Energy scales and scalar evolutions on a string model

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A scale separation between the space-time's and the internal dimensions's degrees of freedom can be obtained by adjusting the parameters of the string geometry. We describe this scale hierarchy and the cosmological evolution of the scalar fields for type IIB string theory on the geometry of the mirror quintic.[b c]

String Theory is a strong candidate to a quantum gravity. In its realm one encounters the gauge interactions: electromagnetism, weak and strong, together with gravity at the Planck scale $M_{Pl} = \sqrt{c^5\hbar/G_N} \sim 10^{-33}$cm. This is the natural scale of a quantum gravity theory depending on the speed of light c, Planck's constant $\hbar$ and Newton's constant G_N. Calabi-Yau (CY) manifolds [2] constitute the extra dimensions of string theory, giving rise to 4-dimensional field theories with one supersymmetry. The CY geometries possess parameters: of shape (complex structure) and size (Kähler structure) denoted moduli.

We will study type IIB string theory with internal dimensions on the mirror of the quintic CY. This geometry is obtained by modding out a ${\mathbb{Z}_5}^3$ symmetry from a one-parameter family of polynomials on $\mathbb{P}^4$ [2]. This family is given by

$$W_\psi = \langle \sum_k {x_k}^5 - 5\psi \prod_k x_k = 0, (x_1, \ldots, x_5) \in \mathbb{P}_4 \rangle. \tag{1}$$

The ${\mathbb{Z}_5}^3$ symmetry is generated by phase rotations on the coordinates: $x_l \to e^{2\pi g(i) i/5} x_l$, with $l = 1, \ldots, 5$ and $g(1) = (0,1,0,0,4)$, $g(2) = (0,0,1,0,4)$, $g(3) = (0,0,0,1,4)$. The symmetry is modded out to obtain the geometry we will employ, the mirror quintic: $W_\psi/{\mathbb{Z}_5}^3$ [2]. The complex structure moduli space $z = (1 - \psi^{-5})$ of a given CY manifold has generic critical points: the conifold, the orbifold and the large complex structure point [1, 2]. In figure 1 the moduli space of the geometry is represented. There are three critical points.

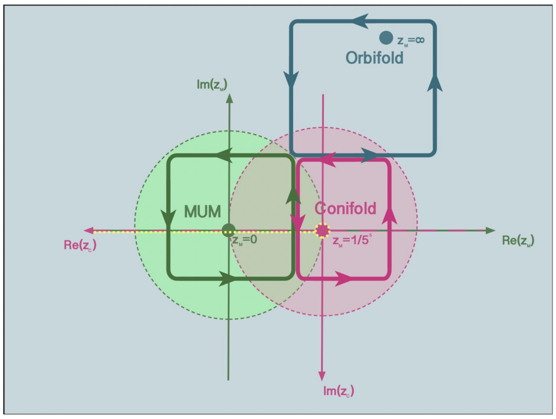

Figure 1: Moduli space of the mirror quintic Calabi-Yau 3-fold, with three critical points. Those points are parameter values at which the geometry develops a singularity.

The solution of the Einstein equations in multiple dimensions leads to a warped metric for the space-time that preserves Poincaré symmetry [3]

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n. \tag{2}$$

The hierarchy between the spacetime (4D) and compactification (6D) physical scales are given by the distance of the complex structure vacuum z_0 to the conifold as $e^A \sim z_0^{1/3}$ [2, 3]. We consider a so called no-scale potential, where the Kähler moduli ϕ^n are un-stabilized and satisfy the restriction: $K^{m\bar{n}} D_m W D_{\bar{n}} \bar{W} - 3|W|^2 = 0$. The effective theory possesses one local supersymmetry, meaning that it constitutes an $\mathcal{N} = 1$ 4D supergravity. The scalar potential, which we consider positive definite, can be written as

$$V(\tau, z, \phi^i) = \frac{1}{2\kappa_{10}^2 g_s} e^K \left[K^{a\bar{b}} D_a W \overline{D}_{\bar{b}} \overline{W} \right], \tag{3}$$

where g_s is the string coupling constant and $\kappa_{10}^2 = \frac{{l_s}^8}{4\pi}$ with l_s the string length. V depends on the superpotential W and the Kähler potential K. The indices a and b denote the moduli fields. In this case we have two complex moduli fields: axio-dilaton and complex structure, scalar fields that are modes with zero mass in the string spectrum. There is a metric in the space of fields with inverse given by $K^{a\bar{b}}$, this occurs because the kinetic term in the Lagrangian reads $K_{a\bar{b}} \partial_\mu \phi^a \partial^\mu \bar{\phi}^b$. The quantity $D_a W = \partial_a W + \partial_a K W$ constitutes the susy covariant derivative. The superpotential [5] and Kähler potential read:

$$\begin{aligned} W &= \int_{CY} G_{(3)} \wedge \Omega = (F - \tau H)\, \Sigma\, \Pi\,, \\ K &= -\ln\left[-i(\tau - \bar{\tau})\right] - \ln\left[-i\, \bar{\Pi}^T\, \Sigma\, \Pi\right] - 2\ln\left[\mathcal{V}\right], \end{aligned} \tag{4}$$

Every CY manifold with 3 complex dimensions has an holomorphic nowhere-vanishing 3-form. One can construct periods $\Pi(z) = \int_A \Omega$ as integrals of the holomorphic 3-form Ω on the 3-cycles. Cycles are dimension-3 sets of submanifolds of the CY with no boundary. Periods satisfy differential equations, denoted by Piccard-Fuchs (PF) equations [2].

Scale hierarchies

Here we describe the techniques employed to find Minkowski vacua and to evaluate their scale hierarchies. We obtain a formula for the vacuum expectation values of the axio-dilaton and the complex structure, for vacua solutions of the scalar potential (3). Minkowski vacua require the vanishing of both convariant derivatives $D_\tau W = D_z W = 0$. These formulas are solved simultaneously to obtain

$$\tau(z) = \frac{F\Sigma\bar{\Pi}}{H\Sigma\bar{\Pi}} = \frac{F\Sigma(\widetilde{\Pi}\Sigma\bar{\Pi})}{H\Sigma(\widetilde{\Pi}\Sigma\bar{\Pi})}. \tag{5}$$

Solving this equation one can find generic Minkowski vacua. We do so considering as many terms in the period series expansion, in order to reach convergence. The approximate value of the complex structure z_0 for a vacuum near the conifold is obtained to leading order as

$$\tau_0 = \frac{F\Sigma\bar{\Pi}^0}{H\Sigma\bar{\Pi}^0}, \qquad W_0 = (F - \tau_0 H)\Sigma\Pi^0 . \tag{6}$$

The solution for a particular set of non-zero fluxes F_1, H_3, H_4 is obtained to be

$$z_0 \quad \sim \quad \exp\frac{H_3\partial\Pi_1\bar{\Pi}_3}{H_4\beta\bar{\Pi}_2}. \tag{7}$$

We found in [1] a correction to the hierarchies of [3]. The result is due to the compactness, as well as from considering the complete period series. Adjusting the fluxes, the warping parameter can lead to big hierarchies.

Scalar field evolution

We analyze the cosmological multifield evolution of the scalar fields τ and z. The setup is the Friedmann–Lemaître–Robertson–Walker background in space-time. One can compute parameters ϵ and η. Their size determines in which regions there can be scalar field displacements that fulfill a slow-roll evolution. These parameters are given by

$$\epsilon \quad = \quad \frac{K^{i\bar{j}}\nabla_i V\bar{\nabla}_j V}{V^2}, \tag{8}$$

$$\eta \quad = \quad \min \text{eigenvector} \frac{K^{i\bar{j}}\nabla_i\bar{\nabla}_j V}{V^2}. \tag{9}$$

We computed them numerically in terms of the scalar potential (3). There are configurations of fluxes for which large inflationary regions with $\epsilon \ll 1$ and $\eta < 1$ are encountered. The results for a particular flux configurations are shown in figure 1. In all of the explored examples, the possible scalar field displacements are below the Planck scale, fulfilling the recent bounds obtained by the discussions of the species scale [4]. This indicates that the scalar potentials dicsussed are valid as an effective theory of strings, without incorporating higher oder curvature contributions.

Conclusions

We explored a string theory model. First we studied the existence of vacua solutions with a scale hierarchy between the internal and the space-time scale. We checked that they appear in the compact case, near conifold sigularities, finding a correction with respect to the generic argument, which can be of more than an order of magnitude. We determined that for the multi-field evolution the scalar field displacements are bounded by M_{Pl} in agreement with the recently discussed bound of the species scale [5]. These compactifications constitute a fertile realm of the string landscape, with potential applications to cosmological models.

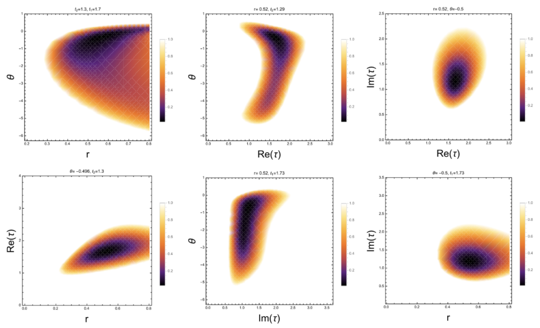

Figure 2: The evolution of the multi-field potential agrees with the so called bound of the species scale. The width of the flat regions is given by: $\Delta\phi_r = 0.124M_{pl}$, $\Delta\phi_\theta = 0.58M_{Pl}$, $\Delta\phi_{\tau_1} = 0.69M_{Pl}$, $\Delta\phi_{\tau_2} = 0.65M_{Pl}$.

Notes

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b. The research work of this note can be found in Ref. [1]

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