On the origins of intermittency in real turbulent flows

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Abstract

A set of data is obtained for all the nine components of the velocity gradient tensor $A_{ij} = \partial u_i / \partial x_j$ along with all the three velocity components in a field experiment at the Reynolds numbers (based on the Taylor microscale) $\mathrm{Re}_\lambda \approx 10^4$. The data is used in order to demonstrate that the small scales - both from the inertial range and from the dissipation range – are *not decoupled* from the energy containing range even at Re_λ as large as 10^4. It is argued that this *direct* interaction/coupling of large and small scales seems to be a generic property of all turbulent flows and the main reason for small scale intermittency, non-universality, and quite modest manifestations of scaling.

1 Introductory notes, motivation

There is no concensus on the meaning of the term *intermittency* even in the community working in the field of fluid turbulence. We shall use it in connection with the small scale (SS) – another not well defined term – structure of turbulent flows and 'misbehaviours' of SS. We will understand the term 'scale' in it simplest geometrical meaning in physical space and/or use the field of velocity derivatives $A_{ij} = \partial u_i / \partial x_j$ as the one objectively (i.e. decomposition/representation independent) representing the small scales, e.g. dissipation happens to occur on $s^2 \equiv s_{ij}s_{ij}$, where $s_{ij} = \frac{1}{2}(A_{ij} + A_{ji})$ and vorticity is known as a basic SS quantity (Tsinober (1999b)).

There are several reasons for the SS intermittency in turbulent flows (see, e. g. Sreenivasan and Antonia (1997), Tsinober (1993, 1998), Yeung et al.(1995), Warhaft (2000) and references therein).

Our concern in this communication is with the *direct coupling* between the large and small scales. The evidence for such a coupling is quite massive (see references in Sreenivasan and Antonia (1997), Tsinober (1993), (1998), Warhaft (2000)). In particular, it goes back to the fact that the skewness of the derivative of temperature fluctuations is not small and is of order 1 (Stewart, (1969); Gibson et al. (1970)). For example, in the experiments of Gibson et al. (1970) it was about -0.5, in conditions as in our experiment decsribed below, whereas for a locally isotropic flow it should be close to zero[2]. Similar observations were made in laboratory flows at $Re_\lambda = 500$ (see

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[2] For additional references see Sreenivasan and Antonia (1997), also Figure 7 therein

INTERMITTENCY IN TURBULENT FLOWS AND OTHER DYNAMICAL SYSTEMS
CAMBRIDGE, 21-24 JUNE, 1999.
ISAAC NEWTON INSTITUTE FOR MATHEMATICAL SCIENCES
PROGRAMME ON TURBULENCE, January 6 - June 30, 1999

Warhaft, 2000 and references therein). Quite recently similar observations were made for the velocity derivatives in the direction of the shear (Garg and Warhaft (1998) and references therein).

An important observation was made by Praskovsky et al. (1993) (see also Sreenivasan and Dhruva (1998)). The specific feature of these works is rather large (Taylor microscale) Reynolds number $Re_\lambda \sim 10^4$. In spite of such large Reynolds number there is clear evidence of strong coupling between large and small scales, which accoring to 'common wisdom' should not exist. This fact only calls for experimental verification. However, our results - except of preformed at $Re_\lambda \sim 10^4$ – have an additional important feature. Namely, in our experiment all the nine velocity derivatives $\partial u_i/\partial x_j$ were evaluated along with all the three velocity components u_i. This was done via implementation in a field experiment of the multihotwire technique used by Tsinober et al. (1992, 1997) in laboratory. This allowed to use invariant quantities (i.e. independent of the system of reference) such as energy, full dissipation (not its surrogate), enstrophy, enstrophy generation, etc., as being most appropriate to describe physical processes.

In the sequel we describe shortly the experiment followed by some results from it relevant to this communication, discussion and concluding remarks. More detailed description of the experiment along with various results will be published elsewhere.

2 The experiment

The choice of the site was one of the most complicated problems. A site was found that answered most of the requirements after a year of search. It is located in a field just outside the Kfar Glikson kibbuts and belongs to the kibbuts. It is a flat grass covered ground with 4.5 km fetch downwind starting with a grove of trees about 15 m hight. Concrete foundations were casted at the station for a mast 10 m hight, diesel generator, and a light prefabricated building for a laboratory with the equipment necessary for the experiment. This included a PC computer, sample & hold, anemometers and other electronics, various controls and auxiliary equipment. A special high precision calibration unit was designed and manufactured for computer controlled three-dimensional calibration of the 20 hot-wire probe consisting of five four-wire arrays forming a cross. Several essential technological innovations were introduced in the manufacturing process of the probe in view of specific requirements of a field experiment as compared with the probe used by Tsinober et al. (1992, 1997) in laboratory experiments. These innovations improved the reliability of the probes and fastened the production process. Considerable amount of work was required to prepare the software for data acquisition and calibration. The preparation of the experiment took 3.5 years. This is mainly due to the specific aspects of a field experiment, in general, and special requirements to the precision, in particular, which apart of huge investments in the 'hardware' required much larger variety and amounts of work to be made.

The reported experiment involves recording and processing of large amounts of data. The results presented here are based on one chosen measurements run, taken at the height of 10 m in approximately neutral, slightly unstable conditions. The duration of the run was 15 min (i.e it was a 6.3 km long sample) and it contained about 8.5×10^6 *simultaneous* samples *of 20 channels* taken at sampling rate nearly 10,000 kHz per channel.

The basic data on this particular run are given in table 1.

U_1	u_1'	u_2'	u_3'	λ	η	r_{uw}	C	Re_λ
m/s	m/s	m/s	m/s	m	m			
7.0	1.0	1.0	0.6	0.14	$8 \cdot 10^{-4}$	-0.33	0.53	10^4

Table 1. Basic information on the experimental run.

The notations are as follows: x_1 - horizontal streamwise, x_2 - horizontal spanwise, and x_3 - vertical coordinates respectively; u_i - corresponding components of velocity fluctuations, u_i' - their rms values; $r_{u_1u_3} = \langle u_1u_3\rangle/\sigma_{u_1}\sigma_{u_3}$ - correlation coefficient between the streamwise and vertical components of velocity fluctuations; C - Kolmogorov constant.

A number of tests were performed regarding the properties of the field of velocity fluctuations itself and velocity derivarives. These include those as in Tsinober et al. (1992) as well as several additional ones. Spectra for all the three velocity components possess wide inertial range with a power law $(k\eta)^{-5/3}$ extending over about 3.5 decades, ending at longitudinal scale about 7 cm, where the spectra deviate from the power law due to the influence of viscosity. It is noteworthy that the *compensated* spectra look not that 'nice', so that the inertial range is considerably shorter. Similar behavior is observed when looking at r – dependence of structure functions. All this seems to be related to a much broader issue on the very existence of scaling in turbulent flows.

Velocity derivatives in the mean flow direction, x_1, were calculated according to Taylor hypothesis. In calculating the x_1-derivative we used both the mean and the instantaneous velocity. The difference was insignificant.

The derivatives in the spanwise and vertical directions were calculated taking velocity differences between the suitable arrays and dividing by the distance between these arrays. Data from five arrays allow to calculate the spatial derivatives in several ways, so that we used nine possible combinations in order to check the reliability of evaluating of velocity derivatives.

Skewness and flattness of velocity derivatives are shown in table 2.

Skewness

$\frac{\partial u_1}{\partial x_1}$	$\frac{\partial u_2}{\partial x_2}$	$\frac{\partial u_3}{\partial x_3}$	$\frac{\partial u_i}{\partial x_k}$, $i=k$	$\frac{\langle \omega_i \omega_k s_{ik}\rangle}{\langle \omega^2\rangle^{3/2}}$	$\frac{\langle s_{ij} s_{jk} s_{ki}\rangle}{\langle s^2\rangle^{3/2}}$
0.73	0.65	0.65	$0.05 \div 0.1$	0.18	0.38
				$(0.21)_{S_{\frac{\partial u_1}{\partial x_1}}=0.7}$	$(0.42)_{S_{\frac{\partial u_1}{\partial x_1}}=0.7}$

Flattness

	$\frac{\partial u_i}{\partial x_k}$	$\frac{15}{7}\frac{\langle s^4\rangle}{\langle s^2\rangle^2}$	$\frac{9}{5}\frac{\langle \omega^4\rangle}{\langle \omega^2\rangle^2}$	$\frac{\langle \omega^2 s^2\rangle}{\langle \omega^2\rangle\langle s^2\rangle}$	$3\frac{\langle(\omega_k s_{ik})^2\rangle}{\langle \omega^2\rangle\langle s^2\rangle}$
Real	$20 \div 25$	17.5	27.6	6.7	3.6
Gaussian	3	3	3	1	1

Table 2. Skewness and flatness (kurtosis) of velocity derivatives. The third line in the table for *skewness* contains values of $\frac{\langle \omega_i \omega_k s_{ik}\rangle}{\langle \omega^2\rangle^{3/2}}$ and $\frac{\langle s_{ij} s_{jk} s_{ki}\rangle}{\langle s^2\rangle^{3/2}}$ obtained assuming isotropy and $S_{\frac{\partial u_1}{\partial x_1}} = 0.7$.

It is noteworthy that our results for the skewness $S_{\frac{\partial u_1}{\partial x_3}} \approx 0.05$ is significantly larger than $S_{\frac{\partial u_1}{\partial x_2}} \approx 0.03$. Similarly $S_{\frac{\partial u_3}{\partial x_2}} \approx -0.2$ is even larger. This is consistent with the results on the anisotropy of small scales resulting from the presence of mean shear (Garg and Warhaft (1998) and references therein).

It is seen that the skewness of the derivatives $\partial u_2/\partial x_2$ and $\partial u_3/\partial x_3$ is close to the one of $\partial u_1/\partial x_3$. Also, noteworthy is the agreement of these values ($\sim 0.6 \div 0.7$) and of the flatness ($\sim 20 \div 25$) with the one known from literature (e. g. see the review by Sreenivasan and Antonia (1997)).

3 Direct coupling of large and small scales.

Here to a large extent we follow the approach of Praskovsky et al. (1993). The main difference is that we have access to invariant quantities such as energy, dissipation (not the usually used surrogate), enstrophy, etc.

We start with the correlation coefficients between the large scale quantities like u_i, v (the centered magnitude of the vector of velocity fluctuations) and the moments of different velocity differences from the inertial and dissipative range. Obviously we have the latter only in the streamwise direction with the exception of those used to estimate the derivative in the vertical and spanwise direction. It is noteworthy that the former are quite reliable. Some results are shown in Table 3.

It is seen that the correlation between the large and small scales definitely is not negligible. The results shown are in agreement with those obtained by Praskovsky et al. (1993).

r/η	1	35	110	350	1000
$\langle u_1 \cdot \delta u_1\rangle / (\sigma_{u_1}\sigma_{\delta u_1})$	0.0028	0.0415	0.0646	0.0991	0.1424
$\langle u_2 \cdot \delta u_2\rangle / (\sigma_{u_2}\sigma_{\delta u_2})$	0.0041	0.0528	0.0806	0.1231	0.1755
$\langle u_3 \cdot \delta u_3\rangle / (\sigma_{u_3}\sigma_{\delta u_3})$	0.0065	0.0876	0.1345	0.2062	0.2974
$\langle u \cdot \delta u_1\rangle / (\sigma_{u}\sigma_{\delta u_1})$	0.0001	-0.0023	-0.0033	-0.0033	-0.0012
$\langle u \cdot \delta u_2\rangle / (\sigma_{u}\sigma_{\delta u_2})$	-0.0007	-0.0038	-0.0061	-0.0089	-0.0112
$\langle u \cdot \delta u_3\rangle / (\sigma_{u}\sigma_{\delta u_3})$	-0.0001	0.0046	0.0064	0.0094	0.0045
$\langle u_1 \cdot \delta u_2\rangle / (\sigma_{u_1}\sigma_{\delta u_2})$	-0.0003	-0.0001	0.0008	0.0026	0.0070
$\langle u_1 \cdot \delta u_3\rangle / (\sigma_{u_1}\sigma_{\delta u_3})$	0.0009	0.0014	0.0024	0.0035	0.0066
$\left\langle u_1 \cdot (\delta u_1)^2\right\rangle / (\sigma_{u_1}\sigma^2_{\delta u_1})$	0.0070	-0.0333	-0.0427	-0.0520	-0.0545
$\left\langle u_2 \cdot (\delta u_2)^2\right\rangle / (\sigma_{u_2}\sigma^2_{\delta u_2})$	-0.0150	-0.0199	-0.0212	-0.0217	-0.0312
$\left\langle u_3 \cdot (\delta u_3)^2\right\rangle / (\sigma_{u_3}\sigma^2_{\delta u_3})$	0.0506	0.0769	0.0817	0.0906	0.0706
$\left\langle u \cdot (\delta u_1)^2\right\rangle / (\sigma_{u}\sigma^2_{\delta u_1})$	0.0282	0.0386	0.0403	0.0466	0.0531
$\left\langle u \cdot (\delta u_2)^2\right\rangle / (\sigma_{u}\sigma^2_{\delta u_2})$	0.0202	0.0276	0.0308	0.0364	0.0485
$\left\langle u \cdot (\delta u_3)^2\right\rangle / (\sigma_{u}\sigma^2_{\delta u_3})$	0.0222	0.0336	0.0388	0.0466	0.0635
$\left\langle u_1 \cdot (\delta u_2)^2\right\rangle / (\sigma_{u_1}\sigma^2_{\delta u_2})$	-0.0099	-0.0564	-0.0575	-0.0574	-0.0630
$\left\langle u_1 \cdot (\delta u_3)^2\right\rangle / (\sigma_{u_1}\sigma^2_{\delta u_3})$	-0.0061	-0.0429	-0.0479	-0.0527	-0.0534
$\left\langle u_1 \cdot (\delta u_1)^3\right\rangle / (\sigma_{u_1}\sigma^3_{\delta u_1})$	0.0004	0.0223	0.0374	0.0610	0.0930
$\left\langle u_2 \cdot (\delta u_2)^3\right\rangle / (\sigma_{u_2}\sigma^3_{\delta u_2})$	0.0009	0.0236	0.0419	0.0669	0.1119
$\left\langle u_3 \cdot (\delta u_3)^3\right\rangle / (\sigma_{u_3}\sigma^3_{\delta u_3})$	0.0025	0.0436	0.0734	0.1092	0.1722
$\left\langle u_1 \cdot (\delta u_1)^4\right\rangle / (\sigma_{u_1}\sigma_{\delta u_1})$	0.0026	-0.0054	-0.0124	-0.0199	-0.0231
$\left\langle u_2 \cdot (\delta u_2)^4\right\rangle / (\sigma_{u_2}\sigma_{\delta u_2})$	-0.0022	-0.0015	-0.0074	-0.0071	-0.0225
$\left\langle u_3 \cdot (\delta u_3)^4\right\rangle / (\sigma_{u_3}\sigma_{\delta u_3})$	0.0044	0.0153	0.0186	0.0118	0.0100

Table 3. Correlation coefficient between u_i, $\delta u_i = u_i(x+r)-u_i(x)$, and the centered magnitude of the vector of velocity fluctuations $v = u - \langle u\rangle, u^2 = u_1^2 + u_2^2 + u_3^2$.

The direct coupling of large and small scales is seen most clearly from the conditional averages of velocity differences conditioned on large scale quantities (see Figures 1 and 2). The main result is that all conditional statistics are *not independent* of large scale quantities as is expected without coupling between large and small scales.

In Figure 1 we show some results similar to ones obtained by Praskovsky et al. (1993) (at the left column)) in parallel with those conditioned on the *centered magnitude* of the vector of velocity fluctuations $v = u - \langle u\rangle$, where $u^2 = u_1^2 + u_2^2 + u_3^2$. Similar behaviour is observed for conditonal statsistics of $\langle \delta u_i^n\rangle$ for all $i = 1, 2, 3$ and $n = 2, 3, 4$.

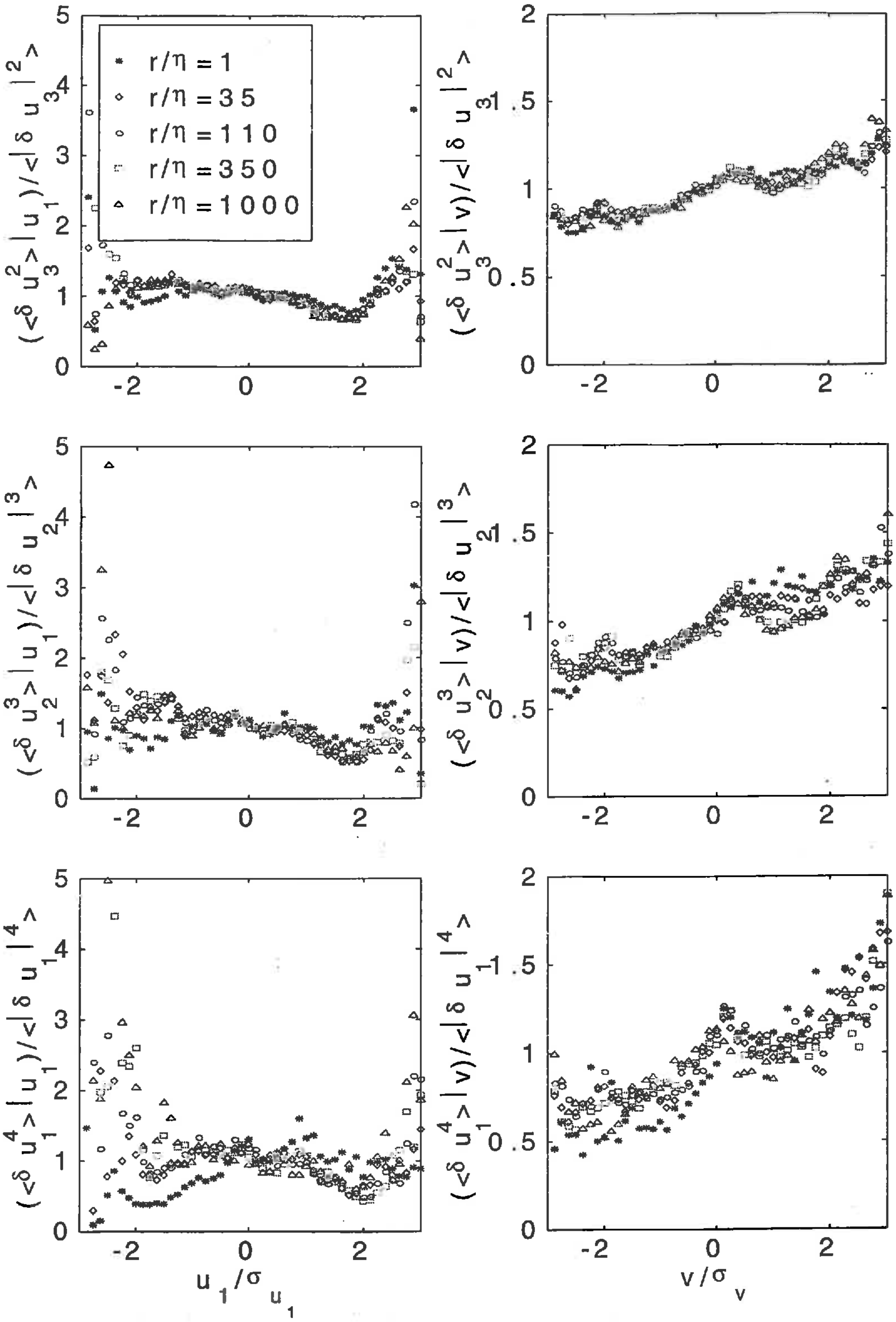

Figure 1. Conditional averages of velocity increments conditioned: left – on the u_1 fluctuation, right – on the centered magnitude of the vector of velocity fluctuations v.

Two aspects deserve special comment.

First, there is a clear tendency of increase of the conditional averages of the structure functions with the *energy* of fluctuations as is seen from the right column of the Figure 1. Second, such a tendency, that is the direct coupling, is observed also for the smallest distance $\sim \eta$, which was used for estimates of the derivatives in the streamwise direction. This result is quite reliable due to the absence of problems in estimating of the derivatives in the streamwise direction like those in the other two directions.

In Figure 2 we show also similar conditional statistics for the enstrophy ω^2 and the total strain $s_{ij}s_{ij}$. The result is quite similar to the one shown in Figure 1 for the smallest distance $\sim \eta$. This result is definitely correct *qualitatively*.

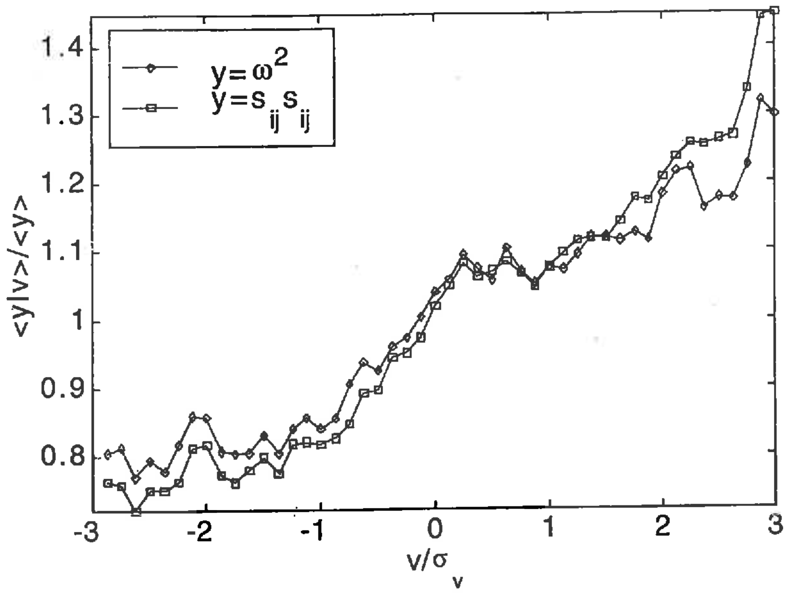

Figure 2. Conditional averages of enstrophy ω^2 and total strain $s_{ij}s_{ij}$ conditioned on the centered magnitude of the vector of velocity fluctuations v.

Special mentioning deserves the PDF of the angle between velocity and vorticity, since it reflects the correlation beween the two vectors (Figure 3). Though the alignment is weak, it is significant and is another manifestation of the direct coupling between large and small scales. It is not surprising that the correlation between a large scale and a small scale quantity is small, in our case $\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle / u\omega = 5 \cdot 10^{-4}$. However, this does not mean that the coupling between them is small as well.

It is noteworthy that two correlations involving large and small scales are of particular importance for shear flows. These are $\langle \omega_2 u_3 \rangle$ and $\langle \omega_3 u_2 \rangle$, since they are directly related to the derivative[3] of the Reynolds stress [4]:

$$\frac{d\langle u_1u_3\rangle}{dz} \sim \langle \omega_2 u_3\rangle - \langle \omega_3 u_2\rangle$$

The important point is that the corresponding correlation coefficients $C_{\omega_2 u_3} = 7.7 \cdot 10^{-3}$ and $C_{\omega_3 u_2} = 1.3 \cdot 10^{-2}$ are small, but significant: if they vanished precisely the mean flow would not 'know' anything about the fluctuative part of the turbulent flow (Tsinober (1998, 1999a)).

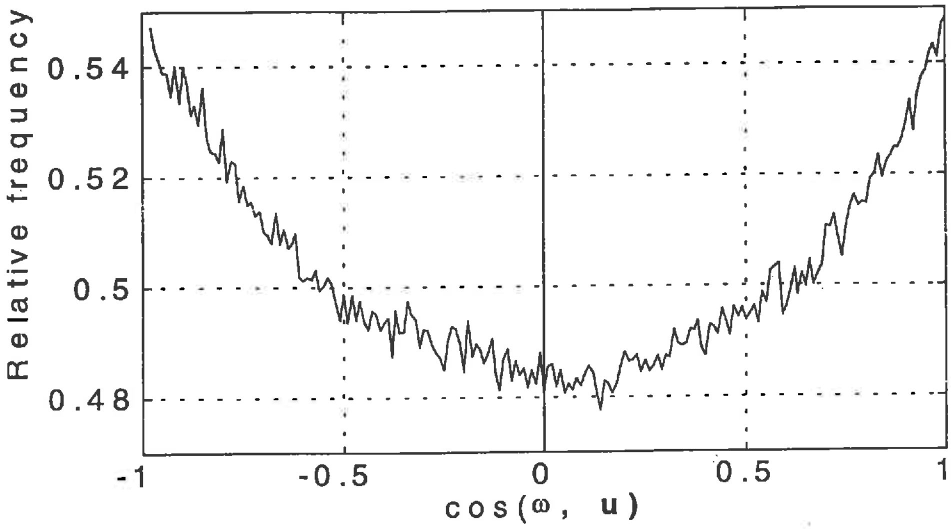

Figure 3. PDF of the cosine of the angle between the vectors of velocity and vorticity.

4 Concluding remarks

The results obtained in this research are the first ones in which explicit information is obtained on the field of velocity derivatives (all the nine components of the tensor $\frac{\partial u_i}{\partial x_j}$) and velocity differences along with all the tree components of velocity fluctuations at $Re_\lambda \sim 10^4$. Up to present the field of velocity derivatives was accessible at $Re_\lambda \sim 10^2$.

The general conclusion drawn from the whole experiment is that the basic physics of turbulent flow at high Reynolds number $Re_\lambda \sim 10^4$, at least qualitatively, is the same as at moderate Reynolds numbers, $Re_\lambda \sim 10^2$. This is true of such basic processes as enstrophy and strain production, geometrical statistics, the role of concentrated vorticity and strain, and depression of nonlinearity.

[3]This is the quantity entering in the equation for the mean flow.

[4]This relation is precise if $\partial/\partial x \langle\cdots\rangle = 0$, e. g. in a channel flow.

One of the specific conclusions is that the small scales - both from the inertial range and from the dissipation range – are *not decoupled* from the energy containing range even at Reynolds numbers (based on the Taylor microscale) as large as 10^4. In other words the Galilean invariance is broken in the restricted sense that the properties of small scale turbulence are *not* independent of parameters characterizing the large scales, such as, e.g. the energy of velocity fluctuations. This *direct* interaction/coupling of large and small scales seems to be a generic property of all turbulent flows and one of the main reasons for small scale intermittency, non-universality, and quite modest manifestations of scaling.

This 'contamination' of small scales by the large ones seems to be unavoidable even in homogeneous and isotropic turbulence, since there are many ways to produce such a flow (i.e. many ways to produce the large scales).[5] It is the difference in the mechanisms of large scales production which 'contaminates' the small scales. Hence, non-universality.

The direct interaction/coupling of large and small scales is in full conformity and is the consequence of the generic property of Navier-Stokes dynamics (as well as some kinematics) - strong non-locality. This includes also another aspect of the coupling between the small and large scales, the bidirectional nature of this coupling, i. e. the 'reaction back' of the small scales (Tsinober (1998, 1999b)). But this is another issue.

Acknowledgements

This research was supported by the U.S.–Israeli Binational Scientific Foundation, Belfer Center for Energy Research and The Israel Science Foundation. Participation of S. Yorish was indispensable: he manufactured the new improved version of the 20-hotwire probe and took active part in the experiments. Our special thanks are to the people of Kfar Glikson kibbuts. Without their warm help the experiment would never been realized.

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[5]For this reason we did not attempt to introduce any corrections (as in Sreenivasan and Dhruva (1998)) due to the presence of shear (in our case $\sim 0.1s^{-1}$), since we don't see any rational way of doing so due to the nonlinear nature of the problem.

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