Differential Rotation and Turbulence in Extended H I Disks

J. A. Sellwood[1]

Department of Physics and Astronomy, Rutgers, The State University of New Jersey, 136 Frelinghuysen Road, Piscataway, NJ 08854-8019

sellwood@physics.rutgers.edu

Steven A. Balbus[1]

Virginia Institute of Theoretical Astronomy, Department of Astronomy, University of Virginia, Charlottesville, VA 22903-0818

sb@virginia.edu

ABSTRACT

When present, extended disks of neutral hydrogen around spiral galaxies show a remarkably uniform velocity dispersion of $\sim 6\ \mathrm{km\ s^{-1}}$. Since stellar winds and supernovae are largely absent in such regions, neither the magnitude nor the constancy of this number can be accounted for in the classical picture in which interstellar turbulence is driven by stellar energy sources. Here we suggest that magnetic fields with strengths of a few microgauss in these extended disks allow energy to be extracted from galactic differential rotation through MHD driven turbulence. The magnitude and constancy of the observed velocity dispersion may be understood if its value is Alfvénic. Moreover, by providing a simple explanation for a lower bound to the gaseous velocity fluctuations, MHD processes may account for the sharp outer edge to star formation in galaxy disks.

Subject headings: galaxies: ISM — galaxies: kinematics and dynamics — hydrodynamics — instabilities — radio lines: galaxies — turbulence

[1] *Also, Isaac Newton Institute for Mathematical Sciences, Cambridge University, 20 Clarkson Rd., Cambridge CB3 0EH, UK*

1. Introduction

It has long been recognized that the interstellar medium (ISM) in galactic disks is turbulent (e.g. Scalo 1987, Dickey & Lockman 1990). Within our own Galaxy, line widths of individual molecular and H I clouds, and cloud complexes greatly exceed the expected thermal width, when kinetic temperatures can be estimated (Miesch & Bally 1994; Caselli & Myers 1995). There is also evidence for a high velocity dispersion population of H I clouds in both the Milky Way Galaxy (Radhakrishnan & Srinivasan 1980, Kulkarni & Fich 1985), and for a high velocity tail in the gas where the disk is optically bright in some external galaxies (Dickey, Hanson, & Helou 1990; Boulanger & Viallefond 1992; Kamphuis & Sancisi 1993; Schulman et al. 1996; Braun 1997).

The classical picture (e.g. Spitzer 1978) is that supernovae and other stellar processes (e.g. winds, outflows, *etc.*) supply the requisite energy to maintain turbulent cloud velocities against dissipative losses. In the absence of such sources, one would therefore expect interstellar turbulence to decay rapidly. In particular, turbulent motion should be negligible beyond the outer optical edges of spiral galaxies, where there are few stars.

This expectation is not in accord with observations. Radio observers report that 21 cm dispersions do not drop below $5-7$ km s^{-1}, regardless of how low the optical surface brightness becomes (e.g. Dickey et al. 1990; Kamphuis 1993). If this line width is a measure of the turbulent motion in an ensemble of cool clouds, the medium is being stirred by other means. We here propose that MHD instabilities (Balbus & Hawley 1991) in the differentially rotating gas layer are responsible for a minimum level of turbulence in all gas disks.

The need for MHD mediation of interstellar turbulence has been noted by a number of other investigators. Magnetic fields in galactic disks tend to be partly tangled on small scales, where lines of force have been observed to thread and be compressed by dense clouds, but also to retain some large-scale coherence (Heiles et al. 1993; Heiles 1996). On the largest Galactic scales, the field energy density is less than that of the random cloud motions, but probably in excess of the average thermal energy density. Within molecular cloud complexes, the field strength can be dominant, and MHD processes in these systems are central to understanding their dynamics and evolution (e.g. McKee & Zweibel 1995; Gammie & Ostriker 1996).

The extended H I disks of some galaxies are a relatively clean laboratory in which interstellar turbulence can be observed on its largest scales. It is clearly present in these systems, even though star formation and the associated supernova rates are very low. If the turbulent motions in this regime are Alfvénic, we show that they are maintained through magnetically mediated dynamical heating from differential rotation. We present a derivation of this heating rate when magnetic fields *and* self-gravity are both present, though in our specific application the process is magnetically dominated. Our result is applied to the particular case of NGC 1058, a face-on disk galaxy with a well-studied extended H I disk. Both the constancy and magnitude of the observed velocity dispersion follow naturally from an MHD explanation.

2. Turbulence in extended H I disks

A compilation of high-quality neutral hydrogen line width data on several galaxies may be found in Kamphuis (1993). The principal conclusions from this study are that 21 cm dispersions are $\sigma \sim 10-12$ km s^{-1} in the bright optical disk (with evidence for still higher values in spiral arms), and that the dispersions decline to no less than $5-7$ km s^{-1} when the optical surface brightness falls below 25 B mag arcsec^{-2}. The higher spatial resolution data presented by Braun (1997) indicate that broad lines are found in a "high brightness network" absent in the optically faint outer disks.

Dickey et al. (1990) drew particular attention to the remarkable lack of spatial variation of the H I line width in the gas layer beyond the optical disk in NGC 1058. Data from other galaxies show a similar trend, and indicate that the velocity dispersion puzzle is widespread. Both NGC 1058 and NGC 5474 (Rownd et al. 1994) are almost face-on, allowing clean measurements of line widths with little contribution from beam smearing. The velocity dispersions of these galaxies appear to approach a steady value of $\sigma \sim 6$ km s^{-1} at radii outside the bright disk. (The missing flux in their observations could, in principle, affect this result, but a similar dispersion is obtained for other galaxies (e.g. Kamphuis 1993) in which the full flux is recovered.) Dickey et al. stress that where the line width is small, the H I line profiles are almost perfect Gaussians.

Let us focus now upon NGC 1058, a small galaxy with an adopted distance of 10 Mpc. Its optical radius $R_{25} = 90''$ or 4.5 kpc and the H I layer extends to $220''$ or 11 kpc. Since it is almost face-on, its rotation curve is poorly known; we adopt a flat circular velocity of 150 km s^{-1}, consistent with the H I velocity field with $0.05 \lesssim \sin i \lesssim 0.1$. Dickey et al. report a H I column density $\sim 3 \times 10^{20}$ cm^{-2} at $R \sim 5$ kpc falling to 2×10^{20} cm^{-2} at $R \sim 10$ kpc. These are likely to be underestimates, however, since their interferometric data miss about half the flux determined from single dish observations. Therefore, as fiducial values, we double the column density numbers in our calculations below, adjust for helium content and also assume there is no significant quantity of molecular gas in the outer layer. We adopt a total thickness of the outer H I layer of $h = 400$ pc – about twice the thickness of the gaseous disk of the Milky Way in the Solar neighborhood, since we expect the layer to flare to some extent. We summarise the properties we adopt in Table 1.

The constant internal H I line width, Dickey et al. argue, could represent either the thermal temperature of the gas or the turbulent motion of many small, cooler clouds. Neither interpretation is free of puzzles.

If the observed 1-D dispersion of $\sim 5 - 7$ km s^{-1} is thermal, it would imply a gas temperature of $\sim 3\,000$–$6\,000$ K, perhaps less if there are some contributions to this width from beam smearing (small in a nearly face-on galaxy) and turbulence. Dickey (1996) points out that atomic hydrogen at a temperature of a few thousand degrees would be in an unstable state. Braun (1998) expects the low ambient pressure to cause all gas to be in the warm phase at a temperature of $\sim 10\,000$ K ($\sigma \sim 9$ km s^{-1}), which is inconsistent with the observed dispersion. While the principal heating and cooling processes at work in the Solar neighborhood (Wolfire et al. 1995) are likely to be different in the far outer disk, they would have to be drastically so for gas to be in a stable phase at $\sim 5\,000$ K. Moreover, stars *are* forming at a very low rate in at least some of these gas layers (Ferguson et al. 1998), which must imply the existence of cooler, denser gas from which they can form.

We therefore prefer to attribute the observed line width largely to turbulent motion in an ensemble of small, cool clouds, which immediately raises two related questions: i) What process accelerates the clouds, maintaining velocities well in excess of the internal sound speed of individual clouds?; and ii) Why should the turbulent velocities be so uniform? We offer answers to both these questions in §4.

2.1. Supernova heating

Supernovae (SNe) are generally assumed to be the dominant energy source for the interstellar turbulent cascade (e.g. Spitzer 1978; Norman & Ferrara 1996). The stellar density and supernova rate (SNR) in the gas layer beyond the Holmberg radius is clearly much lower than in the bright inner parts of galaxies, but could it still be sufficient to drive the turbulence?

Deep CCD images of NGC 1058 (Ferguson et al. 1998) have confirmed the very faint outer spiral arms, first detected by Tamman (1974) in his search for the stellar population responsible for the two SNe detected during the 1960s in the outer parts of this galaxy. Ferguson (1997) estimates an azimuthally averaged surface brightness declining to 28 B mag arcsec^{-2}, with a regular spiral pattern. Ferguson et al. also detect small H II regions along these spiral arms, but the inter-arm region seems to be almost devoid of any star formation.

From the Hα flux, Ferguson (private communication) estimates the azimuthally averaged star formation rate (SFR) per unit area at ~ 1.5 times the optical radius to be $\sim 5 \times 10^{-11}$ M$_\odot$ pc^{-2} yr^{-1}. A high estimate for the SNR for this SFR is 10^{-12} SN pc^{-2} yr^{-1} (e.g. Leitherer & Heckman 1995). Assuming mechanical energy input of $\epsilon 10^{51}$ ergs per SN, where ϵ is the highly uncertain efficiency factor, we obtain an energy input rate of $\sim \epsilon 3 \times 10^{-27}(400\mathrm{pc}/h)$ ergs cm^{-3} s^{-1}.

For $\epsilon \sim 0.01$ (Chevalier 1998, private communication), this energy input rate, while highly uncertain, is not decisively less than that of the differential rotation source we consider below. Nevertheless, it seems unlikely to be the principal source of cloud motions, because there is no correlation whatsoever between the SFR and H I velocity dispersions across the extended disk. Dickey et al. (1990) stress the uniformity of the dispersion, while Ferguson et al. observe H II regions to lie almost exclusively in very narrow arms.

Braun's (1997) high spatial resolution studies of H I in the bright inner parts of other galaxies show a "high brightness network" of distinctly non-Gaussian line profiles with broad, high velocity tails. He further reports (Braun 1998) a loose correspondence between the network of extended H I linewidths in M31 and the observed Hα emission, and argues for a causal connection between linewidth and energy injection from stellar activity. Thus turbulence driven by stellar activity appears to create broad wings in the line profiles in localized regions where the energy is deposited. It is hard to see how this activity could also produce the observed uniform level of near Gaussian line profiles on larger scales.

2.2. Other sources of turbulence

Other sources of turbulence can be imagined. Infall models include the returns from a galactic fountain (Bregman 1980; Schulman et al. 1996), or chimney (Norman & Ikeuchi 1989), or simply the direct accretion of external intergalactic matter (Tóth & Ostriker 1992; Kamphuis & Sancisi 1993). Gravitational scattering by transient spiral waves (Carlberg & Sellwood 1985; Jenkins & Binney 1990; Toomre & Kalnajs 1991) is yet another possible energy source.

Dickey et al. (1990) already remarked that the very low SFR in NGC 1058 is almost certainly inadequate to drive a vigorous galactic fountain. Even granting the presence of a weak fountain, the measured H I dispersion is no lower than in the outer parts of other galaxies with considerably more active star formation (Kamphuis 1993). Furthermore, stirring of the H I layer by infalling dwarf galaxies and debris is unlikely to maintain a uniform level of mild turbulence everywhere, and should result in radiative emission of a substantial fraction of the infall energy.

Transient spiral waves produce a choppy potential sea which scatters stars, and any other material in the disk, causing random motion to rise. Where most of the disk mass is in the collisionless component – i.e. the stars – the process is self-limiting, since the spiral waves quickly become weaker as the velocity dispersion of the stars rises. Because gas is able to dissipate turbulent energy through inelastic collisions, its fate will be different, however. It seems unlikely that gas would settle to a smooth distribution on large scales with a level of turbulence resulting from a balance between scattering and dissipation; it is more likely that stellar spiral arms promote the formation of large gas concentrations in which further gravitational instability will lead to star formation. In parts of the disk in which gas is the dominant mass component, if gravitational instability is present on large scales, we might expect it to cascade directly to forming stars.

The importance of self-gravity in a disk may be determined by evaluating the usual local stability parameter (e.g. Binney & Tremaine 1987)

$$Q = \frac{\sigma\kappa}{\pi G\Sigma}, \qquad (1)$$

where thickness corrections have been neglected. Here σ is the velocity dispersion, the epicyclic frequency $\kappa = \sqrt{2}V_{\rm circ}/R$ for a flat rotation curve, and Σ is the surface density. Values of this parameter for the axially symmetrized gas only in the outer H I layer of NGC 1058 are evaluated in Table 1. The Q value for the extremely faint stellar disk seems to be huge: $Q \sim 60(6\ {\rm kpc}/R)(\sigma_u/10\ {\rm km\ s^{-1}})/\Upsilon_B$, where Υ_B is the mass to B-band luminosity in Solar units; any supporting response from the stars must be utterly negligible in this region.

The values of Q given in Table 1 are uncertain and rather modest to argue strongly that the gas layer is clearly stable. We note, however, that the gas surface density varies by perhaps a factor two between the peaks in the spiral arms and the inter-arm level. Ferguson et al. (1998) report mild star formation, a clear indicator of local Jeans instability, in the arms but the paucity of detectable star formation between the arms suggests that the layer is stable there. The origin of the spiral arms in this outer layer is unclear, but gravitational instability seems quite unlikely since the Jeans length ($\lambda_{\rm crit} = 4\pi^2 G\Sigma_{\rm gas}/\kappa^2$) is much too small for such a large-scale and symmetric two-armed spiral (Table 1). We conclude that gravitational instabilities are unlikely to be stirring the gas layer to maintain the low level of turbulence in the gas.

3. Turbulent heating by coupling to differential rotation

In this section we consider the local turbulent dynamics of a galactic disk. Disk material orbits in the global potential of the galaxy, but is locally subject both to a magnetic field and (in principle) to its own self-gravity. We require the form of the turbulent volumetric heating rate under these conditions. For the reasons just given, self-gravity is not of direct importance for the gaseous disks under consideration here but we include it in the analysis of this section for the sake of completeness, and because it is an interesting problem in its own right. In the presence of self-gravity, the volumetric turbulent heating rate of the gas due to differential rotation is a direct generalization of the nonself-gravitating case: a coupling between the effective stress tensor and the large scale angular velocity gradient.

In a homogeneous gas, if the field is subthermal, the free energy of differential rotation drives a dynamical instability (Balbus & Hawley 1991, 1998), extracting rotational energy and depositing it in turbulent motions. The physical mechanism for the generation of turbulent motion is clear. Mass elements orbiting in the fluid at slightly differing radii, but coupled magnetically, pull on each other as the shear attempts to separate them as if connected by a weak spring. The effect of their mutual forces is to remove angular momentum from the one which has less and donate it to the one which has more. As a result, the radial separation of the elements increases, which increases the difference in angular velocity and the instability runs away – provided the spring is not strong enough to resist. We have a rather different case in mind here: a highly inhomogeneous gas in which the relevant "thermal" motions refer to the macroscopic velocity dispersion. It is likely that random magnetic stresses will continue to tap into the differential rotation as a source of turbulence for relatively strong fields as well (e.g. Eardley & Lightman 1975). Indeed, the expected outcome of differential rotation and any radial magnetic field component is a positive radial-azimuthal Maxwell stress; this alone is sufficient to drive noncircular velocity fluctuations (cf. below).

Adopting a standard (R, ϕ, z) cylindrical coordinate system, we denote the circular velocity as $R\Omega\hat{\mathbf{e}}_\phi$, where $\hat{\mathbf{e}}_\phi$ is a unit vector in the azimuthal direction. The velocity vector $\mathbf{u}$ is the difference between the true velocity and the azimuthal circular velocity; it is a fluctuation velocity satisfying $u \ll R\Omega$. We allow for the possibility of a mean, slowly varying drift velocity, denoted $\langle\mathbf{u}\rangle$, which is much less than the RMS fluctuation $\langle u^2\rangle^{1/2}$. The angle brackets $\langle\ \rangle$ denote local averages in radius and height, but a complete average in azimuth. Thus

$$|\langle\mathbf{u}\rangle| \ll \langle u^2\rangle^{1/2} \ll R\Omega. \tag{2}$$

The Alfvén velocity associated with the magnetic field $\mathbf{B}$ is given by

$$\mathbf{u_A} = \frac{\mathbf{B}}{\sqrt{4\pi\rho}} \tag{3}$$

where ρ is the mass density. We do not make a formal distinction between the mean magnetic field and its fluctuating component, but like its kinetic counterpart, the magnitude u_A is assumed to satisfy $u_A \ll R\Omega$.

We work in the standard *local approximation* in which R is assumed large enough that we can ignore geometrical curvature terms. Our starting point is the energy equation for the u fluctuations with the gravitational contribution written explicitly as a power term (see equation [89] in the review of Balbus & Hawley 1998):

$$\begin{aligned}\frac{\partial}{\partial t}\left\langle \tfrac{1}{2}\rho(u^2+u_A^2)\right\rangle + \nabla\cdot\langle\,\rangle = \\ -\frac{d\Omega}{d\ln R}\left\langle\rho(u_R u_\phi - u_{AR}u_{A\phi})\right\rangle \\ -\langle\rho\mathbf{u}\cdot\nabla\Phi\rangle + \langle P\nabla\cdot\mathbf{u}\rangle \\ -\sum_i\left\langle\rho\nu|\nabla u_i|^2 - \frac{\eta}{4\pi}|\nabla B_i|^2\right\rangle\end{aligned} \tag{4}$$

Here P is the gas pressure, ν the kinematic viscosity, η the resistivity and the operand in the divergence term $\nabla\cdot\langle\,\rangle$ is the energy flux

$$\left(\tfrac{1}{2}\rho u^2 + P\right)\mathbf{u} + \frac{\mathbf{B}}{4\pi}\times(\mathbf{u}\times\mathbf{B}). \tag{5}$$

The gravitational potential Φ includes both a contribution from self-gravity, plus a contribution from the external large scale potential. Locally, the latter may be taken to depend upon z only,

$$\Phi = \Phi_{\rm sg}(R,\phi,z)\ \text{(self gravity)} + \Phi(z)_{\rm ex}\ \text{(external)}. \tag{6}$$

Equation (4) balances changes in the energy density against the net flux, and explicit sources and sinks. The first term on the right side is the energy released by the Reynolds-Maxwell stress, and is the key term in this paper. The other terms are the power extracted from the gravitational potential, the work done by pressure forces, and the viscous and resistive losses. Internal sources for interstellar energy fluctuations (e.g. SNe) can be included in the $P\nabla\cdot\mathbf{u}$ source term.

We now show that the gravitational term can be manipulated into the form of a coupling to the stress tensor. Start with

$$\rho\mathbf{u}\cdot\nabla\Phi = \nabla\cdot(\rho\mathbf{u}\Phi) - \Phi\nabla\cdot(\rho\mathbf{u}). \tag{7}$$

Mass conservation and the Poisson equation then imply,

$$-\Phi\nabla\cdot(\rho\mathbf{u}) = \Phi\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\frac{\nabla^2\Phi}{4\pi G}. \tag{8}$$

(N.B. The second term in the brackets arises because the total velocity is $\mathbf{u} + R\Omega\hat{\mathbf{e}}_\phi$.) Interchanging the order of partial derivatives and integrating by parts leads to

$$\Phi\frac{\partial}{\partial t}\frac{\nabla^2\Phi}{4\pi G} = \frac{1}{4\pi G}\left[\nabla\cdot\left(\Phi\frac{\partial\nabla\Phi}{\partial t}\right) - \frac{1}{2}\frac{\partial}{\partial t}|\nabla\Phi|^2\right]. \tag{9}$$

Similar, but more lengthy, manipulations lead to

$$\Phi\Omega\frac{\partial}{\partial\phi}\frac{\nabla^2\Phi}{4\pi G} = \frac{1}{4\pi G}\left[\nabla\cdot\left(\Phi\Omega\frac{\partial\nabla\Phi}{\partial\phi}\right) + \left(\frac{\partial\Phi}{\partial R}\right)\left(\frac{\partial\Phi}{R\partial\phi}\right)\left(\frac{d\Omega}{d\ln R}\right)\right] - \frac{\partial}{\partial\phi}(\cdots) \tag{10}$$

The final partial ϕ derivative will vanish upon averaging, and is not explicitly written. Carrying this average through and returning to equation (7) leads to

$$\langle\rho\mathbf{u}\cdot\nabla\Phi\rangle = -\frac{\partial}{\partial t}\left\langle\frac{|\nabla\Phi|^2}{8\pi G}\right\rangle + \nabla\cdot\left\langle\rho\mathbf{u}\Phi + \Phi\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\frac{\nabla\Phi}{4\pi G}\right\rangle + \langle\rho u_{GR}u_{G\phi}\rangle\frac{d\Omega}{d\ln R} \tag{11}$$

where the gravitational velocity $\mathbf{u_G}$ is defined by

$$\mathbf{u_G} = \frac{\nabla\Phi}{\sqrt{4\pi G\rho}} \tag{12}$$

If we now place our findings back into the energy equation (4), we may write the result as

$$\frac{\partial}{\partial t}\langle\tfrac{1}{2}\rho(u^2 + u_A^2 - u_G^2)\rangle + \nabla\cdot\langle\,\rangle = -\frac{d\Omega}{d\ln R}T_{R\phi} + \langle P\nabla\cdot\mathbf{u}\rangle - \sum_i\left\langle\rho\nu|\nabla u_i|^2 - \frac{\eta}{4\pi}|\nabla B_i|^2\right\rangle \tag{13}$$

where the stress tensor $T_{R\phi}$ is defined as

$$T_{R\phi} \equiv \langle\rho(u_R u_\phi - u_{AR}u_{A\phi} + u_{GR}u_{G\phi})\rangle \tag{14}$$

and the supressed energy flux has become

$$\left(\frac{1}{2}\rho u^2 + P\right)\mathbf{u} + \frac{\mathbf{B}}{4\pi}\times(\mathbf{u}\times\mathbf{B}) + \rho\mathbf{u}\Phi + \Phi\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\frac{\nabla\Phi}{4\pi G}. \tag{15}$$

The utility of writing the self-gravity in the form of a stress tensor was first noted by Lynden-Bell & Kalnajs (1972). We see that the most general stress tensor is a simple sum of the Reynolds, Maxwell, and Newtonian stresses. Free energy can be extracted from differential rotation at the rate per unit volume $T_{R\phi}d\Omega/d\ln R$.

It should be noted that the instabilities, both MHD and gravitational, drive the in-plane components of the velocity dispersion and do not couple directly to motions normal to the plane. Some degree of velocity anisotropy is therefore expected in turbulence driven by these stresses.

4. Application to NGC 1058

Differential rotation supplies energy to fluctuations at a rate given by

$$-T_{R\phi}\frac{d\Omega}{d\ln R} = \Omega T_{R\phi} \tag{16}$$

for a flat rotation curve. We do not know $T_{R\phi}$ *a priori,* but numerical simulations of magnetized nonself-gravitating disks, carried out under a wide variety of field geometries, equations of state, and numerical grids consistently yield

$$T_{R\phi} \simeq 0.6\,\frac{B^2}{8\pi} \tag{17}$$

(Hawley, Gammie, & Balbus 1995), where the magnetic field is evaluated at the time of saturation. A somewhat delicate point is how to relate B^2 to the mean magnetic field. The simulations reveal a field dominated by its largest scales, and we shall assume that mean field and fluctuations are comparable. Assuming the outer disk in NGC 1058 is not self-gravitating (as argued in §2.2), equation (17) gives a simple, physically sensible estimate for the stress tensor, which we shall adopt.

The scale for the fluctuating kinetic velocities is set by the Alfvén velocity:

$$\langle\rho u_R u_\phi\rangle \simeq 0.6\frac{B^2}{8\pi} + \langle\rho u_{AR}u_{A\phi}\rangle \tag{18}$$

Setting the left hand side to a fraction f_1 of $\overline{\rho}\sigma^2$, where $\overline{\rho}$ is the mean density and σ^2 the measured one-dimensional velocity disperion, and the entire right hand side to a fraction f_2 of $\overline{B}^2/8\pi$, where $\overline{B}$ is the mean magnetic field, we find for a velocity dispersion of 6 km s^{-1}

$$\overline{B} = 3\,\mu\text{G}\left(\frac{f_1}{f_2}\right)^{1/2}\left(\frac{\overline{\rho}}{10^{-24}\text{ g cm}^{-3}}\right)^{1/2}. \tag{19}$$

The implied field strength is a few microgauss – a reasonable number for a spiral galaxy (e.g. Beck et al. 1996). Observational confirmation of this value would be challenging. [The radio continuum emission from NGC 1058 is very weak (van der Kruit & Shostak 1984) because of its very low SFR.]

If we turn the problem around and ask how σ should vary with $\overline{B}$ and $\overline{\rho}$, as the density varies between and within the spiral arms say, a natural explanation for its near constancy emerges. Large scale but otherwise random field lines tend to follow a $\overline{B} \propto \overline{\rho}^n$ scaling, by making the usual assumption of flux freezing. The index $n = 2/3$ for isotropic spherical compression, while $n = 1$ for compression in a plane. Since σ is of order the Alfvén speed, this implies

$$\sigma \sim (\overline{\rho})^{n-1/2} \tag{20}$$

i.e. a rather weak dependence on density. Therefore, a magnetic basis for the velocity dispersions gives, in addition to a sensible inferred field strength, a very simple basis for understanding the near constancy of σ in gas-dominated galactic disks.

Finally, let us estimate the energy deposition rate, using the numerical result (17), and compare it with the SNR from §2.1. Inserting the above derived field strength of 3 μG, we obtain

$$T_{R\phi}\Omega \simeq 1.4 \times 10^{-28} \text{ ergs cm}^{-3} \text{ s}^{-1} \tag{21}$$

at $R = 7$ kpc. Thus differential heating is certainly competitive with the 1% efficient SNR of §2.1, and provides a more natural explanation for the observed uniformity of the turbulence.

5. Summary

We have argued that energy to drive turbulence in the ISM of a galaxy can be extracted from the differential rotation either by way of a well-established MHD instability (Balbus & Hawley 1991), or directly from the $T_{R\phi}$ Maxwell stresses if the field is too strong to be formally unstable. In the bright inner disk, where stars are forming and the SNR is high, the classical picture of turbulence driven by stellar processes prevails, but where the stellar density is low, the energy deposition rate from MHD driven turbulence becomes important.

A quite reasonable field strength (a few microgauss) is required for the turbulence to be Alfvénic, and its observed striking uniformity is, we argue, a simple consequence of flux freezing. But turbulence will decay without a constant energy source. We argue that MHD instabilities are the preferred energy source in the outer H I layer because they are inevitable and will maintain the characteristic turbulent velocity near the Alfvén speed. Other sources of energy, SNe, infall, etc. are also available, and may be important locally, but do not provide a natural explanation for the magnitude and uniformity of the turbulence.

By providing a mechanism for maintaining turbulence in the absence of mechanical energy input from stellar processes, we can claim some theoretical understanding of the semi-empirical star formation threshold advanced by Kennicutt (1989). He argues that (significant) star formation in a galactic disk is truncated abruptly at the outer edge by the rising value of Q, *assuming* the observed constant velocity dispersion of 6 km s^{-1}. He shows that this simple criterion accounts for the extent of most star formation in many galaxies. Here, we are able to provide a possible explanation for the constant velocity dispersion by linking it directly with an Alfvén speed (cf. eq. [20]). We are unaware of any previous explanation for the maintenance of the observed turbulence required for Kennicutt's empirical rule.

It must be stressed, however, that we have not explained why the value $\sigma = 6$ km s^{-1} should be so universal for disk galaxies. Linking it to the Alfvén velocity implies a magnetic field of about 3 μG – close to the field value inferred for many galaxies. Our achievement is to show that MHD instabilities can then maintain turbulent velocity fluctuations of the order of the the Alfvén speed, but why this velocity should always be $\sim$ 6 km s^{-1} is clearly related to why the magnetic field is always a few microgauss. This, we have yet to understand.

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Table 1. Adopted properties in NGC 1058

Radius	5 kpc (100″)	10 kpc (200″)
H I column density	6×10^{20} cm^{-2}	4×10^{20} cm^{-2}
Total gas surface density Σ_g	1.5×10^{-3} g cm^{-2}	10^{-3} g cm^{-2}
Full vertical thickness h	400 pc	400 pc
Mean gas density $\bar{\rho}$	1.2×10^{-24} g cm^{-3}	0.8×10^{-24} g cm^{-3}
Circular velocity $V_{\rm circ}$	150 km s^{-1}	150 km s^{-1}
Velocity dispersion in the gas σ	6 km s^{-1}	6 km s^{-1}
Epicyclic frequency κ	1.4×10^{-15} s^{-1}	0.7×10^{-15} s^{-1}
Local stability parameter Q_g .	2.7	2
Jeans length λ_{rmcrit}	670 pc	1,800 pc

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