psi_a + \{\chi, D_+\Psi_a\}$. For the case $\{\chi, D_+\Psi_a\} = 0$ the $X_a$ is a tcsf, and $\bar{X}_a$ is an anti-tcsf.

Let us discuss the dualization of a simple case [1], when one has $SU(2)$ global symmetry. The original model has a $\mathbb{CP}^1$ vacuum. The dual effective scalar potential obtained from (2) after integrating the $U(1)$ gauge field in the Higgs branch is given by:

$$U = 2Q^2e^2|x_an_a - t/(2Q)|^2 + B_a(x_a + \bar{x}_a), \quad (3)$$

where the $x_a$'s are the scalar components of the tcsf. The constant $B_a$ depends on the vector superfield components. After fixing the gauge, the new vacuum is parametrized by the two-dimensional space:

$$\sum_a x_an_a = \frac{t}{2Q} - \frac{B_1}{2An_1},\ x_2 + \bar{x}_2 = x_3 + \bar{x}_3 = 0. \quad (4)$$

At the quantum level the theory has the symmetry $x_a \to x_a + \frac{2\pi i k_a}{2n_aQ}$, $k_a \in \mathbb{Z}$ coming from the periodicity of $t$, obtaining $T^2$ as the dual target space. For an Abelian direction inside of $SU(2)$ at fixed $n_a = \text{const}$, instanton corrections to the action are identified as $\tilde{W} = e^{-X_an_a}$. Dual models are matched, by checking that the effective potential for the $U(1)$ gauged field is the same in both theories.

## Conclusions

We developed a method to describe T-duality in SUSY $(2,2)$ 2D GLSMs. The Abelian T-duality gives rise to mirror symmetric models [3]. The non-Abelian T-dual theories give rise to new geometries [1]. For the case of a GLSM with $SU(2)$ global symmetry and geometry $\mathbb{CP}^1$, one obtains the dual geometry $T^2$.

There are many aspects of the dual geometries to be explored, for example: solving the duality for the full non-Abelian group. For instance, one has to determine the required non-perturbative effects on the dual theory, e.g. instanton effects. It would be relevant to compute the partition function with localization techniques. We also plan to dualize non-Abelian GLSMs, studying the connection between non-Abelian T-duality and Mirror Symmetry in Pfaffian and determinantal CY. It would be interesting to study compact geometries, i.e. a non-trivial superpotential in the original theory; and to perform a survey of dual geometries. We have studied new correspondences that can be relevant to explore the string theory landscape.

## Notes

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b. The research work of this note can be found in Ref. [1]

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