Vortex stretching versus production of strain/dissipation

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Abstract

A comparison between vortex stretching (VS) and production of strain/dissipation (PD) is made with the emphaiss on the latter. These two processes are nonlocally intreconnected, but weakly correlated. The energy cascade and its final result - dissipation are associated with the latter, i.e. with the quantity $-s_{ij}s_{jk}s_{ki}$ rather than with the enstrophy production $\omega_i\omega_j s_{ij}$. Moreover, vortex stretching suppresses the cascade and does not aid it, at least in a *direct* manner. On the contrary, it is the vortex *compression,* i.e $\omega_i\omega_j s_{ij} < 0$, that aids the production of strain/disspation and in this sense the 'cascade'.

Relation of VS and PD as well as of various alignments to the flow map of invariants of velocity derivatives tensor is given in qualitative terms.

1 Introductory notes, motivation

Velocity derivatives play an outstanding role in the dynamics of turbulence for a number of reasons. Their importance became especially clear since the papers by Taylor (1937, 1938)[1] and Kolmogorov (1941ab). Taylor emphasized the role of vorticity, whereas Kolmogorov stressed the importance of dissipation (strain).

Apart of vorticity and dissipation looking at velocity derivatives is useful in a number of aspects as follows.

- The field of velocity derivatives is much more sensitive to the non-Gaussian nature of turbulence or more generally to its structure, and hence reflects more of its physics (Tsinober 1998c).

[1]Taylor (1937, 1938) was motivated by the assumption of von Karman (1937) *that the expression* $\sum_i \sum_k \omega_i\omega_k \frac{\partial u_i}{\partial u_k}$ (i.e. enstrophy production) *is zero* in the mean and that he *cannot see any physical reason for such a correlation.* Taylor (1937) has conjectured that there is a *strong correlation between* ω_3^2 *and* $\frac{\partial u_3}{\partial u_3}$ *so that* (the mean of) $\omega_3^2\,\frac{\partial u_3}{\partial u_3}$ *is not equal to zero* (x_3 is directed along $\boldsymbol{\omega}$). He has shown that this is really the case (Taylor (1938)), and also expressed the view that *stretching of vortex filaments must be regarded as the principal mechanical cause of the the higher rate of disspation which is associated with turbulent motion.*

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- In Lagrangian description in a frame following a fluid particle, each point is a critical one, i.e. the direction of velocity is not determined. So everything happening in its proximity is characterized by the velocity gradient tensor $A_{ij} = \partial u_i/\partial x_j$. For instance, local geometry/topology is naturally described in terms of critical points terminology (see Chertkov et al (1999), Ooi et al (1999) and references therein).
- There is a generic ambiguity in defining the meaning of the term *small scales* (or more generally scales) and consequently the meaning of the term *cascade* in turbulence research. The specific meaning of this term and associated interscale energy exchange/'cascade' (e.g. spectral energy transfer) is essentially decompostion/reperesentation dependent (for more details/discussion of this issue see Apendix I). Perhaps, the only common in all decompostions/representations (D/R) is that the small scales are associated with the field of velocity derivatives. Therefore, it is naturally to look at this field as the one *objectively* (i.e. D/R independent) representing the small scales. Indeed, the dissipation is associated precisely with the strain field s_{ij} both in Newtonian and non-Newtonian fluids.

The above mentioned reasons prompted us to study in some detail the processes associated with the field of velocity derivatives. In particular, along with vortex stretching and enstrophy production of special interest is the production of strain. There are several reasons for this. First, though formally all the flow field is determined entirely by the field of vorticity the relation between the strain and vorticity is strongly nonlocal (e.g. Constantin (1994), Novikov (1968), Ohkitani (1994)): in many cases they are only weakly correlated. Second, energy dissipation is directly associated with strain and not with vorticity. Third, vortex stretching is essentially a proccess of interaction of vorticity and strain. Four, strain dominated regions appear to be the most active/nonlinear in a number of aspects (Tsinober (1998ab), Tsinober et al (1999)). Finally, the energy cascade (whatever this means) and its final result - dissipation are associated with predominant self-amplification of the rate of strain/production of dissipation and vortex compression rather than with vortex stretching. This last aspect is the main theme of this presentation.

2 Equations, notations, and some previous results

2.1 Equations and related

In the sequel we will need the equations for vorticity, ω_i, and enstrophy, ω^2,

$$\frac{D\omega_i}{Dt} = \omega_i s_{ij} + \nu\nabla^2\omega_i. \tag{1}$$

$$\frac{1}{2}\frac{D\omega^2}{Dt} = \omega_j s_{ij} + \nu\omega_j\nabla^2\omega_j. \tag{2}$$

and the rate of strain tensor, s_{ik}, (Yanitsky (1982)) and the total strain, $s^2 \equiv s_{ij}s_{ij}$ (Brasseur and Lin (1995), Tsinober (1995))

$$\frac{Ds_{ij}}{Dt} = -s_{ik}s_{kj} - \frac{1}{4}(\omega_i\omega_j - \omega^2\delta_{ij}) - \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu\nabla^2 s_{ij}, \tag{3}$$

$$\frac{1}{2}\frac{D2s^2}{Dt} = -2\left\{s_{ik}s_{kj}s_{ji} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}\right\} + 2\nu s_{ij}\nabla^2 s_{ij}. \tag{4}$$

These equations clearly indicate that along with enstrophy, ω^2, and strain, $s^2 \equiv s_{ij}s_{ij}$, the third moments $\omega_i\omega_j s_{ij}$, $s_{ij}s_{jk}s_{ki}$, are the key quantities of turbulence dynamics. It is noteworthy that many aspects of the dynamics of velocity gradient tensor $\partial u_i/\partial x_j$ can be addressed via looking at its invariants: the second – $Q = 1/4(\omega^2 - 2s_{ij}s_{ij})$, and the third – $R = -1/3(s_{ij}s_{jk}s_{ki} + 3/4\omega_i\omega_j s_{ij})$, the first one – $P = \partial u_k/\partial x_k$ is vanishing due to incompressibility (see Chertkov et al (1999), Ooi et al (1999) and references therein). However, it is not sufficient and along with using the Q, R invariants it is more transparent and physically meaningful in several respects to look directly at ω^2, s^2, $\omega_i\omega_j s_{ij}$, and $s_{ij}s_{jk}s_{ki}$. This is seen from the equations (3), (4), which also show that the quantity $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$, i.e. interaction of strain with pressure hessian is of importance (there are two more – $\omega_i\omega_j\frac{\partial^2 p}{\partial x_i \partial x_j}$ and $s_{ik}s_{kj}\frac{\partial^2 p}{\partial x_i \partial x_j}$ – in the equations (5), (6) below). Of course, formally the flow is determined entirely by the field of vorticity. However, due to the nonlocal relation between the rate of strain tensor and vorticity (e.g. Constantin (1994), Novikov (1968), Ohkitani (1994)) it is useful to look at the above mentioned quantities in parallel. Moreover, it appears that the dynamical equations for $\omega_i\omega_j s_{ij}$ and $s_{ij}s_{jk}s_{ki}$

$$\frac{D\omega_i\omega_j s_{ij}}{Dt} = \omega_j s_{ij}\omega_k s_{ik} - \omega_i\omega_j\frac{\partial^2 p}{\partial x_i \partial x_j} + \nu(2\omega_i s_{ij}\nabla^2\omega_j + \omega_i\omega_j\nabla^2 s_{ij}). \tag{5}$$

$$\frac{Ds_{ij}s_{jk}s_{ki}}{Dt} = 3\{-s_{ik}s_{kj}s_{il}s_{lj} + \frac{s_{ij}s_{ij}\omega^2 - \omega_i s_{ij}\omega_k s_{ik}}{4} - \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu s_{ik}s_{kj}\nabla^2 s_{ij}\}. \tag{6}$$

are also instructive in several respects.

Since for homogeneous flows $\langle s_{ij}s_{ik}s_{kj}\rangle = -\frac{3}{4}\langle\omega_i\omega_j s_{ij}\rangle$ and $\langle s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}\rangle = 0$ due to incompressibility it follows that the mean rate of production of strain/dissipation $-\langle 2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})\rangle = \langle\omega_i\omega_j s_{ij}\rangle$ is equal to that of enstrophy. Hence (and also due to $\langle\omega^2\rangle = 2\langle s_{ij}s_{ij}\rangle$) the choice of the coefficient 2 in (4), etc.

2.2 Vortex stretching and enstrophy production

We touch this aspect briefly. More details and references are given in Tsinober (1998ab) and Tsinober et al (1999).

Since Taylor (1938) it is known that $\langle\omega_i\omega_j s_{ij}\rangle > 0$, i.e. vortex stretching prevails on vortex compressing (see also Betchov (1976), Tsinober (1998a) and references therein, and Appendix I). This basic phenomenon is closely associated with subtle geometrical relations such as strict alignment between vorticity ω_i and vortex streching vector $W_i \equiv \omega_j s_{ij}$, alignment between vorticity ω_i and the eigenvector, $\boldsymbol{\lambda}_2$, of the rate of strain tensor, s_{ij}, corresponding to its intermediate eigenvalue Λ_2 (briefly intermediate eigenvector) and some others. However, enstrophy production is associated with two regions, characterised by alignment between between vorticity, ω_i, and the intermediate eigenvector, $\boldsymbol{\lambda}_2$, and between vorticity, ω_i, and the largest eigenvector, $\boldsymbol{\lambda}_1$. Moreover, the largest contribution to the enstrophy production comes from the regions with strong alignment between vorticity, ω_i, and the largest eigenvector, $\boldsymbol{\lambda}_1$, and is associated with large cuvature of vorticity lines and vorticity tilting, and large strain rather than with large enstrophy. The latter is true of all nonlinearities. It is noteworthy that the (approximate) balance between the *mean* enstrophy generation and its *mean* destruction via viscosity holds also in the enstrophy dominated regions. However, in the regions dominated by strain the enstrophy generation is an order of magintute larger than both its destruction via viscosity and the enstrophy generation in the enstrophy dominated regions. Therefore most of enstrophy generation occurs in the regions dominated by strain.

3 Generation of strain/dissipation

The appropriate level of disispation moderating the growth of energy is achieved by the build up of strain of sufficient magnitude which is described by the equation (4). It is seen from this equation that in the mean *the only term contributing postively to the production of strain/dissipation*, s^2, is the term $-s_{ij}s_{jk}s_{ki} = -(\Lambda_1^3+\Lambda_2^3+\Lambda_3^3) = -3\Lambda_1\Lambda_2\Lambda_3$, since $\langle s_{ij}s_{jk}s_{ki}\rangle = -3/4\langle\omega_i\omega_j s_{ij}\rangle$, and $\langle s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}\rangle = 0$ due to homogeneity and incompressibilty. Moreoever, since, $\Lambda_1 > 0$ and Λ_2 is positively skewed, i. e. $\langle\Lambda_2^3\rangle > 0$ the postivenes of $-\langle s_{ij}s_{jk}s_{ki}\rangle$ comes from the term $-\langle\Lambda_3^3\rangle$. In other words, Λ_3 is doing most of the 'cascade', at least, one of the final results of the "cascade" - disspation of energy, which is directly associated with s_{ij} and not with ω_i. Hence, the cascade is directly associated with compressing/squeezing of fluid elements and not with (vortex) stretching. It is noteworthy that this idea is not entirely new: '*It is clear, therefore, that production of vorticity is associated essentially with* Λ_3 *and production of* ω_1 *and* ω_2. *This suggests that the most of important processes associated with production of vorticity and energy transfer resemble a jet collision and not the swirling of a contracting jet* (Betchov (1956)). Betchov arrived to this conclusion analysing the *means* $\langle s_{ij}s_{jk}s_{ki}\rangle$ and $\langle \omega_i\omega_j s_{ij}\rangle$. Looking at the equation (4) it is seen that the above conclusion is true of production of strain, which is associated with Λ_3, and with the '*jet collision*' regions such as sheetlike structures as obseved in laboratory (Frederiksen et al(1996), Schwarz (1990)), and numerical experiments (Brachet et al (1992), Boratav and Pelz (1997), Chen (1997)). As for enstrophy production it is true in part: roughly two thirds of its positive contribution occur in the '*jet collision*' regions, the remaining third happens in the '*swirling of a contraction jet*' regions . Also production of ω^2 requires s_{ij} and interaction between the two, but production of s_{ij} is in some sense less dependent on ω, though without vorticity it is impossible.

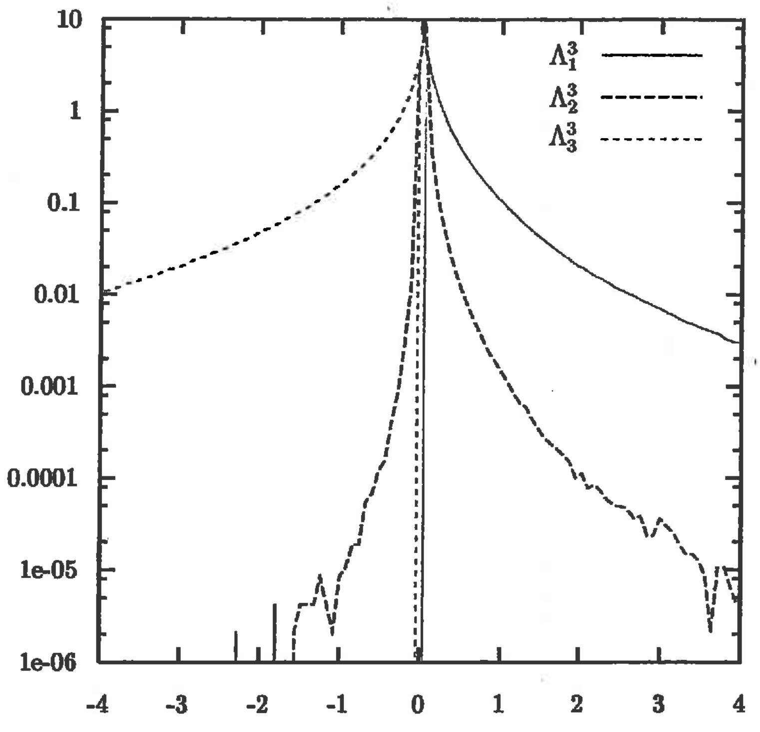

Figure 1. PDFs of cubed eigenvalues, Λ_i^3 of the rate of the strain tensor s_{ij}, normalized on $\langle s^2\rangle^{3/2}$.

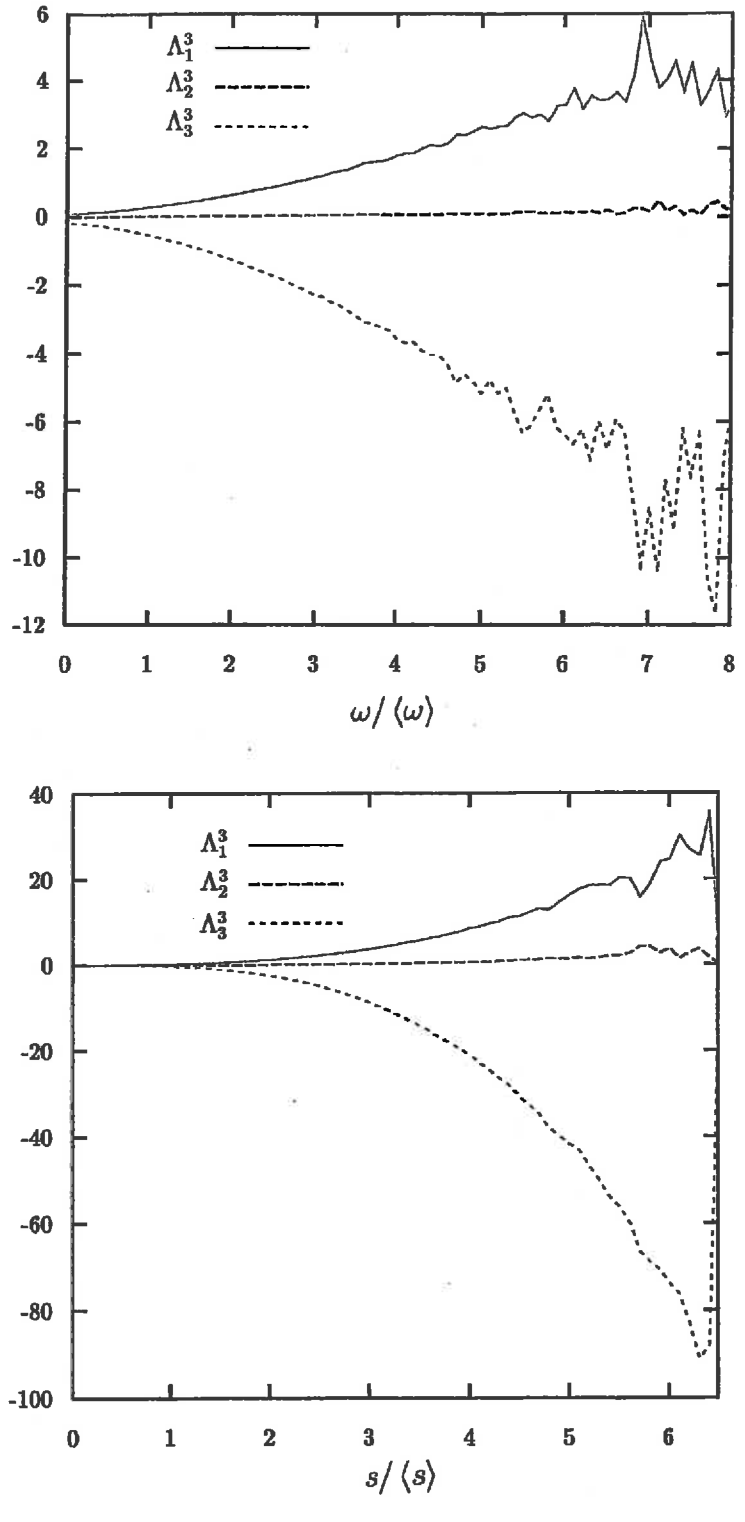

Figure 2. Conditional averages of cubed eigenvalues, Λ_i^3 of the rate of strain tensor, normalized on $\langle s^2 \rangle^{3/2}$, in slots of ω and strain s.

All the results shown in the sequel refer to a DNS simulation of NSE decaying turbulence in a peridic box for the time moment(s) at which the total enstrophy is (close to) maximal and $\mathrm{Re}_\lambda \approx 80$. They are very similar to those for a grid turbulent flow in which the Taylor hypothesis was used for computation of derivatives in the streamwise direction(Tsinober et al(1997)). This similarity indicates that the results below are not entitely local in time. Likewise, they reflect some aspects of nonlocality in space as well mostly due to use of conditional statistics and, of course, due to nonlocal relations between vorticity and strain and between pressure hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$ and velocity derivatives.

The ratios between the $\langle \Lambda_i^3 \rangle$ are as follows: $\langle \Lambda_1^3 \rangle : \langle \Lambda_2^3 \rangle : \langle \Lambda_3^3 \rangle = 1.2 : 0.05 : -2.25$. Their PDFs are shown in figure 1 and their conditional averages in slots of ω and $s \equiv (s_{ij}s_{ij})^{1/2}$ are shown in figure 2. One can see from the latter that Λ_i^3 are an order of magnitude larger in the strain dominated regions than in regions of strong vorticity.

A related phenomenon is shown in figure 3. Namely, it is clearly seen that $s_{ij}s_{jk}s_{ki}$ is strongly correlated with the total strain $s_{ij}s_{ij}$, and is only weakly correlated with the enstrophy ω^2.

Similarly the production of strain $-2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$ is correlated with the total strain $s_{ij}s_{ij}$, and is almost not correlated with the enstrophy ω^2 (figure 4). It is noteworhty that the the production of strain $-2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$ assumes its largest values in the regions with largest Λ_3^3 (see section 4, figure 9). On the other hand the enstrophy production is correlated with both the total strain $s_{ij}s_{ij}$ and with the enstrophy ω^2, but much more with the former (not shown - they are similar to those obtained by Jimenez et al (1993)).

The behavior of conditional averages of the total production of strain $-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ and separate its terms is shown in figure 5. The main feature is that the total inviscid rate of generation of strain/dissipation, $-2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$, is more than an order of magnitude larger in the regions dominated by strain than in the enstrophy dominated regions. As seen from the figure 6 the PDFs of $-2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$ are fully consistent with the behaviour of their conditional averages in slots of ω and s. Again the main feature is the strong positive shift of the PDF of $-2(s_{ij}s_{ik}s_{kj} + \frac{1}{4}\omega_i\omega_j s_{ij} + s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$ in the regions dominated by strain ($s^2 > 2.5\langle s^2 \rangle$).

It is noteworthy that though the mean $\langle s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j} \rangle = 0$ the PDF of $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ is positively skewed at *large* strain (figures 7 and 8), i.e. the interaction of strain and the pressure hessian is such that it is opposing the production of strain when it becomes large. This is also seen from figures 5 and 6 and from conditional averages of $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ in slots of ω and strain s (Tsinober et al (1999).

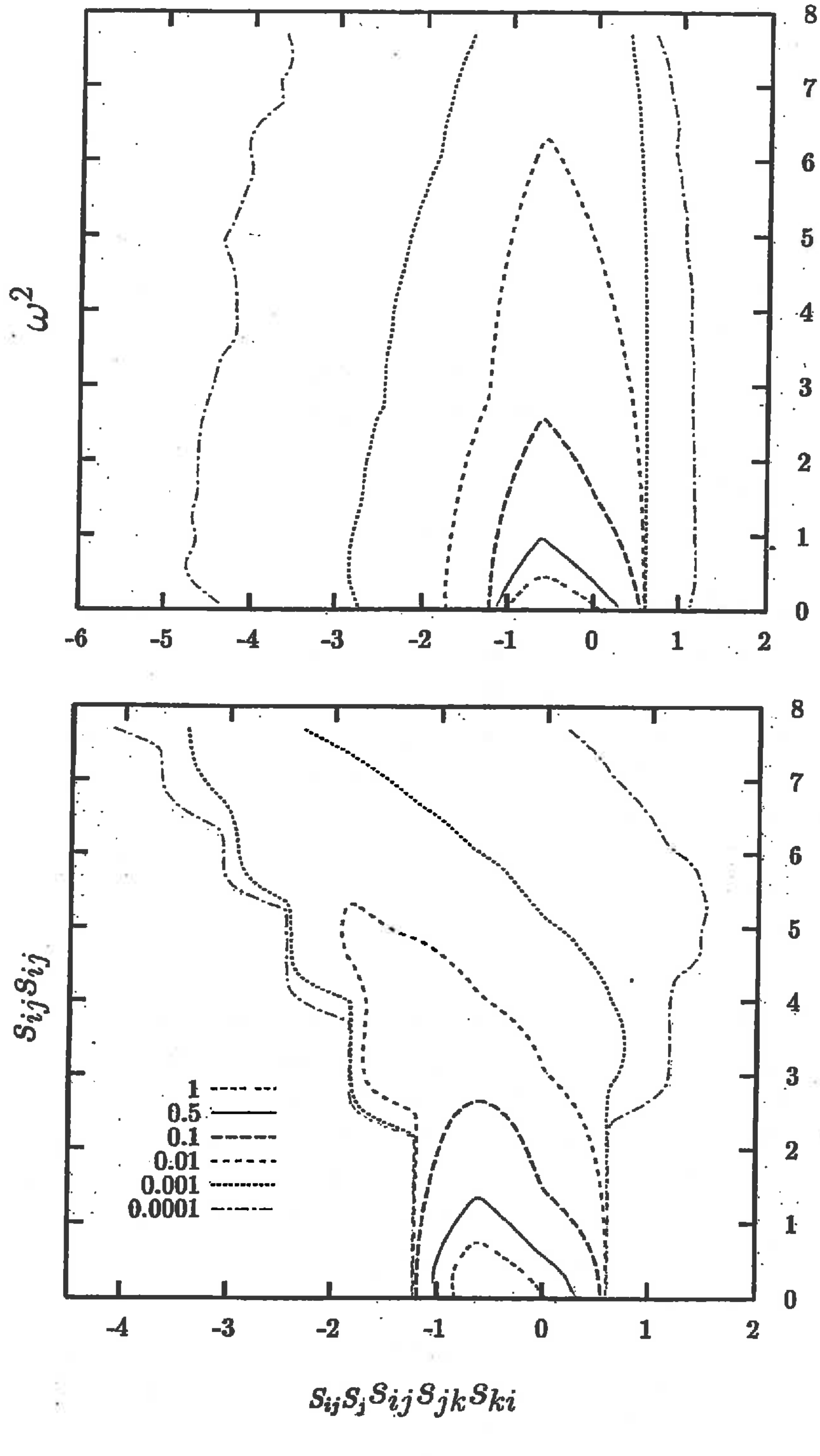

Figure 3. Joint PDF of $-s_{ij}s_{jk}s_{ki}$, normalized on $\langle s^2\rangle^{3/2}$, versus total strain s^2 and enstrophy ω^2, both normalized on $\langle s^2\rangle$.

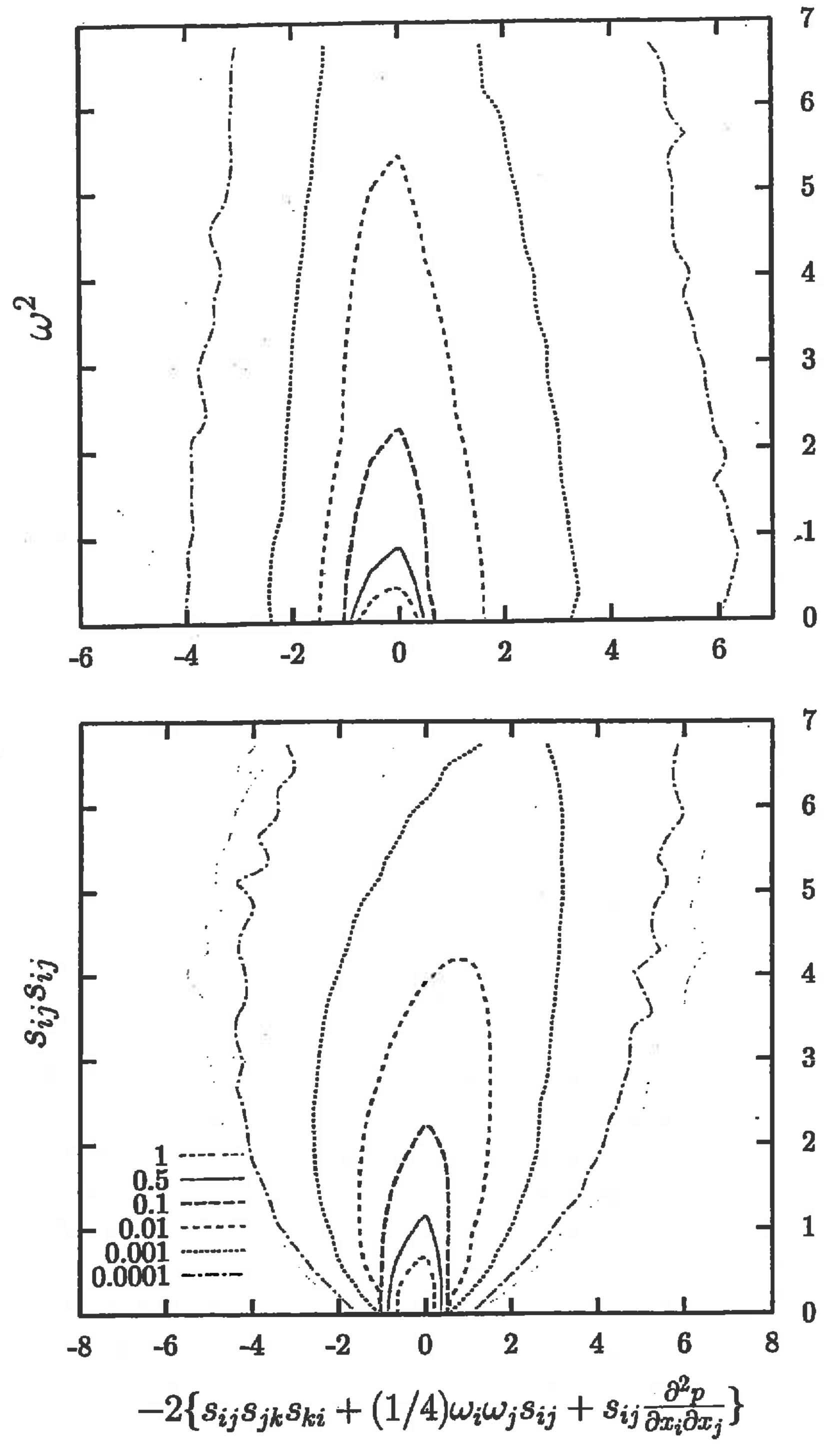

Figure 4. Joint PDFs of production of strain $-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$, normalized on $\langle s^2 \rangle^{3/2}$ versus s^2 and strain ω^2, both normalized on $\langle s^2 \rangle$.

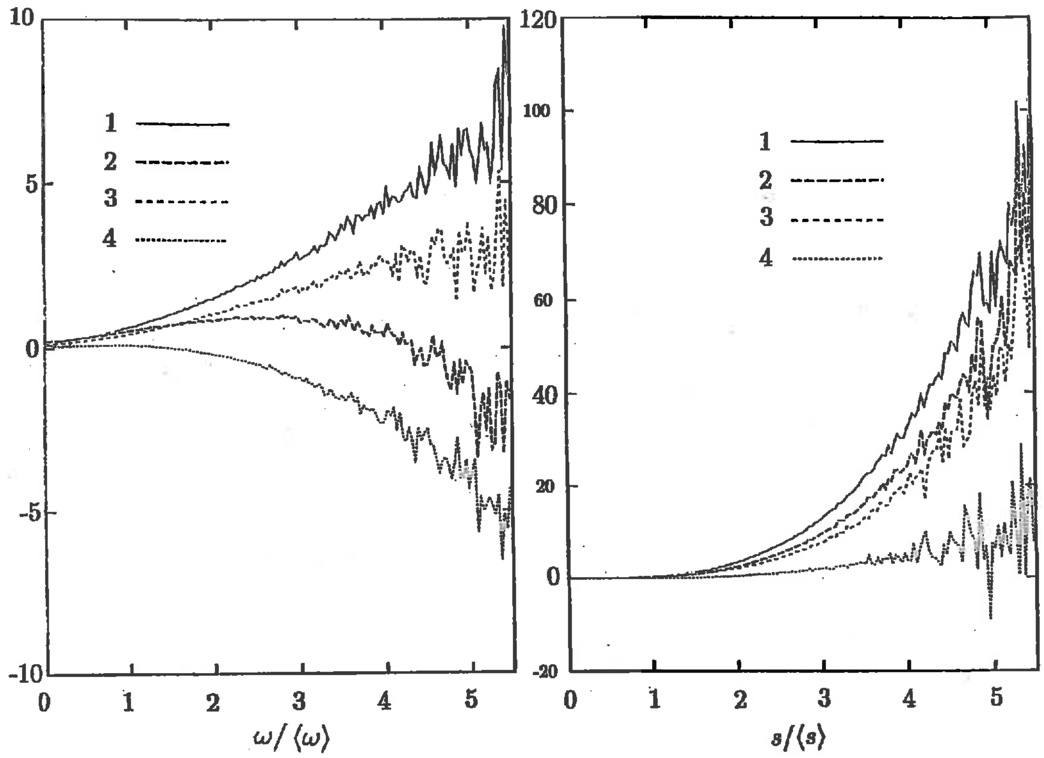

Figure 5. Conditional averages of production of strain and its separate terms in slots of ω and strain s. $1 - -s_{ij}s_{jk}s_{ki}$; $2 - -s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij}$; $3. - -s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$; $4 - \frac{\partial^2 p}{\partial x_i \partial x_j}$. All normalized on $\langle\omega_i\omega_j s_{ij}\rangle$.

The next important point is that the enstrophy production $\omega_i\omega_j s_{ij}$ appears in the equation (4) with the negative sign, so that the vortex stretching is *opposing* the production of dissipation/strain. Indeed, since $\omega_i\omega_j s_{ij}$ is essentially a positively skewed quantity all instantaneous positive values of $\omega_i\omega_j s_{ij}$ make a negative contribution to the right hand side of (4). In other words the energy cascade (whatever this means) is associated primarily with the quantity $-s_{ij}s_{jk}s_{ki}$ rather than with the enstrophy production $\omega_i\omega_j s_{ij}$ and that vortex stretching suppresses the cascade and does not aid it, at least in a *direct* manner $\omega_i\omega_j s_{ij}$ (Tsinober, Ortenberg and Shtilman (1999)). On the contrary it is the vortex *compression,* i.e $\omega_i\omega_j s_{ij} < 0$, that aids the production of strain/disspation and in this sense the 'cascade'. Negative enstrophy production is associated with strong tilting of th vorticity vector and large curvatute of vortex lines, which in turn are associated with large magnitudes of the negative eigenvalue, Λ_3,of the rate of strain tensor (Tisnober 1998ab, Tsinober et al 1998). This is in full conformity with the baove mentioned fact that Λ_3 is doing most of the 'cascade'.

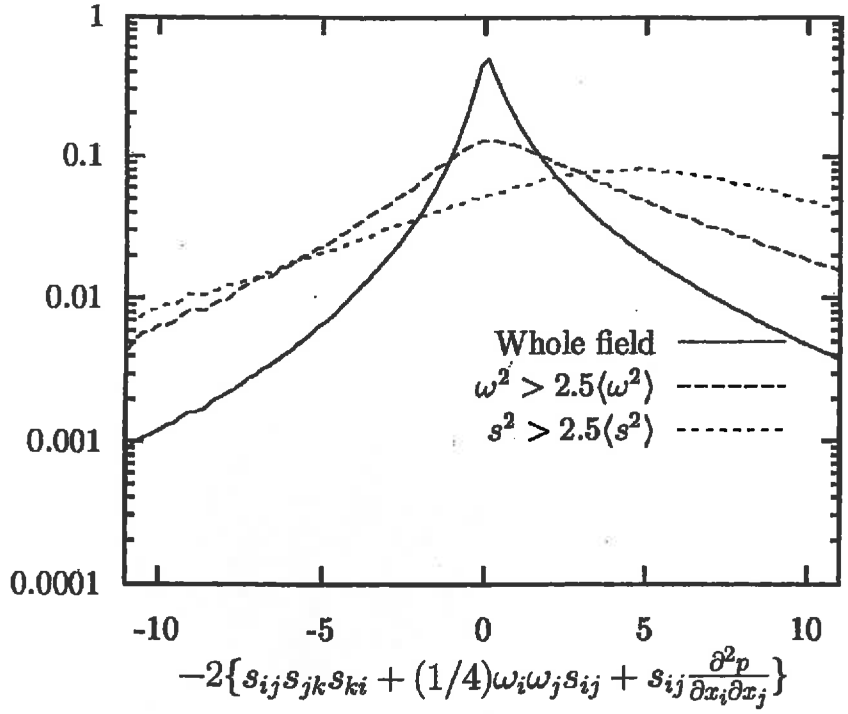

Figure 6. PDF of production of strain $(-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})$ for the whole field and for $\omega^2 > 2.5\,\langle\omega^2\rangle$ and $s^2 > 2.5\,\langle s^2\rangle$.

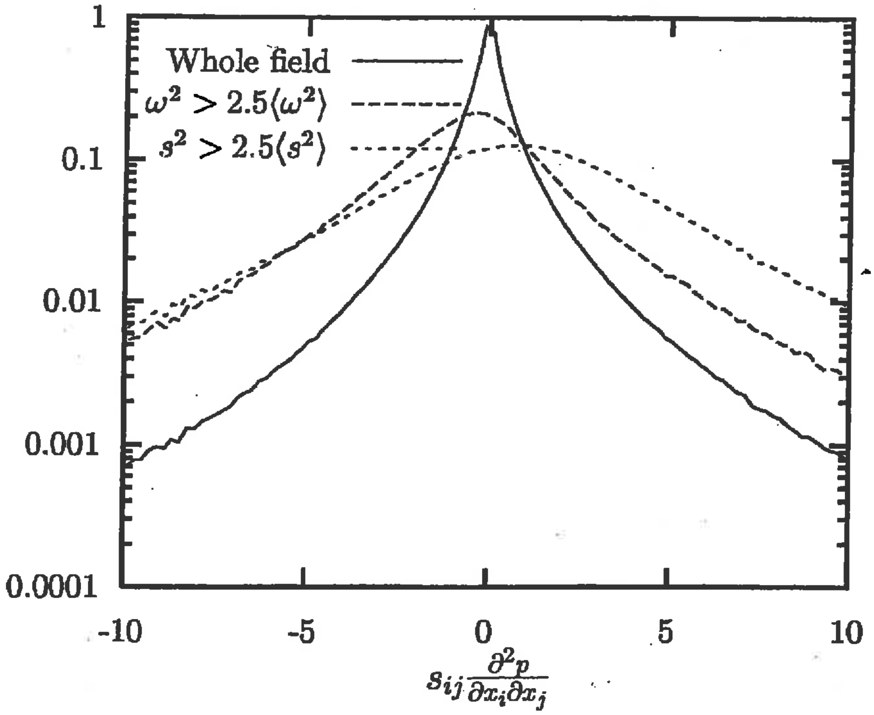

Figure 7. PDF of $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ for the whole field and for $\omega^2 > 2.5\,\langle\omega^2\rangle$ and $s^2 > 2.5\,\langle s^2\rangle$;

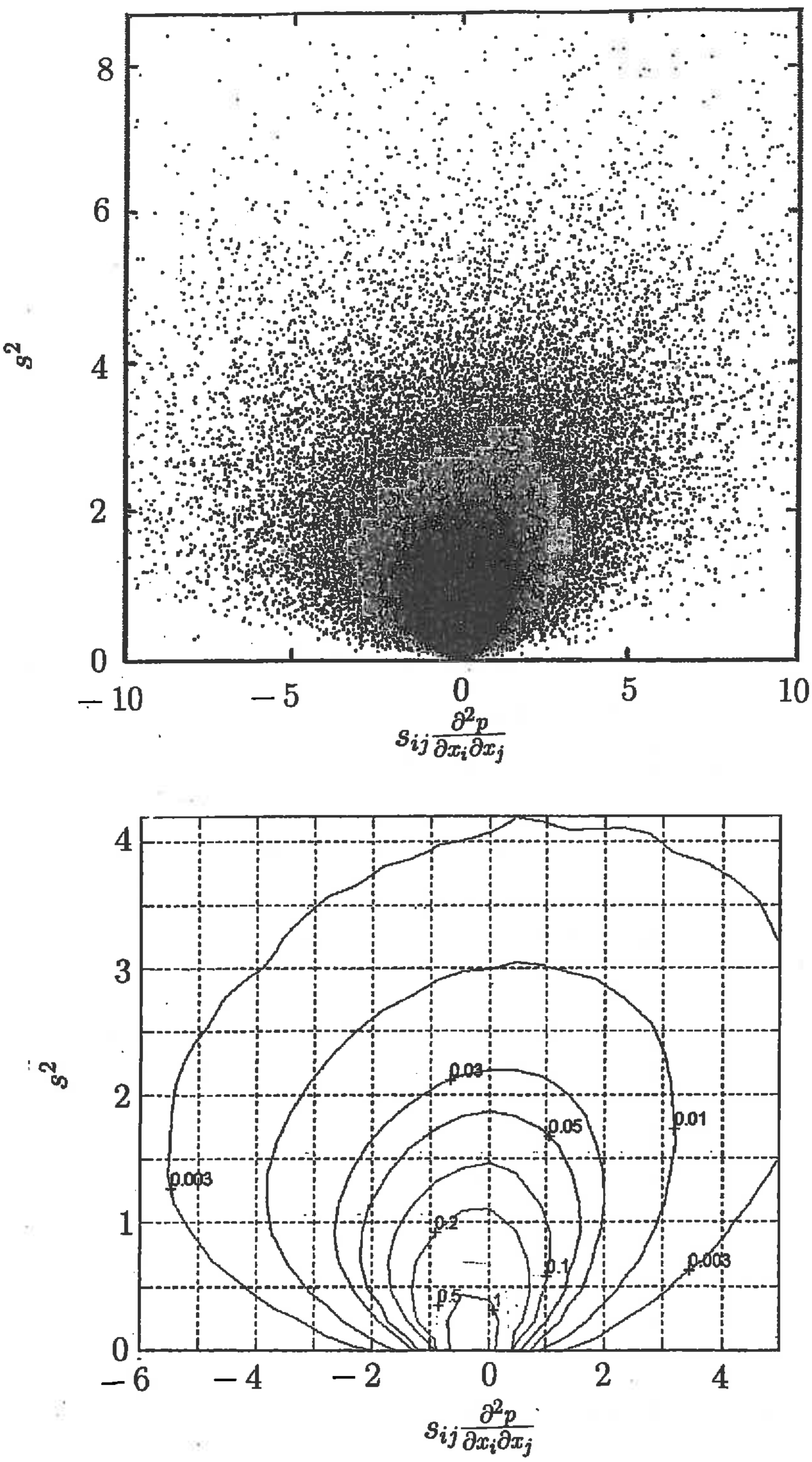

Figure 8. JPDF and scatter plot of $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ and s^2.

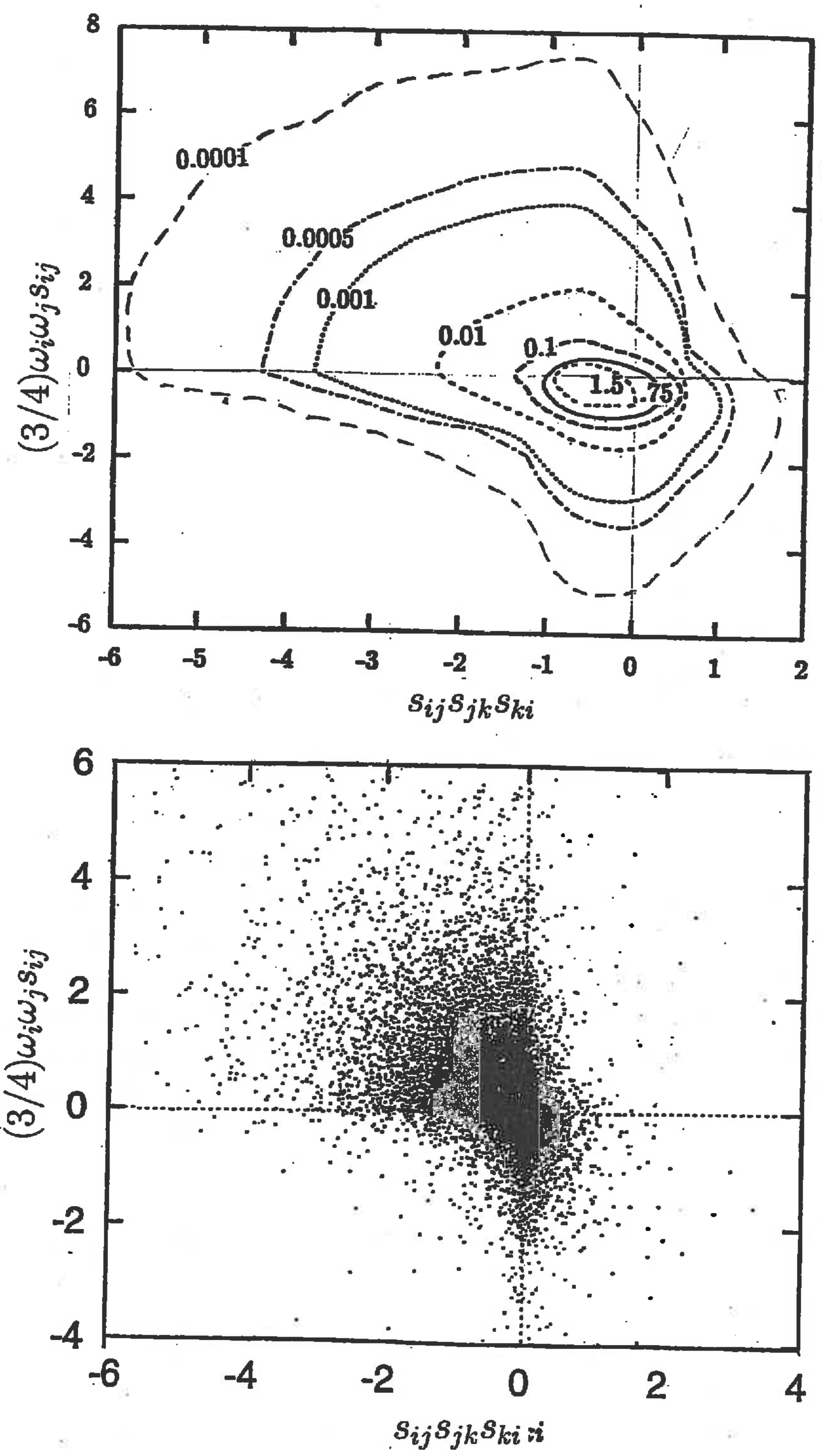

Figure 9. Joint PDF and scatter plot of $\frac{3}{4}\omega_i\omega_j s_{ij}$ versus $-s_{ij}s_{jk}s_{ki}$.

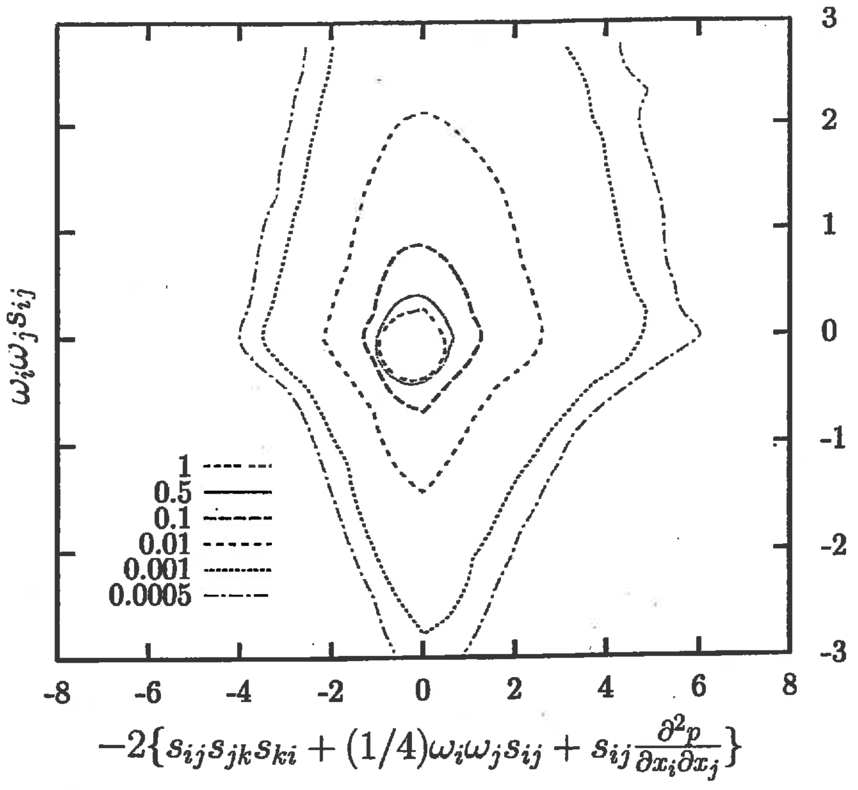

Figure 10. Joint PDF of of $\omega_i\omega_j s_{ij}$ versus $-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$.

One does not have to be confused by the equality $\langle -s_{ij}s_{jk}s_{ki}\rangle = 3/4\langle \omega_i\omega_j s_{ij}\rangle$: though in the mean they are equal, their pointwise relation is strongly nonlocal due to the nonlocal relation between vorticity and strain via a singular integral transform (see, e.g. Ohkitani (1994)). Consequently, locally they are very different as can be seen from their JPDF and scatter plots (figure 9): they are only weakly correlated and there are great many points with small $\omega_i\omega_j s_{ij}$ and large $-s_{ij}s_{jk}s_{ki}$ and vice versa. The same is true of $-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ and $\omega_i\omega_j s_{ij}$ (figure 10).

4 Local flow properties in the $R - Q$ plane

It is convenient to summarize the local flow propreties in the $R - Q$ plane of the invariants of the velocity derivatives tensor $\partial u_i/\partial x_j$. In particular, this allows to see the results given above in a diffrent perspective. The results are given in three separate figures in order to avoid overloading of a single figure with too much of information. The main points are as follows.

Most of the (positive) enstrophy production occurs in the region $D > 0, R < 0$, whereas most of (positive) strain production occurs in the region $D > 0, R > 0$ (see figure 11, where some more details are given). Here $D = Q^3 + (27/4)R^2$ – is the discriminant of the (cubic) equation defining the eigen values of $\partial u_i/\partial x_j$.

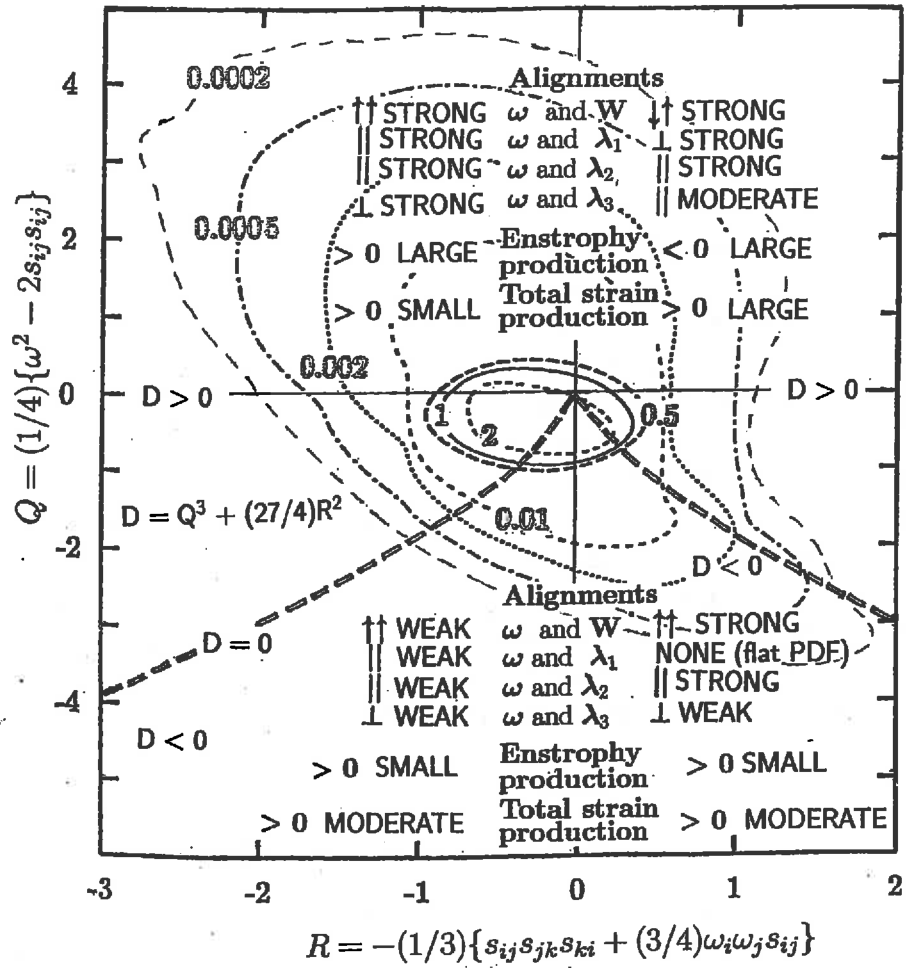

Figure 11. A qualitative summary of local flow properties in the $R-Q$ plane together with the joint PDF of R versus Q, corresponding to averaged quantitities over the four regions $D>0, R>0$; $D>0, R<0$; $D<0, R>0$ and $D<0, R<0$.
I - Alignments, enstrophy production and total strain production. Some additional features: i – the largest *rate* of enstrophy production, $\omega_i\omega_j s_{ij}/\omega^2$ occurrs in the region $D>0, R<0$ and $Q<0$; ii – the largrest production of strain occurrs in the proximity of the curve $D=0$ (from both sides); and $R>0$ and with the largest magnitude of the negative eigenvalue, Λ_3, of the rate of strain; iii – the largrest *rate* of production of strain, $(-s_{ij}s_{jk}s_{ki}-\frac{1}{4}\omega_i\omega_j s_{ij}-s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j})/s^2$, occurrs in the region $D>0, R>0$ and $Q<0$; iv – the PDF of $\cos(\omega, \lambda_2)$ is flat in the region $D>0, R<0$ and $Q<0$.

The region $D > 0, R > 0$ is characterized by large *negative* enstrophy production. Note that strong alignment between vorticity ω_i and the vortex stretcing vector $W_i \equiv \omega_j s_{ij}$ occurrs not only in the region with most of the (positive) enstrophy production ($D > 0, R < 0$), but also in the region $D < 0, R > 0$, where the (positive) enstrophy production is relatively small. Naturally, vorticity, ω_i, and the vortex stretcing vector, $W_i \equiv \omega_j s_{ij}$, *antialign* in the region region $D > 0, R > 0$ with large *negative* enstrophy production. Also is noteworthy that there is strong alignment between vorticity, ω_i, and *both* the largest, $\boldsymbol{\lambda}_1$, and the inermediate, $\boldsymbol{\lambda}_2$, eigenvectors of the rate of strain tensor s_{ij} in the region $D > 0, R < 0$. This is possible, since these alignments happen on *different* sets of points. Likewise strong alignment between vorticity ω_i and and the inermediate eigenvector, $\boldsymbol{\lambda}_2$, occurrs in three *qualitatively* different regions: $D > 0, R < 0$; $D > 0, R > 0$ and $D < 0, R > 0$. This shows that this most popular alignment is caused by different physical reasons in diffrent flow regions (see also figure 12). The above results are in conformity with those regarding curvature of vorticity lines and tilting of $\boldsymbol{\omega}$-vector as shown in figure 13. An additional aspect shown in this figure concerns the interaction of strain an pressure hessian: it is large and positive in the region $D < 0, R > 0$, and it is large and negative in the region $D > 0, R < 0$.

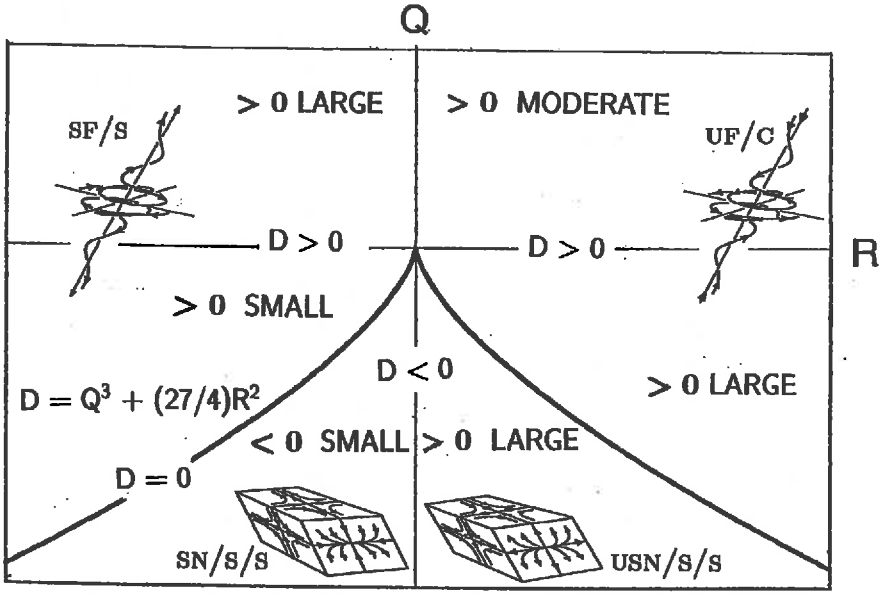

Figure 12. A qualitative summary of the behaviour of the second eigenvalue, Λ_2, of the rate of strain tensor in the $R-Q$ plane corresponding to averages quantitities over the six regions $Q > 0,\ R > 0$; $Q > 0,\ R < 0$; $D < 0,\ R < 0$; $D < 0,\ R > 0$; $Q < 0,\ R > 0$ and $D > 0$; $Q < 0,\ R < 0$ and $D > 0$. Also shown the schematic local flow fields (e.g. Ooi et al (1999)): SF/S – stable focus/stretching, UF/C – stable focus/compressing, SN/S/S – stable node/saddle/saddle, USN/S/S – unstable node/saddle/saddle.

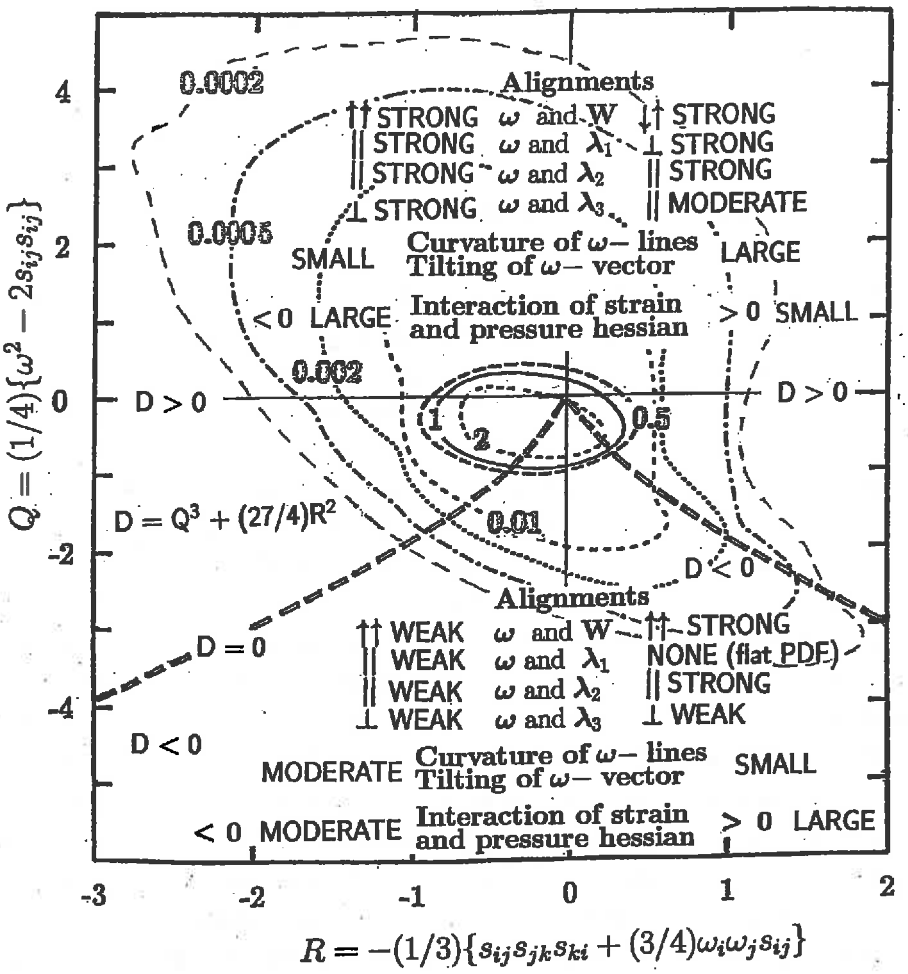

Figure 13. A qualitative summary of local flow properties in the $R-Q$ plane together with the joint PDF of R versus Q, corresponding to averaged quantitities over the four regions $D>0, R>0$; $D>0, R<0$; $D<0, R>0$ and $D<0, R<0$.
II. Curvature of ω - lines, tilting of ω - vector and interaction of strain and pressure hessian $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$.
Some additional features: i – in the region $Q<0$, $R<0$ and $D>0$ the curvature of ω - lines and the tilting of ω - vector are both large, they are both moderate in the region $Q<0$, $R>0$ and $D>0$; ii – the interaction of strain and pressure hessian $s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}$ assumes largest positive values in the proximity of the curve $D=0$ (from both sides) and $R>0$.

It is noteworthy that some similar results in terms of energy flux using conditional averages in the plane of invariants of velocity derivatives , $Q-R$, were reported by Chertkov, Pumir and Shraiman (1999); see also Borue and Orszag (1998)[2].

5 Concluding remarks

•– There exist two nonlocally intreconnected weakly correlated processes: *i* – predominant vortex stretching/ enstrophy production and *ii* – predominant self-amplification of the rate of strain/production of total strain. The energy cascade (whatever this means) and its final result - dissipation are associated with the latter, i.e. with the quantity $-s_{ij}s_{jk}s_{ki}$ rather than with the enstrophy production $\omega_i\omega_j s_{ij}$. Moreover, vortex stretching suppresses the cascade and does not aid it, at least in a *direct* manner [3]. On the contrary it is the vortex *compression*, i.e $\omega_i\omega_j s_{ij} < 0$, that aids the production of strain/disspation and in this sense the 'cascade'. The predominant vortex stretching/ enstrophy production is associated mostly with the largest positive eignevalue, Λ_1, of the rate of stain tensor and also with its intermediate eigenvector, Λ_2. The predominant self-amplification of the rate of strain/production of total strain is associated totally with the largest in magnitude negative eignevalue, Λ_3.

•–The nonlinearities such as enstrophy production, production of strain and many others (Tsinober (1998), Tsinober et al (1999)) are an order of magnitude larger in the regions dominated by strain than in the enstrophy dominated regions. In this sense the enstrophy dominated regions are characterized by reduced nonlinearities including the energy cascade whatever this means. In other words the most intense nonlinear processes occur in the strain dominated regions.[4] This supports the view that regions of con-

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[2]These authors used a parameterization of the SGS energy flux/dissipation in the form

$$-\langle s_{ij}\rangle_l \tau_{ij} \approx l^2\{-\langle s_{ij}\rangle_l\langle s_{jk}\rangle_l\langle s_{ki}\rangle_l + 1/4\langle\omega_i\rangle_l\langle\omega_j\rangle_l\langle s_{ij}\rangle\} \qquad (7)$$

They arrived to the conclusion that sugrid energy transfer over the scales (SGS-dissipation) takes place in regions with negative skewness of the filtered strain tensor, i.e. $\langle s_{ij}\rangle_l\langle s_{jk}\rangle_l\langle s_{ki}\rangle_l)$ or where the vorticity stretching term is positive. The latter is due to the *positive* sign in front of $1/4\langle\omega_i\rangle_l\langle\omega_j\rangle_l\langle s_{ij}\rangle$ in the equation (7), contrary to the *negative* sign in the equation (4). This shows how dangerous is drawing conclusions regarding the physics of turbulence from *models*. Also, though $\langle s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}\rangle = 0$ due to homogeneity and incompressibility it is unlikely that either $\langle s_{ij}\frac{\partial^2 p}{\partial x_i \partial x_j}\rangle_l$ or $\langle s_{ij}\rangle_l\langle\frac{\partial^2 p}{\partial x_i \partial x_j}\rangle_l$ or both will vanish.

[3]Contrary to the common belief: '*It seems that the stretching of vortex filaments must be regarded as the principal mechanical cause of the high rate of dissipation which is associated with turbulent motion*' (Taylor (1938)).

[4]This reduction of nonlinearities is in some respect analogous to the processes occurring in the so called elliptical regions (corresponding to the enstrophy dominated regions) in two-dimensional turbulent flows (Weiss (1991)), and in a turbulent flow in the proximity

centrated vorticity in turbulent flows are not that important as previously thought (Jimenez et al (1993), Dernoucourt et al (1998), Roux et al (1998), Tsinober (1998a)).

6 Appendix I. Why mean enstrophy and strain production are positive?

So far there have been given no theoretical arguments in favor of positiveness of $\langle \omega_i \omega_j s_{ij} \rangle$ (and also $\langle -s_{ij} s_{jk} s_{ki} \rangle$). [5]

The argument that the reason is the (approximate) balance between the enstrophy generation and its destruction via viscosity is misleading and puts the consequences before the reasons, since it is known that for Euler equations the enstrophy generation increases with time very fast – apparently without limit (Yudovich (1974), Betchov (1976), Chorin (1982), Bell and Marcus (1992), Brachet et al. (1992), Fernandez et al. (1995), Grauer and Sideris (1995), Green and Boratav (1997), Kerr (1993),). It is noteworthy that there is another aspect in which the (approximate) balance between the *mean* enstrophy generation and its *mean* destruction via viscosity can be misleading as well. Namely, this balance holds also in the enstrophy dominated regions, but fails in the regions dominated by strain. The important point is that most of enstrophy generation occurs in the regions dominated by strain (see section 2 and Tsinober (1998), Tsinober et al (1999)).

Another rather common view that the prevalence of vortex stretching is due to the predominance of stretching of material lines is - at best - true in part only, since, there exist several qualitative differences between the two processes. For example, for a Gaussian isotropic velocity field $\langle \omega_i \omega_j s_{ij} \rangle \equiv 0$, whereas the mean rate of stretching of material lines is essentially positive, i.e. the nature of vortex stretching process is to a large extent dynamical and not just a kinematic one (for more details on the differences between the two, see Tsinober (1998)).

A following simple theoretical argument can be given for the case with a Gaussian velocity field [6] at the initial moment, $t = 0$. Let us look at the equation for the mean enstrophy production $\langle \omega_i \omega_j s_{ij} \rangle$ (dropping the viscous terms)

$$\frac{D}{Dt} \langle \omega_i \omega_j s_{ij} \rangle = \langle \omega_j s_{ij} \omega_k s_{ik} \rangle - \left\langle \omega_i \omega_j \frac{\partial^2 p}{\partial x_i \partial x_j} \right\rangle , \qquad (8)$$

of a large strained vortex (Andreotti et al (1998)).

[5] Rigorous results on this issue would comprise a major contribution to the understanding of *physics* of turbulence.

[6] It is sufficient that the velocity field satisfies the zero-fourth-cumulant relation, i.e. is quasi-normal.

For a Gaussian velocity field $\langle\omega_i\omega_j s_{ij}\rangle_G = 0$, $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle_G = 0$ and $\langle\omega_j s_{ij}\omega_k s_{ik}\rangle_G = \frac{1}{6}\langle\omega^2\rangle^2 > 0$ (the quantity $\omega_j s_{ij}\omega_k s_{ik} \equiv W^2, W_i = \omega_j s_{ij}$, so it is positive pointwise for any vector field). Hence at $t = 0$

$$\left\{\frac{D}{Dt}\langle\omega_i\omega_j s_{ij}\rangle\right\}_{t=o} = \{\langle\omega_j s_{ij}\omega_k s_{ik}\rangle\}_{t=0} > 0, \tag{9}$$

It follows from the equation (9) that at least for a short time interval t the mean enstrophy production will become positive. For later moments the vorticity–pressure hessian correlation $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle$ becomes finite, an nothing is known rigorously. As folows from DNS of NSE in a periodic box the corelation $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle$ is positive, but is smaller than $\langle\omega_j s_{ij}\omega_k s_{ik}\rangle \equiv \langle W^2\rangle$. Namely, $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle \sim \frac{1}{3}\langle W^2\rangle$, so that the RHS of (8) remains positive (Tsinober et al (1995)). It is noteworthy that the equation (8) with $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle = 0$ is precisely the one arising using the quasinormal approximation $\frac{D^2}{Dt}\langle\omega^2\rangle = \frac{1}{3}\langle\omega^2\rangle^2$ (Proudman and Reid (1954), see also Kaneda (1993)), since $\frac{D}{Dt}\langle\omega_i\omega_j s_{ij}\rangle = \frac{1}{2}\frac{D^2}{Dt}\langle\omega^2\rangle$ and under quasi-normal approximation $\langle\omega_j s_{ij}\omega_k s_{ik}\rangle = \frac{1}{6}\langle\omega^2\rangle^2$ and $\left\langle\omega_i\omega_j\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle = 0$. The essential point is that at $t = 0$ the relation (9) is precise due to the freedom of the choice of the intitial condition.

In a similar way one can see from the equation (6) that the mean rate of production of strain $-2\left\{\langle s_{ij}s_{jk}s_{ki}\rangle - \frac{1}{4}\langle\omega_i\omega_j s_{ij}\rangle\right\}$ ($\left\langle s_{ij}\frac{\partial^2 p}{\partial x_i\partial x_j}\right\rangle = 0$ due to incompressibility) becomes positive at small times (at $t = 0$ it is vanishing) for an initially Gaussian velocity field. It is seen also from the equation for $\langle\omega_i\omega_j s_{ij}\rangle$ that an initially Gaussian and nearly potential velocity field with small seeding of vorticity will produce - at least for a short time - an essentially positive enstrophy generation as well. This process seems to be of importance in the phenomenon of entrainment of nonturbulent fluid into the turbulent region in the proximity of the region separating turbulent and nonturbulent fluid.

It is noteworthy that the equations (8,9) and similar ones for $\langle s_{ij}s_{jk}s_{ki}\rangle$ is one of the manifestations of the statistical irreversibililty[7] of turbulent flows (Betchov (1974), Novikov (1974)). There exist, at least, two different aspects of this problem. The first one is related to purely inertial behaviour governed by the Euler equations as mentioned above. It is closely related to the (possible) formation of singularities in 3D Euler flows in finite or infinite time. The above example (equations (8-9) and similar ones for $\langle s_{ij}s_{jk}s_{ki}\rangle$) is closely related to this aspect. The second aspect is associated with the dissipative nature of turbulent flows. Viscosity provides a sink of energy, enstrophy, etc. moderating their unbounded growth in the inviscid case.

[7]The corresponding dynamical *instantaneous* (inviscid) equations are reversible. Hence, the term *statistical.*

7 Appendix II. On the difficulties in defining scales, cascades and related matters.

As mentioned above there is a generic ambiguity in defining the meaning of the term *small scales* (or more generally scales) and consequently the meaning of the term *cascade* in turbulence research. The specific meaning of this term and associated interscale energy exchange/'cascade' (e. g. spectral energy transfer) is essentially decompostion/reperesentation dependent (see, for example, Borue & Orszag (1998), Frick and Zimin (1993), Fuehrer and Friehe (1999), Germano(1999), Holmes et al (1996), Mahrt & Howell (1994), Meneveau (1991), Pullin and Saffman (1997), Sirovich (1997), Tsuge (1984), Waleffe (1993) and references therein) [8]. Perhaps, the only common in all decompostions/representations is that the small scales are *always* associated with the field of velocity derivatives. Therefore, it is naturally to look at this field as the one *objectively* (i.e. decompostion/reperesenation **in**dependent) representing the small scales. Indeed, the dissipation is associated precisely with the strain field s_{ij}. The advantage of this 'definition' of small scales can be seen from the following example. It is well known that there is no contribution from the nonlinear term in the total energy balance equation (in a homogeneous/periodic flows it's contribution is null as well in the mean) since the nonlinear terms has the form of a spatial flux, $\partial\{...\}/\partial x_j$. In other words the nonlinear term redistributes the energy in physical space, but does it do more than that? [9] The usual claim is that *the nonlinear term redistributes the energy among the scales of motion* (Frisch (1995), p.22), whereas in reality the nonlinear term redistributes the energy, e.g. in the Fourier space between the Fourier components of the turbulent field. However, the nonlinear term does it in a *different* way between the components of *different* decompositions, such as the Fourier – or wavelet representations, the POD, and so on (Frick and Zimin (1993), Holmes et al (1996), Mahrt & Howell (1994), Meneveau (1991), Sirovich (1997), Waleffe (1993) and references therein). In other words the term 'cascade' corresponds to a process of interaction/exchange of (not necessarily only) energy between components of some *particular* decomposition/representation of a turbulent field associated with the nonlinearity and the nonlocality of the turbulence phenomenon (the two **n**'s out of three: **n**onlinearity, **n**onlocality and **n**onintegrability, which make the problem so

[8] Indeed, the meaning of *scales* is different for different reperesentations: it is not the same for *Fourier* ('regular' and helical) and similar (Fourier Weierstrass, Gabor, Littlewood-Paley) decompositions; *Wavelets* (wavepackets, solitons); *POD; LES.* It is also different in various heuristic representations, e.g. 'two-fluid' (Organized/Incoherent, Deterministic/Random and some other two-fluid models); intermittency-prompted (breakdown coefficients/multipliers, (multi)fractals); Moffat's 'smart decomposition' and the 'punctuated' conservative dynamics.

[9] It is straightforward to see that in a homogeneous turbulent flow the mean energy of volume of any scale (Lagrangian and/or Eulerian) is changing due to viscous dissipation and external forcing only.

impossibly difficult). On the other hand, the energy transfer as any physical process should be invariant of particular decompositions/representations of a turbulent field.[10] Taking the velocity derivatives as a basic notion allows one to resolve/clarify (to some extent) the ambiguities associated with the terms 'energy transfer', 'scale', and so on in the following way. While the mean contribution of the nonlinearity in the energy balance is vanishing, the nonlinear term definitely *creates* vorticity (and strain, see sections 2 and 3) in *physical space*, since the enstrophy production $\langle \omega_i \omega_j s_{ij} \rangle > 0$ is strictly positive as well the corresponding term for the production of strain. As mentioned above it is naturally and justified from the *physical* point of view to associate the field of velocity derivatives with *small scales*. It is immediately seen that 3-D turbulent flows have a natural tendency to create small scales. Namely, the velocity field (and its energy) arising in the process of (self) production of the field of velocity derivatives is the one which is associated with the small scales. This process is what can be called as energy transfer from large to small scales in physical space. The latter are not necessarily created via a stepwise turbulent "cascade": it can be bypassed (and most probably this is the case in turbulent flows), e.g. via broad-band instabilities with highest growth rate at short wavelengths (e. g. Pierrehumbert & Widnall 1982, Smith & Wei 1994) or some other approximately single step process (Betchov (1976), Douady, Couder & Brachet (1991), Vincent & Meneguzzi (1994), Garg & Warhaft (1998); the problem goes back to Townsend (1951): *...the postulated process differs from the ordinary type of turbulent energy transfer being fundamentally a single process.*; see also Corrsin (1962), Tennekes (1968)). Indeed two large neighbouring eddies can dissipate energy directly by rubbing each other on a very small scale. Note that the process of vorticity production is not just creation of the field of velocity derivatives. It involves literally creation

[10]The difficulty is not a trivial one and seems to be 'generic'. Under turbulent motion/dynamics the interaction of 'modes' (whatever they are) is strong. The resulting structure(s) is(are) not represented by the modes (of whatever decomposition), e.g. Fourier-decomposition of a flow in a periodic box. Hence the it's ambiguity (for another aspect of Fourier-Transform ambiguity see Tennekes (1976)). Recall the suggestion by Dryden (1948): '*It is necessary to separate the random processes from the nonrandom processes*'. The implication is that such a separation is possible. But this is not obvious at all, as it is seen from the futility of the enourmous efforts to do so. For example, it is even not known how to separate random gravity-wave motion (which does not produce vertical transport) and genuine turbulence (which does) in a stably stratified fluid (Stewart (1959)). All the attempts to find a 'good' decomposition are related to what R. Betchov (1993) called the '*dream of linearized physicists*', i.e. a *superposition* of some (desirably simple) elements). The dream is, of course, to find sets (consisting of small number) of *simple weakly interacting* elements/objects *adequately* representing the turbulent field. Those are known so far are strongly interacting (most of them nonlocally) - a fact reflecting one of the central difficulties in 'solving the turbulence problem' as a whole, in general, and the 'closure problem' (such as LES and other reduced descriptions of turbulence (Kraichnan (1988)) , in particular, as well as in construction of a kind of statistical mechanics of turbulence (Kraichnan and Chen (1989)).

of small scale structure in the following sense. Namely, an inevitable concomittant process to vortex stretching is tilting and folding of vorticity due to the energy constraint (Chorin (1982), Tsinober (1998)). Simultaneosly, the strain field is built up at the same rate too (see section 3 and Tsinober et al (1999)). This together with limitations on the volume scale leads to formation of fine small scale structure. Since the flow field (including velocity, which is mostly a large scale object) is determined entirely by the field of vorticity, i.e. the velocity field is a functional of vorticity $\mathbf{v} = F\{\boldsymbol{\omega}(\mathbf{x}, t)\}$), the production of vorticity 'reacts back' in creating the corresponding velocity field., the prodction of vorticity 'reacts back' in creating the corresponding velocity field.[11] It is noteworthy that due to nonlocality of the relation $\mathbf{v} = F\{\boldsymbol{\omega}(\mathbf{x}, t)\}$ mostly small scale vorticity is, generally, creating also some large scale velocity. Therefore from the *physical point* it seems incorrect to treate small scales as a kind of "passive" objects as well as it seems impossible to "eliminate" them (as is done in many theories) reducing their reacton back to some eddy viscosity or similar things only. In view of the above arguments it seems that in *physical space* the energy is dissipated not necessarily via a multistep cascade-like process. Instead, there is an exchange of energy (and everything else) in both directions, whereas the dissipation occurs in "small scales". So it is quite possible that in the physical space the famous verse by Richardson (1922, p.66)

Big whirls have little whorls,
Which feed on their velocity.
And little whorls have lesser whorls
And so on to viscosity
(In the molecular sense)

should be replaced by the one by Betchov (1976, p. 845)

Big whirls lack smaller whirls,
To feed on their velocity.
They crash and form the finest curls
Permitted by viscosity.

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