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Intersecting Branes

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Abstract

BPS configurations of intersecting branes have many applications in string theory. We attempt to provide an introductory and pedagogical review of supergravity solutions corresponding to orthogonal BPS intersections of branes with an emphasis on eleven and ten space-time dimensions. Recent work on BPS solutions corresponding to non-orthogonally intersecting branes is also discussed. These notes are based on lectures given at the APCTP Winter School "Dualities of Gauge and String Theories", Korea, February 1997.

1 Introduction

There is now very strong evidence that an eleven dimensional M-theory plays a fundamental role in string theory (see [1] for a recent review). The low-energy limit of M-theory is D=11 supergravity but it is not yet known what the correct underlying microscopic theory is*. It is known that D=11 supergravity and hence M-theory contains solitonic membranes, "$M2$-branes", and fivebranes, "$M5$-branes", which play an important role in the dynamics of the theory. Both of these solitons preserve 1/2 of the supersymmetry and hence are BPS states. BPS states are states that preserve some supersymmetry and are an important class of states as we have some control over their behaviour as various moduli are allowed to vary. It is an important issue to understand the spectrum of BPS states in M-theory and we will see that there is a large class of states corresponding to intersecting $M2$-branes and $M5$-branes.

String theory in D=10 also contains a rich spectrum of BPS branes. In the type IIA and IIB theories there are branes that carry charges arising from both the Neveu-Schwarz-Neveu-Schwarz ($NSNS$) and the Ramond-Ramond (RR) sectors of the world-sheet theory. The former class consists of the fundamental strings and the solitonic fivebranes, "$NS5$-branes". The second class of branes, the "D-branes", have a simple perturbative description as surfaces in flat space where open strings can end, which has played a central role in recent string theory developments [3]. By dimensionally reducing the intersecting brane solutions of M-theory we obtain type IIA solutions corresponding to intersecting NS- and D-branes. Various string dualities then enable one to construct all of the supergravity solutions corresponding to intersecting branes in both the type IIA and IIB theories. The properties of these supergravity solutions complement what we can learn about the various branes using string perturbation theory.

*It has been proposed that M-theory in the infinite momentum frame is given by the large N limit of a certain quantum mechanics based on $N \times N$ matrices [2]. This interesting development was discussed by H. Verlinde in his lectures at the School and we refer the reader to his article for more details.

Since solitons are a key ingredient in duality studies, a great deal of effort has been devoted to constructing general soliton solutions of supergravity theories in various dimensions. The intersecting brane solutions in D=11 and D=10 provide a unified viewpoint since many of the other soliton solutions can be obtained by dimensional reduction and duality transformations. Understanding the general structure of intersecting brane solutions is an involved task: a partial list of references is: [4]-[41]. In these lectures we will only consider BPS intersections, but we note here that non-BPS solutions have also been studied. BPS intersecting branes fall into two categories which have been termed "marginal" and "non-marginal" [34]. Roughly speaking the mass (or tension) M and charges Q_i of marginal configurations satisfy $M = \Sigma Q_i$ while for the non-marginal cases one has $M^2 = \Sigma Q_i^2$, corresponding to non-zero binding energy. For the most part we will be focusing on the marginal intersections. The non-marginal solutions can be obtained from the marginal solutions by dimensional reduction and/or duality transformations. They were discussed by J. Russo in his lectures at this school.

Our main focus will be on supergravity solutions with an emphasis on M-theory. It is worth pointing out in advance that there are a number of important applications of intersecting brane configurations which we will not be discussing in much detail. Let us briefly highlight just two here. The first is to provide a microscopic state counting interpretation of black hole entropy[42]†. One can construct classical solutions corresponding to intersecting D-branes that give rise upon dimensional reduction to black holes with non-zero Bekenstein-Hawking entropy. By exploiting the perturbative D-brane point of view one can count the number of open string microstates that give rise to the same macroscopic quantum numbers that the black hole carries and one finds perfect agreement. It should be noted that while the perturbative calculation is valid at weak coupling the supergravity black hole spacetime is valid at strong coupling and one must invoke supersymmetry to argue that the state counting calculation is unchanged as one varies the coupling. Although this is an exciting development there is still more

†See S. Das's contribution to the proceedings for more details and references.

to be understood on how these two complementary views of black holes are related.

A second application is to use BPS intersecting branes to study the infrared dynamics of supersymmetric gauge theories [43, 44] (and references therein). One considers different types of branes intersecting in an appropriately chosen arrangement. The low-energy dynamics on the world-volume of one type of brane is associated with a supersymmetric quantum field theory that one wishes to study. By considering the low-energy dynamics from the point of view of different branes and allowing the branes to move around, enables one, in certain cases, to determine the low-energy effective dynamics of the field theory. This has proven to be a very powerful tool to study supersymmetric gauge theories in three and four spacetime dimensions. It is worth noting that in a recent development some aspects of the supergravity solutions of branes in M-theory played an important role [45].

The plan of the rest of the paper is as follows. In section 2 we discuss orthogonal intersections of branes in M-theory. In section 3 we discuss the intersections of NS- and D-branes in type IIA and IIB string theory. Section 4 reviews recent solutions on supersymmetric configurations of branes that intersect non-orthogonally and section 5 concludes.

2 Intersecting M-Branes

2.1 $M2$-branes and $M5$-branes

The low-energy effective action of M-theory is D=11 supergravity. The bosonic field content consists of a metric, g_{MN}, and a three-form potential, A_{MNP}, with four-form field strength $F_{MNPQ} = 24\nabla_{[M}A_{NPQ]}$. The action for the bosonic fields is given by

$$S = \int \sqrt{-g}\left\{R - \frac{1}{12}F^2 - \frac{1}{432}\epsilon^{M_1\ldots M_{11}}F_{M_1\ldots M_4}F_{M_5\ldots M_8}A_{M_9\ldots M_{11}}\right\}. \tag{1}$$

Supersymmetric solutions to the corresponding equations of motion can be constructed by looking for bosonic backgrounds that admit Killing spinors i.e., backgrounds which admit 32-component Majorana spinors ϵ such that the supersymmetry variation of the gravitino field ψ_M vanishes:

$$\left[D_M + \frac{1}{144}({\Gamma_M}^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}\right]\epsilon = 0. \tag{2}$$

The $M2$-brane solution [46] takes the form

$$\begin{aligned} ds^2 &= H^{1/3}[H^{-1}\left(-dt^2 + dx_1^2 + dx_2^2\right) + \left(dx_3^2 + \ldots dx_{10}^2\right)] \\ F_{t12\alpha} &= \frac{c}{2}\frac{\partial_\alpha H}{H^2}, \qquad H = H(x_3,\ldots,x_{10}), \qquad \nabla^2 H = 0, \qquad c = \pm 1. \end{aligned} \tag{3}$$

We have written the metric with an overall conformal factor as this form will be convenient when we discuss intersecting M-branes. The solution admits Killing spinors of the form $\epsilon = H^{-1/6}\eta$ with the constant spinor η satisfying

$$\hat\Gamma_{012}\eta = c\eta, \tag{4}$$

where $\hat\Gamma_{0\ldots p} \equiv \hat\Gamma_0\ldots\hat\Gamma_p$ is the product of $p{+}1$ distinct Gamma matrices in an orthonormal frame. Using the fact that $(\hat\Gamma_{012})^2 = 1$ and that $\mathrm{Tr}\hat\Gamma_{012} = 0$ we conclude that the $M2$-brane solution has 16 Killing spinors and preserves (breaks) half of the supersymmetry. The solution is governed by a single harmonic function that depends on the coordinates $\vec{x} = \{x_3,\ldots,x_{10}\}$ and we first take it to be of the form

$$H = 1 + \frac{a}{r^6}, \qquad r = |\vec{x}|\,. \tag{5}$$

The solution then describes a single $M2$-brane with world-volume oriented along the $\{0,1,2\}$ hyperplane located at $r = 0$. The $M2$-brane carries electric four-form charge Q_e which is defined as the integral of the seven-form[‡] $*F$ around a seven-sphere that

[‡]To be more precise we should integrate $*F + A\wedge F$, since the field equation is $d*F + F\wedge F = 0$.

surrounds the brane and is proportional to ca. If $c = 1$ we have an $M2$-brane, while if $c = -1$ we have an anti-$M2$-brane. We will often not distinguish between branes and antibranes in the following. The ADM mass per unit area or ADM tension T can be calculated and is proportional to $|Q_e|$ as one requires for a BPS state. The metric appears to be singular at $r = 0$. However, it has been shown that this surface is in fact a regular degenerate event horizon [47]. The metric can be continued into an interior region and it is here that a real curvature singularity is located. By generalising the harmonic function to have many centres

$$H = 1 + \sum_{I=1}^{k} \frac{a_I}{r_I^4}, \qquad r_I = |\vec{x} - \vec{x}_I|, \tag{6}$$

we obtain k parallel $M2$-branes located at positions $\vec{x}_I$.

The construction of the $M5$-brane solution [48] runs along similar lines. The solution is given by

$$\begin{aligned} ds^2 &= H^{2/3}\left[H^{-1}\left(-dt^2 + dx_1^2 + \ldots dx_5^2\right) + \left(dx_6^2 + \ldots + dx_{10}^2\right)\right] \\ F_{\alpha_1\ldots\alpha_4} &= \frac{c}{2}\epsilon_{\alpha_1\ldots\alpha_5}\partial_{\alpha_5}H, \qquad H = H_i(x_6, \ldots, x_{10}), \qquad c = \pm 1, \end{aligned} \tag{7}$$

where $\epsilon_{\alpha_1\ldots\alpha_5}$ is the flat D=5 alternating symbol. It again admits 16 Killing spinors given by $\epsilon = H^{-1/12}\eta$ where η now satisfies the projection:

$$\hat{\Gamma}_{012345}\eta = c\eta. \tag{8}$$

For a single $M5$-brane we choose the harmonic functions to be

$$H = 1 + \frac{a}{r^4}, \qquad r = |\vec{x}|, \tag{9}$$

where $\vec{x} = \{x_6, \ldots, x_{10}\}$. The $M5$-brane carries magnetic four-form charge Q_m which is obtained by integrating F around a four-sphere that surrounds the $M5$-brane and is proportional to ca. $c = \pm 1$ correspond to an $M5$- and an anti-$M5$-brane respectively. The ADM tension is again proportional to $|Q_m|$ in line with unbroken supersymmetry. The $M5$-brane is a completely regular solution as was shown in [49]. A configuration of parallel multi-$M5$-branes is obtained by generalising the single centre harmonic function to have many centres.

The dimensional reduction of D=11 supergravity on a circle leads to D=10 type IIA supergravity. Indeed this is necessary for the type IIA string theory to be dual to M-theory. There are two distinct ways in which the M-brane solutions can be dimensionally reduced to D=10: they can be "wrapped" or "reduced", as we now explain (we will also return to this in section 3). Since both the $M2$-brane and the $M5$-brane solutions are independent of the coordinates tangent to the world-volume of the branes we can demand that one of them is a periodic spatial coordinate upon which we compactify. The result of this wrapping leads to the fundamental string and the $D4$-brane solutions of the type IIA theory, respectively. If we denote the compactified direction as x_{10} and the other coordinates by x_μ, we find that the membrane carries electric two-form $A_{\mu\nu 10}$ charge while the four-brane carries magnetic three-form $A_{\mu\nu\rho}$ charge. The process of reducing along a direction transverse to the world-volume is slightly more involved. To obtain a solution that is periodic in such a direction, x_{10} say, we construct a periodic array of either $M2$- or $M5$-branes i.e., we take a multi M-brane solution with the branes lined up along the x_{10} direction and equally spaced by a distance $2\pi R$. The solution obtained by dimensional reduction along the x_{10} direction will have non-trivial dependence on the compactified coordinate or equivalently the D=10 solution will have massive Kaluza-Klein modes excited. If we average over the compact coordinate, i.e., if we ignore the massive modes, then we obtain the $D2$-brane and the $NS5$-brane solutions of type IIA supergravity, respectively. The former carries electric $A_{\mu\nu\rho}$ charge and the latter magnetic $A_{\mu\nu 10}$ charge. A more direct way to get these IIA solutions is simply to take the harmonic functions for the D=11 $M2$- or the $M5$-brane to be independent of one of the transverse directions. A brane solution whose harmonic function is independent of a number of transverse coordinates is sometimes said to be "delocalised", "averaged" or "smeared" over those directions. Delocalised branes will appear when we discuss intersecting brane solutions.

2.2 Intersecting M-branes

We now turn to solutions corresponding to intersecting M-branes. We begin by presenting the generalized supersymmetric solution for two $M2$-branes orthogonally "overlapping" in a point [5, 7] which we will denote by $M2 - M2(0)$:

$$
\begin{aligned}
ds^2 &= (H_1 H_2)^{1/3} [-(H_1 H_2)^{-1} dt^2 + H_1^{-1} \left(dx_1^2 + dx_2^2\right) + H_2^{-1} \left(dx_3^2 + dx_4^2\right) \\
&\qquad + \left(dx_5^2 + \ldots + dx_{10}^2\right)], \\
F_{t12\alpha} &= \frac{c_1}{2} \frac{\partial_\alpha H_1}{H_1^2}, \qquad F_{t34\alpha} = \frac{c_2}{2} \frac{\partial_\alpha H_2}{H_2^2}, \qquad \alpha = 5, \ldots, 10. \\
H_i &= H_i(x_5, \ldots, x_{10}), \qquad \nabla^2 H_i = 0, \qquad c_i = \pm 1, \qquad i = 1, 2.
\end{aligned} \tag{10}
$$

There are Killing spinors of the form $\epsilon = (H_1 H_2)^{-1/6} \eta$, where η is constant and satisfies the algebraic constraints

$$
\begin{aligned}
\hat{\Gamma}_{012} \eta &= c_1 \eta \\
\hat{\Gamma}_{034} \eta &= c_2 \eta.
\end{aligned} \tag{11}
$$

Since $[\hat{\Gamma}_{012}, \hat{\Gamma}_{034}] = 0$ and $\mathrm{Tr}(\hat{\Gamma}_{012})(\hat{\Gamma}_{034}) = 0$, each condition projects out an independent half of the spinors and we conclude that there are eight Killing spinors and hence the solution preserves $1/4$ of the supersymmetry.

The functions H_i are harmonic in the coordinates $\vec{x} = \{x_5, \ldots, x_{10}\}$ and we first take them to be of the form

$$
H_i = 1 + \frac{a_i}{r_i^4}, \qquad r_i = |\vec{x} - \vec{x}_i| \,. \tag{12}
$$

The solution then describes an $M2$-brane oriented in the $\{1, 2\}$ plane with position $\vec{x}_1$ and another oriented in the $\{3, 4\}$ plane with position $\vec{x}_2$ orthogonally overlapping in a point. To see this we note that the solution is a kind of superposition of each individual $M2$-brane solution. For the directions tangent to the ith $M2$-brane the metric appears with the inverse of the harmonic function i.e., H_i^{-1}, and the directions transverse to the ith $M2$-brane are independent of H_i exactly as in (3). Moreover, the overall conformal factor is the product of the two harmonic functions to the appropriate power, as one expects for $M2$-branes. In the degenerate case that $\vec{x}_1 = \vec{x}_2$, an $M2$-brane with $\{1,2\}$ orientation intersects one with $\{3,4\}$ orientation. Note that the special case when $H_1 = H_2$ was first constructed by Güven [48][§].

A more general solution has harmonic functions of the form

$$H_i = 1 + \sum_{I=1}^{k_i} \frac{a_{i,I}}{r_{i,I}^4}, \qquad r_{i,I} = |\vec{x} - \vec{x}_{i,I}|\,. \tag{13}$$

The solution then describes k_1 parallel $M2$-branes with $\{1,2\}$ orientation and positions $\vec{x}_{1,I}$, and k_2 parallel $M2$-branes with $\{3,4\}$ orientation and positions $\vec{x}_{2,I}$. Each $M2$-brane of one set orthogonally overlaps all of the $M2$-branes in the other set in a point. An $M2$-brane with $\{1,2\}$ orientation intersects one with $\{3,4\}$ orientation in the case that $\vec{x}_{1,I} = \vec{x}_{2,J}$, for some combination I, J. Note that in describing the solutions in the rest of the paper we will implicitly take the harmonic functions to be that of a single brane as in (12) for ease of exposition.

There is a potentially confusing point with our interpretation of (10). To explain this lets first introduce some terminology: we refer to *common tangent* directions as being tangent directions common to all branes. In the case that the branes intersect rather than overlap these are the intersection directions. *Relative transverse* directions are those tangent to at least one but not all branes and *overall transverse* directions are those orthogonal to all branes. The two harmonic functions in (10) are invariant under the common tangent direction, i.e., the time direction in this case, and also under translations in all the relative transverse directions $x_1, \ldots, x_4$. In particular, we note that H_1 does not fall off in the x_3, x_4 directions, as one would expect for a D=11 $M2$-brane spatially oriented in the $\{1,2\}$ plane. i.e., the H_1 $M2$-brane is delocalised in the directions tangent to the other $M2$-brane. Similarly the H_2 $M2$-brane in the $\{3,4\}$ plane

[§]This was described as a "4-brane" solution in [48] because of the $SO(4)$ invariance in the (x_1, x_2, x_3, x_4) plane. The problem with this interpretation is the absence of boost invariance that single branes possess and it is best interpreted as a special case of the $M2 \perp M2(0)$ solution.

is delocalised in the directions tangent to the $M2$-brane lying in the $\{1,2\}$ plane. It is natural to conclude that our interpretation of the solutions as describing intersecting branes is valid but that we have not found the most general fully localised solutions. In a later subsection we will discuss more general solutions that make some progress in this direction.

Since the $M2$-branes are delocalised in the directions tangent to the other brane, we can immediately consider the solution (10) in a dimensionally reduced context with all relative transverse directions periodically identified. This implies, e.g., that the $M2$-brane with spatial orientation in the $\{1,2\}$ plane has been reduced in the $\{3,4\}$ directions to give a membrane in D=9 and then wrapped in the $\{1,2\}$ directions to give a point object in D=7 that carries electric charge of the D=7 gauge field $A_{\mu 12}$. Similarly the other $M2$-brane is a point object in D=7 carrying electric charge with respect to the gauge field $A_{\mu 34}$. Thus, the dimensionally reduced solution may be regarded as two charged D=7 black holes, each carrying an electric charge with respect to different $U(1)$'s. In the intersecting case with $\vec{x}_1 = \vec{x}_2$ the two black holes are coincident and we can interpret it as a single black hole that carries two charges. These BPS black holes solutions are extremal and in fact have naked singularities. Later we will describe how extremal black holes with non-zero horizon area can be constructed from intersecting branes.

The solutions (10) are generically singular on the surfaces $\vec{x} - \vec{x}_i = 0$, with the scalar curvature diverging. This behavior is different from that of a single $M2$-brane where, as we have noted, these surfaces are regular event horizons. The singularity in the present case arises because the $M2$-branes are delocalised in the relative transverse dimensions. It is possible that more general localised solutions will exhibit a similar singularity structure to that of a single $M2$-brane.

Lets now turn to configurations involving $M5$-branes. We will present a solution describing an $M2$-brane intersecting an $M5$-brane in a one-brane, $M2 - M5(1)$, and another describing an $M5$-brane intersecting another $M5$-brane in a threebrane, $M5-M5(3)$. Both solutions are constructed as a kind of superposition of their constituents. There is another solution involving $M5 - M5(1)$ which is qualitatively different and will be discussed in a later subsection. The $M2 - M5(1)$ solution is given by [5, 7]

$$
\begin{aligned}
ds^2 &= H_1^{2/3}H_2^{1/3}[H_1^{-1}H_2^{-1}(-dt^2+dx_1^2)+H_1^{-1}\left(dx_2^2+dx_3^2+dx_4^2+dx_5^2\right) \\
&\qquad +H_2^{-1}\left(dx_6^2\right)+\left(dx_7^2+dx_8^2+dx_9^2+dx_{10}^2\right)], \\
F_{6\alpha\beta\gamma} &= \frac{c_1}{2}\epsilon_{\alpha\beta\gamma\delta}\partial_\delta H_1, \qquad F_{t16\alpha}=\frac{c_2}{2}\frac{\partial_\alpha H_2}{H_2^2}, \qquad H_i=H_i(x_7,\ldots,x_{10}),
\end{aligned} \tag{14}
$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the D=4 flat space alternating symbol. The eight Killing spinors have the form $\epsilon = H_1^{-1/12}H_2^{-1/6}\eta$ with the constant spinor η satisfying

$$
\begin{aligned}
\hat\Gamma_{016}\eta &= c_1\eta \\
\hat\Gamma_{012345}\eta &= c_2\eta.
\end{aligned} \tag{15}
$$

and we have used the fact that $\hat\Gamma_{10} = \hat\Gamma_0\hat\Gamma_1\ldots\hat\Gamma_9$. If we choose the harmonic functions to have single coincident centres then the solution describes an $M5$-brane in the $\{1,2,3,4,5\}$ direction intersecting an $M2$-brane in the $\{1,6\}$ direction.

The solution corresponding to $M5 - M5(3)$ is given by [4, 5, 7]

$$
\begin{aligned}
ds^2 &= (H_1H_2)^{2/3}\left[(H_1H_2)^{-1}\left(-dt^2+dx_1^2+dx_2^2+dx_3^2\right)+H_1^{-1}\left(dx_4^2+dx_5^2\right)\right. \\
&\qquad \left.+H_2^{-1}\left(dx_6^2+dx_7^2\right)+\left(dx_8^2+dx_9^2+dx_{10}^2\right)\right], \\
F_{67\alpha\beta} &= \frac{c_1}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma H_1, \qquad F_{45\alpha\beta}=\frac{c_2}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma H_2, \qquad H_i=H_i(x_8,x_9,x_{10}),
\end{aligned} \tag{16}
$$

where $\epsilon_{\alpha\beta\gamma}$ is the D=3 flat space alternating symbol. The solution preserves 1/4 of the supersymmetry and the Killing spinors are given by $\epsilon = (H_1H_2)^{-1/12}\eta$ with the constant spinor η satisfying the constraints

$$
\begin{aligned}
\hat\Gamma_{012345}\eta &= c_1\eta \\
\hat\Gamma_{012367}\eta &= c_2\eta.
\end{aligned} \tag{17}
$$

If we choose the harmonic functions to have single coincident centres then the solution describes an $M5$-brane in the $\{1,2,3,4,5\}$ direction intersecting an $M5$-brane in the $\{3,4,5,6,7\}$ direction.

Note that in both solutions (14), (16) the harmonic functions again just depend on the overall transverse directions. Thus, just as in the $M2-M2(0)$ solution above, each of the branes are delocalised along the directions tangent to the other. We will see later how the solutions (14) and (16) can be obtained from (10) after dimensional reduction, duality transformations and then uplifting back to D=11.

2.3 Multi-Intersections and Black Holes

In the last section we presented three basic intersections of two M-branes, each preserving 1/4 of the supersymmetry. We can construct solutions of n orthogonally intersecting M-branes by simply ensuring that the branes are aligned along hyperplanes in such a way that the pairwise intersections are amongst the allowed set. The solutions are then constructed by superposing the solutions in a way that we have already seen: there is a harmonic function H for each constituent brane that depends on the overall transverse coordinates. It appears in the metric only as H^{-1} multiplying the directions tangent to that brane and in the overall conformal factor with the appropriate power depending on whether it is an $M2$- or an $M5$-brane. The four-form field strength has non-zero components corresponding to those of each of the M-branes. This procedure [5, 7] has been called the "harmonic function rule".

Generically a configuration of n intersecting branes will preserve 2^{-n} of the supersymmetry [4, 5, 7]. This is because the Killing spinors are projected out by products of Gamma matrices with indices tangent to each brane, and generically these projections are independent. There are, however, important exceptions when the projections are not independent [6, 7]. Let us illustrate this by discussing the cases for three intersecting

M-branes which all preserve 1/8 of the supersymmetry. There is a unique configuration corresponding to three $M2$-branes. If the $M2$-branes are orientated along the $\{1,2\}$, $\{3,4\}$ and $\{5,6\}$ hyperplanes the metric is given by [5, 7]:

$$\begin{aligned} ds^2 &= (H_1H_2H_3)^{1/3}[-(H_1H_2H_3)^{-1}dt^2 + H_1^{-1}\left(dx_1^2+dx_2^2\right) + H_3^{-1}\left(dx_3^2+dx_4^2\right) \\ &\qquad +H_3^{-1}\left(dx_5^2+dx_6^2\right) + \left(dx_7^2+dx_8^2+dx_9^2+dx_{10}^2\right)], \end{aligned} \tag{18}$$

with the harmonic functions $H_i = H_i(x_7, x_8, x_9, x_{10})$.

There is also a unique configuration corresponding to two $M2$-branes and one $M5$-brane and we can take the orientations of the branes to be in the $\{1,2,3,4,5\}$, $\{1,6\}$ and $\{2,7\}$ hyperplanes. This solution provides us with the first special triple intersection. To see this note that the product of the three Gamma matrix projections gives another projection corresponding to an $M5$-brane in the $\{3,4,5,6,7\}$ direction. This means that we can obtain an $M2-M2-M5-M5$ configuration that breaks 1/8 of the supersymmetry (and not 1/16 as one might naively expect) as long as we choose the polarisation of the fourth $M5$-brane (i.e., whether it is a brane or anti-brane) to be determined by the polarisations of the first three. The metric for this solution is given by [6]

$$\begin{aligned} ds^2 = {} & (H_1H_2)^{1/3}(H_3H_4)^{2/3}[-(H_1H_2H_3H_4)^{-1}dt^2 + (H_1H_3)^{-1}\left(dx_1^2\right) + (H_2H_3)^{-1}\left(dx_2^2\right) \\ & +(H_3H_4)^{-1}\left(dx_3^2+dx_4^2+dx_5^2\right) + (H_1H_4)^{-1}\left(dx_6^2\right) + (H_2H_4)^{-1}\left(dx_7^2\right) \\ & + \left(dx_8^2+dx_9^2+dx_{10}^2\right)], \end{aligned} \tag{19}$$

with the harmonic functions $H_i = H_i(x_8, x_9, x_{10})$.

There are two ways in which two $M5$-branes and one $M2$-brane can intersect. The first is when they are oriented along the $\{1,2,3,4,5\}$, $\{3,4,5,6,7\}$ and $\{1,6\}$ planes. Note that this is again a special intersection as we can add an $M2$-brane in the $\{2,7\}$ plane to return to the solution (19). The other intersection has the $M2$-brane lying in the $\{3,8\}$ plane and the three branes intersect in a common string. For this solution there are only two overall transverse directions and so the three harmonic functions have logarithmic divergences.

Finally there are three ways in which three $M5$-branes can intersect. Take the first two to lie in the $\{1,2,3,4,5\}$ and $\{3,4,5,6,7\}$ planes. The third $M5$-brane can be placed in the $\{1,2,3,6,7\}$ direction in which case there is an overall string intersection. We shall return to this configuration in a moment. If the third $M5$-brane is placed in the $\{1,3,4,6,8\}$ plane there is a common 2-brane intersection and we obtain a third special triple intersection since we can add a fourth $M5$-brane in the $\{2,3,4,7,8\}$ plane and still preserve 1/8 of the supersymmetry. Note that this configuration has only two overall transverse dimensions. The third case has the $M5$-brane lying in the $\{3,4,5,8,9\}$ plane and now there is only one overall transverse dimension.

Although conceptually clear it is slightly involved to list all of the supersymmetric intersecting M-brane configurations and determine the amount of supersymmetry preserved taking into account the three special triple intersections. This was undertaken in [25].

We now turn to intersecting brane configurations corresponding to BPS black holes in D=4,5 that have non-zero horizon area. To obtain such a black hole in D=5 we can dimensionally reduce the $M2-M2-M2$ solution (18) along the six relative transverse directions $x_1,\ldots,x_6$. If we take the harmonic functions H_i to have a single coincident centre we are led [5] to a black hole solution in D=5 that carries three electric charges corresponding to three $U(1)$'s coming from the three-form components $A_{\mu 12}$, $A_{\mu 34}$ and $A_{\mu 56}$. One can show that the BPS black hole is extremal and has non-zero horizon area. There is another way to obtain such a D=5 black hole. One considers the $M2-M5(1)$ solution (14) and adds momentum along the string direction. The procedure for doing this is well known and the solution one gets is [5]

$$ds^2 = \quad H_1^{2/3}H_2^{1/3}[H_1^{-1}H_2^{-1}(dudv + Kdu^2) + H_1^{-1}\left(dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2\right)$$

$$+H_2^{-1}\left(dx_6^2\right)+\left(dx_7^2+dx_8^2+dx_9^2+dx_{10}^2\right)], \tag{20}$$

where $u, v = x_1 \pm t$ and the function K is harmonic in the overall transverse coordinates: in the simplest case of a single centre it corresponds to a "pp-wave" carrying momentum in the string direction. The wave in the $\{1\}$ direction imposes the constraint

$$\epsilon = \pm\hat{\Gamma}_{01}\epsilon \tag{21}$$

on the Killing spinors ($\pm$ depending on which direction it is travelling). It thus breaks a further 1/2 of the supersymmetry and hence the solution preserves 1/8 of the supersymmetry. Reducing this to D=5 along the relative transverse directions and the string intersection, we obtain a black hole that carries electric $A_{\mu 16}$ charge, magnetic $A_{\mu\nu 6}$ charge (note that in D=5 this is dual to a vector field) and electric Kaluza-Klein $g_{\mu 1}$ charge corresponding to the momentum running along the string.

Let us now discuss how D=4 black holes can be constructed from intersecting M-branes. One way is to dimensionally reduce the $M2-M2-M5-M5$ solution (19) along the relative transverse directions [6]. In this way one obtains a black hole carrying two electric and two magnetic charges. Another way is to consider three $M5$-branes all overlapping in a common string, with momentum running along the common string direction [6]:

$$\begin{aligned} ds^2 &= (H_1H_2H_3)^{2/3}[(H_1H_2H_3)^{-1}\left(dudv+Kdu^2\right)+(H_1H_2)^{-1}\left(dx_2^2+dx_3^2\right) \\ &+ (H_1H_3)^{-1}\left(dx_4^2+dx_5^2\right)+(H_2H_3)^{-1}\left(dx_6^2+dx_7^2\right)+\left(dx_8^2+\ldots dx_{10}^2\right)]. \end{aligned} \tag{22}$$

Note that as long as the direction of the wave is chosen appropriately, it does not impose any additional constraints on the Killing spinors and hence the solution preserves 1/8 of the supersymmetry.

It should become clear in the next section that the different configurations of M-branes giving black holes in either D=4 or D=5 can be related to each other by dimensional reduction and duality. There we will also discuss ways in which intersecting

D-branes give rise to black holes. The perturbative D-brane point view has been very successfully exploited in giving a microscopic interpretation of black hole entropy. As less is understood about M-brane dynamics it is harder to do this in M-theory. However, one can turn this around and see what can be learned about M-theory dynamics if we demand that it is consistent with black hole entropy. This has been pursued in [6].

2.4 Dynamics of Intersections

As we have noted all of the solutions we have considered so far are delocalised along the relative transverse directions i.e., in the directions tangent to all of the branes. As such, the properties and dynamics of the intersection are somewhat occluded. Addressing this directly at the level of finding more general solutions is an interesting open question but we can also obtain a great deal of insight using more general arguments [50, 51].

Lets begin by considering the possibility of an $M2$-brane ending on an $M5$-brane in a string. One immediately faces a potential problem with charge conservation: consider a seven-sphere surrounding the $M2$-brane. The integral of $*F$, along this seven sphere gives the $M2$-brane charge Q_e, where F is the four-form field strength. It might seem that we could smoothly deform the sphere to a point by slipping it off the end past the $M5$-brane and hence conclude that Q_e must vanish. However, this argument ignores what happens when the sphere is passed through the $M5$-brane. The argument can fail if the charge can somehow be carried by the string boundary inside the $M5$-brane.

One way to study this is to include the world-volume dynamics of the $M5$-brane in the supergravity equations of motion. The low-energy dynamics of an $M5$-brane and its coupling to the spacetime supergravity fields can be described by a low-energy effective action on the world-volume of the brane. This can be constructed from first principles by determining the zero modes in the small fluctuations around a classical solution. The dynamics for the $M5$-brane is governed by a D=6 $(0,2)$ supermultiplet multiplet whose bosonic fields consist of 5 scalars and a two-form V_2 that has self dual field strength [54]. The world-volume action contains the coupling $|dV_2 - A|^2$ where A is the supergravity three-form pulled back to the world-volume: $A_{ijk}{=}A_{\mu\nu\rho}\partial_i X^\mu \partial_j X^\nu \partial_k X^\rho$, where $X^\mu(\sigma^i)$ are the world-volume scalar coordinates. This modifies the A equation of motion to include a world-volume source term. After integrating over an asymptotic seven sphere we deduce that $Q_e = \int_{S^3} *dV_2$ where the integral is a world-volume integral and $*$ is the world-volume Hodge-dual. In the world-volume theory this integral is non-zero if there is a self-dual string inside the six-dimensional world-volume. Thus we conclude that it is possible for an $M2$-brane to end in a string on an $M5$-brane if the $M2$-brane charge is carried by a self-dual string inside the world-volume theory. Note that it is also possible to reach an identical conclusion without having to introduce world-volume dynamics if one takes into account the contribution of Chern-Simons couplings in the supergravity [51].

This conclusion indicates that the $M5$-brane is a natural generalisation of a D-brane in string theory to M-theory. It also suggests that we can think of the $M2 - M5(1)$ solution (14) as being associated with these configurations. It is possible that more general supergravity solutions exist that have localised $M2$-branes ending on $M5$-branes. They would be very interesting as they would illuminate the geometry of the boundary of the $M2$-brane and the dynamics of the self-dual string. These solutions will probably be highly non-trivial to construct but perhaps progress can be made by looking for localised solutions with an $M2$-brane ending on the $M5$-brane from either side.

Similar arguments can be developed for self intersections of M-branes. The following argument in fact works for all p-branes [4]. If we assume that we can consider a q-brane intersection within a given p-brane as a dynamical object in the $p+1$-dimensional world-volume field theory, then the condition that the p-brane can support a dynamical q-brane intersection would be that its world volume contains a $(q+1)$-form potential to which the q-intersection can couple. The effective action of all p-branes contain scalar fields which are the Goldstone modes arising from the fact that the classical p-brane solution breaks translation invariance. These scalar fields have one-form field strengths which can be dualised in the world-volume to give $(p-1)$-form dual potentials which can couple to a $q=(p-2)$-dimensional intersection. Hence we conclude that a p-brane can have a dynamical self intersection in $(p-2)$ dimensions. The $M2-M2(0)$ and $M5-M5(3)$ solutions (10), (16) are both consistent with this rule.

2.5 $M5 \perp M5(1)$

There is another solution corresponding to two $M5$ branes overlapping in a string [7]:

$$\begin{aligned}
ds^2 &= (H_1H_2)^{2/3}\,[(H_1H_2)^{-1}(-dt^2+dx_1^2)+H_2^{-1}(dx_2^2+dx_3^2+dx_4^2+dx_5^2) \\
&\qquad +H_1^{-1}(dx_6^2+dx_7^2+dx_8^2+dx_9^2)+dx_{10}^2] \\
F_{mnp10} &= -\frac{c_1}{2}\epsilon_{mnpq}\partial_q H_1, \qquad F_{\mu\nu\lambda 10}=-\frac{c_2}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\rho H_2, \\
H_1 &= H_1(X_m^1), \qquad H_2=H_2(X_\mu^2), \qquad \nabla^2 H_i=0,
\end{aligned} \tag{23}$$

where $X_m^1=(x_2,x_3,x_4,x_5)$ and $X_\mu^2=(x_6,x_7,x_8,x_9)$. For single centre harmonic functions this corresponds to an $M5$-brane with orientation $\{1,2,3,4,5\}$ overlapping another with orientation $\{1,6,7,8,9\}$. There are 16 Killing spinors of the form $\epsilon=(H_1H_2)^{-1/12}\eta$ with the constant spinor η satisfying

$$\begin{aligned}
\hat\Gamma_{016789}\eta &= c_1\eta \\
\hat\Gamma_{012345}\eta &= c_2\eta,
\end{aligned} \tag{24}$$

It satisfies the harmonic function rule but with a key difference: the harmonic functions are now independent of the single overall transverse direction and only depend on the relative transverse directions. That is, the $M5$-branes are now localised inside the directions tangent to the other $M5$-brane but are delocalised in the overall transverse direction that separates them.

Another interesting feature of this solution is that it does not satisfy the $(p-2)$ dimensional self-intersection rule for p-branes that we discussed in the last subsection. The resolution of this puzzle is quite interesting. A consequence of the Gamma-matrix projections (24) is that $\hat{\Gamma}_{0110}\eta = c_1 c_2 \eta$. This suggests that we can add an $M2$-brane in the $\{1, 10\}$ plane without breaking further supersymmetry. Note that such an $M2$-brane overlaps each of the $M5$-branes in a string which is allowed. The solution is given by [34, 52]

$$\begin{aligned} ds^2 &= (H_1 H_2)^{2/3} H_3^{1/3} [(H_1 H_2 H_3)^{-1}(-dt^2 + dx_1^2) + H_2^{-1}(dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2) \\ &\qquad + H_1^{-1}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) + H_3^{-1} dx_{10}^2] \\ F_{mnp10} &= -\frac{c_1}{2}\epsilon_{mnpq}\partial_q H_1, \qquad F_{\mu\nu\lambda 10} = -\frac{c_2}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\rho H_2 \qquad F_{t110I} = \frac{c_1 c_2}{2}\frac{\partial_I H_3}{H_3^2}, \end{aligned} \tag{25}$$

where $x_I = (X^1_m, X^2_\mu)$ and the function $H_3(X^1, X^2)$ corresponding to the $M2$-brane satisfies the equation

$$\left[H_1^{-1}(X^1)\nabla^2_{(X^1)} + H_2^{-1}(X^2)\nabla^2_{(X^2)}\right] H_3 = 0. \tag{26}$$

Functions of the form

$$H_3(X^1, X^2) = h_1(X^1) + h_2(X^2), \tag{27}$$

solve this equation if the h_i are harmonic on $\mathbb{E}^4$, but point singularities of h_1 or h_2 would represent $M2$-branes that are delocalized in four more directions. We expect that there exist solutions of (26) representing localized $M2$-branes although explicit solutions may be difficult to find. In the same way the solution $M2 - M5(1)$ can be thought of as being related to an $M2$-brane ending on an $M5$-brane we can think of the solution (25) as corresponding to an $M2$-brane being stretched between two $M5$-branes. This interpretation and the fact that we can add the extra $M2$-brane without breaking any more supersymmetry also provides a resolution of the fact that the solution (23) violates the $(p-2)$ self-intersection rule: when two $M5$-branes are brought together to intersect on a string, one should think of the intersection as being a collapsed $M2$-brane.

This observation suggests the following nomenclature: the solutions $M2 - M2(0)$, $M2 - M5(1)$ and $M5 - M5(3)$ can be called intersecting brane solutions, since when they do intersect (as opposed to overlap) they describe dynamical intersections. On the otherhand the $M5-M5(1)$ solution should be described as an overlap since it is not until we add an extra $M2$-brane that we get a dynamical intersection.

It is worth noting that if we remove one of the $M5$-branes in (25) we obtain a more general solution than the previous $M2-M5(1)$ solution (14) in that the equation for the $M2$-brane coming from (26) is more general than just a harmonic function in the overall transverse coordinates. Again we do not know of any interesting solutions in closed form. There are also generalisations of the $M2-M2(0)$ and $M5-M5(3)$ solutions where one of the M-branes satisfies a more general equation. These can be obtained by dimensional reduction and duality using the results of the next section. Finally we note that more general configurations of multi-intersecting M-branes can be obtained by combing these types of intersections with the previous ones. See [25] for some results in this direction.

3 Intersecting Branes in Type II String Theory

3.1 NS and D-branes

The D=10 type IIA supergravity action can be obtained from dimensional reduction on a circle of D=11 supergravity. The Kaluza-Klein ansatz for the bosonic fields leading to the string-frame 10-metric is

$$\begin{aligned} ds^2_{(11)} &= e^{-\frac{2}{3}\phi(x)}dx^\mu dx^\nu g_{\mu\nu}(x) + e^{\frac{4}{3}\phi(x)}(dy + dx^\mu C_\mu(x))^2 \\ A_{(11)} &= A(x) + B(x)\wedge dy \ , \end{aligned} \tag{28}$$

where $A_{(11)}$ is the D=11 three-form potential and x^μ are the D=10 spacetime coordinates. We read off from the right hand side the bosonic fields of D=10 IIA supergravity; these are the $NSNS$ fields $(\phi, g_{\mu\nu}, B_{\mu\nu})$ and the RR fields $(C_\mu, A_{\mu\nu\rho})$. The bosonic fields of the D=10 type IIB supergravity coming from the $NSNS$ sector are identical to that of the type IIA theory, $(\phi, g_{\mu\nu}, B^{(1)}_{\mu\nu})$. From the RR sector of the IIB theory there is an axion, another two-form and a four-form that has a self-dual field strength $(l, B^{(1)}_{\mu\nu}, A^{+}_{\mu_1\mu_2\mu_3\mu_4})$.

The rank of the various form potentials immediately suggests what the spectrum of BPS branes is. A potential of rank r has a field strength of rank $(r+1)$ that can be integrated along an $(r+1)$-sphere which in D spacetime dimensions surrounds a $(D-3-r)$-brane. The value of the integral gives the magnetic r-form charge carried by the the $(D-3-r)$-brane. Similarly, the field strength of rank $(D-1-r)$ that is Poincare dual to the $(r+1)$-form field strength can be integrated along a $(D-1-r)$ sphere that surrounds an $(r-1)$-brane. Now the integral gives the electric r-form charge carried by the $(r-1)$-brane. Of course one still needs to check that such solutions to the non-linear field equations exist and moreover to check if they admit any Killing spinors. This has been carried out and we record here the metric and dilaton behaviour of the various BPS solutions.

The IIA and IIB NS-strings are electrically charged with respect to the NS two-form. For both the type IIA and IIB theory we have:

$$\begin{aligned} ds^2 &= H^{-1}\left(-dt^2 + dx_1^2\right) + dx_2^2 + \ldots + dx_9^2 \\ e^{2\phi} &= H^{-1}, \qquad H = H(x_2, \ldots, x_9), \qquad \nabla^2 H = 0. \end{aligned} \tag{29}$$

The IIA and IIB $NS5$-branes carry magnetic NS two-form charge and we have:

$$\begin{aligned} ds^2 &= -dt^2 + dx_1^2 + \ldots + dx_5^2 + H\left(dx_6^2 + \ldots + dx_9^2\right) \\ e^{2\phi} &= H, \qquad H = H(x_6, \ldots, x_9), \qquad \nabla^2 H = 0. \end{aligned} \tag{30}$$

Finally, the Dp-branes carry either electric or magnetic charge with respect to the RR fields and the metric and dilaton are given by:

$$\begin{aligned} ds^2 &= H^{-1/2}\left(-dt^2 + dx_1^2 + \ldots + dx_p^2\right) + H^{1/2}\left(dx_{p+1}^2 + \ldots + dx_9^2\right), \\ e^{2\phi} &= H^{-\frac{(p-3)}{2}}, \qquad H = H(x_{p+1}, \ldots, x_9), \qquad \nabla^2 H = 0. \end{aligned} \tag{31}$$

Given the rank of the RR-forms that we mentioned above, we see that the type IIA theory has Dp-branes with $p = 0, 2, 4, 6$. There is an additional $D8$-brane which is related to massive type IIA supergravity and we refer the reader to [53] for more details. For the IIB theory we have $p = -1, 1, 3, 5, 7$. $p = -1$ corresponds to an instanton [60] and we won't include it in our discussions of intersecting branes. Note that we have written all of the above solutions in the sigma-model string metric which is related to the Einstein metric via $g_E = e^{-\phi/2} g_\sigma$.

All of these type II branes preserve 1/2 of the supersymmetry. The type II theories have two spacetime supersymmetries parameters given by Majorana-Weyl spinors ϵ_L, ϵ_R. In the type IIA theory they have opposite chirality and we choose $\Gamma_{10}\epsilon_L = \epsilon_L$ and $\Gamma_{10}\epsilon_R = -\epsilon_R$. In the type IIB theory they have the same chirality and we choose $\Gamma_{10}\epsilon_L, \epsilon_R = \epsilon_L, \epsilon_R$. The solutions have 16 Killing spinors which satisfy the following projections:

$$
\begin{aligned}
&\text{IIA/IIB} \quad NS\text{–strings}: && \epsilon_L = \hat{\Gamma}_{01}\epsilon_L && \epsilon_R = -\hat{\Gamma}_{01}\epsilon_R \\
&\text{IIA} \quad NS5\text{–branes}: && \epsilon_L = \hat{\Gamma}_{012345}\epsilon_L && \epsilon_R = \hat{\Gamma}_{012345}\epsilon_R \\
&\text{IIB} \quad NS5\text{–branes}: && \epsilon_L = \hat{\Gamma}_{012345}\epsilon_L && \epsilon_R = -\hat{\Gamma}_{012345}\epsilon_R \\
&\text{IIA/IIB} \quad Dp\text{–branes}: && \epsilon_L = \hat{\Gamma}_{01\ldots p}\epsilon_R. &&
\end{aligned}
\tag{32}
$$

The Gamma-matrix projections will have an extra minus sign for the corresponding anti-branes.

Let us first make a few brief comments on these different branes. The NS-string solutions that exist for each theory are simply identified with the fundamental string of each theory [56, 57, 58, 59]. The IIA and IIB $NS5$-branes of each theory are like solitons in quantum field theory in the sense that their tension T is related to the string coupling g and magnetic three-form charge Q via $T \sim Q/g^2$. These solitons have an elegant $(4, 4)$ superconformal field theory description which is illuminating, but incomplete [55]. Although the IIA and IIB $NS5$-branes have the same supergravity solution the world-volume theories that govern the low-energy dynamics of these solitons are quite different [54]. The IIA $NS5$-brane has $(0,2)$ supersymmetry on the six-dimensional world-volume just as the $M5$-brane. The bosonic fields consist of five real scalars and a two-form with self dual field strength. The world-volume theory of the IIB $NS5$-brane has $(1,1)$ supersymmetry whose bosonic field content is four scalars and a vector field.

The branes that carry charge with respect to the RR fields are the D-branes. These branes differ from the NS-branes in that their tension is related to the string coupling and charge via $T \sim Q/g$. This fact is closely related to the fact that D-branes have a very simple perturbative description in string theory [3]. At weak coupling, they are surfaces in flat spacetime where open strings can end i.e., if we let X^μ, $\mu = 0, \ldots, p$, be the coordinates tangent and X^T, $T = p+1, \ldots, 10$, be the coordinates transverse to the brane, then the strings coordinates $X^\mu(\tau, \sigma)$ satisfy Neumann boundary conditions and $X^T(\tau, \sigma)$ satisfy Dirichlet boundary conditions. This perturbative description has played a central role in recent developments in string theory. Note that $D9$-branes fill all of space and a closer analysis leads one to the type I theory. They are not associated with any supergravity solution. The world-volume theory for all Dp-branes is given by the dimensional reduction of ten-dimensional superYang-Mills theory to $(p+1)$ dimensions. The bosonic fields are $9-p$ scalars and a single vector field.

It will be convenient when we come to discussing intersecting brane solutions to know how the above solutions are related. We noted earlier that if we wrap the $M2$- or $M5$-brane on a circle we are led to the type IIA NS-string and $D4$-brane, respectively, while if we reduce the M-branes then we get the $D2$-brane and the $NS5$-brane, respectively. The second observation is that the type II brane solutions are related by various symmetries of the supergravity equations of motion. The type IIB supergravity has an $SL(2,\mathbb{R})$ symmetry of which an $SL(2,\mathbb{Z})$ is conjectured to survive as a non-perturbative symmetry of the string theory. The action on the low-energy fields is as follows: the $NSNS$ and RR two-forms $B^{(i)}_{\mu\nu}$ transform as a doublet, the self dual four-form $A^+_{\mu_1\mu_2\mu_3\mu_4}$ and the

Einstein metric are invariant and the dilaton and RR scalar can be packaged into a complex scalar $\tau = l + ie^{-\phi}$ which undergoes fractional liner transformations. The Z_2 "S-duality" transformation that interchanges the two-forms and acts as $\tau \to -1/\tau$ allows us to construct the $NS5$-brane and NS-string solutions from the $D5$-brane and $D1$-brane solutions, respectively, and vice-versa. Note that the $D3$-brane is left inert under this and all $SL(2,\mathbb{Z})$ transformations. For the behaviour of the $D7$-brane see [60]. If we employ more general $SL(2,\mathbb{Z})$ transformations then we obtain "non-marginal" BPS branes in the type IIB theory. Specifically if we start with a $NS5$-brane we obtain a (p,q) 5-brane that is a bound state of p $NS5$-branes and q $D5$-branes, with p and q relatively prime integers. Similarly from the NS-string we get (p,q) strings [61]. Since the $SL(2,\mathbb{Z})$ transformations do not break supersymmetry, all of these solutions preserve 1/2 of the supersymmetry: the projections on the Killing spinors are the $SL(2,\mathbb{Z})$ rotations of those in (32). The (p,q) 5-branes will play a role when we discuss branes intersecting at angles in the next section.

The other basic tool to relate various branes is T-duality. The type IIA theory compactified on a circle of radius R is T-dual to the IIB theory compactified on a circle of radius $1/R$. This can be established exactly in perturbation theory. Since T-duality interchanges Dirichlet with Neumann boundary conditions [3], if we perform T-duality in a direction transverse (tangent) to a Dp-brane we obtain a $D(p+1)$-brane ($D(p-1)$-brane). This can also be seen at the level of classical supergravity solutions using the fact that T-duality manifests itself as the ability to map a solution with an isometry into another solution. The action of T-duality on the NS fields with respect to a symmetry direction z, mapping string-frame metric to string-frame metric,is

$$
\begin{aligned}
d\tilde{s}^2 &= [g_{\mu\nu} - g_{zz}^{-1}(g_{\mu z}g_{z\nu} + B_{\mu z}B_{z\nu})]dx^\mu dx^\nu + 2g_{zz}^{-1}B_{z\mu}dzdx^\mu + g_{zz}^{-1}dzdz \\
\tilde{B} &= \frac{1}{2}dx^\mu \wedge dx^\nu[B_{\mu\nu} - g_{zz}^{-1}(g_{\mu z}B_{z\nu} + B_{\mu z}g_{z\nu})] + g_{zz}^{-1}g_{z\mu}dz \wedge dx^\mu \\
\tilde{\phi} &= \phi - \frac{1}{2}\log g_{zz}
\end{aligned} \tag{33}
$$

where x^μ are the rest of the coordinates and we have indicated the transformed fields by a tilde. These rules may be read as a map either from IIA to IIB or vice-versa. The action on the RR gauge fields can be found in [75]. For our applications the new solution obtained by T-duality will preserve the same amount of supersymmetry as the original one. Using these transformations we again conclude that if we perform T-duality on a direction tangent to a Dp-brane solution (31) then we are led to a $D(p-1)$-brane solution. For example, the metric component $g_{zz}=H^{-1/2}\to H^{1/2}$. But note that we do not arrive at the most general solution as the harmonic function of the $D(p-1)$-brane is invariant under the z direction. Similarly, if we take a Dp-brane that is delocalised in a transverse direction and perform T-duality in that direction we get a $D(p+1)$-brane solution. Performing T-duality on a direction transverse to a IIA/B fundamental string (29) delocalised in that direction will transform it into a IIB/A fundamental string. Acting on a direction tangent to the IIA/B string, say in the $\{1\}$ direction, will replace the B_{01} component of the NS two-form in the string solution with an off diagonal term in the metric g_{01}. The final solution is a pp-wave of the IIB/A theory. Note that this is is the spacetime manifestation of the fact that in perturbation theory T-duality interchanges winding and momentum modes. The supersymmetry projections for pp-waves travelling in a given direction for both the IIA and IIB theories are:

$$\mathrm{IIA/IIB}\quad \mathrm{pp-wave:}\qquad \epsilon_L=\hat{\Gamma}_{01}\epsilon_L\qquad\qquad \epsilon_R=\hat{\Gamma}_{01}\epsilon_R. \tag{34}$$

Acting with T-duality in a direction tangent to a IIA/B $NS5$-brane (30) will lead to a IIB/A $NS5$-brane. If it is in a direction transverse to a delocalised $NS5$-brane then we again get off diagonal terms in the metric. One finds that the non-trivial part of the metric is given by Taub-NUT space, which we will review in the next section. These T-duality results are summarised in Table 1.

Using these duality transformations we can essentially obtain all others starting from, say the $M2$-brane. Reducing the $M2$-brane to D=10 we obtain the IIA $D2$-brane. T-dualising this solution leads to all Dp-brane solutions of the IIA and IIB theory. To implement this for $p\geq 7$ one must use the massive IIA supergravity [53]. On the otherhand S-duality on the $D1$ and $D5$-branes gives the IIB NS-string and the $NS5$-

	Tangent	Transverse
$NS1$	$\mathrm{pp-wave}$	$NS1$
$NS5$	$NS5$	$\mathrm{Taub-NUT}$
Dp	$D(p-1)$	$D(p+1)$

Table 1: T-Duality Rules For Type II Branes

brane solutions, respectively. The corresponding IIA NS-branes are then obtained by T-dualising on a transverse or tangent direction, respectively. Similarly, we obtain the IIA/B pp-waves (Taub-NUT) from the IIB/A NS-string ($NS5$-brane) by T-dualising on a tangent (transverse) direction. The $M5$-brane solution can be obtained by "uplifting" either the $D4$-brane or the IIA $NS5$-brane to D=11. Uplifting the IIA $D0$-brane gives a $D = 11$ pp-wave and, as we shall discuss in the next section, uplifting the $D6$-brane gives Taub-NUT space. Note that in performing these transformations we will be led to the correct BPS solutions but possibly not the most general solution as the harmonic function may become delocalised in the procedure, as we noted above.

3.2 Intersecting NS and D-Branes

We can use the duality transformations discussed in the last subsection to obtain all intersecting brane solutions in type II string theory. Lets start with the $M2 - M2(0)$ solution with the $M2$-branes oriented along say the $\{1,2\}$ and $\{3,4\}$ directions. Reducing this configuration along an overall transverse direction we obtain a $D2 - D2(0)$ solution with the $D2$-branes having the same orientations. If we now perform T-duality in a direction parallel to one of the $D2$-branes, say the $\{2\}$ direction, we transform it into a $D1$-brane in the $\{1\}$ direction and the other $D2$-brane into a $D3$-brane with orientation $\{2,3,4\}$. The final configuration is thus $D1 - D3(0)$. One can continue T-dualising in all possible ways and one generates the following list of intersecting D-branes [7, 8]:

$$
\begin{aligned}
IIA: \quad & 2-2(0); \quad 4-4(2); \quad 6-6(4); \quad 2-4(1); \quad 4-6(3); \\
& 6-8(5); \quad 0-4(0); \quad 2-6(2); \quad 4-8(4); \\
IIB: \quad & 3-3(1); \quad 5-5(3); \quad 7-7(5); \quad 1-3(0); \quad 3-5(2); \\
& 5-7(4); \quad 1-5(1); \quad 3-7(3); \quad 5-9(5);
\end{aligned} \tag{35}
$$

These solutions all preserve 1/4 of the supersymmetry and can be directly constructed using the analogue of the harmonic function rule we used for M-branes. The harmonic functions for each brane depends on the overall transverse coordinates. Note that for the cases where the branes overlap in a 5-brane the overall transverse directions have shrunk to a point and the derived solution is just Minkowski space. Their could however, be more general solutions for these cases since, for example, the case $5-9(5)$ corresponds to a type I $D5$-brane which does correspond to a classical solution.

There is an elegant way of characterising these D-brane configurations in perturbation theory. Consider open strings with one end on each of the two intersecting branes. The string coordinates can either be NN, DD or ND depending on whether the coordinate has Neumann (N) or Dirichlet (D) boundary conditions at each end. The number of coordinates with mixed ND boundary conditions is equal to the number of relative transverse directions and is four in all of the above cases. One can also show in perturbation theory that these configurations preserve 1/4 of the supersymmetry [3].

If we also act with S-duality in the type IIB theory we can generate solutions containing NS-branes. Further acting with T-duality gives:

$$
\begin{aligned}
& Dp-NS1(0), \qquad 0 \leq p \leq 8; \\
& Dp-NS5(p-1), \qquad 1 \leq p \leq 6; \\
& NS1-NS5(1); \qquad NS5-NS5(3); \\
& NS1+W; \quad NS5+W; \quad Dp+W, \quad 1 \leq p \leq 9,
\end{aligned} \tag{36}
$$

where the configurations in the last line correspond to pp-waves which travel in one direction tangent to the brane and the solutions with NS-branes only are valid in both IIA and IIB. Note that we have not included Taub-NUT configurations. We also have not included "non-marginal" configurations that are obtainable by employing more general $SL(2,\mathbb{Z})$ transformations. The $M2-M5(1)$ and the $M5-M5(3)$ solutions can both be obtained from the above IIA solutions, as claimed earlier. For example, one can uplift $NS1-NS5(1)$ and $D4-D4(2)$, respectively.

These configurations can be broadly classified into three categories: self-intersections, branes ending on branes and branes within branes. Let us make some general comments on each of these. The D-brane self intersections in (35), $Dp-Dp(p-2)$, and the $NS5$ self intersection for IIA and IIB in (36), $NS5-NS5(3)$, all satisfy the $(p-2)$ self-intersection rule that we described earlier. The second category is where branes can end on branes, $p-q(p-1)$. Although the solutions are too general to directly describe this setup we expect that this is the physical situation they are naturally associated with in the same way that we explained for the $M2-M5(1)$ configuration (14). The best understood example of branes ending on branes is the case $NS1-Dp(0)$ which corresponds to a fundamental string ending on a Dp-brane. Note that the end of the fundamental string appears as either an electric or magnetic point source in the D-brane world-volume. All other cases of the form $p-q(p-1)$ can be obtained by S- and T- duality on these configurations (which thus supports the brane ending on brane interpretation). For example S-duality immediately gives the $D1-D3(0)$ and $D1-NS5(0)$ configurations in the IIB theory. It is perhaps worth highlighting some other cases: the $D3-NS5(2)$ case in the IIB theory was used in [43] to study three dimensional gauge theories on the 2+1-dimensional intersection. The case of a $D4$-brane ending on a $NS5$-brane, $NS5-D4(3)$ in IIA was used to study four-dimensional gauge theories [62]. It is interesting that this can be lifted to M-theory as a single $M5$-brane [45]. The third category of configurations are the branes inside of branes which have the form $p-q(p)$. These correspond to brane soliton solutions inside the world-volume theory. As an example consider $D0-D4(0)$. If we consider N parallel $D4$-branes then we should consider a $U(N)$ super-Yang-Mills theory on the $4+1$ dimensional world-volume [63]. Four dimensional euclidean $U(N)$ instantons correspond to static solitons in the world-volume theory which can also be interpreted as $D0$-branes [64].

We now comment on some different configurations of branes that give rise to D=4 and D=5 black holes. Start with the $M2-M5(1)$ configuration with a pp-wave along the intersection (20) which gives a D=5 black hole upon dimensional reduction. One can now perform the following steps:

$$
\begin{array}{rlllllll}
M5: & 1 & 2 & 3 & 4 & 5 & & \\
M2: & 1 & & & & & 10 & \stackrel{R10}{\longrightarrow} \\
W: & 1 & & & & & &
\end{array}
\quad
\begin{array}{rllllll}
NS5: & 1 & 2 & 3 & 4 & 5 & \\
NS1: & 1 & & & & & \stackrel{T1}{\longrightarrow} \\
W: & 1 & & & & &
\end{array}
$$

$$
\begin{array}{rllllll}
NS5: & 1 & 2 & 3 & 4 & 5 & \\
W: & 1 & & & & & \stackrel{S}{\longrightarrow} \\
NS1: & 1 & & & & &
\end{array}
\quad
\begin{array}{rlllll}
D5: & 1 & 2 & 3 & 4 & 5 \\
W: & 1 & & & & \\
D1: & 1 & & & &
\end{array}
\qquad (37)
$$

where we have dimensionally reduced on the 10 direction to get a IIA solution, T-dualised on the 1 direction to get a IIB solution and then performed S-duality. The resulting IIB configuration, $D5-D5(1)$ plus a pp-wave [65], is the case that has been most studied in black hole entropy studies. We noted that the $M2-M2-M2$ solution (18) can also give a D=5 black hole. It can be related to the above configuration by dimensional reduction and duality:

$$
\begin{array}{rlllllll}
M2: & 1 & & & & & 10 & \\
M2: & & 2 & 3 & & & & \stackrel{R10}{\longrightarrow} \\
M2: & & & & 4 & 5 & &
\end{array}
\quad
\begin{array}{rllllll}
N1: & 1 & & & & & \\
D2: & & 2 & 3 & & & \stackrel{T145}{\longrightarrow} \\
D2: & & & & 4 & 5 &
\end{array}
$$

$$
\begin{array}{rlllll}
W: & 1 & & & & \\
D5: & 1 & 2 & 3 & 4 & 5 \\
D1: & 1 & & & &
\end{array}
\quad . \qquad (38)
$$

We mentioned two ways in which D=4 black holes can be obtained from intersecting

M-branes: $M5-M5-M5$ with momentum flowing along a common string direction (22), and $M5 - M5 - M2 - M2$ (19). Both of these can be related to a very symmetrical configuration of four $D3$-branes [6, 9]. The second case works as follows:

$$\begin{array}{lccccccc}
M5: & 1 & 2 & 3 & 4 & 5 & & \\
M5: & 1 & 2 & 3 & & & 6 & 10 \\
M2: & & & & 4 & & 6 & \\
M2: & & & & & 5 & & 10
\end{array}
\quad \xrightarrow{R10} \quad
\begin{array}{lcccccc}
NS5: & 1 & 2 & 3 & 4 & 5 & \\
D4: & 1 & 2 & 3 & & & 6 \\
D2: & & & & 4 & & 6 \\
NS1: & & & & & 5 &
\end{array}
\quad \xrightarrow{ST1}$$

$$\begin{array}{lcccccc}
D5: & 1 & 2 & 3 & 4 & 5 & \\
D3: & & 2 & 3 & & & 6 \\
D3: & 1 & & & 4 & & 6 \\
D1: & & & & & 5 &
\end{array}
\quad \xrightarrow{T34} \quad
\begin{array}{lcccccc}
D3: & 1 & 2 & & & 5 & \\
D3: & & 2 & & 4 & & 6 \\
D3: & 1 & & 3 & & & 6 \\
D3: & & & 3 & 4 & 5 &
\end{array}. \tag{39}$$

We now turn to the overlapping brane solutions that can be generated from the $M5-M5(1)$ overlap (23). Reducing on the common string direction we get $D4 - D4(0)$ and T-duality generates the list of overlapping D-branes:

$$\begin{array}{llllll}
IIA: & 0-8(0); & 2-6(0); & 2-8(1); & 4-4(0); & 4-6(1); \\
IIB: & 1-7(0); & 1-9(1); & 3-5(0); & 3-7(1); & 5-5(1).
\end{array} \tag{40}$$

These solutions all break 1/4 of the supersymmetry and can also be directly constructed using the harmonic function rule but taking into account that the harmonic functions depend on the relative transverse coordinates not the overall transverse coordinates. At the level of string perturbation theory, these configurations correspond to D-branes that have eight string coordinates with mixed ND boundary conditions. Employing S-duality we obtain the following configurations with NS branes:

$$\begin{array}{ll}
NS5 - Dp(p-3) & 3 \leq p \leq 8; \\
NS5 - NS5(1). &
\end{array} \tag{41}$$

Recall that an extra $M2$-brane can be added to the $M5 - M5(1)$ solution without breaking any more supersymmetry and the resulting configuration can be interpreted as an $M2$-brane stretched between the two $M5$-branes. After dimensional reduction of this solution and performing dualities we find analogous generalisations of the above solutions. For the D-brane intersections in (40) that intersect in a point, we find that we can add a fundamental string without breaking any more supersymmetry. This can then be interpreted as a fundamental string being stretched between the two D-branes. If the D-branes intersect in a string then we find that we can add a pp-wave along this string intersection without breaking any more supersymmetry. One interesting example of this is the $1-9(1)$ case. Since the IIB $D9$-brane leads to the type I theory, our interpretation translates into the fact that a type I D-string can carry momentum without breaking any more supersymmetry. This D-string can be interpreted as a heterotic string soliton [66] and the properties of the heterotic string with momentum were studied in detail in [58, 59]. For the cases with NS- and D-branes (41) we find that we can add a $D(p-2)$-brane that can be thought of as being stretched between the $NS5$-brane and the Dp-brane that ends on each in a $D(p-3)$ brane, cases that were considered above in (36). One example of this is a $D3$-brane stretched between a $NS5$-brane and a $D5$-brane, intersecting each on a two-brane. This setup was considered in [43]. The $NS5-NS5(1)$ case is considered below.

In the next section we will be considering some of these configurations but generalised so that the branes intersect at angles. In preparation for this let us be a little more explicit about some cases. Start with the $NS5-D5(2)$ configuration of IIB with the extra $D3$-brane which preserves $1/4$ of the supersymmetry and perform T-duality in one of the common intersection directions:

$$
\begin{array}{lccccccccccclcccccccccc}
N5: & 1 & 2 & 3 & 4 & 5 & & & & & & N5: & 1 & 2 & 3 & 4 & 5 & & & & \\
D5: & 1 & 2 & & & & & 7 & 8 & 9 & \xrightarrow{T2} & D4: & 1 & & & & & & 7 & 8 & 9 \\
D3: & 1 & 2 & & & & 6 & & & & & D2: & 1 & & & & & 6 & & &
\end{array}
\qquad (42)
$$

We can now uplift this IIA solution to give the M-theory solution (25):

$$\begin{array}{lllllllllll} M5: & 1 & 2 & 3 & 4 & 5 & & & & & \\ M5: & 1 & & & & & & 7 & 8 & 9 & 10\,. \\ M2: & 1 & & & & & 6 & & & & \end{array} \tag{43}$$

Reducing on the 6 direction and then relabeling the 10 direction as the 6 direction we get the IIA solution

$$\begin{array}{llllllllll} NS5: & 1 & 2 & 3 & 4 & 5 & & & & \\ NS5: & 1 & & & & & 6 & 7 & 8 & 9\,. \\ NS1: & 1 & & & & & & & & \end{array} \tag{44}$$

Performing T-duality on the 1 direction we get the IIB configuration

$$\begin{array}{llllllllll} NS5: & 1 & 2 & 3 & 4 & 5 & & & & \\ NS5: & 1 & & & & & 6 & 7 & 8 & 9\,. \\ W: & 1 & & & & & & & & \end{array} \tag{45}$$

It is interesting to note that in the IIA theory we can add a fundamental string to the $NS5-NS5(1)$ configuration without breaking any more supersymmetry, while in the IIB theory we can add a pp-wave. Since the IIA and IIB theories have the same NS-fields the configuration (44) does give a solution of the IIB theory, but it breaks 1/8 of the supersymmetry not 1/4. Finally carrying out S-duality on (45) we get

$$\begin{array}{llllllllll} D5: & 1 & 2 & 3 & 4 & 5 & & & & \\ D5: & 1 & & & & & 6 & 7 & 8 & 9\,. \\ W: & 1 & & & & & & & & \end{array} \tag{46}$$

4 Branes Intersecting at Angles

In the configurations that we have studied so far all of the branes have orthogonal intersections. In a perturbative D-brane context it has been pointed out that certain rotations away from orthogonality lead to configurations that still preserve some supersymmetry [67]. In this section we will summarise some recent work on constructing classical supergravity solutions that describe such intersections [37]. Other recent work on finding configurations with non-orthogonal intersections will not be discussed [36][38]-[41]. The solutions in [37] which we shall describe are much more complicated than the ones we have seen so far. They have a common origin in D=11 using toric Hyper-Kähler manifolds. To motivate the solutions we shall first begin by recasting some of the orthogonal solutions in a similar language.

4.1 Taub-NUT space and Overlapping Branes

We begin by reviewing the construction of the $D6$-brane solution of the type IIA theory in terms of Taub-NUT space [70]. Taub-NUT space is a four-dimensional Hyper-Kähler manifold. That is, the manifold admits three covariantly constant complex structures $J^{(m)}$ and the metric is Kahler with respect to each. Consider the Hyper-Kähler metrics

$$\begin{aligned} ds^2 &= V(\mathbf{x})d\mathbf{x}\cdot d\mathbf{x} + V^{-1}(\mathbf{x})(d\psi + \mathbf{A}(\mathbf{x})\cdot d\mathbf{x})^2, \\ \nabla\times\mathbf{A} &= \nabla V. \end{aligned} \tag{47}$$

Choosing the harmonic function V to have single centre, $V = 1 + m/r$, and hence $\mathbf{A} = m\cos\theta d\phi$, where (r,θ,ϕ) are spherical polar coordinates on $\mathbb{E}^3$, gives Taub-NUT space. The metric appears singular at $r = 0$ but this is in fact a coordinate singularity if we choose ψ to be a periodic coordinate with period $4\pi m$. The $U(1)$ isometry corresponding to shifts in the coordinate ψ is tri-holomorphic i.e., the Lie derivative of the complex structures with respect to the $U(1)$ killing field vanishes. The topology of each surface with fixed r is a three sphere which is a circle bundle over a two-sphere base with ψ being the coordinate of the fibre. The global topology of the manifold is $\mathbb{E}^4$. From the metric we note that as $r\to\infty$ the radius of the circle approaches $4\pi m$ which suggests that we can use Taub-NUT space in a Kaluza-Klein setting [68, 69]. Since it is Hyper-Kähler the manifold is automatically Ricci-flat and hence will solve Einstein's equations. We can use this to give an D=11 supergravity solution by adding in 6+1 Minkowski space:

$$ds^2 \;\; = \;\; -dt^2 + dx_1^2 + \ldots + dx_6^2 + ds^2_{\rm TN}$$

$$A = 0. \tag{48}$$

If we now reduce this solution along the $U(1)$ Killing-vector using (28) we obtain the $D6$-brane solution with metric and dilaton as in (31) with the non-trivial RR one-form coming from the off diagonal terms in the metric. It is worth emphasising that while the $D6$-brane is a singular solution in ten-dimensions, it has a non-singular resolution in M-theory. If V is multicentred, $V = 1 + \Sigma m_i/r_i$ with $r_i = |\mathbf{x} - \mathbf{x}_i|$, then we obtain the multicentre Hyper-Kähler manifolds. They are non-singular provided that no two centres coincide. Upon dimensional reduction they give rise to parallel $D6$-branes.

If we relabel the Taub-NUT coordinates $(\mathbf{x}, \psi)$=$(x_7, \ldots, x_{10})$, then the solution (48) has 16 Killing spinors which satisfy the constraints

$$\epsilon = \hat{\Gamma}_{78910}\epsilon. \tag{49}$$

This is equivalent to the $D6$-brane constraint (32). if we reduce on x_{10}. Consider now the Taub-NUT space to lie along the $(x_3, \ldots, x_6)$ directions with x_6 being the coordinate on the circle. If we now reduce along x_{10} then we arrive at the IIA Taub-NUT configuration. Recall that if we now T-dualise in the circle direction x_6 we obtain the IIB $NS5$-brane (delocalised in the x_6 direction transverse to the brane). This will be useful in a moment. For completeness we note here that the supersymmetry projections for a IIA or IIB Taub-NUT configuration in the $(x_3, \ldots, x_6)$ direction is

$$\mathrm{IIA/B} \quad \mathrm{Taub-NUT:} \qquad \epsilon_L = \hat{\Gamma}_{3456}\epsilon_L \qquad \epsilon_R = \hat{\Gamma}_{3456}\epsilon_R. \tag{50}$$

A natural generalisation of the above construction of the $D6$-brane is to consider an eight dimensional Ricci-flat manifold obtained as the product of two Taub-NUTS. By adding in 2+1 dimensional Minkowski space we get a D=11 supergravity solution:

$$\begin{aligned} ds^2 &= ds^2(\mathbb{E}^{1,2}) + ds^2_{\mathrm{TN}_1} + ds^2_{\mathrm{TN}_2} \\ A &= 0. \end{aligned} \tag{51}$$

Label the coordinates of the circles of the two Taub-NUT metrics by x_6 and x_{10}, respectively. Reduce on the x_{10} direction to get a IIA configuration and then T-dualise on the x_6 direction to get a type IIB solution. Reducing the second Taub-NUT on x_{10} leads to a $D6$-brane in the $1, \ldots, 6$ directions and T-dualising in the x_6 direction transforms it into a $D5$-brane. On the other hand, reducing the first Taub-NUT on x_{10} gives IIA Taub-NUT and the T-duality turns it into a IIB $NS5$-brane. Since both branes share the 2+1 dimensional space we see that the final configuration is a $D5$-brane orthogonally overlapping a $NS5$-brane in a two brane, $D5 - NS5(2)$, which is a solution we have already considered. Recall that by a sequence of dualities we can relate it to the first pair of branes in (42)-(46).

4.2 Toric Hyper-Kähler Manifolds and Branes Intersecting at Angles

To obtain solutions corresponding to non-orthogonally overlapping branes we replace Taub-NUT×Taub-NUT by an eight-dimensional toric hyper-Kähler manifold i.e., one that admits a $U(1) \times U(1)$ triholomorphic isometry:

$$
\begin{aligned}
ds^2 &= ds^2(\mathbb{E}^{1,2}) + ds^2_{HK} \\
A &= 0.
\end{aligned}
\tag{52}
$$

After dimensional reduction and dualities we shall get solutions with branes as in (42)-(46) (ignoring the last entry) that overlap non-orthogonally. We shall discuss the inclusion of the other brane later. One interesting aspect of these solutions is that they all come from completely regular D=11 metrics.

All of these solutions will generically preserve 3/16 of the supersymmetry. The proof of this is essentially an application of the methods used previously in the context of KK compactifications of D=11 supergravity (see, for example, [71]). We first decompose the 32-component Majorana spinor of the D=11 Lorentz group into representations of $SL(2;\mathbb{R}) \times SO(8)$:

$$\mathbf{32} \to (\mathbf{2}, \mathbf{8}_s) \oplus (\mathbf{2}, \mathbf{8}_c) \ . \tag{53}$$

The two different 8-component spinors of $SO(8)$ correspond to the two possible $SO(8)$ chiralities. The unbroken supersymmetries correspond to singlets in the decomposition of the above $SO(8)$ representations with respect to the holonomy group $\mathcal{H}$ of $\mathcal{M}$. Consider for example, D=11 Minkowski space for which $\mathcal{H}$ is trivial; in this case both 8-dimensional spinor representations decompose into 8 singlets, so that all supersymmetries are preserved. The generic holonomy group for an eight-dimensional hyper-Kähler manifold is $Sp(2)$, for which we have the following decomposition of the $SO(8)$ spinor representations:

$$\begin{aligned} \mathbf{8}_s &\to \mathbf{5} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \\ \mathbf{8}_c &\to \mathbf{4} \oplus \mathbf{4} \ . \end{aligned} \tag{54}$$

There are now a total of 6 singlets (three $SL(2;\mathbb{R})$ doublets) instead of 32, so that the D=11 supergravity solution preserves 3/16 of the supersymmetry, unless the holonomy happens to be a proper subgroup of $Sp(2)$ in which case the above representations must be further decomposed. For example, the **5** and **4** representations of $Sp(2)$ have the decomposition

$$\begin{aligned} \mathbf{5} &\to (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1}) \\ \mathbf{4} &\to (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}) \end{aligned} \tag{55}$$

into representations of $Sp(1) \times Sp(1)$. We see in this case that there are two more singlets (one $SL(2;\mathbb{R})$ doublet), from which it follows that the solution preserves 1/4 of the supersymmetry whenever the holonomy is $Sp(1) \times Sp(1)$. Since this is the holomony group for Taub-NUT$\times$Taub-NUT space, we recover our previous result.

4.3 Toric hyper-Kähler manifolds

To proceed we need to be more concrete about the properties of eight dimensional toric hyper-Kähler manifolds. The most general metric has the local form

$$ds^2 = U_{ij}\, d\mathbf{x}^i \cdot d\mathbf{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j), \tag{56}$$

where U_{ij} are the entries of a positive definite symmetric 2×2 matrix function U of the 2 sets of coordinates $\mathbf{x}^i = \{x^i_r\,; r = 1,2,3\}$ on each of 2 copies of $\mathbb{E}^3$, and U^{ij} are the entries of U^{-1}. The two one-forms A_i have the form $A_i = d\mathbf{x}^j \cdot \boldsymbol{\omega}_{ji}$ where $\boldsymbol{\omega}$ is a triplet of 2×2 matrix functions of coordinates on $\mathbb{E}^6$ and are determined by the matrix U_{ij}. Specifically, the two-forms $F_i = dA_i$ with components

$$F^{rs}_{jk\,i} = \partial^r_j \omega^s_{ki} - \partial^s_k \omega^r_{ji}\ , \tag{57}$$

must satisfy

$$F^{rs}_{jk\,i} = \varepsilon^{rst}\partial^t_j U_{ki}, \tag{58}$$

where we have introduced the notation

$$\frac{\partial}{\partial x^i_r} = \partial^r_i\ . \tag{59}$$

Note that $dF_i = 0$ implies

$$\partial_i \cdot \partial_j U = 0 \qquad (i,j = 1,2)\ . \tag{60}$$

The simplest hyper-Kähler manifold, which may be considered to represent the 'vacuum', is constant U which implies $A_i = 0$. We shall denote this constant 'vacuum matrix' by $U^{(\infty)}$. For our applications we may restrict $U^{(\infty)}$ to be such that

$$\det U^{(\infty)} = 1. \tag{61}$$

Regular non-vacuum hyper-Kähler metrics can be found by superposing this with some linear combination of matrices of the form

$$U_{ij}[\{p\}, \mathbf{a}] = \frac{p_i p_j}{2|\sum_k p_k \mathbf{x}^k - \mathbf{a}|}, \tag{62}$$

where the 'p-vector' $\{p_1, p_2\}$ is an ordered set of coprime integers and $\mathbf{a}$ is an arbitrary 3-vector. Any matrix of this form may be associated with a 3-plane in $\mathbb{E}^6$, specified by the 3-vector equation

$$p_1 \mathbf{x}^1 + p_2 \mathbf{x}^2 = \mathbf{a}. \tag{63}$$

If we have two p-vectors the angle between the two 3-planes can be determined and is given by:

$$\cos\theta = \frac{p \cdot p'}{\sqrt{p^2 p'^2}}, \tag{64}$$

with inner product

$$p \cdot q = (U^{(\infty)})^{ij} p_i q_j. \tag{65}$$

The general non-singular metric may now be found by linear superposition. For a given p-vector we may superpose any finite number $N(\{p\})$ of solutions with various distinct 3-vectors $\{\mathbf{a}_m(\{p\});\ m = 1, \ldots, N\}$. We may then superpose any finite number of such solutions. This construction yields a solution of the hyper-Kähler conditions of the form

$$U_{ij} = U_{ij}^{(\infty)} + \sum_{\{p\}} \sum_{m=1}^{N(\{p\})} U_{ij}[\{p\}, \mathbf{a}_m(\{p\})]. \tag{66}$$

Since each term in the sum is associated with a 3-plane in $\mathbb{E}^6$, any given solution is specified by the angles and distances between some finite number of mutually intersecting 3-planes [72]. It can be shown that the resulting hyper-Kähler 8-metric is complete provided that no two intersection points, and no two planes, coincide. This is the analogue of the four dimensional multicentre metrics being singular when two centres coincide and has been demonstrated by means of the hyper-Kähler quotient construction in [37].

The simplest examples of these manifolds are found by supposing $\Delta U \equiv U - U^{(\infty)}$ to be diagonal. For example,

$$U_{ij} = U_{ij}^{(\infty)} + \delta_{ij} \frac{1}{2|\mathbf{x}^i|}. \tag{67}$$

which is constructed from the p-vectors $(1,0)$ and $(0,1)$. Hyper-Kähler metrics with U of this form were found previously on the moduli space of 2 distinct fundamental BPS monopoles in maximally-broken rank four gauge theories [73] (see also [74]). For this reason we shall refer to them as 'LWY metrics'. Whenever ΔU is diagonal we may choose the two one-forms A_i to be one-forms on the ith Euclidean 3-space satisfying

$$F_i = \star dU_{ii} \qquad (i=1,2), \tag{68}$$

where $\star$ is the Hodge dual on $\mathbb{E}^3$.

For the special case in which not only ΔU but also $U^{(\infty)}$ is diagonal then U is diagonal and the LWY metrics reduce to the metric product of 2 Taub-Nut metrics with $Sp(1)\times Sp(1)$ holonomy. Note that for the LWY metrics the angle between the 3-planes, (64) reduces to

$$\cos\theta = -\frac{U_{12}^{(\infty)}}{\sqrt{U_{11}^{(\infty)}U_{22}^{(\infty)}}}, \tag{69}$$

and we see that $Sp(1)\times Sp(1)$ holonomy occurs when the 3-planes intersect orthogonally. In general one can argue that the holonomy of a general toric Hyper-Kähler manifold is $Sp(2)$ and is only a proper subgroup of $Sp(2)$ when there are only two 3-planes or two sets of parallel 3-planes intersecting orthogonally, in which case the metric is a product of two Hyper-Kähler 4-metrics.

4.4 Overlapping branes from hyper-Kähler manifolds

Let us return to the interpretation of our D=11 solution (52) for a general hyper-Kähler manifold specified by a matrix U as in the last subsection. We follow the steps that we considered when we discussed Taub-NUT×Taub-NUT. We first reduce the solution along one of the $U(1)$ Killing vectors to obtain a IIA solution that preserves 3/16 of the supersymmetry and then T-dualise along the other $U(1)$ Killing vector. Using the T-duality rules of [75] we get a IIB solution with Einstein metric and other fields given by

$$
\begin{aligned}
ds_E^2 &= (\det U)^{\frac{3}{4}}[(\det U)^{-1}ds^2(\mathbb{E}^{2,1}) + (\det U)^{-1}U_{ij}d\mathbf{x}^i \cdot d\mathbf{x}^j + dz^2] \\
B_{(i)} &= A_i \wedge dz \\
\tau &= -\frac{U_{12}}{U_{11}} + i\frac{\sqrt{\det U}}{U_{11}}.
\end{aligned}
\tag{70}
$$

As the interpretation of this solution is rather subtle lets first consider continuing with the transformations as in (42)-(46): T-dualising on one of the $\mathbb{E}^{2,1}$ directions leads to a IIA solution which we shall not write down. If we uplift it to D=11 one obtains:

$$
\begin{aligned}
ds_{11}^2 &= (\det U)^{\frac{2}{3}}[(\det U)^{-1}ds^2(\mathbb{E}^{1,1}) + (\det U)^{-1}U_{ij}\, dX^i \cdot dX^j + dy^2] \\
F &= F_i \wedge d\varphi^i \wedge dy,
\end{aligned}
\tag{71}
$$

where $X^i = (\mathbf{x}^i, \phi^i)$ with ϕ^i the coordinates of the torus that is (essentially) dual to the one with coordinates ϕ_i. We shall start by considering the case in which U is diagonal. In the simplest of these cases the 8-metric is the metric product of two Euclidean Taub-Nut metrics, each of which is determined by a harmonic function with a single pointlike singularity. Let $H_i = [1 + (2|\mathbf{x}^i|)^{-1}]$ be the two harmonic functions; then

$$
U = \begin{pmatrix} H_1(\mathbf{x}^1) & 0 \\ 0 & H_2(\mathbf{x}^2) \end{pmatrix},
\tag{72}
$$

and we return to the $M5 - M5(1)$ solution (23). Note that in this derivation, the H_i are harmonic on the ith copy of $\mathbb{E}^3$, rather than on the ith copy of $\mathbb{E}^4$ and hence each of the $M5$-branes are delocalised in the direction between them and in one direction tangent to the other $M5$-brane. Next generalising to the LWY metrics (67) we still interpret the singularities in U to be the locations of the two (delocalised) $M5$-branes. Since the $M5$-branes have a string direction in common, the configuration is determined by the relative orientation of two 4-planes in the 8-dimensional space spanned by both. Because of the delocalisation the angle between the two four planes is taken to be the angle between the singular three planes (69). It can be argued that this rotation can be thought of as an $Sp(2)$ rotation of one $M5$-brane relative to the other in $\mathbb{E}^8$ [37]. We thus conclude that the process of rotating one $M5$-brane away from another by an $Sp(2)$ rotation preserves 3/16 supersymmetry. In the more general case in which ΔU is non-diagonal the solution can be interpreted as an arbitrary number of $M5$-branes intersecting at angles determined by the associated p-vectors; these angles are restricted only by the condition that the pairs of integers p_i be coprime¶. It is an interesting open question whether these 3/16 supersymmetric solutions can be generalized to allow U to depend on all eight coordinates $\{X^{(i)},\, i=1,2\}$.

Reducing on the overall transverse coordinate we obtain a IIA solution which we shall omit. In the simplest case that U is diagonal as in (72) it is the $NS5-NS5(1)$ solution in (44). For more general U there is an arbitrary number of $NS5$-branes intersecting at angles determined by their p-vectors as in the $M5$-brane case. Again there is a delocalisation in one direction tangent to each of the $NS5$-branes. If we now T-dualize in the common string direction we obtain a solution involving IIB $NS5$-branes with an identical interpretation. This may be mapped to a similar configuration involving only $D5$-branes by S-duality. In this way we deduce that

$$\begin{aligned} ds^2_E &= (\det U)^{\frac{1}{4}}[ds^2(\mathbb{E}^{1,1}) + U_{ij}\, dX^i \cdot dX^j] \\ B' &= A_i \wedge d\varphi^i \\ \tau &= i\sqrt{\det U}\ , \end{aligned} \tag{73}$$

is also solution of IIB supergravity preserving 3/16 supersymmetry. In the simplest case, in which U is of LWY type, this solution represents the intersection on a string of two $D5$-branes, with one rotated relative to the other by an $Sp(2)$ rotation with angle θ, given by (69). We are now in a position to make contact with the work of Berkooz, Douglas and Leigh [67]. They considered two intersecting Dirichlet (p+q)-branes with

¶Note that this condition comes from demanding that the Hyper-Kählermanifold is regular. If we just wanted to have solutions to the supergravity equations of motion then we could allow the p_i to be arbitrary real numbers.

a common q-brane overlap in perturbation theory. According to their analysis, each configuration of this type is associated with an element of $SO(2p)$ describing the rotation of one (p+q)-brane relative to the other in the 2p-dimensional relative transverse space. The identity element of $SO(2p)$ corresponds to parallel branes, which preserve 1/2 the supersymmetry. Other elements correspond to rotated branes. The only case considered explicitly in [67] was an $SU(p)$ rotation, but it was noted that the condition for unbroken supersymmetries was analogous to the reduced holonomy condition arising in KK compactifications. The case we are considering corresponds to an $Sp(2)$ rotation in $SO(8)$. The analysis of [67] was generalised in [37] to show that this setup preserves 3/16 supersymmetry. In addition the solution (73) shows that at least for the $Sp(2)$ case the analogy with holonomy is exact since this IIB solution is dual to a non-singular D=11 spacetime of $Sp(2)$ holonomy.

Let us now return to the interpretation of the IIB solution (70). When U is diagonal we obtain the $NS5 - D5(2)$ solution. Since the two fivebranes share two common directions, the singular three planes correspond to the location of the fivebranes in the six-dimension space. For this case the fivebranes are just delocalised in the extra direction that separates them i.e., there is no further delocalisation in directions tangent to the other brane as above. By studying the action of $SL(2,\mathbb{Z})$ on the solution and recalling that a IIB (p,q) 5-brane can be constructed using $SL(2,\mathbb{Z})$ transformations, we come to the following interpretation for a general Hyper-Kähler metric: a 'single 3-plane solution' of the hyper-Kähler conditions with p-vector (p_1,p_2) is associated with a IIB superstring 5-brane with 5-brane charge vector (p_1,p_2). This implies that there is a direct correlation between the angle at which any given 5-brane is rotated, relative to a $D5$-brane, and its 5-brane charge. An instructive case to consider is the three 5-brane solution involving a $D5$-brane and an NS-5-brane, having orthogonal overlap, and one other 5-brane. As the orientation of the third 5-brane is changed from parallel to the $D5$-brane to parallel to the NS-5-brane it changes, chameleon-like, from a D-brane to an NS-brane.

4.5 Intersecting branes from hyper-Kähler manifolds

There is a generalisation of (52) called a 'generalized membrane' solution which takes the form

$$\begin{aligned} ds^2 &= H^{-\frac{2}{3}} ds^2(\mathbb{E}^{2,1}) + H^{\frac{1}{3}} ds^2_{HK} \\ F &= \pm \omega_3 \wedge dH^{-1}, \end{aligned} \tag{74}$$

where ω_3 is the volume form on $\mathbb{E}^{2,1}$ and H is a T^2-invariant[‖] harmonic function on the hyper-Kähler 8-manifold. Provided the sign of the expression for the four-form F in (74) is chosen appropriately it can be shown that the solution with $F \neq 0$ breaks no more supersymmetries than the solution (52) with $F = 0$. Point singularities of H are naturally interpreted as the positions of parallel $M2$-branes. For our purposes we require H to be independent of the two φ coordinates, so singularities of H will correspond to $M2$-branes delocalized on T^2. Such functions satisfy

$$U^{ij} \partial_i \cdot \partial_j H = 0. \tag{75}$$

Proceeding as before we can now convert this D=11 configuration into various intersecting brane configurations. Lets first consider the case of the Hyper-Kähler manifold being a product of two Taub-NUT manifolds. Recall that we first reduced on one of the Taub-NUT circles and then T-dualised on the other circle and for $H = 1$ we obtained the $NS5 - D5(2)$ configuration. For $H \neq 1$, the reduction gives a $D2$-brane and the T-duality converts it into a $D3$-brane and we arrive at the first configuration in (42). Continuing with the various dualities we arrive at all of the configurations in (42)-(46) In the case of (43) we recover the M-theory solution considered in (25). Note that substituting (72) into (75) produces (26).

If we now consider a general toric Hyper-Kähler manifold in (74) we obtain solutions

[‖]This condition on H is needed for our applications; it is not needed to solve the D=11 supergravity equations.

corresponding to the configurations in (42)-(46) with the first two branes overlapping non-orthogonally and the third brane stretched between them.

5 Conclusions

In this paper we have reviewed various supergravity solutions corresponding to BPS intersecting branes in M-theory and in type II string theory. We first discussed three basic intersections of two M-branes which all break 1/4 of the supersymmetry: $M2-M2(0)$, $M2-M5(1)$ and $M5-M5(3)$. Upon dimensional reduction these configurations were related to type II intersecting D-branes with four ND string coordinates, as well as to other configurations involving NS-branes. We argued that these solutions could be interpreted in one of three ways: self intersections of branes in $(p-2)$ dimensions, branes within branes, or branes ending on branes. All of the solutions have the property that they are delocalised along the relative transverse directions. We pointed out that the harmonic function of one of these branes can be generalised to have a dependence on the coordinates tangent to the other brane. It would be interesting if more general solutions of (26) could be found. More generally it would be of interest to construct fully localised intersecting brane solutions.

We noted that multi intersections of n-branes are allowed and that they generically break 2^{-n} of the supersymmetry. An interesting exception to this are various special triple overlaps that allow an extra brane to be added without breaking more supersymmetry. We showed that intersecting branes can be dimensionally reduced to give black holes with non-zero horizon area in D=4 and D=5. Considering the intersecting D brane configurations from a perturbative point of view has had remarkable success at reproducing the black hole entropy from state counting.

We also discussed the $M5-M5(1)$ overlap. This solution has the interesting property that the $M5$-branes are localised inside the world-volume of the other brane, but are delocalised in the direction that separates them. We noted that these configurations violate the $(p-2)$ self intersection rule and that the resolution of this is the fact that there are more general solutions with an extra $M2$-brane that still preserve 1/4 of the supersymmetry. The extra $M2$-brane is interpreted as being stretched between the two $M5$-branes. After dimensional reduction these are related to D-brane intersections with the number of ND string coordinates being eight.

We showed that toric hyper-Kähler manifolds can be used to construct generalisations of the $M5-M5(1)$ solution which preserve 3/16 supersymmetry where the $M5$-branes overlap non-orthogonally. Similar configurations can be obtained by dimensional reduction and duality. One interesting case is two $D5$-branes intersecting non-orthogonally. The two $D5$-branes are related by an Sp(2) rotation in the eight relative transverse directions. Since the solution is related by duality to a non-singular D=11 spacetime of $Sp(2)$ holonomy it makes precise the analogy between the fraction of supersymmetry preserved by non-orthogonal D-branes and the standard holonomy argument in Kaluza-Klein compactifications that was discussed in [67]. In view of this it would be of interest to consider other subgroups of $SO(8)$. As pointed out in [67], the holonomy analogy would lead one to expect the existence of intersecting D-brane configurations in which one D-brane is rotated relative to another by an $SU(4)$, G_2 or Spin(7) rotation matrix. If so, there presumably exist corresponding solutions of IIB supergravity preserving 1/8, 1/8 and 1/16 of the supersymmetry, respectively. These IIB solutions would presumably have M-theory duals, in which case one is led to wonder whether they could be non-singular (and non-compact) D=11 spacetimes of holonomy $SU(4)$, G_2 or Spin(7).

By considering a generalised membrane solution involving the toric Hyper-Kähler manifold allowed us to construct solutions corresponding to branes overlapping non-orthogonally with an additional brane stretched in between them. In the case in which a $D3$-brane intersects overlapping IIB 5-branes, the fact that the solution preserves 3/16 of the supersymmetry implies that the field theory on the 2-brane intersection has N=3 supersymmetry. When the 5-branes overlap orthogonally the field theory has $N = 4$ supersymmetry and for this case Hanany and Witten have shown that the brane point of view can be used to determine the low-energy effective actions of these field theories [43]. It would be interesting if these techniques could be adapated to the $N = 3$ case when the 5-branes overlap non-orthogonally.

We hope to have given the impression that although much is known about intersecting branes there is still much to be understood.

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