

## F-theory and the Gimon-Polchinski Orientifold

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### Abstract

We establish the equivalence of the Gimon-Polchinski orientifold and F-theory on an elliptically fibered Calabi-Yau three fold on base  $CP^1 \times CP^1$  by comparing the gauge symmetry breaking pattern, local deformations in the moduli space, as well as the axion-dilaton background in the weak coupling limit in the two theories. We also provide an explanation for an apparent discrepancy between the F-theory and the orientifold results for constant coupling configuration.

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# 1 Introduction

Conventional compactification of type IIB string theory is characterized by the property that both, the dilaton and the scalar field arising in the Ramond-Ramond (RR) sector of the theory (which we shall refer to as the axion), are constant on the internal space. F-theory[1, 2] provides us with a novel way of compactifying type IIB string theory that does not suffer from this restriction. The starting point in an F-theory compactification is a manifold  $\mathcal{M}$  that admits elliptic fibration over a base  $B$ . F-theory on  $\mathcal{M}$  is by definition type IIB on  $B$ , with the axion-dilaton modulus set equal to the complex structure modulus of the fiber torus at every point on the base. In a generic F-theory compactification the scalar fields undergo non-trivial  $SL(2,Z)$  monodromy around closed cycles of the internal manifold. Since  $SL(2,Z)$  is a non-perturbative symmetry of the type IIB theory, there is no conventional perturbative description of F-theory compactification.

It was conjectured by Vafa[1] that F-theory on a K3 surface elliptically fibered over a base  $CP^1$  is dual to heterotic string theory on  $T^2$ . This was established in [3] by examining the F-theory background at a special point in the moduli space corresponding to the orbifold limit of K3, where the dilaton-axion field becomes constant on the base, and hence the theory describes a conventional string compactification. This was found to be an orientifold[4, 5] of type IIB theory on  $T^2$ , which is related by T-duality to type I on  $T^2$ . The conjectured equivalence between type I and the  $SO(32)$  heterotic string theory in ten dimensions[6] then establishes the duality between F-theory on K3 and heterotic string theory on  $T^2$ .

It was also shown in [3] that away from this special point in the moduli space, the axion-dilaton background in the F-theory agrees with that in the orientifold theory in the weak coupling limit, but they differ by non-perturbative terms. This was interpreted as due to the quantum corrections to the orientifold background which modifies it to the F-theory background. This was proved in [7] by using a three brane to probe the orientifold background.

In this paper we shall carry out a similar analysis for F-theory on a Calabi-Yau 3-fold with Hodge numbers  $(h_{11} = 3, h_{12} = 243)$  which admits an elliptic fibration over the base  $CP^1 \times CP^1$ [1, 2]. (Some aspect of this compactification has recently been discussed in ref.[8] following ref.[9].) Thus this describes a compactification of type IIB theory on  $CP^1 \times CP^1$  with varying axion-dilaton field. This model has been conjectured[2] to be

dual to  $E_8 \times E_8$  heterotic string theory on K3, with the total instanton number 24 equally divided among the two  $E_8$  subgroups. This, in turn, has been conjectured[10] to be dual to an orientifold of type IIB theory compactified on  $T^4$ , constructed by Gimon and Polchinski[11] (see also refs.[12] for earlier construction of this model at special points in the moduli space). The main purpose of this paper will be to establish the duality between this particular F-theory compactification and a T-dual of the Gimon-Polchinski (GP) model directly following methods similar to that in [3]. Some attempts in this direction were made earlier in refs.[13, 14], and duality between a different pair of orientifold and F-theory vacua in six dimensions was established in refs.[15, 16].

The particular T-dual of the GP model that we shall consider may be described as type IIB on  $CP^1 \times CP^1$  with a configuration of four orientifold seven planes and eight pairs of Dirichlet seven branes transverse to each  $CP^1$ . For such a configuration the non-abelian gauge group is  $SU(2)^8 \times (SU(2)')^8$ . We identify a similar configuration in F-theory which has identical gauge group and for which the background  $\lambda$  agrees with that of the GP model in the weak coupling limit. We then consider various deformations away from this point in the GP model and identify corresponding deformations in F-theory by matching a) the unbroken gauge group and b) the background axion-dilaton field in the weak coupling limit. We find exact one to one correspondence between deformations in the GP model and those in the F-theory. We also consider special subspaces in the moduli space where the gauge group is enhanced on the GP side, typically giving  $Sp(2k)$  or  $SU(2k)$  gauge groups. We identify the corresponding symmetry enhancement points on the F-theory side, and again find exact agreement between codimensions of these subspaces in the two theories.

In a previous paper[17] we had analysed part of this problem by studying the physics near one pair of intersecting orientifold planes accompanied by four pairs of intersecting seven branes. It was found that non-perturbative effects in the orientifold theory converts the background axion-dilaton field into an F-theory like configuration. The present paper generalizes this result in two important ways. First it shows how to put sixteen copies of this structure together to get the full GP model. Second it shows how to describe in F-theory switching on of the vacuum expectation value of the massless fields associated with the open string states stretched between an intersecting pair of D-branes. We also provide an explanation for an apparent discrepancy between the enhanced gauge symmetries in the two theories for a configuration where the axion-dilaton field is constant on the base.

The plan of this paper is as follows. In section 2 we give a brief review of F-theory on the elliptically fibered Calabi-Yau manifold over the base  $CP^1 \times CP^1$ , and also of the T-dual of the GP model. In section 3 we identify the  $SU(2)^8 \times (SU(2)')^8$  family of points in the GP model with a specific family of points in the moduli space of the F-theory, and also compare deformations away from these points in both theories. In this context we consider both, deformations that are neutral under the  $SU(2)^8 \times (SU(2)')^8$  gauge group, and deformations that are charged under the gauge group, and find exact correspondence between these deformations in the two theories. The axion-dilaton background in the two theories also agree in the weak coupling limit. Later in this section we consider the reverse problem, namely finding subspaces of the moduli space in both theories where the gauge symmetry is enhanced. Again we find exact one to one correspondence between these subspaces in the two theories. In section 4 we summarize our results with some concluding remarks.

## 2 Review of F-theory and Gimon-Polchinski Model

We start with a review of F-theory on the Calabi-Yau manifold with elliptic fibration over  $CP^1 \times CP^1$ . Let  $u$  and  $v$  denote the complex coordinates of the two  $CP^1$ 's. Also let us define the complex scalar field  $\lambda$  as

$$\lambda = a + ie^{-\Phi}, \quad (2.1)$$

where  $a$  is the axion field and  $\Phi$  is the dilaton field. Then this particular F-theory compactification may be described as type IIB theory compactified on  $CP^1 \times CP^1$ , with a background  $\lambda(u, v)$  equal to the complex structure modulus of the torus described by the equation:

$$y^2 = x^3 + f(u, v)x + g(u, v). \quad (2.2)$$

Here  $f$  and  $g$  are polynomials in  $u$  and  $v$  of degree (8,8) and (12,12) respectively. The coefficients appearing in  $f$  and  $g$  are part of the moduli of this F-theory compactification. From (2.2) we can write down a more explicit form of  $\lambda$  as function of  $u$  and  $v$ :

$$j(\lambda(u, v)) = \frac{4 \cdot (24f)^3}{4f^3 + 27g^2}, \quad (2.3)$$

where  $j(\lambda)$  denotes the modular invariant function of  $\lambda$  with a single pole at  $\lambda = i\infty$ , zero at  $\lambda = e^{i\pi/3}$  and normalized as in ref.[3]. Thus at the zeroes of the denominator

$$\Delta \equiv 4f^3 + 27g^2 \quad (2.4)$$

$\lambda$  goes to  $i\infty$  up to an  $SL(2,Z)$  transformation. These surfaces of (complex) codimension one<sup>3</sup> can be identified with the locations of the seven-branes in this background.

Generically, an F-theory compactification on a Calabi-Yau manifold with Hodge numbers  $(h_{11}, h_{12})$  has  $h_{12} + 1$  neutral hypermultiplets. This gives a  $2(h_{12} + 1)$  (complex) dimensional hypermultiplet moduli space. However, of these only a  $h_{12}$  dimensional subspace is visible as complex structure deformation of the Calabi-Yau manifold,<sup>4</sup> and can be identified with the coefficients appearing in the polynomials  $f$  and  $g$ . The rest of the deformations in the hypermultiplet moduli space is non-geometrical and is not visible as complex structure deformations of the Calabi-Yau manifold. We need to keep this in mind when we try to compare the moduli space deformations in the GP model with those in F-theory.

Let us now turn to a brief description of the T-dual version of the GP model that we shall study. This may be described as type IIB string theory  $T^2 \times (T^2)' \times R^6 / (Z_2 \times Z_2')$  where we label  $T^2$  and  $(T^2)'$  by  $(x^6, x^7)$ , and  $(x^8, x^9)$  respectively, and  $Z_2$  and  $Z_2'$  are generated by

$$g = (-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_{67}, \quad h = (-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_{89}. \quad (2.5)$$

Here  $\mathcal{I}_{67}$  and  $\mathcal{I}_{89}$  denote the transformations  $(x^6 \rightarrow -x^6, x^7 \rightarrow -x^7)$  and  $(x^8 \rightarrow -x^8, x^9 \rightarrow -x^9)$  respectively,  $(-1)^{F_L}$  denotes the transformation that changes the sign of all the Ramond sector states on the left moving sector of the world sheet of string theory, and  $\Omega$  denotes the world-sheet parity transformation.<sup>5</sup> If we define

$$w = x^6 + ix^7, \quad z = x^8 + ix^9, \quad (2.6)$$

and if  $\tau$  and  $\tau'$  denote the complex structure moduli of  $T^2$  and  $(T^2)'$  respectively, then the seven planes at

$$w = 0, \frac{1}{2}, \frac{\tau}{2}, \frac{\tau+1}{2}, \quad (2.7)$$

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<sup>3</sup>Throughout this paper we shall count complex codimension / dimension of various subspaces unless specified otherwise.

<sup>4</sup>This point has also been emphasized recently in refs.[18, 19].

<sup>5</sup>Eq.(2.5) does not completely specify the action of  $g$  and  $h$  on twisted sector / open string states. These are determined by demanding that this model is related to the GP model by T-duality. This was discussed at length in [17].

and

$$z = 0, \frac{1}{2}, \frac{\tau'}{2}, \frac{\tau' + 1}{2}, \quad (2.8)$$

are fixed under  $g$  and  $h$  respectively. These are known as orientifold seven planes. We can introduce  $g$  and  $h$  invariant coordinates  $u$  and  $v$  through the relations

$$dw = du \prod_{m=1}^4 (u - \tilde{u}_m)^{-\frac{1}{2}}, \quad dz = dv \prod_{m=1}^4 (v - \tilde{v}_m)^{-\frac{1}{2}}. \quad (2.9)$$

The numbers  $\{\tilde{u}_m\}$  ( $\{\tilde{v}_m\}$ ) are to be chosen such that their images in the  $w$  ( $z$ ) plane correspond to the values given in (2.7) ((2.8)). This relates the parameters  $\tau$  and  $\tau'$  to the cross ratios

$$\frac{(\tilde{u}_1 - \tilde{u}_2)(\tilde{u}_3 - \tilde{u}_4)}{(\tilde{u}_1 - \tilde{u}_3)(\tilde{u}_2 - \tilde{u}_4)} \quad \text{and} \quad \frac{(\tilde{v}_1 - \tilde{v}_2)(\tilde{v}_3 - \tilde{v}_4)}{(\tilde{v}_1 - \tilde{v}_3)(\tilde{v}_2 - \tilde{v}_4)}, \quad (2.10)$$

respectively. In the  $(u, v)$  coordinate system the orientifold seven planes are located at  $u = \tilde{u}_m$  and  $v = \tilde{v}_m$  respectively.

It is known from standard analysis (see *e.g.* [11]) that each orientifold plane carries  $-4$  units of RR charge. In other words, along any closed contour  $C$  enclosing an orientifold seven plane,

$$\oint_C d\lambda = -4, \quad (2.11)$$

signalling that the axion  $a$  changes by  $-4$  units as we move around an orientifold plane. Since the  $u$  and  $v$  planes are compact ( $CP^1$ ) this RR charge must be neutralized. This is done by putting sixteen Dirichlet seven branes transverse to the  $u$  plane and sixteen Dirichlet seven branes transverse to the  $v$  plane[20]. For reasons explained in [17] these seven branes move only in pairs. Thus a generic configuration will have eight pairs of D-branes placed at  $u = u_i$  ( $1 \leq i \leq 8$ ) and eight pairs of D-branes placed at  $v = v_i$  ( $1 \leq i \leq 8$ ). For any contour  $C$  enclosing such a D-brane pair, we have

$$\oint_C d\lambda = 2. \quad (2.12)$$

This particular orientifold has  $N = 1$  supersymmetry in six dimensions. The massless spectrum is as follows:

1. The untwisted sector closed string states contribute

(a) The  $N=1$  supergravity multiplet,

- (b) A massless tensor multiplet, and
- (c) Four hypermultiplets. These contain the axion-dilaton field  $\lambda$ , and the moduli  $\tau$  and  $\tau'$ . The other moduli associated with these hypermultiplets will not be visible as deformations of the complex structure moduli of the Calabi-Yau manifold on the F-theory side. This is a reflection of the fact that of the  $2(h_{12} + 1)$  hypermultiplet moduli in F-theory, only  $h_{12}$  are visible as complex structure deformations of the Calabi-Yau manifold.

2. The closed string states twisted by the element

$$gh = (-1)^{F_L + F_R} \mathcal{I}_{67} \mathcal{I}_{89}, \quad (2.13)$$

contribute 16 hypermultiplets from the sixteen fixed points of  $\mathcal{I}_{67} \mathcal{I}_{89}$ . These contain the blow up modes of the corresponding orbifold singularities.

3. The open string states with ends lying on D-branes that are parallel to each other contribute

- (a) Massless vector multiplets corresponding to the gauge group  $SU(2)^8 \times (SU(2)')^8$ . Each  $SU(2)$  is associated with a D-brane pair, with the first eight being associated with the pairs at  $u = u_i$  and the last eight being associated with the pairs at  $v = v_i$ .
- (b) Sixteen gauge neutral hypermultiplets, containing the locations  $u_i, v_i$  of the D-brane pairs. Note that as in the case of F-theory, only half of each hypermultiplet represents geometrical modulus.

4. Finally open string states whose ends lie on intersecting D-branes contribute massless hypermultiplets in the  $(2,2)$  representation of  $SU(2)_i \times SU(2)'_j$  for all pairs  $(i, j)$  ( $1 \leq i, j \leq 8$ ). Here  $SU(2)_i$  denotes the  $SU(2)$  group associated with the D-brane pair at  $u = u_i$  and  $SU(2)'_j$  denotes the  $SU(2)$  group associated with the D-brane pair at  $v = v_j$ . We shall denote these hypermultiplets as  $(2_i, 2'_j)$  states.

In this model we can further break the  $SU(2)^8 \times (SU(2)')^8$  group by giving vev to the  $(2_i, 2'_j)$  hypermultiplets. We shall now analyze some specific breaking patterns which will be useful for later study. Consider giving vev to the  $(2_i, 2'_j)$  states for all  $(i, j)$  with  $1 \leq i \leq p, 1 \leq j \leq q$ . This leaves  $SU(2)^{8-p} \times (SU(2)')^{8-q}$  unbroken. Let  $SU(2)_d$  denotes

the diagonal subgroup of the first  $p$   $SU(2)$  and the first  $q$   $SU(2)'$  groups. Then under  $SU(2)_d$  the  $(2_i, 2'_j)$  state decomposes as  $\mathbf{3} \oplus \mathbf{1}$ . Two of the possible symmetry breaking patterns are the following:

1. We can consider giving vev to only the singlet components of all  $(2_i, 2'_j)$  states. This breaks the first  $p$   $SU(2)$  and the first  $q$   $SU(2)'$  to  $SU(2)_d$  in general. The resulting massless spectrum consists of

$$pq \tag{2.14}$$

neutral hypermultiplets, and

$$(p-1)(q-1) \tag{2.15}$$

hypermultiplets in the triplet of  $SU(2)_d$ , taking into account the fact that  $(p+q-1)$  of the triplet hypermultiplets become massive by Higgs mechanism.

2. For  $p > 1, q > 1$ , we can further break  $SU(2)_d$  by giving vev to the surviving massless triplets of  $SU(2)_d$ . For  $p = q = 2$  we have only one triplet of  $SU(2)_d$  and hence vev of this triplet breaks  $SU(2)_d$  to  $U(1)$  leaving one extra neutral hypermultiplet. On the other hand for  $p \geq 3$  or  $q \geq 3$  we have more than one triplet of  $SU(2)_d$  and giving vev to these triplets we can break  $SU(2)_d$  completely. The number of extra neutral hypermultiplets, obtained after this symmetry breaking, is given by

$$3(p-1)(q-1) - 3 = 3(pq - p - q). \tag{2.16}$$

In particular, by taking  $p = q = 8$ , and carrying out the second step in the above description, we can break the gauge group completely.

Instead of breaking the  $SU(2)^8 \times (SU(2)')^8$  group, we can also enhance the gauge group further by bringing the D-brane pairs on top of each other, and/or on top of an orientifold plane. If  $k$  D-brane pairs are on top of each other the gauge group is  $Sp(2k)$ , whereas if  $k$  D-brane pairs are on top of an orientifold plane, the non-abelian part of the gauge group is  $SU(2k)$ .

### 3 Comparison of the Two Theories

In this section we shall carry out a detailed comparison between the two theories discussed in the previous section. Naively one would have thought that a convenient starting point

would be a configuration where the RR charge is neutralized locally in the internal space, and hence the axion-dilaton field is a constant, as was the case in [3]. In the GP model this would correspond to placing two D-brane pairs on top of each orientifold plane, thereby giving an  $(SU(4))^8$  non-abelian gauge group. On the other hand, in F-theory, this would correspond to choosing

$$f(u, v) = \alpha \prod_{m=1}^4 (u - \tilde{u}_m)^2 (v - \tilde{v}_m)^2, \quad g(u, v) = \prod_{m=1}^4 (u - \tilde{u}_m)^3 (v - \tilde{v}_m)^3, \quad (3.1)$$

where  $\alpha$  is an arbitrary constant. This corresponds to a set of intersecting  $D_4$  singularities giving rise to  $SO(8)^8$  gauge group as well as tensionless strings[21, 22]. Thus the two theories would seem to disagree. It turns out that there is a subtle reason for this discrepancy that will be explained later in this section. However, due to this subtlety, this will not be a convenient starting point for our analysis.

Instead we shall start from the point in the moduli space of the GP model with  $SU(2)^8 \times (SU(2)')^8$  gauge symmetry and identify the corresponding point in the F-theory moduli space. We shall then consider various symmetry breaking as well as symmetry enhancement patterns as we move in the moduli spaces of the two theories and compare the results.

### 3.1 The $SU(2)^8 \times (SU(2)')^8$ Point

In the GP model this corresponds to a configuration of sixteen pairs of D-branes situated at  $u = u_i$  and  $v = v_i$  ( $1 \leq i \leq 8$ ) and eight orientifold planes situated at  $u = \tilde{u}_m$  and  $v = \tilde{v}_m$  ( $1 \leq m \leq 4$ ). Requiring  $\lambda$  to be an analytic function of  $u$  and  $v$  (for preservation of space-time supersymmetry) and using eqs.(2.11) and (2.12) we get the following behaviour of  $\lambda$  near the D-branes and the orientifold planes:

$$\begin{aligned} \lambda &\simeq \frac{2}{2\pi i} \ln(u - u_i) \quad \text{as } u \rightarrow u_i, \\ \lambda &\simeq \frac{2}{2\pi i} \ln(v - v_i) \quad \text{as } v \rightarrow v_i, \end{aligned} \quad (3.2)$$

$$\begin{aligned} \lambda &\simeq -\frac{4}{2\pi i} \ln(u - \tilde{u}_m) \quad \text{as } u \rightarrow \tilde{u}_m, \\ \lambda &\simeq -\frac{4}{2\pi i} \ln(v - \tilde{v}_m) \quad \text{as } v \rightarrow \tilde{v}_m. \end{aligned} \quad (3.3)$$

From (3.2) we see that as  $u \rightarrow u_i$  or  $v \rightarrow v_i$ ,  $\lambda$  approaches  $i\infty$ . This corresponds to weak coupling and hence this behaviour is not expected to be modified due to quantum corrections (except possible corrections to the locations of the D-branes). If we continue to denote by  $u_i$  and  $v_i$  the quantum corrected locations of the D-branes, then we expect the following behaviour of  $j(\lambda)$  near  $u = u_i$  and  $v = v_i$ :

$$\begin{aligned} j(\lambda) &\simeq \frac{1}{(u - u_i)^2} \quad \text{as } u \rightarrow u_i, \\ &\simeq \frac{1}{(v - v_i)^2} \quad \text{as } v \rightarrow v_i. \end{aligned} \quad (3.4)$$

On the other hand as  $u \rightarrow \tilde{u}_m$  or  $v \rightarrow \tilde{v}_m$ ,  $\lambda$  approaches  $-i\infty$ . This is inconsistent with the definition (2.1) of  $\lambda$  according to which the imaginary part of  $\lambda$  is positive definite. Thus strong coupling effects must modify this behaviour. From our analysis of ref.[3] we know what kind of modification we should expect. Away from the intersection points, each orientifold plane should split into two seven branes related to the D-brane by  $SL(2, Z)$  transformation, such that the total monodromy of  $\lambda$  as we go around both these branes is given by (2.11). In the weak coupling limit the splitting between these two branes is small. Thus after taking into account these quantum corrections, we should see a pair of poles of  $j(\lambda)$  around each of the surfaces  $u = \tilde{u}_m$  and  $v = \tilde{v}_m$ .

In order to reproduce this behaviour in F-theory, we need to adjust the functions  $f$  and  $g$  such that  $j(\lambda)$  calculated from eq.(2.3) has these properties. First of all, in order to reproduce (3.4),  $\Delta$  defined in eq.(2.4) should be of the form:

$$\Delta = \left( \prod_{i=1}^8 (u - u_i)^2 (v - v_i)^2 \right) \delta(u, v), \quad (3.5)$$

where  $\delta$  is a polynomial of degree (8,8). At the first sight it would seem extremely unlikely that it will be possible to find such  $f$  and  $g$ . Since  $f$  and  $g$  are polynomials of degree (8,8) and (12,12) respectively, the total number of adjustable parameters is given by

$$9^2 + (13)^2 = 250. \quad (3.6)$$

$\Delta$  is a polynomial of degree (24,24) in  $f$  and  $g$ . Thus demanding that  $\Delta$  has the form (3.5) with  $u_i$  and  $v_i$  fixed according to our choice gives

$$(25)^2 - 9^2 = 544 \quad (3.7)$$

constraints on the coefficients appearing in  $f$  and  $g$ . Here  $9^2$  reflects the number of coefficients appearing in  $\delta$  which can be adjusted in trying to solve eq.(3.5). This is clearly a highly overdetermined system of equations.

Nevertheless eq.(3.5) has a simple solution. Let us choose:

$$f = \eta - 3h^2, \quad (3.8)$$

and

$$g = h(\eta - 2h^2), \quad (3.9)$$

where  $h$  is a polynomial of degree (4,4) and  $\eta$  is a polynomial of degree (8,8) in  $u$  and  $v$  respectively. In this case,  $\Delta$  calculated from eq.(2.4) is given by

$$\Delta = (4\eta - 9h^2)\eta^2. \quad (3.10)$$

Thus we can now easily satisfy the requirement (3.5) by choosing

$$\eta(u, v) = C \prod_{i=1}^8 (u - u_i)(v - v_i), \quad (3.11)$$

where  $C$  is an arbitrary constant. This gives,

$$\Delta(u, v) = C^2 \prod_{i=1}^8 (u - u_i)^2 (v - v_i)^2 (4C \prod_{i=1}^8 (u - u_i)(v - v_i) - 9h^2), \quad (3.12)$$

and

$$j(\lambda) = \frac{4 \cdot (24)^3 \cdot (C \prod_{i=1}^8 (u - u_i)(v - v_i) - 3h^2)^3}{C^2 \prod_{i=1}^8 (u - u_i)^2 (v - v_i)^2 (4C \prod_{i=1}^8 (u - u_i)(v - v_i) - 9h^2)}. \quad (3.13)$$

From this expression we see that as  $C \rightarrow 0$ , with  $\{u_i\}$ ,  $\{v_i\}$  and  $h(u, v)$  fixed,  $j(\lambda) \rightarrow \infty$  almost everywhere in the  $(u, v)$  space except on subspaces of codimension one where the numerator vanishes. Thus  $C \rightarrow 0$  corresponds to the weak coupling limit of the theory. In this limit, we can associate the deformations associated with  $C$  to the deformations in the GP model associated with the vev of  $\lambda$ .

In order to correctly reproduce the configuration in the GP model, we must also examine the other zeroes of  $\Delta$  and ensure that they lie pairwise near the surfaces  $u = \tilde{u}_m$  and  $v = \tilde{v}_m$  for small  $C$ . From (3.12) we indeed observe that for small  $C$  the other zeroes of  $\Delta$  lie pairwise around the surface

$$h(u, v) = 0. \quad (3.14)$$

Thus we must adjust  $h$  such that the surface  $h = 0$  coincides with the locations of the orientifold planes. This is easily done by choosing:

$$h(u, v) = K \prod_{m=1}^4 (u - \tilde{u}_m)(v - \tilde{v}_m), \quad (3.15)$$

where  $K$  is an arbitrary constant. There is however some redundancy in this parametrization of  $h$ . First of all, note that  $j(\lambda)$  defined in (2.3) is invariant under a rescaling of the form:

$$f \rightarrow s^2 f, \quad g \rightarrow s^3 g, \quad (3.16)$$

for any constant  $s$ . Using this freedom we can choose  $K$  to be unity. Furthermore, since both  $u$  and  $v$  parametrize  $CP^1$ , there is a pair of  $SL(2, C)$  transformations on  $u$  and  $v$  which simply reflect different choice of coordinates on the base. Using these transformations we can fix three of the  $\tilde{u}_m$  and three of the  $\tilde{v}_m$  to any value we like. Thus at the end the relevant parameters appearing in  $h$  are the cross ratios defined in eq.(2.10). As has already been pointed out, these cross ratios correspond to the moduli  $\tau$  and  $\tau'$  in the GP model.

It is however clear that in F-theory, if we want to get  $SU(2)^8 \times (SU(2)')^8$  gauge group, then all we need is sixteen  $A_1$  singularities where  $\Delta$  has double zeroes without  $f$  and  $g$  vanishing. This means that we maintain the  $SU(2) \times (SU(2)')^8$  symmetry even when we choose  $h$  to be completely arbitrary, since according to eq.(3.12) this does not affect the double zeroes of  $\Delta$ . If we are to establish a one to one correspondence between the deformations in F-theory and those in the GP model, then we must be able to interpret the deformations associated with  $h$  as some deformations in the GP model. For this it will be convenient to choose a specific parametrization of  $h$ .  $h$ , to begin with, has  $5^2 = 25$  coefficients, but the freedom of rescaling  $f$  and  $g$  and  $SL(2, C)$  transformations on  $u$  and  $v$  remove seven of these parameters. Two of the remaining eighteen parameters are already present in the form of  $h$  given in (3.15) in the cross ratios (2.10). Thus we need to introduce sixteen more parameters in  $h$ . We choose the following parametrization:

$$h(u, v) = \prod_{m=1}^4 (u - \tilde{u}_m)(v - \tilde{v}_m) + \sum_{k,l=1}^4 \alpha_{kl} \prod_{m \neq k} (u - \tilde{u}_m) \prod_{n \neq l} (v - \tilde{v}_n), \quad (3.17)$$

with  $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{v}_1, \tilde{v}_2$  and  $\tilde{v}_3$  chosen to be some fixed numbers.  $\alpha_{kl}$  are the new parameters introduced for labelling the most general  $h$  up to the redundancy discussed above.

We shall now show that the deformations associated with  $\alpha_{kl}$  correspond in GP model to the effect of blowing up the sixteen orbifold singularities by switching on the twisted

sector closed string states. For this let us consider the deformation where  $\alpha_{11} \neq 0$ , and all other  $\alpha_{ij}$  are zero. Then we have

$$h(u, v) = \prod_{m=2}^4 (u - \tilde{u}_m)(v - \tilde{v}_m) \{ (u - \tilde{u}_1)(v - \tilde{v}_1) + \alpha_{11} \}. \quad (3.18)$$

As we have already seen from eq.(3.12), in the weak coupling limit ( $C \rightarrow 0$ ) the approximate locations of the orientifold planes are at the zeroes of  $h$ . Thus the orientifold planes are now situated at:

$$\begin{aligned} u &= \tilde{u}_m, & \text{for } 2 \leq m \leq 4, \\ v &= \tilde{v}_m, & \text{for } 2 \leq m \leq 4, \\ \text{and } & (u - \tilde{u}_1)(v - \tilde{v}_1) + \alpha_{11} = 0. \end{aligned} \quad (3.19)$$

In other words, the orientifold planes at  $u = \tilde{u}_1$  and  $v = \tilde{v}_1$  have joined together to become a smooth complex hyperbola.

Thus we now need to show that this can be interpreted as the result of blowing up the orbifold singularity at the intersection of  $u = \tilde{u}_1$  and  $v = \tilde{v}_1$ . For this it will be useful to take the generators of the  $Z_2 \times Z'_2$  group as:

$$gh = \mathcal{I}_{67} \mathcal{I}_{89} (-1)^F, \quad \text{and} \quad g = (-1)^{FL} \cdot \Omega \cdot \mathcal{I}_{67}. \quad (3.20)$$

We shall first mod out the type IIB theory on  $T^4$  by the  $Z_2$  group generated by  $gh$ , then blow up the orbifold singularity at  $u = \tilde{u}_1, v = \tilde{v}_1$ , and finally mod out the theory by the  $Z_2$  group generated by  $g$  and study the location of the orientifold planes. Let the image of the point  $(\tilde{u}_1, \tilde{v}_1)$  in the  $(w, z)$  plane be at  $(w = 0, z = 0)$  with  $z$  and  $w$  as defined in eq.(2.6). Since  $gh$  takes  $z$  to  $-z$  and  $w$  to  $-w$ , close to  $(w = 0, z = 0)$  the single valued coordinates on the resulting space are:

$$u - \tilde{u}_1 = w^2, \quad v - \tilde{v}_1 = z^2, \quad \xi = zw, \quad (3.21)$$

with the restriction

$$(u - \tilde{u}_1)(v - \tilde{v}_1) = \xi^2. \quad (3.22)$$

The surface described by this equation is singular at  $(u = \tilde{u}_1, v = \tilde{v}_1)$  reflecting the orbifold singularity present there. We can blow up the singularity by replacing eq.(3.22) by

$$(u - \tilde{u}_1)(v - \tilde{v}_1) - \xi^2 = a^2, \quad (3.23)$$

where  $a$  is the blow up parameter.

Now we mod out type IIB theory on this blown up space by the  $Z_2$  group generated by  $g$ . Since  $g$  changes the sign of  $w$  without changing the sign of  $z$ , we see from eq.(3.21) that under this transformation  $u$  and  $v$  remain unchanged but  $\xi$  changes sign. Thus the orientifold plane is situated at  $\xi = 0$ , which using eq.(3.23) can be rewritten as

$$(u - \tilde{u}_1)(v - \tilde{v}_1) = a^2. \quad (3.24)$$

Comparing with (3.19) we see that this is precisely what we got on the F-theory side provided we identify  $\alpha_{11}$  with  $-a^2$ . Thus the deformation associated with  $\alpha_{11}$  indeed represents the blow up mode in the GP model of the orbifold singularity at  $(\tilde{u}_1, \tilde{v}_1)$ . Similar analysis shows that  $\alpha_{kl}$  represents the blow up mode of the orbifold singularity at  $(\tilde{u}_k, \tilde{v}_l)$ .

This exhausts all deformations that are neutral under  $SU(2)^8 \times (SU(2)')^8$ . We shall now turn to the effect of switching on the vev of the hypermultiplets which are charged under this gauge group.

### 3.2 Symmetry Breaking Pattern

We now turn to the deformations in the GP model due to switching on the vev of the  $(2_i, 2'_j)$  states, and identify the corresponding deformations on the F-theory side. (Similar analysis has been carried out in ref.[21] in the context of intersecting singularities in F-theory.) In particular we shall consider a specific symmetry breaking pattern in which we switch on the vev of all the  $(2_i, 2'_j)$  states for  $(1 \leq i \leq p)$ ,  $(1 \leq j \leq q)$ . As pointed out in the last section, by aligning these vev's properly one can break the first  $p$   $SU(2)$  and the first  $q$   $SU(2)'$  into their diagonal subgroup  $SU(2)_d$ . The last  $(8 - p)$   $SU(2)$  and the last  $(8 - q)$   $SU(2)'$  remain unbroken. We shall first try to understand this subspace in F-theory. At the end we shall also study the case where we destroy the alignment of the vev's and break  $SU(2)_d$  as well.

First of all, since  $SU(2)^{8-p} \times (SU(2)')^{8-q}$  is preserved  $\Delta$  should still possess a factor

$$\prod_{i=p+1}^8 (u - u_i)^2 \prod_{j=q+1}^8 (v - v_j)^2. \quad (3.25)$$

Second, since  $SU(2)_d$  is preserved, one would expect that the deformation on the F-theory

side should convert the

$$\prod_{i=1}^p (u - u_i)^2 \prod_{j=1}^q (v - v_j)^2$$

factor in (3.12) into a perfect square:

$$(\phi_{p,q}(u, v))^2, \quad (3.26)$$

where  $\phi_{p,q}$  is a polynomial of degree  $(p, q)$  in  $(u, v)$ . This would ensure the presence of an  $A_1$  singularity and hence unbroken  $SU(2)_d$ . From eqs.(3.8)-(3.10) we see that this form of  $\Delta$  can easily be achieved if we choose  $f$  and  $g$  of the form (3.8) and (3.9) respectively, with arbitrary  $h$  and

$$\eta(u, v) = C \phi_{p,q}(u, v) \prod_{i=p+1}^8 (u - u_i) \prod_{j=q+1}^8 (v - v_j). \quad (3.27)$$

We shall now compare the number of extra deformation parameters that appear in the two theories.<sup>6</sup> In GP model it is given by (2.14). On the F-theory side the extra deformation parameters are those appearing in  $\phi_{p,q}$ , modulo the parameters that were already present earlier. Since  $\phi_{p,q}$  is a polynomial of degree  $(p, q)$ , it has  $(p+1)(q+1)$  parameters to start with. Of this the coefficient of  $u^p v^q$  can be absorbed into the parameter  $C$ . Furthermore  $(p+q)$  of these parameters were present before in the form of  $u_i$  ( $1 \leq i \leq p$ ) and  $v_j$  ( $1 \leq j \leq q$ ). Thus the net number of extra parameters is

$$(p+1)(q+1) - 1 - p - q = pq, \quad (3.28)$$

in perfect agreement with the answer (2.14) in the GP model.

For the special case  $p = q = 1$  we can take

$$\phi_{1,1}(u, v) = (u - u_1)(v - v_1) + \beta, \quad (3.29)$$

where  $\beta$  is an arbitrary constant.  $\phi_{1,1} = 0$  is the location of the D-brane after switching on the vev of the  $(2, 2')$  state. Thus we see that the effect of this deformation is to fuse two D-branes into one smooth complex hyperbola (times a five dimensional manifold).

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<sup>6</sup>Note that the agreement between the numbers of neutral hypermultiplets in the two theories would be a trivial consequence of the anomaly cancellation condition in six dimensions if we could establish that these two theories have the same gauge group and same charged matter content. But in F-theory we cannot easily identify U(1) factors in the gauge group. There is also some ambiguity in determining the spectrum of charged matter in F-theory although much progress in this direction has been made[23].

Finally in the GP model we consider deformations that break  $SU(2)_d$  as well by switching on vev of the hypermultiplets in the triplet of  $SU(2)_d$ . The number of such triplets is  $(p-1)(q-1)$  as given in eq.(2.15) and hence this breaking is possible only for  $p > 1$  and  $q > 1$ . For  $p = q = 2$   $SU(2)_d$  is broken to  $U(1)$ , and we get an extra neutral hypermultiplet after the breaking. For  $p \geq 3$  or  $q \geq 3$ ,  $SU(2)_d$  can be broken completely, and we get  $3(pq - p - q)$  extra neutral hypermultiplets after this breaking, as given in eq.(2.16). On the F-theory side breaking of  $SU(2)_d$  would mean that we no longer require  $\Delta$  to contain a factor of the form  $\phi_{p,q}^2$ . Thus we can now relax (3.8), (3.9). However, we would still want  $\Delta$  to contain a factor of

$$\eta_0^2 \equiv \prod_{i=p+1}^8 (u - u_i)^2 \prod_{j=q+1}^8 (v - v_j)^2 \quad (3.30)$$

since  $SU(2)^{8-p} \times (SU(2))^{8-q}$  is still unbroken. For this let us consider the following deformations of eqs.(3.8), (3.9)

$$\begin{aligned} f &= \eta - 3h^2 + \delta f \\ g &= h(\eta - 2h^2) + \delta g, \end{aligned} \quad (3.31)$$

where  $h$  is an arbitrary polynomial of degree (4,4) in  $(u, v)$  and  $\eta$ , according to eqs.(3.27), (3.30) can be expressed as

$$\eta = C\eta_0\phi_{p,q}. \quad (3.32)$$

We shall work to first order in  $\delta f$  and  $\delta g$  and find the number of independent deformations for which

$$\delta\Delta = 12f^2\delta f + 54g\delta g, \quad (3.33)$$

has a factor of  $\eta_0^2$ . Using eqs.(3.31) we can reexpress (3.33) as

$$\delta\Delta = 108h^3(h\delta f - \delta g) + 18h\eta_0\phi_{p,q}(3\delta g - 4h\delta f) + 12\eta_0^2\phi_{p,q}^2\delta f. \quad (3.34)$$

The last term already has a factor of  $\eta_0^2$  and hence can be ignored during the rest of the analysis. Since the second term has an explicit factor of  $\eta_0$ , a necessary condition for the full expression to have a factor of  $\eta_0^2$  is that the first term must also have a factor of  $\eta_0$ . Thus we have the requirement:

$$\delta g - h\delta f = \eta_0\delta\chi_{p+4,q+4}, \quad (3.35)$$

where  $\delta\chi_{p+4,q+4}$  is a polynomial of degree  $(p+4, q+4)$  in  $(u, v)$  since  $\eta_0$  is of degree  $(8-p, 8-q)$ . Substituting this in (3.34) and ignoring terms of order  $\eta_0^2$  we get

$$\delta\Delta = -18h^2\eta_0(\phi_{p,q}\delta f + 6h\delta\chi_{p+4,q+4}). \quad (3.36)$$

Thus in order that  $\delta\Delta$  is proportional to  $\eta_0^2$ , we require that the expression inside ( ) be proportional to  $\eta_0$ :

$$\phi_{p,q}\delta f + 6h\delta\chi_{p+4,q+4} = \eta_0\delta\xi_{2p,2q}, \quad (3.37)$$

for some polynomial  $\delta\xi_{2p,2q}$  of degree  $(2p, 2q)$  in  $(u, v)$ .

Thus the question to be addressed is: how many independent solutions of the above equation do we have? Since  $\delta f$ ,  $\delta\chi_{p+4,q+4}$  and  $\delta\xi_{2p,2q}$  are of degree  $(8,8)$ ,  $(p+4, q+4)$  and  $(2p, 2q)$  in  $(u, v)$  respectively, the total number of adjustable parameters is

$$N_{par} = 81 + (p+5)(q+5) + (2p+1)(2q+1). \quad (3.38)$$

Both sides of eq.(3.37) are polynomials of degree  $(8+p, 8+q)$ . Thus the total number of constraints is

$$N_{con} = (p+9)(q+9). \quad (3.39)$$

Naively, we would have  $N_{par} - N_{con}$  solutions of eq.(3.37). However, one should keep in mind that some of these deformations may simply correspond to deformations of  $h$ ,  $C$  and  $\phi_{p,q}$ , which move us inside the subspace of unbroken  $SU(2)_d$ , and some may simply correspond to the redundant deformations associated with the  $SL(2, C)$  transformations on  $u$  and  $v$  or rescaling of  $f$  and  $g$  given in (3.16). All such deformations can be described by deforming  $h$  by an arbitrary polynomial of degree  $(4,4)$  and deforming  $C\phi_{p,q}$  by an arbitrary polynomial of degree  $(p, q)$ . Thus the total number of redundant deformations is given by

$$N_{red} = 25 + (p+1)(q+1). \quad (3.40)$$

This gives the total number of independent deformations that take us out of the subspace of unbroken  $SU(2)_d$  as:

$$N_{par} - N_{con} - N_{red} = 3(pq - p - q), \quad (3.41)$$

in exact agreement with the GP model answer (2.16).

Note however that for  $p = q = 2$  (3.41) vanishes, whereas in the GP model this corresponds to a one parameter family of deformations. This seems to be a contradiction, but

this is resolved by the fact that in this case all the  $N_{con}$  constraints are not independent. Indeed since now  $\eta_0$  is of degree (6,6), one can explicitly construct a one parameter family of solutions for  $\delta f$  and  $\delta g$  satisfying all the requirements as follows:

$$\delta f = 0, \quad \delta g = \gamma \eta_0^2, \quad (3.42)$$

where  $\gamma$  is an arbitrary constant. Thus we see that the F-theory results are again consistent with the results from the GP model.

### 3.3 Symmetry Enhancement

We shall now start from the generic  $SU(2)^8 \times (SU(2)')^8$  configuration in the two theories, and consider special subspaces of this moduli space where there is enhanced gauge symmetry. In the GP model the starting point is a configuration of sixteen intersecting pairs of D-branes at arbitrary locations  $u = u_i$ , and  $v = v_i$  ( $1 \leq i \leq 8$ ) with all the sixteen orbifold singularities blown up. In F-theory we start from  $f$  and  $g$  of the form given in (3.8) and (3.9), with arbitrary  $h$ , but  $\eta$  restricted to be of the form (3.11).

Now in the GP model two of the  $SU(2)$  groups can combine to give an enhanced  $Sp(4)$  gauge group if the locations of two of the D-brane pairs (say  $u_1$  and  $u_2$ ) coincide. This is a subspace of (complex) codimension one. There also appears a hypermultiplet in the  $\mathfrak{5}$  representation of  $Sp(4)$ . Breaking of  $Sp(4)$  to  $SU(2)^2$ , which corresponds to separating the two D-brane pairs, is achieved by giving vev to this hypermultiplet. Four of the five components become massive due to Higgs mechanism, and the remaining component gives the extra singlet of  $SU(2) \times SU(2)$  that measures the separation between the D-brane pairs.

In F-theory this process also has a simple (and obvious) description. It simply corresponds to the codimension one subspace of the moduli space where we choose  $u_1 = u_2$  in the expression (3.11) for  $\eta$ . As a result,

$$\Delta \propto (u - u_1)^4. \quad (3.43)$$

Furthermore it can easily be seen that neither  $f$  nor  $g$  vanish there. Thus this corresponds to an  $A_3$  singularity. Naively one might have thought that this signals the appearance of an  $SU(4)$  gauge group, but it can be seen that in the language of ref.[21] (see also [24]) this singularity is generically non-split and hence  $SU(4)$  is broken to  $Sp(4)$  due to the

monodromy in the  $u = u_1$  plane. To see this note that we have the following expansion of  $f$  and  $g$  in a power series in  $(u - u_1) \equiv \tilde{u}$ :

$$\begin{aligned}
f &= -3h_1^2(v) - 3h_1(v)h_2(v)\tilde{u} - 3h_3(v)\tilde{u}^2 - h_4(v)\tilde{u}^3 + O(\tilde{u}^4) \\
g &= 2h_1^3(v) + 3h_1^2(v)h_2(v)\tilde{u} + 3(h_3(v) + \frac{1}{4}h_2^2(v))h_1(v)\tilde{u}^2 \\
&\quad + \left[ \left( \frac{3}{2}h_3(v) - \frac{1}{8}h_2^2(v) \right) h_2(v) + h_1(v)h_4(v) \right] \tilde{u}^3 + O(\tilde{u}^4)
\end{aligned} \tag{3.44}$$

where,

$$\begin{aligned}
h_1(v) &= h(u_1, v) \\
h_2(v) &= 2\partial_{u_1} h(u_1, v) \\
h_3(v) &= -\frac{1}{3}C \prod_{i=3}^8 (u_1 - u_i) \prod_{j=1}^8 (v - v_j) + \frac{1}{2}\partial_{u_1}^2 h^2(u_1, v) \\
h_4(v) &= -C \prod_{i=3}^8 (u_1 - u_i) \prod_{j=1}^8 (v - v_j) \sum_{k=3}^8 \frac{1}{u_1 - u_k} + \frac{1}{2}\partial_{u_1}^3 h^2(u_1, v).
\end{aligned} \tag{3.45}$$

In order that the singularity is split we need  $h_1(v)$  to be a perfect square. Since this is not so in general, we get a non-split singularity and hence an enhanced  $Sp(4)$  gauge group.

Let us now proceed further. In the GP model if we place these two pairs of D-branes on top of an orientifold plane, we expect to get an enhanced  $SU(4)$  non-abelian gauge group. Naively, this is a subspace of complex codimension one in the previous subspace with enhanced  $Sp(4)$  gauge symmetry, since we need to adjust only one parameter  $u_1$  to be equal to the location  $\tilde{u}_1$  (say) of the orientifold plane. But as was shown in ref.[10], due to one loop anomaly effects, in this configuration one of the sixteen blow up modes becomes massive. Hence this is really a subspace of codimension two in the previous subspace. In the language of Higgs mechanism this phenomenon is explained as follows. There are two hypermultiplets in the  $\mathbf{6}$  representation of  $SU(4)$ . Giving vev to these hypermultiplets with appropriate alignment we can break  $SU(4)$  to  $Sp(4)$ . Each  $\mathbf{6}$  representation of  $SU(4)$  decomposes into a  $\mathbf{5}$  and a singlet of  $Sp(4)$ . One of the  $\mathbf{5}$ 's become massive due to Higgs mechanism, and we are left with two extra singlets and a  $\mathbf{5}$  of  $Sp(4)$ .

What is the corresponding subspace of the moduli space of F-theory with enhanced  $SU(4)$  gauge symmetry? It is clear that this would correspond to the subspace where the

$A_3$  singularity is split. As has been stated before, this requires  $h_1(v) \equiv h(u_1, v)$  to be a perfect square. Since  $h_1(v)$  is a polynomial of degree four in  $v$ , requiring it to be a perfect square gives two constraints on the parameters. Thus we see that in the F-theory the subspace of the moduli space where  $Sp(4)$  gets enhanced to  $SU(4)$  is of codimension two, again in agreement with the answer in the GP model.

Let us now consider in F-theory a subspace of this moduli space where  $h(u_1, v)$  vanishes. This new subspace is of codimension three in the previous subspace. In this case,

$$h(u, v) = (u - u_1)P_{3,4}(u, v), \quad (3.46)$$

where  $P_{3,4}(u, v)$  is a polynomial of degree (3,4) in  $(u, v)$ . This corresponds to choosing in (3.11), (3.17):

$$\begin{aligned} \tilde{u}_1 &= u_1 = u_2 \\ \alpha_{1l} &= 0 \quad \text{for} \quad 1 \leq l \leq 4. \end{aligned} \quad (3.47)$$

This gives

$$\begin{aligned} f(u, v) &\simeq (u - u_1)^2, \\ g(u, v) &\simeq (u - u_1)^3, \end{aligned} \quad (3.48)$$

near  $u = u_1$ . This is a  $D_4$  type singularity. In the language of ref.[21] this singularity can be shown to be semi-split, thereby giving rise to an  $SO(7)$  gauge group. Physically this can be understood as follows. Since the D-branes which intersect the plane  $u = u_1$  move only in pairs, the  $SL(2, Z)$  monodromy in the  $u = u_1$  plane around these D-brane pairs is given by  $T^2$  (with  $T$  and  $S$  denoting the standard generators of  $SL(2, Z)$ ). According to ref.[25] this does not induce any triality action in  $SO(8)$ . On the other hand the monodromies around the zeroes of  $\Delta$  representing the split orientifold plane are given by  $STS^{-1}$  and  $-T^{-4}ST^{-1}S^{-1}$ [25] both of which induce  $SO(8)$  triality action[25]

$$\mathbf{8}_c \leftrightarrow \mathbf{8}_v, \quad \mathbf{8}_s \rightarrow \mathbf{8}_s \quad (3.49)$$

where  $\mathbf{8}_v$ ,  $\mathbf{8}_s$  and  $\mathbf{8}_c$  represent the vector, spinor and the conjugate spinor representations of  $SO(8)$  respectively. Now there is a (non-standard) embedding of  $SO(7)$  in  $SO(8)$  under which these different representations of  $SO(8)$  decompose as:

$$\mathbf{8}_v = \mathbf{8}, \quad \mathbf{8}_c = \mathbf{8}, \quad \mathbf{8}_s = \mathbf{7} + \mathbf{1}, \quad (3.50)$$

where  $\mathbf{7}$  denotes the vector representation and  $\mathbf{8}$  denotes the unique spinor representation of  $SO(7)$ . From eq.(3.49) and (3.50) we see that the monodromy in the  $u = u_1$  plane acts trivially on this  $SO(7)$  subgroup of  $SO(8)$ . Thus  $SO(8)$  is broken to  $SO(7)$ .

By requiring that the breaking of  $SO(7)$  to  $SU(4)$  be describable by conventional Higgs mechanism, we can also infer the spectrum of massless charged hypermultiplets in this special subspace of the F-theory moduli space. In particular, it must contain three hypermultiplets in the  $\mathbf{7}$  representation. (This is consistent with the counting described in ref.[26].) Vacuum expectation values of these hypermultiplets, when appropriately aligned, can break  $SO(7)$  to  $SU(4) \equiv SO(6)$ . This gives two hypermultiplets in the 6 representation of  $SU(4)$ , and three neutral hypermultiplets, thereby showing that the  $SO(7)$  symmetry enhancement takes place inside a codimension three subspace.

Now that we have found this subspace of enhanced  $SO(7)$  symmetry in the F-theory, we would like to ask, how do we reach these enhanced symmetry points in the GP model? In order to analyze this question, we examine closely the F-theory background, and try to identify the corresponding configuration in the GP model. First, setting  $u_1 = u_2 = \tilde{u}_1$  implies that in the GP model we must have two pairs of D-branes on top of a single orientifold plane. Second, setting  $\alpha_{1i}$  to zero means that we set to zero all the blow up modes associated with the orbifold singularities lying on the  $u = \tilde{u}_1$  plane. However, examining the corresponding configuration in the GP model we still find that the gauge group is  $SU(4)$  and not  $SO(7)$ !

In order to understand the source of this discrepancy, let us note that according to our previous analysis, the three blow up modes, which had to be switched off in order to recover the unbroken  $SO(7)$  symmetry, are each part of a vector representation of  $SO(7)$ . In other words there are three hypermultiplets in the vector representation of  $SO(7)$ , and the components of these which are singlet under  $SU(4) \equiv SO(6)$  are the three blow up modes. Let us recall however, that each hypermultiplet contains two complex scalars. In the present example, the partners of a blow up mode are the flux of two tensor fields  $B_{\mu\nu}$  and  $B'_{\mu\nu}$  through the two cycle that is being blown up.<sup>7</sup> Being super-partners of the blow up mode, these are also part of the vector multiplet of  $SO(7)$  and hence must be switched off in order to recover the unbroken  $SO(7)$  symmetry. However, as was pointed out by Aspinwall[27], the conformal field theory orbifold has half unit of  $B$  (and possibly  $B'$ )

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<sup>7</sup>Both the tensor fields, as well as the two cycle, are odd under the  $Z_2$  generator  $g$ , and as a result the fluxes are even.

flux through the two cycle switched on. This would break  $SO(7)$  to  $SO(6) \equiv SU(4)$ , and thus it is not surprising that in the GP model we do not see the  $SO(7)$  gauge symmetry restored even when all the blow up modes are switched off.

This analysis shows that we can recover the  $SO(7)$  gauge symmetry in the GP model by continuously turning off the tensor field flux, but this configuration will not, in general, be describable in terms of a solvable conformal field theory. This result also shows that even though GP model and F-theory on the elliptically fibered Calabi-Yau on  $CP^1 \times CP^1$  are in the same moduli space, they represent different slices of the moduli space. In our analysis thus far we did not discover it because the deformations (flux of  $B$  and possibly  $B'$  fields) which take us from one slice to the other are neutral under the gauge group of the GP model.

We can proceed further and consider a configuration in  $F$ -theory of the form where:

$$h(u, v) = (u - u_1)(v - v_1)P_{3,3}(u, v), \quad (3.51)$$

where  $P_{3,3}$  is a polynomial of degree (3,3) in  $(u, v)$ . In this case, near  $u = u_1, v = v_1$ :

$$f(u, v) \sim (u - u_1)^2(v - v_1)^2, \quad g(u, v) \sim (u - u_1)^3(v - v_1)^3. \quad (3.52)$$

This corresponds to an intersection of two  $D_4$  singularities. According to the result of ref.[21, 22] the physics of this situation cannot be described by a local quantum field theory. In particular we expect to get tensionless strings from three branes wrapped around the collapsed two cycle.

In the GP model, this configuration corresponds to having two pairs of branes on top of the orientifold plane at  $u = \tilde{u}_1 (= u_1)$ , two pairs of branes on top of an orientifold plane at  $v = \tilde{v}_1 (= v_1)$ , and switching off the blow up modes for all the orbifold singularities lying in the  $u = \tilde{u}_1$  and  $v = \tilde{v}_1$  plane. But we do not see any singular behaviour in the GP model at this point in the moduli space. This mismatch can again be attributed to the presence of the flux of the  $B$  field through the collapsed two cycles. The presence of this flux breaks the gauge group to  $SU(4) \times SU(4)$  due to reasons outlined before. Also, as shown in [27], in type IIA theory on an orbifold, presence of this flux prevents a two brane wrapped around this two cycle to become massless. A simple argument involving T-duality will tell us that the same mechanism will prevent a three brane in type IIB theory, wrapped around the two cycle, to become tensionless. This then is the promised explanation of the apparent discrepancy between the F-theory and GP model results for constant  $\lambda$  configuration.

### 3.4 Geometry of Intersecting Orientifold Planes and D-branes

Now that we have established the equivalence of F-theory and the GP model, we can use the F-theory result to study how non-perturbative corrections modify the geometry of the intersecting orientifold planes and D-branes. Of these the fate of intersecting orientifold planes was already discussed in ref.[17] where it was shown that the split orientifold planes join smoothly near their would be intersection point to give a pair of complex hyperbola. On the other hand, since the coupling constant vanishes on the D-brane, non-perturbative corrections do not modify the geometry of intersecting D-branes. This is seen from eq.(3.12); the parameter  $C$  that controls the coupling constant does not affect the locations  $u = u_i$  or  $v = v_i$  of the D-branes. However switching on vev of hypermultiplets associated with open string states stretched between intersecting D-branes join a pair of intersecting D-branes into a complex hyperbola as seen from eq.(3.29).

It remains to study the fate of intersecting  $D$ -branes and orientifold planes. First we consider the intersection of an  $A_3$  singularity and an orientifold plane. For this let us consider the  $\Delta$  given in (3.12) with  $u_1 = u_2$ , and analyse the second factor (which gives the location of the split orientifold plane) near the  $A_3$  singularity  $u = u_1$ . The locations of the zeroes of  $\Delta$  from the second factor in this approximation is given by

$$h^2 - K^2(u - u_1)^2 = 0, \quad (3.53)$$

for some constant  $K$ . This can be written as two surfaces:

$$h \pm K(u - u_1) = 0. \quad (3.54)$$

Since this surface is reducible, we see that the two branches into which the orientifold plane splits intersect at  $u = u_1$ , but do not join smoothly.

Next we consider intersection of the orientifold plane with an  $A_1$  singularity. This will correspond to taking  $u_i, v_j$  arbitrary in eq.(3.12). Thus now near the  $A_1$  singularity at  $u = u_1$ , the zeroes of the second factor of  $\Delta$  are given by

$$h^2 - K'(u - u_1) = 0, \quad (3.55)$$

for some other constant  $K'$ . This surface is not reducible, showing that the two branches of the split orientifold plane join smoothly at  $u = u_1$ .

Finally if we destroy the  $A_1$  singularity at  $u = u_1$  by switching on more general deformations of the form (3.31), then the factorization of  $\Delta$  into the D-brane part and the

orientifold part, as given in (3.10), is destroyed in general. Thus under such deformation the D-branes and the two components of the orientifold plane all join smoothly near their would be intersection point.

## 4 Summary and Conclusion

In this paper we have established the equivalence between the Gimon-Polchinski orientifold, and F-theory on a Calabi-Yau manifold with Hodge numbers (3,243) by comparing the gauge symmetry breaking pattern, local deformations in the moduli space, and background axion-dilaton fields in the weak coupling limit. It was also found that these two models represent two different slices of the full moduli space of the theory. This difference is not visible for most purpose, but becomes relevant when we analyze subspaces of the F-theory moduli space with  $D_4$  type singularities.

We expect that the techniques used in this paper can be easily generalised to establish the equivalence between other F-theory - orientifold pairs, notably in four dimensions. In particular it might be used in establishing the conjectured duality[14] between the orientifold constructed in ref.[28] and F-theory on an elliptically fibered Calabi-Yau four fold with base  $(CP^1)^3$ .

Our result can also be interpreted as giving non-perturbative information about the dynamics of a three brane probe on this orientifold. In particular the background  $\lambda$  in F-theory has the interpretation as the inverse coupling of the U(1) gauge field living on the three brane in the infrared. The tree level Lagrangian describing the dynamics of this probe has been constructed recently in [29].

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