

Report of the programme on *Random Geometry* Isaac Newton Institute, January – July 2015

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1 Overview

Over the past twenty or thirty years, *two-dimensional random geometry* has emerged as a large and fascinating subject at the interface between analysis, probability, geometry and combinatorics. The mathematical theory is rich, with new problems and connections emerging regularly to other areas of mathematics, and the subject has been growing and changing at a tremendous speed. These developments have driven the theory far beyond its traditional boundaries, and have led to the development of radically new concepts and intuitions. This relatively recent explosion of ideas was the main motivation for this programme, which was built around the long-term participation of both established experts and *extraordinarily* strong younger researchers, as well as a many short term visitors (260 visitors overall). We cannot describe here the relevant publications of all 260 individuals. However, we can high-

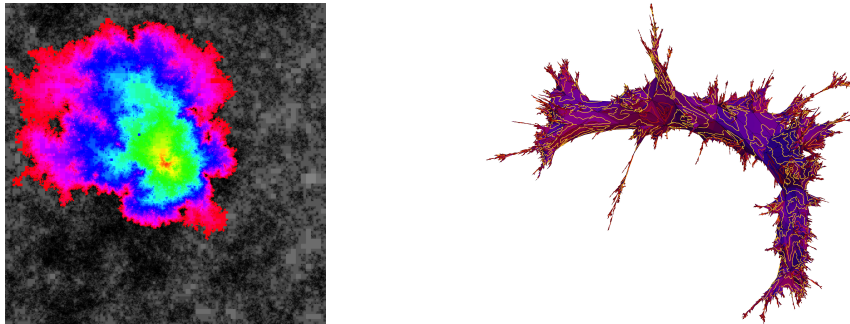


Figure 1: Metric ball in Liouville quantum gravity. Random planar map decorated with FK Ising loops.

light a few striking research results achieved by long-term participants and explain some of the programme's positive effects on the community.

2 Research highlights

For many years, the *unification* of the existing perspectives on random planar geometry — which have grown out of different disciplines (such as conformal field theory, Gaussian multiplicative chaos, combinatorics, random planar maps, statistical physics, and string theory) — has been something of a holy grail for this subject. Thanks to work completed during this programme, through a coordinated effort by many programme participants, much of this unification has been achieved. In particular:

1. **Miller and Sheffield** have posted a series of papers that provides the first direct and rigorous connection between the two primary paradigms for random surface construction: *Liouville quantum gravity* and *the Brownian map*.
2. A team of younger researchers **Aru, Huang and Sun** has rigorously proved the equivalence of two existing random sphere constructions.
3. Papers by (various subsets of) **Gwynne, Mao, Miller and Sun** and **Berestycki, Laslier and Ray** have unified the theory FK-decorated random planar maps and the theory of CLE-decorated Liouville quantum gravity.
4. **Curien and Le Gall** have shown that the metric on random planar maps is very robust in the sense that various distance processes (first passage percolation, Eden model) are identical on the large scale (thereby showing that the behaviour of these processes is surprisingly simpler in random geometry than on deterministic lattices).
5. **Gwynne, Miller and Sun** have settled long open questions about the multifractal behavior of random SLE curves.
6. **Benoist, Dumaz, Werner** have made rigorous sense of *renormalization* in the context of uniform spanning trees and FK random cluster models. Werner has announced fascinating new relationships between Brownian loop soups and the Gaussian free field.
7. **Holden, Gwynne, Miller** have produced a new method to calculate scaling exponents and Hausdorff dimensions for complicated random curves in terms of (potentially simple) Brownian motion exponents.
8. **Gwynne, Holden, Miller, Sun** have derived an important formula concerning Liouville quantum gravity surfaces obtained by gluing trees together. Related work by **Gwynne, Kassel, Miller, Wilson** and by *Angel, Kenyon, Miller, Sheffield, Wilson* has found new statistical physics models that fit into the SLE and quantum gravity framework.

3 Scientific interactivity and pedagogy

1. *Four highly successful workshops* have highlighted different aspects of the subject. These were well attended with more than 100 participants for each workshop.
2. *A school for young researchers* enabled a large crowd of early career researchers to learn about the state of the art of the subject in a unified way. This interconnectedness was cited by many participants as a key benefit of the programme.
3. *Two research seminars per week* exposed participants to cutting edge ideas.
4. *An active and well-attended reading group*, involving weekly meetings and discussions throughout most of the programme, has led us to a deeper understanding of disparate fields. Looking to the future, we spent the last part of the programme discussing recent advances on probabilistic approaches to the famous Yang–Mills problems, which shares some common features with themes already explored within random geometry. For this it was particularly useful to be able to set up a videoconference meeting with Chatterjee in Stanford to discuss his recent work on the subject. We hope that we have inspired some of the bright young researchers attending the programme to think about this outstanding problem.