# Accretion Disk Turbulence

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I review recent developments in the theory of turbulence in centrifugally supported astrophysical disks. Turbulence in disks is astrophysically important because it can transport angular momentum through shear stresses and thus allow disks to evolve and accrete. Turbulence can be initiated by magnetic, gravitational, or purely hydrodynamic instabilities; I give an abbreviated review of the linear and nonlinear theory of each of these possibilities, and conclude with a list of problems.

## 1. Introduction

Spiral galaxies, quasars, active galactic nuclei, X-ray binaries, cataclysmic variables, and young stars: these are a few of the astronomical objects that contain disks. Disks are common in astrophysics because it is usually difficult to change the specific angular momentum of gas, but easy to radiate away its thermal energy. Gas injected into in a spherically symmetric potential thus naturally shocks, radiates, and settles down into a plane normal to its mean angular momentum.

Because they are so common, disks occupy a lot of the astronomical community's time and energy (that would otherwise be entirely dissipated in attempting to measure $\Omega_0$). Although there are enormous differences between individual disk systems in global structure and observational appearance, there are a number of fluid dynamical processes common to all disks. These processes are worth understanding in detail.

The most fundamental process in disks, analogous to nuclear reactions in stars, is angular momentum transport. The disk cannot evolve unless gas in the disk can be persuaded to give up some of its angular momentum and spiral down the gravitational potential. Accretion disks (e.g. CV disks, but not spiral galaxies) are mainly heated by frictional processes associated with this gradual inflow; we would not see them at all absent some process for redistributing angular momentum.

The central role of angular momentum transport is evident if we write down an equation for the evolution of the disk surface density $\Sigma$, obtained directly from the angular momentum and continuity equation in the limit that the disk is thin:

$$\frac{\partial\Sigma(r,t)}{\partial t} = \frac{1}{(2-q)r}\frac{\partial}{\partial r}\left(\frac{1}{r\Omega}\frac{\partial}{\partial r}(r^2 W_{r\phi}) - \frac{\tau}{\Omega}\right) - \dot{\Sigma}_W; \tag{1.1}$$

here $\Omega \equiv$ orbital frequency $\sim r^{-q}$ and $\dot{\Sigma}_W$ is the mass lost in a wind. Angular momentum is either redistributed (diffused) through the disk by the height-integrated and azimuthally averaged shear stress $W_{r\phi} \equiv \int dz d\phi\, w_{r\phi}/(2\pi)$ (using cylindrical coordinates centered on the disk) or else removed directly from the disk by an external torque $\tau$ per unit area, provided perhaps by a magnetohydrodynamic (MHD) wind. Without angular momentum transport (or a wind) the disk does not evolve.

The shear stress $w_{r\phi}$ is caused by turbulence, broadly defined, in the disk; fluid viscosity and radiation viscosity are negligible. Since the disk behaves approximately like a fluid, it is easy to write down a direct expression for the shear stress:

$$w_{r\phi} = \rho v_r \delta v_\phi - \frac{1}{4\pi}B_r B_\phi + \frac{1}{4\pi G}\delta g_r g_\phi. \tag{1.2}$$

Here $\delta$ implies departures from the mean value and $\mathbf{g}$ is the gravitational field. One of the main goals of accretion disk theory is to calculate $w_{r\phi}$.

Disk turbulence is interesting because of its importance to accretion disk evolution, but it is also interesting from a fluid-dynamical viewpoint as a model, self-sustained turbulent system. In this review I will discuss turbulence initiated by magnetohydrodynamical instabilities (Section 2), gravitational instabilities (Section 3), and purely hydrodynamical instabilities (Section 4). In conclusion (Section 5) I discuss directions for future research.

Finally, there are a number of other recent reviews that focus on similar issues: Balbus & Hawley (1998) gives a complete discussion of the literature; Gammie (1998a) focuses on recent numerical experiments; Stone et al. (1998) discuss transport processes in protostellar disks; Brandenburg (1998) gives a different perspective on the numerical experiments with an emphasis on the connection to dynamo theory.

## 2. Magnetohydrodynamic turbulence

### 2.1. *Balbus-Hawley instability: Linear Theory*

Of all the unstable modes discovered to date one stands out as having the largest growth rates in the largest part of disk "phase space." It is a *local, linear, MHD* instability (here local means that it does not depend on the global radial or vertical structure of the disk) first understood in the context of accretion disks by Balbus & Hawley in 1991 (although it was discovered earlier in the context of magnetized Couette flow by Velikhov 1959).

The BH instability is most easily explained using a mechanical analogy first developed by Balbus & Hawley (1992). Consider two equal masses in coplanar orbit in a Keplerian potential. The masses are close together in the sense that $|\delta\mathbf{r}|/|\mathbf{r}| \ll 1$, and they are connected by a spring with natural frequency $\gamma$. The masses represent "fluid elements"; the spring represents the magnetic field if $\gamma^2 = (\mathbf{k}\cdot\mathbf{V}_{\rm A})^2$, where $\mathbf{V}_{\rm A} \equiv \mathbf{B}/\sqrt{4\pi\rho}$ is the Alfvén velocity and $\mathbf{k}$ the wavevector of the perturbation. This analogy is exact in the case of a purely vertical magnetic field and a vertical wavevector in ideal MHD.

In a frame coorbiting with the center of mass, the governing equations are

$$\delta\ddot{\mathbf{r}} = -2\mathbf{\Omega}\times\delta\dot{\mathbf{r}} + 3\Omega^2\delta r\hat{\mathbf{r}} - \gamma^2\delta\mathbf{r}, \tag{2.3}$$

where the terms on the right hand side are the Coriolis, tidal, and spring accelerations, respectively. These equations are already linear and so, taking $\delta\mathbf{r} \sim e^{st}$, we find that

$$s^4 + s^2(2\gamma^2 + \Omega^2) + \gamma^2(\gamma^2 - 3\Omega^2) = 0. \tag{2.4}$$

In the limit of a weak spring ($\gamma^2 \ll \Omega^2$), $s \simeq \pm\sqrt{3}\gamma$ or $s \simeq \pm i\Omega$. The unstable member of the first pair is the BH mode; the second pair are just the usual epicyclic oscillations of the particles.

One can recover several interesting properties of the BH instability from the dispersion relation: its maximum growth rate is $3\Omega/4$; this maximum is achieved at $\gamma = \sqrt{15}\Omega/4$; the instability vanishes if $\gamma > \sqrt{3}\Omega$. Returning to the analogous magnetic problem using $\gamma \to |\mathbf{k}\cdot\mathbf{V}_{\rm A}|$, we see the most remarkable property of the BH instability: no matter what the magnetic field strength, the maximum growth rate is always dynamical! As the field strength decreases, the fastest growing mode is simply pushed to smaller and smaller scales.

The local, linear analysis of a magnetized disk for a general field is somewhat involved (Balbus & Hawley 1992). The main complication is differential rotation, but this can be handled by the shearing plane wave formalism invented by Goldreich & Lynden-Bell (1965). Because of differential rotation there are no nonaxisymmetric local modes, but this does not mean the BH mechanism is absent. Instead it takes the form of transient, finite amplification of nonaxisymmetric, shearing plane waves, but the amplification factor can be made as large as required by adjusting the initial wavevector. It turns out, then, that the BH instability is present for any magnetic field orientation and a broad range of field strengths.

Because the BH instability is so powerful and is present under such general conditions, it is an important matter to discover when the instability is *not* present. Two potentially relevant effects can shut off the BH instability.

(1) The disk is poorly ionized. Then the instability can be damped by either ordinary resistivity (see Balbus & Hawley 1998; Jin 1996; Papaloizou & Terquem 1997) or ambipolar diffusion (Blaes & Balbus 1994). Protostellar disks (Hayashi 1981; Gammie 1996) and CV disks in quiescence (Gammie & Menou 1998) may fall within this regime.

(2) The magnetic field is too strong. In this case one must examine the vertical or radial structure of a specific disk model to determine precise stability conditions (see Section 2.4 below). A convenient rule of thumb, however, is that a strong field can only shut off the instability if the smallest-scale unstable mode has wavelength comparable to the disk scale height, so that the unstable modes no longer "fit" within the disk. This happens when $V_A \gtrsim c_s$. Weaker fields are certainly unstable. It is unclear if any realistic disk model is stabilized by a strong magnetic field; one might expect that a strong field would be difficult to anchor within the disk, i.e. that the system would be unstable to some other global or quasi-global instability. Nevertheless strong fields have been proposed as one way of turning off turbulence in quiescent CV disks (Armitage et al. 1996).

### 2.2. *Balbus-Hawley Instability: Nonlinear Outcome*

Studies of the nonlinear outcome of the BH instability have mainly used a local model of the disk. The local model is a first order expansion of the equations of motion in $H(R)/R$ ($H \equiv$ disk scale height) in a frame comoving with some fiducial point in the disk. Together with a set of boundary conditions called the shearing box (see Hawley et al. 1995), this model allows one to study the nonlinear development of the BH instability in a practical fashion. Experiments typically evolve the equations of compressible MHD, sometimes including the vertical structure of the disk and sometimes not. They have used a variety of vertical boundary conditions.

I have reviewed the results of these numerical experiments in detail elsewhere (Gammie 1998a). To give a brief summary, these experiments show that: (1) the instability leads to fully developed MHD turbulence; (2) this turbulence transports angular momentum outward; (3) in the absence of a mean field, this turbulence is self-sustained even in the presence of explicit or numerical dissipation. It can therefore be described as a dynamo; (4) the mean shear stress in the outcome, $\alpha \equiv 2\langle w_{r\phi}\rangle/(3\rho c_s^2)$, depends on the mean magnetic field:

$$\alpha \sim 0.01 + 4\frac{|\langle V_{A,z}\rangle|}{c_s} + \frac{1}{4}\frac{|\langle V_{A,\phi}\rangle|}{c_s}, \tag{2.5}$$

where $\langle \mathbf{V}_A \rangle$ is the mean Alfvén velocity; (5) Zero-mean field experiments with finite explicit resistivity saturate at a level that depends on the resistivity. At magnetic Reynolds numbers below about $10^3$ turbulence and transport die away completely (Hawley et al. 1996); (6) the slope of the power spectrum of the turbulence is consistent with Kolmogorov, but the turbulence is anisotropic.

### 2.3. *Quasi-Global MHD Instabilities*

Magnetized disks are also subject to quasi-global instabilities, by which I mean instabilities that depend on the local vertical structure of the disk, but not on its radial structure.

The Parker and interchange modes are one example of a linear, quasi-global MHD instability. The original linear analysis is due to Tserkovnikov (1960) for the interchange mode and Newcomb(1961) for the Parker mode. The stability criterion for Parker modes in a stratified atmosphere with uniform gravity $g$, adiabatic index $\gamma$, and a horizontal magnetic field, is

$$-\frac{d\rho}{dz} > \frac{\rho^2 g}{\gamma p}. \qquad \text{(STABLE)} \tag{2.6}$$

This is equivalent to the Schwarzschild criterion written as if the magnetic field were absent (it is, however, implicitly present in the equilibrium). I will only quote the stability criterion for the Parker mode, since it always goes unstable before the interchange mode. Notice that a large radiative diffusivity, as in accretion disks, can erase the stabilizing effects of stable stratification (Acheson 1978, Hughes 1985), driving the Parker mode stability criterion back to that for an adiabatic atmosphere: $d\ln B/dz > 0$.

Brandenburg et al. (1995) and Stone et al. (1996) have studied the nonlinear outcome of the Balbus-Hawley instability in a stratified disk, which is then potentially subject to magnetic Rayleigh-Taylor instabilities. In the experiments of Stone et al. (1996) there was no significant vertical Poynting flux, suggesting that magnetic buoyancy was not dynamically important. The vertical run of magnetic pressure was consistent with marginal stability to the Parker mode.

Radiation-dominated disks, such as might be found in disks around neutron stars and black holes accreting near the Eddington limit, are subject to yet another type of quasi-global instability called "photon bubbles" (Arons 1992, Gammie 1998b). The instability occurs in radiation dominated regions where the magnetic pressure exceeds the gas pressure. The nonlinear outcome of the instability is not yet known, although it has been investigated in the context of neutron star polar cap accretion by Hsu et al. (1997), where it greatly enhances the vertical transport of energy.

### 2.4. *Global MHD Instabilities*

Disks are potentially subject to an enormous variety of global instabilities. Global or quasi-global linear analyses of model astrophysical disks may be found in Papaloizou & Szuszkiewicz (1992), Gammie & Balbus (1994), Ogilvie & Pringle (1996), Terquem & Papaloizou (1996), Curry & Pudritz (1996), and Ogilvie (1998), to name a few. There is also an extensive literature on global instabilities in contexts other than disks (e.g. Couette flows); see Balbus & Hawley (1998) and references therein. Global analyses can exhibit the effects of boundaries and background gradients in the flow, but they lack the generality of local analyses since they depend on particular choices of the equilibrium.

The three-dimensional nonlinear outcome of Balbus-Hawley and other global MHD instabilities in disks has not yet been studied, except in an idealized Couette flow model (Armitage 1998).

## 3. Gravitational Instability

In the outer parts of AGN disks and protostellar disks gravitational instability may compete with or completely dominate the BH instability. Local stability of the disk is determined by Toomre's $Q \equiv c_s\kappa/(\pi G\Sigma)$, where $\kappa \equiv$ epicyclic frequency. For $Q < 1$ the disk is axisymmetrically unstable (the precise value depends on the vertical structure of the disk; 0.676 is the critical value for an isothermal disk finite thickness disk, 1 for a zero thickness disk). In an $\alpha$ disk, the instability criterion can be rewritten in the form

$$\dot{M} > 3\alpha\frac{c_s^3}{G} \simeq 7\times 10^{-3}\alpha\left(\frac{c_s^3}{1\mathrm{kms}^{-1}}\right)^3 \mathrm{M}_\odot\,\mathrm{yr}^{-1}, \qquad \text{(UNSTABLE)} \tag{3.7}$$

a condition easily satisfied in the outer parts of AGN disks for $\alpha \sim 1$, and in YSO disks for $\alpha \ll 1$.

Disks that are grossly unstable do not exist in nature, so the nonlinear theory of such systems is a mathematical exercise. Instead it is likely that disks are driven unstable, either by cooling (lowering $c_s$) or by mass-loading (raising $\Sigma$, possibly via infall), and that stability is partially recovered in the nonlinear outcome either by dissipation (raising $c_s$, possibly by shock heating) or by mass-shedding (lowering $\Sigma$, in AGNs possibly by star formation). For an $\alpha$ disk, cooling gives $d\ln Q/dt \sim \alpha\Omega$. If infall is to dominate this cooling, then it is easy to show that the infall accretion rate per logarithmic interval in radius must exceed the accretion rate within the disk by a factor of order $(R/H)^2$. It thus seems likely that cooling is the main driver of gravitational instability in most circumstances.

The nonlinear outcome of gravitational instability with cooling has been studied in the context of a thin, local model of a gaseous disk by Gammie (1998c). I find that the disk goes unstable due to cooling and that, if certain conditions are satisfied, it then shock heats and returns to marginal stability. In the outcome the disk contains fluctuating surface density variations of order unity, and the density correlation length is of order $2\pi QH$. The density structure transport angular momentum through both Reynolds stress and through gravitational stresses.

Finally, a note on linear theory: it is somewhat underappreciated that self-gravitating disks with constant kinematic viscosity are secularly unstable, a point first noticed by Lynden-Bell and Pringle (1974) and later discussed in the context of differentially rotating disks by Safronov (1991), Willerding (1992), and Gammie (1996). The instability grows on the viscous timescale ("viscosity" is here a proxy for smaller-scale turbulence; molecular viscosity is negligible). In the limit of weak viscosity, the growth rate $s$ of an axisymmetric mode in a zero thickness disk is

$$s \simeq \nu k_r^2 \frac{2\pi G\Sigma|k_r| - c_s^2 k_r^2}{\kappa^2 - 2\pi G\Sigma|k_r| + c_s^2 k_r^2}. \tag{3.8}$$

For an $\alpha$ disk ($\nu_{turb} \simeq \alpha c_s H$), in the limit that $Q \gg 1$ and $\alpha \ll 1$ the maximum growth rate of the instability is $27\alpha\Omega/(16Q^4)$.

Why are self-gravitating disks secularly unstable? It is clearly energetically advantageous for disks to bunch up into long-wavelength rings, thereby increasing their gravitational binding energy at little expense in compressional heating. But in inviscid disks there is an obstacle to this: the conservation of potential vorticity $\xi = (\nabla\times\mathbf{v})/\Sigma$. Once viscosity is introduced then $\xi$ can evolve and the rotational support of the disk at long wavelengths is compromised.

## 4. Hydrodynamic Instabilities

Absent self-gravity and magnetic fields, we are left with purely hydrodynamic mechanisms for generating turbulence. The local linear stability criterion for rotating fluid flow is the Rayleigh criterion: $d(r^2\Omega)^2/dr > 0$, i.e. specific angular momentum should increase outwards. Most disks, and in particular Keplerian disks, satisfy the Rayleigh criterion.

It has been suggested that Keplerian disks are locally nonlinearly unstable because of their high Reynolds number (e.g. Shakura & Sunyaev 1973, Lynden-Bell & Pringle 1974). This idea has been developed in some detail by Dubrulle & Zahn (1991), Dubrulle (1993), and Kato & Yoshizawa (1997). Numerical experiments in the local model (Balbus et al. 1996), however, fail to find any evidence of nonlinear instability in Keplerian shear flows. Nonlinear instability is found in a narrow band near $d\ln\Omega/d\ln r = -2$, i.e. in disks that are marginally stable by the Rayleigh criterion. While one can always ask whether the numerical experiments achieve sufficiently high Reynolds number, Balbus et al. (1996) present an argument based on moments of the momentum equations that suggests, but does not prove, that Keplerian disks are nonlinearly stable.

Disks can also suffer quasi-global instabilities such as convection (see Ruden et al. 1988 for the axisymmetric linear theory). One point that is not generally appreciated is the degree to which ordinary convective instabilities are damped by radiative diffusion in disks (although there are other, inertial, oscillations that become overstable in the presence of radiative diffusion).

Workers had long thought that convection might lead to enhanced turbulent transport of angular momentum in disks, the idea being that turbulence always implies transport. An early sign that this expectation might be incorrect was a quasi-linear calculation (Ryu & Goodman 1992) of the angular momentum flux associated with linear, nonaxisymmetric convective motions; the direction of the flux was found to be inwards rather than outwards. Subsequent numerical experiments (Stone & Balbus 1996; Cabot 1996) showed that in the nonlinear regime the angular momentum flux was small and inwards. This nonintuitive result is a nice illustration of the value of numerical experiments.

Finally, disks are susceptible to a wide variety of global hydrodynamic instabilities. One example is the Papaloizou-Pringle (1984, 1985) instability, subsequently elucidated by Narayan et al. (1986); see Savonije & Heemskerk (1990) for a readable physical account of this and allied global instabilities. A different type of global instability has been discovered by Goodman (1993). It requires a tidal field capable of distorting the disk streamlines into an oval shape. The instability grows from the free energy available in this oval distortion, causing it to decay by parametric instability into small scale inertial oscillations.

## 5. Conclusions

Great progress has been made in the last few years in understanding the origins and development of turbulence in accretion disks. We know that under a broad range of conditions the BH instability can initiate turbulence that transports angular momentum outwards. We also have strong numerical evidence that other types of turbulence in disks, such as convective turbulence, do not necessarily provide the angular momentum transport required for disk evolution. But there are still many interesting open questions about turbulence in disks; I will conclude with three of particular current interest.

*1. Is angular momentum transport local?* Numerical studies of the three dimensional nonlinear outcome of the BH instability have so far been restricted to regions of the disk of order $H$ in size (but see Armitage 1998). It is always found that most of the energy, and angular momentum flux, is contained in structures that are as large as allowed in the experiments. Thus the outcome is limited by the experiment size. What will happen in more realistic, larger-scale experiments?

One possibility is that largest scale structures will have a small fraction of the turbulent energy, with most the turbulent energy being concentrated at scales of order $H$. In this case angular momentum transport would be truly local. But another possibility is that most of the energy is always contained in the largest scale structures allowed. Then angular momentum transport would be mainly due to structure much larger than the disk scale height: it is nonlocal. This would be inconsistent with current approaches to modeling disk evolution embodied in the $\alpha$ model. Numerical experiments may be able to decide between these, and intermediate, alternatives in the near future.

*2. Are unmagnetized Keplerian disks nonlinearly stable?* Numerical experiments have diligently sought nonlinear instability in Keplerian disks and not found it (Balbus et al. 1996). But there remains a pool of skeptics who point out that the numerical experiments do not reach astrophysical Reynolds numbers, and so there is still the possibility of nonlinear instability. Since astrophysical Reynolds numbers will never be computationally accessible, what is needed is either a proof of nonlinear stability– a mathematically challenging problem– or an explicit demonstration of nonlinear instability. But for now the bulk of the evidence seems to favor the nonlinear stability of Keplerian shear flows.

*3. How do waves and turbulence interact in disks?* It is common to model the effect of turbulence on waves as a viscosity. This is done in studies of the tidal interaction between planets and protostellar disks (e.g. Lin & Papaloizou 1993), and in studies of warped disks (e.g. Pringle 1996); in these examples the turbulent viscosity completely governs the evolution of the disk. But the viscous model is completely untested. It could be quite misleading if, for example, it amplifies certain modes, or couples together linear modes of the laminar disk, or even gives the disk gas elastic properties. Numerical experiments that are immediately practical could measure the effects of turbulence on large-scale waves and settle this issue.

I am grateful to Jim Stone, Eve Ostriker, and Gordon Ogilvie for their comments and suggestions. This work was supported in part by NASA grant NAG 52837.

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