ON STATISTICS AND STRUCTURE(S) IN TURBULENCE [1]

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1 Introduction

It is common in the vast literature on turbulence to consider the terms *statistical* and *stuctural* as incompatible or even contradictory. See, for example, the collection of citations by Lumley 1989. Three additional typical examples are added below in which following Lumley the names were suppressed 'to protect the guilty':

'it became obvious that statistical averaging was in fact destroying the most interesting and important phenomena in turbulence – the formation, dynamics and persistence of vortex motion.'

'In place of theory without structure, the result to date has been structure without theory.'

'Its blindeness to these structural facts is precisely the disability of the statistical idea.'

The first aim of this paper is to demonstrate that it is a misconception to contrapose the *statistical* and the *structural* and that they represent different facets/aspects of the same problem, so that there is no gap between structure(s) [2] and statistics, just like it seems impossible to separate the structure(s) from the so called 'random structureless background' or the '*random processes from the nonrandom processes*' (Dryden 1948) due to strong interaction (and nonlocality) both between individual structures, and between structures and the background.

On the other hand there are numerous attempts to relate *some* of the statistical manifestations of turbulence - such as various scaling exponents, *some* PDFs, etc. - to the structure(s) of turbulence - whatever this means.

It is the second aim of this paper to show that there exists no one-to-none relation between simple statistical manifestations and structure(s) of turbulence, e. g. *qualitatively different* phenomena can possess the same set of scaling exponents, and that one needs more subtle statistical characterizations of turbulence structure(s).

Both issues are intimately related to the non-Gaussian nature of turbulence (and *some*

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[2] The term *structure(s)* is used here deliberately in order to emphasize the duality (or even multiplicity) of the meaning of the underlying problem. The first is about how turbulence 'looks like'. The second implies existence of some observable entities. Objective treatment of both requires use statisitcal methods.

Both issues are intimately related to the non-Gaussian nature of turbulence (and *some* of its quasi-gaussian manifestations) and the necessity (and the only objective means) to handle the issues of turbulence structure(s) via statistics. Examples of *structure sensitive statistics*, which can be considered as statistical tools, comprise the third aim of this paper.

2 On what is structure(s) of turbulence

This question is as difficult as the question about what is turbulence itself, but it can be answered via a statement of impotence: speaking about 'structure(s)' in turbulence implies that there exists something 'structureless', e.g. Gaussian random field as representative of full/complete disorder. Hence in principle all non-Gaussian manifestations of turbulent flows can be seen as the *statistical* signature of turbulence structure(s)[3] [4]. The advantage of such an approach is that it allows one to get insights into the *structure* of turbulence without the necessity of knowing how it's *structures* 'look'[5]. This is especially important in view of numerous problems/ambiguities in defintions of *individual* structures in turbulent flows and their identification and statistical characterization as well as their incorporation in "theories" (see Appendix I).
The next most difficult question is about the relevance/significance of some particular aspect of non-Gaussianity to a specific problem in question. It seems that here enters the *subjective* realm: the criteria of significance (which is the matter of physics!) are decided by the researchers. However, the following examples show that objective choice of the structure sensitive statistics is dictated by general dynamical aspects of the problem [6].

For instance, the build up of *odd* moments is an important *specific* manifestation of structure of turbulence along with being the manifestation of its nonlinearity. Two most important examples are the third velocity structure function $S_3(r) = \langle\{[\mathbf{u}(\mathbf{x}+\mathbf{r})-\mathbf{u}(\mathbf{x})]\cdot \mathbf{r}/r\}^3\rangle$ and the mean enstrophy generation $\langle\omega_i\omega_k s_{ik}\rangle$. The first one is associated with the $-4/5$ Kolmogorov law (Kolmogorov 1941b)

$$S_3(r) = -4/5\langle\epsilon\rangle r,$$

[3]This does not imply that an exactly Gaussian field does not necessarily possess any spatial or temporal structures (see, e. g. fig 3 in She et al 1990). This means, however, that an exactly Gaussian field does not possess any *dynamically* relevant structure(s).

[4]This includes all aspects of the so called intermittency problem, though there is no concensus on the meaning of the term *intermittency* (see Frisch 1995, Sreenivasan and Antonia 1997, Tsinober 1998b and references therein).

[5]It is noteworthy that such an approach is not new and is due Kolmogorov 1941a. Note the title of his famous paper *The local structure of turbulence in ncompressible viscous fluid for very large Reynolds numbers.*

[6]In the sequel we are interested in the dynamical aspects of the problem. Various 'kinematic' issues like transport of passive objects (scalars, vectors, etc.), in which Gaussian or other *precribed* velocity fields are used rather successfully are beyond the scope of this paper.

which is the first strong indication of presense of structure in the inertial range showing that both non-Gaussianity and structure of turbulence are directly related to it's dissipative nature. The $-4/5$ Kolmogorov law clearly overrules the claims that '*Kolmogorov's work on the fine-scale properties ignores any structure which may be present in the flow*' (Frisch 1995, p. 182) and that it is associated with *near-Gaussian statistics* (Farge and Guyon 1998, Katul et al 1994, She et al 1991, Chertkov et al 1999). The essentially positive value of the mean enstrophy generation $\langle \omega_i \omega_k s_{ik} \rangle$, discovered by Taylor 1938, is the first indication of presense of structure in the dissipation range, where turbulence is particularly strongly non-Gaussian and intermittent (Kraichnan 1967 , Novikov 1963, Sreenivasan and Antonia 1997). The two examples show that both the essential turbulence dynamics and its structure are associated with those aspects of it's non-Gaussianity which are exhibited in the build up of *odd* moments, which among other things means phase and geometrical coherency, i. e. structure.

3 Examples of statistics weakly sensitive to structure(s)

The first example of this kind are energy spectra in which the phase information is lost. Hence the weak sensitivity to the structure of turbulence. This insensitivity, in particular, is exhibited in the scaling exponents when such exist. For example, the famous $-5/3$ exponent can be obtained for a great variety of *qualitatively* different real systems and theoretical models (Biferale et al 1994, Chorin 1994, Pullin and Saffman 1997, tsinober 1998b), and, of course, one can construct a set of purely Gaussian velocity fields, i.e. statistically structureless, with any desired length of the $-5/3$ 'inertial' range (see, e.g. Elliott and Majda 1995). Vice versa the spectral slope can change, '*yet retaining all the phase information*' (Armi and Flament 1987). Moreover, not only '*the spectral slope alone is inadequate to differentiate between theories*' (Armi and Flament 1987), but *alone* it does not necessarily correspond to any particular structure(s) in turbulence or to absense of it: there is no one-to-one relation between scaling exponents and structure(s) of turbulence. This is true not only of exponents related to Fourier decomposition with its ambiquity (Tennekes 1976), but of many other scaling exponents including those obtained in some wavelet space and in the physical space.

Likewise similar PDFs of *some* quantities can correspond to qualitatively different structure(s) and quantitatively different values of Reynolds number (Kraichnan and Kimura 1994, Tsinober 1998a,b). The emphasis is on *some* quantitities like pressure or some other usually (but not necessarily) even order quantitities in velocities or their derivatives, since the PDFs of *other* approprately chosen quantities are sensitive to structure (see below).

4 Structure sensitive statistics

4.1 Use of odd order structure funcions

This is an example on how structure sensitive statistics can help in looking for the *right* reasons of measured spectra in the lower mesoscale range (Lindborg 1998). This is done by using the third order structure functions which are generally positive in the two dimensional case (contrary to the three-dimensional case) and calculations based on wind data from 5754 airplane flights, reported in the MOZAIC data set. It is argued that the k^{-3}-range is due to two-dimensional turbulence and can be interpreted as an enstrophy inertial range, while the $k^{-5/3}$-range is probably not due to two-dimensional turbulence and should not be interpreted as a two-dimensional energy inertial range [7]. There is a competing hypothesis that the large scale $-5/3$ range is the spectrum of weakly non-linear internal gravity waves with a forward energy cascade (VanZandt 1982).

Another example is the demonstration that the small scale structure (both in the inertial and dissipative range) of a homogeneous turbulent shear flow is essentially *anisotropic* at moderately large Reynolds number (Garg and Warhaft 1998). This is done via looking at the velocity structure functions of third and fifth order of both longitudinal and transverse velocity components and corresponding moments of velocity derivatives. In particular, there is skewness of order 1 of the derivative of the longitudinal velocity in the dirction of the mean gradient. Similar results were obtained in DNS (see references in Garg and Warhaft 1998, Borue and Orszag 1996). [8] This is the right place to comment on two aspects of statistical manifestations of turbulence structure. First, *anisotropy* is a typical *statistical* characteristic of turbulent flows and hardly can be applied to *individual* structures, e. g. a turbulent flow consisting mostly of 'anisotropic' individual structures can be statistically istropic [9]. Second, an ideally *homogeneous in the mean* turbulent *shear* flow is *unphysiscal* in the following sense. The equation for the mean part of such a flow does not contain any information on the turbulent fluctuations, since any turbulent shear flow 'knows' about the turbulent fluctuations via the *gradients* of the (mean) Reynolds stresses. In a homogeneous shear flow these gradients vanish, so that the fluctuating part of the turbulent flow is decoupled from it's mean, and there is no source of energy to sustain turbulent flow. This is a clear indication of an *ill defined mean* - in reality the profiles of properly defined 'mean' velocity (and other quantitites) have a ladder-like

[7] There is also a claim that the spectral slope in the enstrophy range is more shallow than -3 and is close to $-7/3$ (Tsinober 1995). This range (and related anomalous diffusion) is explained in terms of the phenomenon of spontaneous breaking of statistical isotropy (rotational and/or reflexional) symmetry - locally and/or globally.

[8] It is noteworthy that analogous 'misbehaviour' of large Reynolds number turbulence was reported by R. W. Stewart in 1969 regarding the skewness of temperature fluctuations in the atmospheric boudary layer.

[9] Hill 1997 has shown that the -4/5 Komlogorov law is more sensitive to the anisotropy of the third-order structure function (again odd moments) than to anisotropy of the second-order structure function.

structure with regions of large velocity gradients connecting regions with weak velocity gradients. Alternatively, either the mean profile deviates substantially from the basic one (and in that way the flow becomes inhomogeneous and not impotent, i.e. able to support turbulence), or IF it remains statistically homogeneous it should asymptotically be as the basic one (i.e. laminar) since the mean profile Sy is impotent in the sense that it is unable to "feed" turbulence due to $\frac{\partial\langle u_1 u_2\rangle}{\partial x_2} = 0$ in such a flow. This example serves as an illustration that statistics - as any other tool - should be used with caution.

4.2 Geometrical statistics

This example shows how conditional sampling based on *geometrical* statistics can help to get insight into the nature of various regions of turbulent flow, e. g. those associated with strong/weak vorticity, strain, various alignmets, etc. For instance, the PDF of the cosine of the angle between vorticity $\boldsymbol{\omega}$ and the vortex stretching vector $W_i \equiv \omega_k s_{ik}$, $\cos(\boldsymbol{\omega}, \mathbf{W})$ is symmetric for a Gaussian velocity field, whereas it is strongly positively skewed in real turbulent flows. It remains essentially positively skewed for any part of the turbulent field, e. g. in the 'weak background' (whatever the definition: on enstrophy, strain, both and/or any other). Thus, contrary to common beliefs, the so called 'background' is not structureless, dynamically not inactive and essentially non-Gaussian, just like the whole flow field or any part of it. The structure of the apparently random 'background' seems to be rather complicated. The previous qualitative observations (mostly from DNS) about the '*little apparent structure in the low intensity component*' or the '*bulk of the volume*' with '*no particular visible structure*' should be interpreted as meaning that no *simple visible* structure has been observed so far in the bulk of the volume in the flow. It is a reflection of our inability to 'see' more intricate aspects of turbulence structure: intricacy and 'randomness'are not synonyms of absence of structure (for more details and other aspects of geometrical statistics see Tsinober 1998a,b and references therein).

4.3 Pressure hessian

Recently special attention was attracted by the pressure hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$. Among the general reasons for such an interest is that the pressure hessian is intimately related to the nonlocality of turbulence in physical space (Nomura and Post 1998, Ohkitani and Kishiba 1995, Tsinober 1998a,b). This can be seen from the following example with the so called restricted Euler equations (Cantwell 1992). Namely, replacing in the Euler equations the pressure Hessian, which is both nonlocal and non-Lagrangian, by a local quantity $\frac{1}{3}\delta_{ij}\nabla^2 p = \frac{\rho}{6}\{\omega^2 - 2s_{ij}s_{ij}\}$ turns the problem into a local one and allows one to integrate the equations for the invariants of the tensor of velocity derivatives $\partial u_i/\partial x_j$ in terms of a Lagrangian system of coordinates moving with a particle (Cantwell 1992). This means that nonlocality due to pressure and its non-Lagrangian nature are essential for sustaining turbulence without external *random* forcing.

One of the quantities in the context of the theme of this paper directly associated with the pressure hessian is the scalar quantity $\omega_i\omega_k\frac{\partial^2 p}{\partial x_i \partial x_k}$. It is responsible for the nonlocal effects in the rate of change of enstrophy generation $\omega_i\omega_k s_{ik}$ (see Tsinober 1998a and references therein). What is special about this quantity, which is of *even* order in velocity, is that for a Gaussian velocity field $\left\langle \omega_i\omega_k\frac{\partial^2 p}{\partial x_i \partial x_k}\right\rangle \equiv 0$, whereas in a real flow it is essentially positive and $\left\langle \omega_i\omega_k\frac{\partial^2 p}{\partial x_i \partial x_k}\right\rangle \sim \frac{1}{3}\left\langle W^2\right\rangle$, where $W_i \equiv \omega_k s_{ik}$ is the vortex stretching vector. Thus interaction between the pressure hessian and the vorticity is an essential feature of turbulence structure associated with its nonlocality.

It is noteworhty that the pressure hessian represents also the potential part of the Lagrangian particle acceleration and is important in the context of Lagrangian dynamics of fluid particles and stirring properties of (nearly two-dimensional) turbulence (see Hua et al. 1998 and Hua's presentation at this meeting).

5 On universal aspects of turbulence structure

In dynamical systems one looks for structure in *phase space,* (e. g. Chernikov et al. 1989) whereas in turbulence it is common to look for structure in *physical space* with the hope that the structure(s) of turbulence - as we observe it in *physical space* - is (are) the manifestation of the generic structural properties of mathematical objects (*in phase space*) which are called (strange) attractors and which are invariant in some *(statistical)* sense. In other words the structure(s) is assumed to be 'built in' in the turbulence independently of its origin - hence universality. However, the expectation for universal numbers seems to be unjustified in view of infinite dimensionality and (presumably) the extremely complex nature of the phase space of turbulent flows. Therefore, it is more natural to expect *universal qualitative statistical features* in the physical space rather than universal numbers. Indeed some such features have been already observed, which are common for very different - essentially all known - turbulent flows. These are not only the general qualitative features of turbulent flows which are universal (see Appendix 2), but rather specific ones. We mention two recent examples. The first example is related to *geometrical statistics,* namely various alignments such as the alignments between vorticity and the eigenbasis of the rate of strain tensor, and between vorticity and the vortex stretching vector (see Tsinober 1998a for references). The second example, is the so called 'tearing drop' feature observed in the invariant map of the second invariant ($Q = \frac{1}{4}\omega^2 - \frac{1}{2}s_{ik}s_{ik}$) versus the third invariant ($R = -\frac{1}{3}s_{ik}s_{km}s_{mi} - \frac{1}{4}\omega_i\omega_k s_{ik}$) of the velocity gradient tensor. Both features are essentially the same for all known incompressible flows such as grid turbulent flow, periodic flow in a computational box, turbulent boundary layer and channel flow, mixing layer and some others (for references see Martin et al 1998, Nomura and Post 1998, Tsinober 1998a) and also in compressible flows (A. Pouquet, private communication, and references in Tsinober 1998a). Such features can be seen as universal statistical manifestations of the structure of turbulent flows.

6 Concluding remarks

The main point of this communication is to emphasize the distinction between statistics weakly dependent on structure(s) and structure sensitive statistics. The latter allows one to obtain information on the structure of turbulence without knowing how its structures 'look'. Among other things this is likely to result in obtaining the right reasons, e.g. the underlying physics of various observed spectra – a much overstressed aspect of turbulent flows. Another important point is that there is no turbulence *without* structure – every part of the turbulent field (just as the whole) possesses structure. Structureless turbulence (or any its part) contradicts both the experimental evidence and the Navier-Stokes equations. It should be emphasized that the concern here is not with statistical *theories* – all of which use various *ad hoc* assumptions on the nature of turbulence (mostly of the small scale structure) and/or attempt to represent it as a collection of more or less *simple* objects. The focus is on statistical methods of *description* and interpretation of the data on turbulent flows via appropriate *processing* of the data. The latter is likely to be a prerequisite for any 'theory' of turbulence. Quoting Komogorov: *...I soon understood that there was little hope of developing a pure, closed theory, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data.* (Tikhomirov 1991, p.487).

7 Appendix I. On difficulties of a "naive" direct structural approach and related matters

It is impossible to underestimate the observational information on the *instantaneous* structures of turbulent flows. Due to these observations there was (and perhaps still is) a hope that it is possible to construct a theory based on the view that turbulence can be adequately described in terms of collections of 'simple' (weakly interacting) objects (for references see Kraichnan and Chen 1989, Pullin and Saffman 1997, Tsinober 1998a,b), though it is becoming clear that it is a gross inadequate oversimpification (Tsinober 1998a).

However, for a number of reasons it is very difficult, if not impossible, to quantify the information on the *instantaneous* structures of turbulent flows into dynamically relevant/significant form, just like there is little hope to construct above mentioned theory. An overview of these reasons is given below.

Most of the observations are performed with some kind of Lagrangian tracer, which in most cases reflect the cumulative effect of its time history and not the instantaneous field of some dynamical variable. In such a way one can see structure where in reality it is absent, say, in the velocity field (Cimbala et al. 1988). In other words what we 'see' is real - the problem is interpretation. However, even in cases when the required field is available, as in DNS, there exist an intrinsic problem of defining what the *relevant* structures are (see Bonnet 1996 for references and a review of existing techniques - which all are based on statistics anyhow - of defining extracting/educing and characterizing of the so called coherent structures). Simple probability criteria are definitely insufficient since '*...one can find in statistical data irrelevant structures with high probability*' (Lumley 1981), '*you can find structures, essentially arbitrary, which have equal probability to the ones we have latched onto over the years: bursts, streaks, etc....If structures are defined as those objects which can be extracted by conditional sampling criteria, then they are everywhere one looks in turbulence.*' (Keefe 1990). For instance, looking at a snapshot of the enstrophy levels of a *purely Gaussian* velocity field (She et al. 1990) one can see a number of filaments (the irrelevant ones) like those observed in real turbulent flows.

The observed indiviual structures are not simple and are neither weakly interacting between them nor with the background. A similar situation exists with the decompositions of a turbulent field (formal like Fourier, wavelet, POD; heuristic like two-fluid: structures-random, coherent/organized-random, etc.). Recall the suggestion by Dryden in 1943: '*It is necessary to separate the random processes from the nonrandom processes*'. The implication is that such a separation is possible. But this is not obvious at all, as it is seen from the futility of the enourmous efforts to do so. The difficulty is a nontrivial one, e.g. it is even not known how to separate random gravity-wave motion (which does not produce vertical transport) and genuine turbulence (which does) in a stably stratified fluid (Stewart 1959). All the attempts to find a 'good' decomposition are related to what R. Betchov 1993 called the '*dream of linearized physicists*', i.e. a *superposition* of some (desirably simple) elements. The dream is, of course, to find sets (consisting of small number) of *weakly interacting* elements/objects *adequately* representing the turbulent field. Those are known so far are strongly interacting (most of them nonlocally) - a fact reflecting one of the central difficulties in 'solving the turbulence problem' as a whole, in general, and the 'closure problem' (and LES and reduced description of turbulence (Kraichnan 1988)), in particular, as well as in construction of a kind of statistical mechanics of turbulence (Kraichnan and Chen 1989).

8 Appendix II. Major qualitative features of turbulent flows

- \- Intrinsic spatio-temporal randomness, irregularity. Turbulence is definitely chaos. However, vice versa, generally, is not true: many chaotic flow regimes are not turbulent (e.g. Lagrangian/kinematic chaos, laminar "turbulent" flows).
- \- Two initially identical turbulent flows do not remain such on the time scale of dynamical interest, but have the same statitsical properties.
- \- Extremely wide range of strongly interacting scales, i.e. turbulent flows are large systems. In atmospheric flows relevant scales range from hundreds *km* to parts of a *mm*,

i.e. it possess $\sim 10^{18}$ excited degrees of freedom. Hence extreme complexity of turbulence.
• - Highly dissipative. A source of energy is required to maintain turbulence. Continuous energy flux from large to small scales: the energy supply is at mostly large scales [10], its dissipation is at small ones. Statistical irreversibility.
• - Three-dimensional and rotational! It is a "random" field of vorticity with predominant vortex stretching (!), i.e. continuous net production of enstrophy by inertial nonlinear processes, which is dissipated by viscosity. Random potential flows are not turbulence.
• - Strongly diffusive (random waves are not), i.e. it exhibits strongly enhanced transport processes of momentum, energy, passive objects (scalars, e.g. heat, salt; vectors, e.g. material lines, magnetic field). Laminar 'hyperbolic' flows exibit enhanced transport of passive objects only.

These mostly wide known qualitative features of all turbulent flows are essentially the same, i.e. it is meaningful to speak about *qualitative univesality* of turbulent flows as mentioned in section 5.

9 References

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[10]The common view on turbulence dynamics is via the Richardson-Kolmogorov cascade of energy (the famous poem by Richardson). However, there are numerous examples in which turulence develops from small scales into the larger ones, e.g. in all spatially developing turbulent flows, both free such as turbulent jets, wakes, plumes, and wall bounded such as turbulent boundary layers.

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