hep-ph/9706316
NI 97033-NQF

The gluon/charm content of the η' meson and instantons

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Abstract

Motivated by recent CLEO measurements of the $B \to \eta' K$ decay, we evaluate gluon/charm content of the η' meson using the interacting instanton liquid model of the QCD vacuum. Our main result is $\langle 0|g^3 f^{abc} G^a_{\mu\nu} \tilde{G}^b_{\nu\alpha} G^c_{\alpha\mu}|\eta'\rangle = (2.3 \div 3.3) GeV^2 \times \langle 0|g^2 G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}|\eta'\rangle$. It is very large due to the strong field of small-size instantons. We show that it provides quantitative explanations of the CLEO data on the $B \to \eta' K$ decay rate (as well as inclusive process $B \to \eta' + X$), via a virtual Cabbibo-unsuppressed decay into $\bar{c}c$ pair which then becomes η'. If so, a significant charm component should be present in other hadrons also. In particular, we found a large contribution of the charmed quark in the *polarised* deep-inelastic scattering on a proton.

1.Instantons of a small size ($\rho \sim 1/3 fm$) are known for long time to be a very important component of the QCD vacuum [1]. In general, their fields are responsible for a scale 1 GeV which restrict perturbative QCD from below, and effective hadronic Lagrangians from above. Because of fermionic zero modes, they play especially important role for light (u,d,s) quark physics (for recent review see [2]). It was nevertheless believed that they are irrelevant for charm-related physics: and indeed, the instanton-induced spin-dependent and independent potentials between heavy quarks are small compared to standard confining-plus-perturbative one. However, as we show in this paper, the situation is reversed for *virtual* $\bar{c}c$ pairs: they can only appear due to the strongest gluonic fluctuations in vacuum, and those are instantons. (In fact, the gluonic fields in the centre of relevant instantons is so large, that one may even question whether gG/m_c^2 is a good expansion parameter.)

The way to see this is to look at the charm component in hadrons with different quantum numbers. The object of this paper, η', is long known to play a very special role in QCD: separated by a large gap from other pseudo-scalars (the Weinberg's U(1) problem [3, 4]) it serves as a screening mass for the topological charge (see recent detailed discussion in [5]). Thus testing whether the high dimension gluonic operator does or does not couple strongly to the η' we are actually testing whether the strongest vacuum fluctuations do or do not possess the topological charge. No effect of such magnitude should exist e.g. for vector mesons: and indeed, the empirical Zweig rule is very strict in vector channels, allowing only tiny flavor mixing.

2.Recently, CLEO collaboration has reported [6] measurements of inclusive and exclusive production of the η' in B-decays :

$$Br(B \to \eta' + X\ ; 2.2\, GeV < E_{\eta'} < 2.7\, GeV) = (7.5 \pm 1.5 \pm 1.1) \cdot 10^{-4}\ , \qquad (1)$$

$$Br(B \to \eta' + K) = (7.8^{+2.7}_{-2.2} \pm 1.0) \cdot 10^{-5}\ . \qquad (2)$$

Simple estimates [7] show that these data are in severe contradiction with the standard mechanism, the b-quark decay into light quarks, because Cabbibo suppression factor V_{ub} leads to numbers which are by two orders of magnitude smaller than the data (both the inclusive and exclusive cases). Alternative mechanism, suggested in [7] is based on the Cabbibo favored $b \to c\bar{c}s$ process, followed by a transition of virtual $\bar{c}c$ into the η'. The latter transition may be possible, provided there exist large intrinsic charm component of the η'. Its quantitative measure can be expressed through the matrix element

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'(q)\rangle \equiv i f_{\eta'}^{(c)} q_\mu\ . \qquad (3)$$

and one needs $f_{\eta'}^{(c)} \approx 140\ MeV$ in order to explain the CLEO data, see [7]. This value is surprisingly large, being only a few times smaller than the analogously normalised residue $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta_c(q)\rangle = i f_{\eta_c} q_\mu$ with $f_{\eta_c} \simeq 400\ MeV$ known experimentally from the $\eta_c \to \gamma\gamma$ decay.

3.Because the c-quark is heavy, it may only exist in the η' in a virtual loop,and its contribution can be evaluated in terms of gluonic fields. Taking the divergence of the axial current in Eq.(3) one gets

$$f_{\eta'}^{(c)} = \frac{1}{m_{\eta'}^2}\langle 0|2m_c\bar{c}i\gamma_5 c + \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}_{\mu\nu}|\eta'\rangle\ . \qquad (4)$$

which can be further simplified by the Operator Product Expansion in inverse powers of the $c-$quark mass

$$2m_c\bar{c}i\gamma_5 c = -\frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}_{\mu\nu} - \frac{1}{16\pi^2 m_c^2}g^3 f^{abc}G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu} + O(G^4/m_c^4) \qquad (5)$$

(see the appendix in [7] for a detailed derivation of this result. Further terms in expansion (5) are neglected in what follows.) Thus the problem is reduced to the matrix element of a particular dimension-6 pseudo-scalar gluonic operator:

$$f_{\eta'}^{(c)} = -\frac{1}{16\pi^2 m_{\eta'}^2}\frac{1}{m_c^2}\langle 0|g^3 f^{abc}G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}|\eta'\rangle\ . \qquad (6)$$

4.The magnitude of the matrix element (3) was related [7] to the *vacuum* expectation value of similar operators:

$$f_{\eta'}^{(c)} \simeq \frac{3}{4\pi^2 b}\frac{1}{m_c^2}\frac{\langle g^3G^3\rangle_{YM}}{\langle 0|\frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}_{\mu\nu}|\eta'\rangle}\ . \qquad (7)$$

where $\langle G^3 \rangle$ should be evaluated in pure gluodynamics, not QCD. Unfortunately, only indirect order-of-magnitude estimate for this latter quantity was given, and thus in [7] rather wide range of values was given $f^{(c)}_{\eta'} = (50 \div 180)\ MeV$.

5.We have performed direct calculation of this quantity using the Interacting Instanton Liquid Model (IILM). In its present form, this model takes into account instantons coupling to light quarks to *all orders* in t'Hooft effective interaction, which was shown to be crucial for η' physics. It has correctly reproduced multiple mesonic/baryonic/glueball correlation functions, and also has an increasing direct support from lattice studies of instantons (see [2]).

The calculation is based on numerical evaluation of the following two-point Euclidean correlation functions

$$K_{22}(x) = \langle 0|g^2 G^a_{\mu\nu} \tilde{G^a_{\mu\nu}}(x) g^2 G^a_{\mu\nu} \tilde{G^a_{\mu\nu}}(0)|0\rangle \tag{8}$$

$$K_{23}(x) = \langle 0|g^2 G^a_{\mu\nu} \tilde{G^a_{\mu\nu}}(x) g^3 f^{abc} G^a_{\mu\nu} \tilde{G}^b_{\nu\lambda} G^c_{\lambda\mu}(0)|0\rangle \tag{9}$$

$$K_{33}(x) = \langle 0|g^3 f^{abc} G^a_{\mu\nu} \tilde{G}^b_{\nu\lambda} G^c_{\lambda\mu}(x) \quad g^3 f^{abc} G^a_{\mu\nu} \tilde{G}^b_{\nu\lambda} G^c_{\lambda\mu}(0)|0\rangle \tag{10}$$

Studies of $K_{22}(x)$ has been made previously [8], where it was demonstrated that in the "unquenched" ensemble of instantons with dynamical quarks the non-perturbative part change sign at distances $x > 0.6 fm$, displaying a "Debye cloud" of compensating topological charge. It is identified with the η' contribution, and lead to an estimate

$$\langle 0|g^2 G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}|\eta'\rangle = \frac{16\pi^2}{\sqrt{3}} f_{\eta'} m^2_{\eta'} \approx 7\, GeV^3 \tag{11}$$

which agrees reasonably well with other estimates in literature. In this formula we have expressed matrix element (11) in terms of the standard parameter $f_{\eta'} \approx 85 MeV$ which is defined as follows

$$\langle 0|\frac{1}{\sqrt{3}} \sum_{i=u,d,s} \bar{q}_i \gamma_\mu \gamma_5 q_i |\eta'\rangle = i f_{\eta'} q_\mu .$$

Using an anomaly in the chiral limit, $m_u = m_d = m_s = 0$ we arrive to (11).

6. We have calculated the correlators mentioned by numerical simulation, using of the ensemble 16 instantons and 16 anti-instantons, put into a box $4 \times 2^3 fm^4$, with (without) dynamical quarks[9] Unfortunately, the propagation of the gluons in the background non-perturbative fields of instantons was not studied in such details as for light quarks, and so far we do not have the gluon propagator program which could be used for all distances. At small x purely perturbative results (e.g. $K_{22}^{pert}(x) = 384 g^4/\pi^4 x^8$) dominate, while the non-perturbative fields can be included via the operator product expansion (see e.g.[10, 1]). At large x we would argue below that (at least with dynamical quarks) the non-perturbative fields dominate.

The quantity $f^{(c)}_{\eta'}$ (6) can be obtained from the correlation functions (8, 9, 10)[11]:

$$\frac{f^{(c)}_{\eta'} \sqrt{3} m_c^2}{f_{\eta'}} = \frac{K_{23}(x \to \infty)}{K_{22}(x \to \infty)} = \sqrt{\frac{K_{33}(x \to \infty)}{K_{22}(x \to \infty)}} \tag{12}$$

It is expected that at large distances the contribution to two other correlators would also be dominated by non-perturbative field of the instantons. If so, one has a simple estimate for the ratio of matrix elements

$$\frac{\langle 0|g^3 f^{abc} G^a_{\mu\nu} \tilde{G}^b_{\nu\alpha} G^c_{\alpha\mu}|\eta'\rangle}{\langle 0|g^2 G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}|\eta'\rangle} = \frac{12}{5}\langle\frac{1}{\rho^2}\rangle \approx (1 \div 1.5) GeV^2 \tag{13}$$

Two numbers given here correspond to averaging over instanton size distribution for two variants of the instanton-anti-instanton interaction, the so called "streamline" and "ratio-ansatz" ones, and indicate the sytematics involved. The latter (giving smaller average size and larger number above) should be considered preferable, because it better agrees with the size distribution directly obtained from lattice gauge field configurations, see discussion in [2]. (Recent measurements [12] using refined "inverse blocking" method has found somewhat smaller instantons than others, but those seem to belong to correlated instanton-anti-instanton pairs, which would not contribute to the compensating Debye cloud we look for.)

In our measurements of K_{23}, K_{33} *both* ratios entering (12) were found to stabilize at large enough x$> 0.8 fm$ at the *same* numerical value. We take it as an indication that η' contribution does in fact dominate, although we were not able to see that all correlators fall off with the right mass[13]. Numerical values of the ratios about $(1.5 \div 2.2) GeV^2$, for two ensembles mentioned. The numers are somewhat larger than in (13) because the second operator in the correlator makes it more biased toward smaller instantons.

Proceeding to final result, we have to look at radiative corrections. The experimental number mentioned above is defined at the scale $\mu_1^2 \approx m_c^2$, which is different from that obtained in the instanton calculation. In the latter case the charge and fields are normalized at $\mu_2^2 \approx gG$ where G is the typical gauge field at the points which contribute the most to the correlators. Two scales are not too far apart numerically $\mu_2^2 \approx (0.5 \div 1) GeV^2$, but the anomalous dimension of the $g^3 G\tilde{G}G$ operator [14] is large, and it leads to correction

$$f^{(c)}_{\eta'}(\mu_1 \simeq m_c) = [\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)}]^{\frac{-18}{2b}} f^{(c)}_{\eta'}(\mu_2) \simeq 1.5 f^{(c)}_{\eta'}(\mu_2), \tag{14}$$

Here we use $m_c(\mu_1 \simeq m_c) \simeq 1.25 GeV$ for the numerical estimates. This concludes our derivation of the parameter

$$\frac{f^{(c)}_{\eta'}}{f_{\eta'}} \simeq 0.85 \div 1.22, \tag{15}$$

where the second value is preferable, see above. We present our final result as the ratio $f^{(c)}_{\eta'}/f_{\eta'}$ instead of the absolute value of $f^{(c)}_{\eta'}$ because most systematic errors are gone for the ratio. The final uncertainty in eq. (15) comes from the systematic errors of the instanton model, which can be judged from comparison of the instanton size distribution or the scalar glueball size to corresponding lattice results. It will certainly be soon reduced by on-going works. Finally, we compare it with the "experimental" value needed to explain CLEO measurements (3), and conclude that our result obtained in the instanton liquid model agrees with it, inside the uncertainties.

7. The next logical question to ask is whether the unexpectedly large gluon/charm content of η' has profound consequences outside of B physics, for other hadrons. One point we want to make is that that it seems now likely that understanding of the spin problem of the nucleon cannot be done without its "intrinsic charm" as well[15]. Relevant matrix element

$$\langle N|\bar{c}\gamma_\mu\gamma_5 c|N\rangle = g_A^{(c)}\bar{N}\gamma_\mu\gamma_5 N \tag{16}$$

could be generated by the η' "cloud" inside the nucleon. Assuming now the η' dominance in this matrix element[16] one could get the following Goldberger-Treiman type relation[15] $g_A^{(c)} = \frac{1}{2M_N} g_{\eta' NN} f_{\eta'}^{(c)}$. Although the precise value of $g_{\eta' NN}$ is unknown, and phenomenological estimates of the coupling vary significantly $g_{\eta' NN} = 3 - 7$[17], it leads to

$$\langle N|\bar{c}\gamma_\mu\gamma_5 c|N\rangle = (0.2 \div 0.5)\bar{N}\gamma_\mu\gamma_5 N \tag{17}$$

which is comparable to the light quark contribution! We plan to calculate $g_A^{(c)}$ and $g_{\eta' NN}$ in the instanton model as well. Lattice determination of all those quantities would be more than welcome. Ultimately, the contribution of the charmed quarks in polarized deep-inelastic scattering may be tested experimentally, by tagging the charmed quark jets.

8.It is by now widely known that the Zweig rule is badly broken in all scalar/pseudoscalar channels, and that (rather large) mass of the η' is in fact due to light-quark-gluon mixing. Furthermore, all these phenomena are attributed to instantons. In this work we have found that similar phenomena are even more profound for larger-dimension (multi-gluon) operators as well. Moreover, the flavor mixing includes also a significant fraction of $\bar{c}c$ in η'. Perhapse it is not so surprising qualitatively: but the fact that one can actually quantitatively calculate these matrix elements and quantitatively compare it to real data is still rather amasing.

Acknowledgments

This work was done during the program in I.Newton institute in Cambridge, and we use this opportunity to thank P.van Baal for invitation and nice program.This work is partly supported by the US Department of Energy under Grant No. DE-FG02-88ER40388.

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