Scaling limits, rough paths, quantum field theory: September – December 2018

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Our understanding of physics at very small scale is based on Quantum Field Theory (QFT), the offspring of a marriage between quantum mechanics and special relativity. Right from inception QFT challenged mathematics because the interrelations of the infinitely many degrees of freedom of a quantum field are beyond standard approximation schemes. Part of the response to this was the formulation of axioms that capture the essential properties of QFT, but these threw up further questions including: Are the axioms themselves consistent?

Models satisfying these axioms have been constructed for two and three dimensional space-time, but not for the four dimensional space-time of physics. For two and three dimensional space-time these constructions required the invention of ingenious new methods which go under the name of Constructive QFT. By merging those efforts with Wilson's Renormalisation Group, QFT can be related to the probabilistic behaviour of assemblies of very many “almost” independent degrees of freedom, as formalised by the concept of scaling limits. A basic example of a scaling limit appears in the Central Limit Theorem from probability, but scaling limits can be far more interesting than this. Recent progress in understanding the non-Gaussian scaling limits described by non-linear stochastic partial differential equations (SPDE) has rested on Rough Path theory, the final ingredient in our program. This is a systematic calculus for SPDE that incorporates the propagation of nonlinear effects from small to large scales.

Despite their shared interests, Constructive QFT and SPDE evolved separately over an extended period and the goal of our program was to create at the Newton Institute the best possible environment for each of the two communities to learn from one another.

Our 81 participants were drawn more or less evenly from both communities and related areas. Via a series of workshops, mini-courses and regular seminars we fostered the interest of the younger generation in the problems and results of the older, and built foundations of friendship, collaborations and respect that were not there before. Long term participants Moinat (Warwick), Weber (Bath), Gubinelli (Bonn) and Hofmanova (Bielefeld) developed new SPDE proofs of some of the axioms for Euclidean scalar quantum fields in 3 dimensions that originally rested on difficult stability results of the constructive program by Glimm and Jaffe. Gauge invariant QFT has been invigorated by participants Chandra (Imperial), Hairer (Imperial) and Shen (Wisconsin-Madison) using an SPDE point of view, which is very different from the classic work of Balaban. Gauge invariance is key to progress in four dimensions and rough path theory has applications in diverse fields including financial mathematics and fluid mechanics. We reviewed these applications in seminars and a wide-ranging workshop on statistical applications including handwriting recognition. However, the true signal of success of our program will be the progress in coming years.

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